PARTICLES TRAVELING FASTER THAN LIGHT*

by

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Faster and faster go things of human creation and the fastest things of nature including man are taking second place. There are animals that run faster than man but the fleetest animal is easily outdistanced by the automobile, the fastest birds by the bullet. Even the speed of sound is overtaken by the airplane. But nature's triumph in the high speed department is light. No one has yet succeeded in exceeding the speed of light though with subatomic and sub-nuclear particles like beta electrons and particle beams from giant accelerators one has approached that speed within a fraction of a percent. Cosmic rays contain a few particles even closer to the speed of light. But except for light itself in all its guises like gamma rays, X-rays, visible light, heat and radio waves and the various kinds of neutrinos nothing seems to move with this incredible speed of about $3 \times 10^{10}$ cms per second. Can anything move faster? What is the speed of thought?

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Three Classes of Objects: Bradyons, Luxons and Tachyons

The reason for this limitation on the speed of light as a barrier to the speed of material particles is that as you approach the speed of light the amount of energy needed for further accelerating the particle also increases. The apparent inertia of the particle increases without limit and hence the same force would cause less and less acceleration. For a particle traveling with 9/10 speed of light the (longitudinal) inertia is

\[ \frac{v^2}{c^3} \left(\frac{v}{c}\right)^{3/2} \approx 10 \text{ for } v/c = 9/10 \]

while for a particle traveling with 99/100 speed of light it is \(10^3\). This increase in the inertia puts the speed of light as the limit for speed of material particles.

We can recognize this limitation in a somewhat different manner: the relativistic energy momentum relation asserts that the total energy of a material object of rest mass \(m\) moving at a speed \(v\) is given by

\[ E = mc^2 \left(1 - \frac{v^2}{c^2}\right)^{1/2} \]

This quantity increases without limit as \(v\) approaches \(c\). If \(v = c\) the energy "becomes infinite"; in other words we cannot supply enough energy to bring about this speed. The speed of light is indeed a light barrier for material par-
particles which may hence be called "tardyons" or "bradyons".

But if this is so we may ask: How come that neutrinos and light itself can move at this speed forbidden to material particles? How does one speed up these particles to this particular speed? The answer is that no one speeded them up: they were created at that speed. There is no question of speeding up a light quantum or a neutrino: nor is there any question of slowing down one of them. These objects can travel only at one unique speed, namely the speed of light. For material particles as the energy increases the speed increases and vice versa. But for photons (light quanta) or for neutrinos the speed is independent of the energy. Objects of this second class which always move with the speed of light may be collectively called "luxons".

These two classes of objects are distinct: neither can be changed into the other by acceleration or deceleration. But, then, the observation that you cannot have any bradyon cross the light barrier by acceleration is irrelevant to the question of whether a third class of objects exist which move faster than light. This question has to be answered by itself by direct observation. None of the luxons or bradyons can be speeded up into this domain of velocities. These must forever belong to speeds exceeding the speed of light.
These objects may be called "tachyons". As analogy we may consider demographers studying the population patterns of the Indian Subcontinent. They would find that there are a lot of people in India but as you approached the Himalayas there were fewer and fewer people. Climbing the giddying heights is a difficult task and it is well nigh impossible to climb it let alone live there. Suppose a demographer calmly asserts that there are no people north of the Himalayas since none could climb over the mountain ranges! That would be an absurd conclusion. People of Central Asia are born there and live there: they did not have to be born in India and cross the mountain ranges. So with tachyons.

Theory of Relativity

The constancy of the velocity of light for all inertial observers is one of the basic postulates of the theory of relativity. It is a circumstance at variance with our familiar Galilean law of addition of velocities. So the proper framework to discuss particles traveling faster than light is the theory of relativity. The transformation laws of space-time coordinates in this theory is such that the spatial momentum and energy transform like the components of a four vector. If \( p_1, p_2, p_3 \) and \( E \) are these quantities
in one frame and $p_1', p_2', p_3'$ and $E'$ in another frame moving relative to the first with a velocity $u$, in the third direction:

$$p_1' = p_1, \quad p_2' = p_2,$$

$$p_3' = \left( p_3 + \frac{u}{c^2} E \right) \left/ \left( 1 - \frac{u^2}{c^2} \right)^{\frac{1}{2}} \right.$$

$$E' = \left( E + up_3 \right) \left/ \left( 1 - \frac{u^2}{c^2} \right)^{\frac{1}{2}} \right..$$

The relativistic relation between velocity, momentum and energy is given in terms of $c$, the velocity of light, by:

$$v = \frac{c^2 p}{E}. \quad (2)$$

So for a particle whose initial velocity $v$ was in the third direction the velocity $v'$ in the moving frame is given, according to (1) by:

$$v' = \frac{(v+u)}{\left( 1 + \frac{uv}{c^2} \right)} \quad (3)$$

For the relative motion between observers we take a velocity whose magnitude is less than the speed of light (since observers are bradyons!) though some people have considered even superluminal observers.

The velocity addition law (3) has the following consequences: If $v < c$, $v' < c$; if $v = c$, $v' = c$ and if $v > c$, $v' > c$. 
In other words the three classes of objects, bradyons, luxons and tachyons are relativistically invariant. Each observer sees a bradyon as a bradyon, a luxon as a luxon and a tachyon as a tachyon.

From the relation (2) we note that for a bradyon the energy is always larger than c times the momentum; for the luxon they are equal and for the tachyon the energy is less than c times the momentum. In particular, for the bradyon we can find a frame in which it would be at rest. For the luxon and the tachyon we cannot do this. The tachyon can have a frame in which the energy is zero and the momentum nonzero. Such a tachyon travels with infinite speed. These tachyons therefore instantaneously transmit spatial momentum with the transfer of no energy: this is therefore the relativistic realization of impulses transmitted by a perfectly rigid body.

The Strange World of Tachyons.

For tachyons the correspondence between energy and velocity is the opposite of that for bradyons. Usually we think that as the energy of a particle increases its speed also increases: We use the words "fast" and "energetic" almost interchangeably. Supplying energy to a moving
particle is taken to be the same as accelerating it. For tachyons, on the other hand, the more energetic the tachyon the slower it moves. Zero energy tachyons move with infinite speed; the most energetic tachyons approach the speed of light. Accelerating a tachyon is to take energy away from it.

The transformation law (1) implies the invariance of the quantity \( E^2 - c^2 p^2 \) under relativistic frame changes. If we write

\[
E^2 - c^2 p^2 = m^2 c^4
\]

then the quantity \( m \) may be identified with the rest mass of the particle. For the three classes of particles we compute \( m^2 \) and see that \( m^2 \) is positive for bradyons, zero for luxons and negative for tachyons. An imaginary rest mass for the tachyon may look peculiar, but there is no danger of measuring such a quantity in a balance since the tachyons never come to rest. The quantities that are directly measured are the quantities that are additively conserved, namely the momenta and the energies. These are always real for the tachyon as well as for the luxon and the bradyon. And gravitational interactions are really not with the mass but with the energy and so, here again, no troubles arise from imaginary mass.

The velocity addition law (3) leads to some surprises
when $v$ and $u$ are of opposite signs since for a velocity

$$u = -\frac{c^2}{v}$$  \hspace{1cm} (5)

(which is less in magnitude than $c$) the resultant velocity $v'$ is infinite and for still larger velocities the resultant velocity $v'$ is opposite in sign to $v$.

Suppose we go for a walk early in the morning and see a vicious dog chasing us. After taking stock of the situation we would decide that discretion is the better part of valor and take to our heels rapidly and away from the dog, looking back every now and then to verify that we are outdistancing the dog. If we are not we run faster. The implied assumptions are that the effective relative speed of the dog and us is the difference between the respective ground speeds. In other words we employ the Galilean velocity addition law appropriate to the motion of ordinary and usual dogs. Let us however imagine a person who can move with relativistic speeds, but nevertheless less than the speed of light. In his morning sauntering he encounters a tachyon dog which by his estimate is approaching him with a speed faster than light. He puts in a little bit of effort and starts running away from the tachyon dog: when he looks back he is amazed to find that the dog is gaining on him with a greater speed than before; he runs even faster and
finds that the dog runs still faster. If he runs really fast at a certain point he will think he is having a nightmare: He sees the dog running with great speed in the opposite direction, but being reassured by it if he slows down a little bit the dog is running towards him with great speed. If the audience does not like to think of such high speed runners and tachyon dogs think of a galactic patrol rocket ship which is sent out to investigate a (tachyonic) flying saucer. Not only the astronaut but also his computer would be having nightmares unless someone recognizes that the relativistic velocity addition theorem for tachyons leads to such a behaviour.

By the way, if you are ever trying to capture a tachyonic flying saucer please remember that it cannot be brought to rest: it will disappear without trace if you attempt to slow it down much as a moonbeam caught in your fist. This is part of the tragedy of human existence: the most beautiful things, the most fascinating things taunt us, inspire us, make us want to be one with it at any cost and under any condition. But, every such attempt shows up totally incomprehensible behaviour in the idol of our dreams. Perhaps the essence of dreams is to know what cannot be in waking life: beauty like that gives both ecstasy and agony.
Tachyon Paradoxes

There are some people who say tachyons cannot possibly exist because they lead to embarrassing physical situations. There are others who feel that their existence would threaten the whole edifice of relativity theory and perhaps even all of physics including the principle of causality. Some of them even feel that permitting them to exist would create absurd paradoxes of causal cycles and drive us all insane. Let us examine some of these questions.

The energy momentum transformation law (1) shows that for the critical velocity \( u = \frac{-c^2}{v} \) the new energy \( E' \) is zero; for larger values of \( u \) of the same negative sign it follows that the energy \( E' \) will become negative. Now we have not only not seen negative energy particles but the possibility of emitting negative energy particles would seem to violate the spirit of the second law of thermodynamics. And, yet, the transformation laws seem immutable. How do we reconcile ourselves to this paradox?

Another unusual thing happens at this special velocity (5) regarding the time sequence of events. If we have a particle whether it be tachyon, luxon or bradyon if it goes from the point \( A \) to the point \( B \) the time instants \( t_A \) will be earlier than \( t_B \). If we go to a moving frame the precise
values of \( t_A \) and \( t_B \) will change to new values \( t'_A \) and \( t'_B \); but as long as we consider the relative velocity \( u \) between the two frames to be small we expect \( t'_A \) to be earlier than \( t'_B \). In other words the temporal order of events is expected to be relativistically invariant. As long as we restrict attention to bradyons or luxons this expectation can be proved to hold for all relative velocities \( u \). But with tachyons involved this ceases to be true for large enough frame velocities: in suitable frames \( t'_A \) will be later than \( t'_B \). In other words in some frames tachyons will be traveling backward in time.

Closer examination shows that the two paradoxes are closely related: for the critical velocity the energy becomes zero and \( t'_A \) and \( t'_B \) coincide. When the energy becomes negative \( t'_A \) exceeds \( t'_B \). In other words: positive energy particles propagate forwards and negative energy particles propagate backwards.

**Resolution of Tachyon Paradoxes: The Switching Principle**

This connection between the two paradoxes also lead to their resolution. For an observer in the moving frame the energy loss at \( t'_B \) occurs earlier than the energy gain at \( t'_A \); he would, therefore, tend to interpret the two events as cor-
responding to a particle carrying the positive energy \(-E'\) and the momentum \(-p'\) going from B to A. Thus each observer sees only positive energy particles and propagation forwards into time. But the sense of motion is reversed in going to frames with relative velocities exceeding the critical velocity (5). This reversal of the velocity is already contained in the velocity composition law and our story of the brave man and the tachyon dog. So the paradoxes of negative energies and of propagation backward in time are really not paradoxes at all. We do have a reinterpretation program: we may refer to this interchange of the sense of propagation in suitable frames as the Switching Principle.

The Switching Principle interchanges initial and final states of the tachyon between the frames. It follows that the process of emission (or absorption) of a tachyon is not a relativistically invariant notion since in a suitable frame the Switching Principle would tell us that it is to be considered as an absorption (or emission) process. This is in contrast to the more familiar situation with bradyons or luxons where emissions (and absorptions) are relativistically invariant notions. In fact we take this for granted in assuming for example that a perfectly black body would remain totally absorbing in all frames.
We now see that this picture must be revised. If we have a perfect absorber in our frame in that it absorbs everything that falls on it and emits nothing, (say, a black body cavity of infinite heat capacity at absolute zero of temperature) then if it is seen from a moving frame it must absorb almost all tachyons incident on it except those for which the corresponding critical velocities are exceeded by the velocity of this frame. For these velocities we will see that the "blackbody" acts as a perfect emitter but it would not absorb any tachyons incident on it. This is strange and unusual: however it is not inconsistent behaviour.

Attention must be paid to the fact that the Switching Principle deals with "processes" involving tachyons, not "states" involving them. Emission is replaced by absorption but it happens that in the same context an absorption is replaced by emission. So in dealing with tachyonic phenomena we must deal with the entire process to avoid confusions and paradoxes. It would take us too long to dwell on various paradoxes and their resolution; the interested reader may consult my article in Lectures in Theoretical Physics vol. 10: A. Ramakrishnan (Editor), Plenum Publishing Company, (New York, 1970) and the discussions in "Physics Today," May 1969 and Dec. 1969 issues.
Causality

The possibility to interchange emission and absorption leads to certain questions related to statistical irreversibility and the problem of cause and effect. The question has been raised: Is not the Switching Principle inadequate to deal with the difficulty of effects preceding causes? This question is based on the tacit assumption that causes can be distinguished and separated from effects in an intrinsic manner. In a purely mechanical theory there is no such intrinsic discussion. What is earlier of two invariably connected events is called the cause and the later one the effect. If this be the case by definition no possibility exists of causes succeeding effects.

But we may want to invoke statistical considerations: for example we can have spontaneous emissions but no spontaneous absorptions. We can ask: If we consider spontaneous emission of tachyons are the considerations different? Could we not use such an emitter and would not the Switching Principle get into difficulty?

A full and detailed discussion of the relevant statistical mechanical notion is beyond the scope of this lecture and we must content ourselves with a brief discussion. The question of spontaneous emission implies an unstated but
essential "boundary condition": that the space around the emitter is not filled with the radiation that it is trying to emit. The stars can shine in the night sky because the sky is dark. A candle sheds light during the night but in the daytime sunlight it casts a shadow. So, to the extent this boundary condition is not satisfied spontaneous emission is no longer "spontaneous". The statistical mechanics of tachyon processes must be pursued with great care. To see how a moving object behaves we must take proper account of the boundary conditions as "seen" by the objects.

The subtlest form in which statistical questions can be raised concern the "free will" of moving observers. By using two or more who are moving in relation to each other we can attempt to construct logical paradoxes which involve causal cycles: each sequence is cause-effect as seen in a suitable frame but the chain closes. You could, for example arrange to prevent your grandfather and grandmother from ever meeting and thus bring about your not being born. But is this really so? Closer examination shows that no single frame would permit a causal cycle: So one or more links must be acting in the opposite direction as seen by this observer. But suppose the moving observer insists that things go the other way. If you like you can ask him
to deal with the whole cycle when he himself will find yet other links going the opposite way. If they are men of goodwill they conclude that not all their free wills could be allowed to be free at the same time: he sees the other person doing strange things not very satisfactorily understood on the basis of a common sense choice. He concludes that the other person's free will is somewhat of a delusion as long as he is moving relative to him. But no one observer will see any paradox at all; and would be happy to conclude that the only causes which are really invariable are those that are outside physics; and that the only person who really has free will is he himself. But if he is a wise man he will keep this knowledge to himself!

Quantum Theory of Tachyons: Tachyon Fields

Most of the discussions so far have been done with tachyons being treated as classical particles. From the nature of things we anticipate that if a quantum theory of tachyons can be constructed essentially all these considerations should apply to them as well. The necessity for the Switching Principle shows that we will have to deal with complete processes and not just with states. A quantum theory formulation of tachyons has been constructed on this basis but a discussion of this is also beyond our scope here.
One could also consider a classical theory of tachyon fields which obey a differential equation of the form

\[ \left( \frac{\partial^2}{\partial t^2} - \nabla^2 - \kappa^2 \right) \phi(x, t) = j(x, t) \ldots \] (6)

These are linear fields whose excitations have a wave number-frequency correlation similar to the tachyons; the frequency is smaller in magnitude than the wave number multiplied by the velocity of light. In such a case the excitations will propagate with dispersion: the velocity corresponding to each wave number will be different. But each of these will be greater than the velocity of light. This is consistent with the fact that the quanta of such a field when it is quantized are tachyons which move with a speed greater than the speed of light. If such fields are excited then the stress tensor due to the field will be radically different from the stress tensor for a familiar field: the pressure for a classical fluid is always less than the energy density so that the trace of the stress tensor is always positive. But for a tachyonic field the pressure would exceed this limit. This would be of considerable interest in the dynamical evolution of stellar interiors and for cosmological models themselves.

There is a mathematical question which has to be resolved
in this context. We remarked above that excitations of a tachyonic field propagate faster than light and hence any source would influence points at a distance larger than what light can travel in the same time from the same source. We may say: the influence propagates outside the "light cone". But in the theory of differential equations it is shown that the propagation of a field described by second order linear inhomogeneous differential equation (6) from its source \( j(x,t) \) depends only on the geometrical aspects of the highest derivatives in the equation: hence in this case the propagation should be confined to within the light cone. The apparent contradiction is resolved by observing that the propagation function (the "Green's function") for the problem is not defined without specifying the boundary conditions. We have always chosen the boundary conditions so that positive energy propagates forwards in time while negative energy propagates backwards in time. The corresponding propagation is described by the so called "time-ordered Green's function". This Green's function is large outside the light cone. In classical differential equation theory as well as in much of classical relativity and fluid dynamics one deals with the "retarded Green's function" which propagates both positive and negative energies forwards.
We must conclude that tachyon fields do connect points which are outside each other's light cones and can propagate influence between such points.

Do Tachyons Exist?

In the quantum theory of tachyons, if these objects are coupled to the usual bradyons and luxons then they could be exchanged in their mutual interactions. These would show up as sharp peaks in their scattering amplitudes at values of the invariant momentum transfer squared equal to the negative value corresponding to the square of the tachyon mass with a width governed by the natural spread in this tachyon mass. This possibility has been tested successfully on some high energy reactions: but one cannot say that the fit to this mechanism is compelling as contrasted with a number of other ingenious mechanisms suggested for these secondary and tertiary maxima in the diffraction-like patterns produced in high energy scattering. This fit as well as a number of other indirect methods lead to considering a rather elegant dynamical formalism of nonlocal interaction as equivalent to tachyon exchange forces.

A number of authors have experimentally searched for particles traveling faster than light. So far no conclusive evidence for them has been found. We mentioned above the
partial indications for tachyon exchange in high energy physics. There are two kinds of "experiments" that may give a positive indication: the first is a search for intense radiation of low frequency tachyons in any frame moving relative to the local galactic frame, the cut off frequency increasing approximately in direct proportion to the velocity relative to the galactic rest frame. This is a new kind of "Michelson-Morley" experiment to detect "absolute motion" but here it is no longer "absolute" but "preferred", the local galactic frame being taken as the "night sky for tachyons". This is a difficult experiment requiring high precision and much skill. Neither should be lacking in the land of Sir C. V. Raman.

The other "experiment" is really the unbiased analysis of astrophysical and cosmological data to search for motions faster than light, for pressures exceeding a third of the energy density, or for dynamical correlation between points which are outside each other's light cones. Of these the direct observation of relative motion with speeds exceeding that of light is perhaps the most direct and the most convincing. Discoveries of such motions are often announced but rarely confirmed. Yet they are not ruled out.
The search for tachyons must go on. If they exist we must find them. If they do not we ought to find why they do not exist. Not only is the quest fascinating but along the way we are led to reexamination of fundamental questions in physics.

These reexaminations direct our attention to those questions of natural philosophy which are beyond the classification into individual disciplines: among these are the problem of causality, of free will and of the variability of physical description. These are questions which seem to be in the logical sequence starting from any area of fundamental physics, be it quantum mechanics, tachyon physics, the investigation into the nature of light or the second law of thermodynamics. It is perhaps gratifying that physics demonstrates this unity and this essential incompleteness.

Physics too is a glimpse of the thousand-petal lotus. Contemplation of the lotus takes us from the lotus to its presiding transcendent intelligence, the beautiful One among whose countless names are lalita (the vibrant one), māya (the enigmatic one) and priscilla (the pristine one). Contemplation of that which may or may not be takes us to that which must be. In the contemplation of what must be all thoughts merge; and this is the goal of all intellectual endeavor.