Photometric measurements on tracks of very heavy
    cosmic ray particles with $E > 1$ GeV/nucleon

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## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. THE EMULSION STACK</td>
<td>2</td>
</tr>
<tr>
<td>3. EXPOSURE DETAILS</td>
<td>2</td>
</tr>
<tr>
<td>4. PROCESSING AND PREPARATION FOR MEASUREMENT</td>
<td>3</td>
</tr>
<tr>
<td>5. SCANNING AND SCANNING CRITERIA</td>
<td>6</td>
</tr>
<tr>
<td>6. PHOTOMETRIC APPARATUS</td>
<td>6</td>
</tr>
<tr>
<td>6.1. The optical part</td>
<td>7</td>
</tr>
<tr>
<td>6.2. The electronic part</td>
<td>9</td>
</tr>
<tr>
<td>6.2.1. The depth meter</td>
<td>9</td>
</tr>
<tr>
<td>6.2.2. Signal checking</td>
<td>10</td>
</tr>
<tr>
<td>6.2.3. The reading sequences</td>
<td>11</td>
</tr>
<tr>
<td>7. SELECTION OF TRACKS</td>
<td>11</td>
</tr>
<tr>
<td>8. MEASUREMENTS</td>
<td>12</td>
</tr>
<tr>
<td>9. DATA HANDLING</td>
<td>13</td>
</tr>
<tr>
<td>10. GENERAL REMARKS ON THE NORMALIZATION PROCEDURE</td>
<td>14</td>
</tr>
<tr>
<td>10.1. Depth normalization</td>
<td>14</td>
</tr>
<tr>
<td>10.2. Exclusion of disturbed readings</td>
<td>16</td>
</tr>
<tr>
<td>10.3. Time normalization</td>
<td>17</td>
</tr>
<tr>
<td>10.4. Operator normalization</td>
<td>19</td>
</tr>
<tr>
<td>10.5. Plate normalization</td>
<td>19</td>
</tr>
<tr>
<td>10.5.1. First order plate factors</td>
<td>20</td>
</tr>
<tr>
<td>10.5.2. Second order plate factors</td>
<td>20</td>
</tr>
<tr>
<td>10.6. Dip normalization</td>
<td>22</td>
</tr>
<tr>
<td>10.6.1. The effect of overlapping</td>
<td>22</td>
</tr>
<tr>
<td>10.6.2. Defocussing and instrumental effects</td>
<td>23</td>
</tr>
<tr>
<td>10.6.3. Dip factors</td>
<td>24</td>
</tr>
<tr>
<td>10.5.1. First order plate factors</td>
<td>20</td>
</tr>
<tr>
<td>10.5.2. Second order plate factors</td>
<td>20</td>
</tr>
<tr>
<td>10.6. Dip normalization</td>
<td>22</td>
</tr>
<tr>
<td>10.6.1. The effect of overlapping</td>
<td>22</td>
</tr>
<tr>
<td>10.6.2. Defocussing and instrumental effects</td>
<td>23</td>
</tr>
<tr>
<td>10.6.3. Dip factors</td>
<td>24</td>
</tr>
</tbody>
</table>
11. A STATISTICAL MODEL FOR THE ANALYSIS OF TRACK WIDTH READINGS 24
   11.1. Theory 24
   11.2. Estimation of parameters 28
12. THE EXISTENCE OF NON-RELATIVISTIC PARTICLES 31
13. THE RELATION BETWEEN TRACK WIDTH AND VELOCITY 32
14. IDENTIFICATION OF NON-RELATIVISTIC PARTICLES 34
15. IDENTIFICATION OF RELATIVISTIC PARTICLES 36
16. THE CHARGE DISTRIBUTION 37
   16.1. Experimental results 37
   16.2. Analysis of sources of error 40
   16.3. Comparison with other emulsion experiments 46
17. THE INTENSITY OF MH AND VH NUCLEI 48
18. THEORETICAL CALCULATION OF ENERGY OF ARRIVING PARTICLES 53
APPENDIX 55
ACKNOWLEDGEMENTS 57
REFERENCES 58
FIGURE CAPTIONS 60
FIGURES 64
ABSTRACT

About 120 heavy primary particles in the charge interval $16 \leq Z \leq 28$ have been identified by means of photometric measurements in a nuclear emulsion stack exposed in a high altitude balloon flight from Palestine, Texas, U.S. at $2.3 \text{ g cm}^{-2}$ of residual atmosphere. Particles entering the detector at this site should according to earlier data be relativistic or nearly relativistic. Nevertheless, in this study we have found that a considerable part of the registered particles are non-relativistic. The basical quantity used for charge identification has been the photometrically determined track width $W$. To identify the non-relativistic particles this information has been combined, whenever possible, with observations of the rate of change of $W$ along the track, $\frac{\delta W}{\delta R}$. A charge distribution is presented. Complete resolution between consecutive charges has not been obtained. Possible sources of error are discussed with attention to the sources of error associated with measurements of $\frac{\delta W}{\delta R}$. A statistical model for the analysis of errors is presented. The investigation shows that combined measurements of $W$ and $\frac{\delta W}{\delta R}$ can be a useful tool for identification of non-relativistic particles. However, it also shows that this method is very sensitive to irregularities in the emulsions. This means that correct charge identification can only be obtained from flat tracks of particles passing many emulsions. The intensities at the top of the atmosphere of particles with charge numbers $16 \leq Z \leq 19$ and $20 \leq Z \leq 28$ were found to be $0.092 \pm 0.012$ and $0.38 \pm 0.04 \text{ (m}^2\text{sr s})^{-1}$ respectively. These figures are in good agreement with results obtained by Freier and Waddington in the same balloon flight.
1. Introduction

It is commonly accepted that an accurate knowledge of the relative abundances of different elements in the primary cosmic radiation will give valuable information about the nature of the sources of cosmic rays and about the injection and the acceleration mechanisms. It may also tell us something about the propagation of the cosmic ray particles through the interstellar space, the region of confinement and the age of the cosmic ray nuclei.

Although any theory about the origin and the propagation of cosmic rays must include the two principal components of cosmic radiation, hydrogen with the charge number $Z=1$ and helium with $Z=2$, the chemical composition of heavy primary nuclei ($Z>3$), and maybe especially the composition of very heavy nuclei ($Z>16$), will contain much valuable information. The total intensity of all nuclei heavier than helium is only about 2 percent of the total radiation above a given rigidity, and from these 2 percent only a fraction corresponds to the charge interval $Z>16$. Owing to the low intensity of very heavy nuclei most investigations have poor statistical weights, and therefore the results sometimes seem to contradict each other. The controversiality of the results is accentuated by the difficulty to attain complete resolution between consecutive charges. In order to increase the resolution and the statistics a number of different techniques of detection have been developed. The detectors which have been used for identification of cosmic ray nuclei are essentially nuclear research emulsions, solid state track...
detectors and counters. Their properties and the resolutions which can be achieved are carefully discussed in the excellent survey by Shapiro and Silberberg (1).

Up to now there are only a few works in which reasonable resolution has been achieved. In the published charge spectra a peak at iron is usually well established, but often the resolution of individual elements is fuzzy. This fuzziness seems not to be the same in different energy regions. Especially in the energy interval above 1 GeV/nucleon the results obtained by different groups are in disagreement. In the investigations by the Lund group (2,3), acceptable resolution is achieved, but the statistical weights are poor. This work has been undertaken to increase the statistics in the charge interval $Z > 16$. The registration of the particles is made in nuclear emulsions at a high altitude balloon flight over Texas, U.S., and the identification is made by means of photometric measurements. The measured quantity is the track width, $W$.

2. The emulsion stack

The emulsion stack, which has been used in this investigation, consists of 167 stripped Ilford G5 emulsions, each one being $10 \times 20 \times 0.06 \text{ cm}^3$ in size. Thus the volume of the stack amounts to two liters. In this investigation only 79 emulsions have been used.

3. Exposure details

The emulsion stack was carried by a high altitude balloon which was launched from Palestine, Texas, U.S., on July 1, 1966. The time of flight from launch until cut-down
was 12 hours from which 10 were spent at a mean depth corresponding to 2.3 g cm\(^{-2}\) of residual atmosphere. The curve of flight can be seen in Figure 1.

The emulsion camera had no shutter to protect it during ascent. Instead of a shutter the stack was meant to be flipped 90° after that the balloon had reached its floating altitude. On account of an incorrect estimate of the ascent velocity this flip occurred first after that the balloon had spent 1 1/2 hours above 6 g cm\(^{-2}\) of residual atmosphere. More details about the flight can be seen in Table I.

Usually in emulsion experiments the geomagnetic cutoff at Palestine, Texas, is meant to be high enough to discriminate between relativistic and non-relativistic cosmic ray primaries. This assumption is based on the cutoff calculations by Quenby and Wenk (5). However, in many experimental situations this seems to be a rough approximation. It should also be pointed out that the cutoff values calculated by Shea and Smart (6) are lower than those used earlier. In the charge region studied here, 16 ≤ Z ≤ 28, the A/Z - ratio varies from 2.00 to 2.15. This implies different cutoff energies for the same threshold rigidity and causes a spread in the low energy region for different nuclei. It should be remembered that due to the form of the differential energy spectrum the greatest particle flux is to be found at low energies.

4. **Processing and preparation for measurement**

The stripped emulsions were mounted on glass plates before development. The pellicles were processed according to the temperature method. The dry hot stage was + 25°C and
Table I

Data for flight No. 225-P July 1, 1966

<table>
<thead>
<tr>
<th>Type of balloon</th>
<th>Winzen SF-250.2-NSC-01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>1.7 $10^5 \text{ m}^3$</td>
</tr>
<tr>
<td>Material</td>
<td>0.127 mm stratofilm</td>
</tr>
<tr>
<td>Weight</td>
<td>311 kg</td>
</tr>
<tr>
<td>Payload</td>
<td>148 kg</td>
</tr>
<tr>
<td>Gross load</td>
<td>459 kg</td>
</tr>
<tr>
<td>Free lift</td>
<td>10 %</td>
</tr>
<tr>
<td>Ballast</td>
<td>57 kg</td>
</tr>
<tr>
<td>Launch point</td>
<td>$96^\circ \text{ W } 32^\circ \text{ N}$</td>
</tr>
<tr>
<td>Impact point</td>
<td>$112^\circ \text{ W } 33^\circ \text{ N}$</td>
</tr>
<tr>
<td>Vertical threshold rigidity according to Quenby and Wenk (5) for the launch point</td>
<td>4.70 GV</td>
</tr>
<tr>
<td>Effective vertical cutoff according to Shea and Smart (6) for the launch point</td>
<td>4.48 GV</td>
</tr>
<tr>
<td>Launch time</td>
<td>1100 GMT</td>
</tr>
<tr>
<td>Flip time</td>
<td>1430 GMT</td>
</tr>
<tr>
<td>Cut down time</td>
<td>0100 GMT July, 2</td>
</tr>
<tr>
<td>Residual matter, including packing material</td>
<td>$2.8 \pm 0.2 \text{ g cm}^{-2}$</td>
</tr>
</tbody>
</table>

the time of development was 90 minutes. The degree of development is normal, and the plateau value of blob density for singly charged particles was found to be 20 blobs per 100 $\mu$m. More details about the processing procedure can be
Table II
Processing data

<table>
<thead>
<tr>
<th>Moment</th>
<th>Time</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soaking, aq. dest.</td>
<td>2 hours</td>
<td>5</td>
</tr>
<tr>
<td>Soaking, developer</td>
<td>3 hours</td>
<td>5</td>
</tr>
<tr>
<td>Development, dry</td>
<td>1.5 hours</td>
<td>25</td>
</tr>
<tr>
<td>Stopping bath, HAc 2%</td>
<td>2 hours</td>
<td>25</td>
</tr>
<tr>
<td>Fixing</td>
<td>14 days</td>
<td>12</td>
</tr>
</tbody>
</table>

After the processing each plate was divided into two halves, one called the left and the other the right half. The pellicles were mounted in aluminium frames and aligned. Thus it is possible to trace a track from one plate to the next and from one half of the plate to the other.

The emulsions were processed in several batches owing to limitations in equipment and to difficulties in handling too many pellicles at the same time. All batches were treated equally, but still there is a slight difference in average track widths. The average levels of track widths are shown in Table III.

Table III
Batch data

<table>
<thead>
<tr>
<th>Batch number</th>
<th>Number of plates</th>
<th>Average level of W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
<td>1.000 ± 0.044</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>1.072 ± 0.048</td>
</tr>
</tbody>
</table>
5. **Scanning and scanning criteria**

The scanning was done according to the line-scanning method. Each plate was scanned along a line 4.3 mm from the upper edge of the plate. The tracks accepted in the scanning had to fulfil the following conditions:

1. The track must be produced by particles entering into the stack from the upper hemisphere.
2. The angle between the assumed zenith direction and the projection of the track onto the emulsion plane must be \( \leq 60^\circ \).
3. The length of the projection of the track onto the emulsion plane must be \( \geq 1.0 \) mm.
4. The track must have \( \leq 8 \) blobs in 115 \( \mu m \).

The last condition ensures us that all tracks of particles with charge number \( Z > 8 \) are included in the sample. In addition about \( 1/3 \) of the particles with \( Z = 8 \) were found to fulfil this condition.

The scanning efficiency for the left half of the stack has been estimated to be \( \approx 80 \% \) and for the right half \( \geq 90 \% \). The two halves have been scanned by different scanners which may explain the difference in these figures.

All particles have been traced backwards to the point where they entered the stack and forwards to the point where they interacted or left the stack.

6. **Photometric apparatus**

For the measurements presented in this study we have used a nuclear track photometer developed in Lund. The prototype of the apparatus has been described by von Friesen and Kristiansson (7). After that several changes and impro-
vements have been made. Principally the photometer consists of an ordinary microscope on which a slit system and a photomultiplier are mounted replacing the ordinary eyepiece. This part of the apparatus is usually called the optical part in contrast to the electronic part, which is used for taking the readings. The photometer is used to obtain a measure of the obscuration of the light by the particle tracks. To this end, two signals \( S_1 \) and \( S_2 \) from the photomultiplier are registered. The signal \( S_1 \) corresponds to the radiant flux from the central slit containing a section of the track and the signal \( S_2 \) to the radiant flux from the two background slits. In the ideal case the quantity \( 1 - \frac{S_1}{S_2} \), if multiplied by the width of the slit, would give the real width of the track. In the present work this multiplication has not been carried out. We use the quantity

\[
W = (1 - \frac{S_1}{S_2}) \times 10^4
\]

as a measure of the obscuration. The quantity \( W \) will in this work be referred to as the photometrically determined width of the track. It should be pointed out that \( W \) is a dimensionless quantity.

6.1. The optical part

The optical part has in detail been described by Jönsson, Kristiansson and Malmqvist (8) and Rosander (9). Therefore we will only mention differences which separate our apparatus from theirs. Figure 2 is a sketch of the optical part. The microscope has a condenser with \( NA=0.95 \) and is equipped with a Leitz Oleat 22x oil immersion objective. The dimensions of the central photometric slit are 1.99 x x 0.21 mm\(^2\) corresponding to 103 x 11 \( \mu \)m\(^2\) in the emulsion.
plane. As the tracks are moved in steps of 100 µm there will be a negligible effect of overlapping. The background reading is taken between the lateral distances 15 µm and 29 µm from the track.

In front of the reading microscope there is a filter which has its transmission maximum at the wavelength $\lambda = 530$ nm. The transmission curve is shown in Figure 3a. Further an interference filter with half value transmission at 501.5 and 518.5 nm is placed between the slit system and the photomultiplier. The transmission curve of the interference filter is shown in Figure 3b. In Figure 3c the spectral sensitivity of a human eye is shown in order to justify the choice of the filters: The adjustment of the photometer has been done in the same wavelength interval as the one used for taking the readings.

The used photomultiplier is an EMI 9524 S with a spectral response according to Figure 3d.

As can be seen in Figure 2 the apparatus is equipped with a movable mirror and a movable diaphragm. These are activated by rotary solenoids on orders from a logical unit.

The field of view of the reading microscope is the same as that of the main microscope when equipped with an ordinary eyepiece, except for the superimposed image of the slit system. This enables the operator to adjust the track into the central slit. To take a reading the mirror is automatically flipped out of the beam of light to let the light pass up to the photomultiplier.

To register the signal $S_1$ the movable diaphragm is positioned with the central slit on the optical axis of the apparatus. When the reading of the signal from the central slit
9.

has been finished, the solenoid which turns the diaphragm is automatically powered. This brings the background slits into position to register the signal \( S_2 \). The function of the movable diaphragm is described in detail by Jönsson, Kristiansson and Malmqvist (8) and will not be repeated here.

6.2 The electronic part

The photometer is regarded semi-automatic, because the advancement and adjustment of the tracks into the central slit are made manually by the operator. When a track segment is properly adjusted, the operator initiates a reading sequence from a control unit. During this sequence, which is performed automatically, the following data is transferred to a data recording system: The signals \( S_1 \) and \( S_2 \), the depth coordinate of the track and various book-keeping data.

The data recording system, called OP 3417, is capable of recording data from three measuring stations A, B and C. Its main parts are a digital voltmeter and a paper tape punch. The system and the measuring units B and C are described in a report by Mathiesen (10). Figure 4 shows a principal diagram of our measuring unit and its connection to the data recording system. The logical part of our unit, called unit A, will be described in a forthcoming report by Mathiesen (11). Some details of the electronic part will be discussed below.

6.2.1 The depth meter

As the measured parameter \( W \) is correlated to the depth in the emulsion it is very important to know at what depth the measurement is made. To be able to register the depth
coordinate the apparatus was furnished with a depth meter.

The fine focusing screw of the microscope has been connected with the sliding contact of a potentiometer in such a way that the contact is moved when the screw is turned. A constant voltage is applied across the outer poles of the potentiometer. The depth coordinate is obtained by reading out the voltage across the sliding contact and one of the outer poles.

A measurement of a track in an emulsion begins with a reading on the air surface of the emulsion, where the track starts and is followed by a reading on the glass surface at the same place. Thus the coordinates of the air and the glass surface are known. Then the track is measured in steps of 100 µm to the point where it leaves the pellicle. Each reading contains a measure of the depth coordinate of the track. The measurement of a track in a plate is finished by reading out the coordinate of the air and the glass surface of the emulsion. From the known coordinates of the air and the glass surface at both ends of the track we can derive correct depths for all points of the track assuming that the variations of the coordinates of the surfaces between the end points of the track are linear.

6.2.2 Signal checking

The signals from the photomultiplier and the depth potentiometer can be read out on a monitor prior to their registration by the data recording system. The data recorded on the paper tape punch can be simultaneously printed out on a strip printer. These units are shown in Figure 4.
6.2.3 The reading sequences

From the operator control unit four reading sequences with different meanings can be initiated. The contents of them have been described in detail in a report by Andersson and Larsson (12). Therefore we will only briefly mention the main objectives of the four reading sequences.

1. To register the depth coordinate of the air or the glass surface.

2. To register the depth coordinate, the signal from the central slit and the signal from the background slits.

3. To be used if the last reading sequence has to be replaced by the forthcoming.

4. To be used if the actual reading sequence shall be excluded from the analysis.

The different reading sequences are flagged in such a way that they can be identified in the data analysis.

7. Selection of tracks

In addition to the scanning criteria stated in section 5, a track must fulfil the following conditions to be accepted for measurement:

1. No readings are to be made at a distance less than 8 mm from an edge of the emulsion. With this condition we intend to eliminate errors in W caused by edge effects.

2. It must be possible to measure a track in at least 3 plates.

3. The total length of a track accepted for measurement must exceed a minimum length.

The minimum length is determined in such a way that the ex-
pected standard error of the track mean will be small enough to give satisfactory resolution between consecutive charges. In this work we base our track length criterion on the calculations by Kristiansson, Mathiesen and Waldeskog (2). We have stated that the total length of a track must exceed 1.5 times the value given by these authors.

8. Measurements

The photometric measurements have been carried out in three steps. First we have made preliminary measurements in order to assign a tentative charge number to the particles. Second all tracks whose charge was estimated to be \( Z \geq 14 \) were measured according to the selection criteria discussed above. These measurements are called the main measurements. We believe that the main measurements include all particles having \( Z \geq 16 \). Third all tracks whose total projected length was greater than 3 cm were measured over as long a distance as possible. These measurements are called the remeasurements. The three kind of measurements are made at somewhat different occasions for the left and the right halves of the stack. This will be discussed in connection with the time normalization in section 10.3.

In all about 800 tracks were measured in the preliminary run. From these 197 were accepted for further analysis. Among these 197 particles there are some with \( Z < 14 \). They are included in the main measurements only for calibration and normalization purposes. The remeasurements, which were made in order to study the track width as a function of range, include 50 tracks. The numbers of measured tracks are summarized in Table IV.
Table IV

The number of measured tracks from the three measurements

<table>
<thead>
<tr>
<th>Run</th>
<th>Date</th>
<th>Number of tracks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Left half</td>
</tr>
<tr>
<td>main</td>
<td>Dec 1969</td>
<td>96</td>
</tr>
<tr>
<td>measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>main</td>
<td>May 1970</td>
<td></td>
</tr>
<tr>
<td>measurement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>re-</td>
<td>April 1971</td>
<td>22</td>
</tr>
<tr>
<td>measurement</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) In addition 5 tracks from Dec 1969 were remeasured in May 1970.

9. Data handling

Because of the great number of tracks and the great number of plates the data handling would be very complicated unless it could be made on a computer. In this investigation we have used the UNIVAC 1108 of the Computing Centre of the University of Lund.

With a great computer it is possible to make the analysis wider and in various manners. Thus it is possible to reveal effects, which would have been overlooked in a manual computing procedure.

In order to make a computer analysis possible all data from the photometer and all book-keeping data were recorded on magnetic tape. This information was then analyzed...
tic tape to make the analysis more comfortable.

The foundations of the technical part of the data handling and some of the basic programs are described in a report by Andersson and Larsson (12).

10. General remarks on the normalization procedure

It is a well-established fact that the raw data from the photometer must be subjected to a set of corrections and normalizations in order to compensate for measuring and emulsion effects. Some of them which will be discussed are:

1. depth normalization
2. exclusion of disturbed readings which are not representative for the track
3. time normalization
4. operator normalization
5. plate normalization
6. dip normalization

The different corrections and normalizations are not independent of each other, but in order to simplify the treatment, we will discuss them separately.

10.1. Depth normalization

When the level of the raw readings of W from tracks produced by relativistic particles is studied with respect to the measuring depth in the emulsion, a strong correlation between the level of W and depth will be found. There are many and complex reasons why there should be a depth dependence and some of them are:

1. light scattering in the emulsion
2. a development gradient in the emulsion
3. a rinse out gradient near the air surface of the emul-
4. fading of the latent image near the air and the glass surface of the emulsion.

The resultant depth effect must be corrected for. From the list above the greatest distortion may be expected near the air and the glass surfaces. Therefore we have excluded all readings taken from the upper 15% and lower 10% of the emulsion volume in all pellicles.

In this investigation the depth normalization has been obtained in the following way: The measurement of a track section in an emulsion constitutes a sample of readings of $W$. Each reading has been normalized to the sample mean. The readings have been classified in depth intervals. This procedure has been repeated for all tracks produced by relativistic particles. Then we have obtained regression on depth of the means of normalized $W$ of different depth intervals. Thus we have a functional relation between $W$ and the measuring depth in the emulsion. We have used the reciprocal of the value of the depth function as a normalizing factor.

The regression analysis has been carried out in three steps. First we have made linear regression of normalized $W$ on depth. The output of this step has been used to obtain curve linear regression of $W$ on depth. These parts of the analysis have been performed separately for each plate. After having applied these two normalizing functions, curve linear regression analysis of the resulting $W$ on depth has been carried out for four different groups of values of $W$.

The influence of the track width on the amplitude of the depth correction has been discussed by several authors, e.g. Waldeskog and Mathiesen (13). In this investigation we
have found a greater difference in depth effect between different plates than between different levels of W. This is why we first have made depth normalization for each plate separately. Figure 5 shows the distribution of the slopes of the regression lines obtained in the first depth normalization. As can be seen from Table V the corresponding slopes for different levels of W are nearly independent of W.

Table V
The slopes of the regression lines for four intervals of W.

<table>
<thead>
<tr>
<th>Track width</th>
<th>Slope (mm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W &lt; 1200</td>
<td>0.57</td>
</tr>
<tr>
<td>1500 &lt; W &lt; 1800</td>
<td>0.55</td>
</tr>
<tr>
<td>2200 &lt; W &lt; 2500</td>
<td>0.53</td>
</tr>
<tr>
<td>2800 &lt; W</td>
<td>0.54</td>
</tr>
</tbody>
</table>

The distribution of readings of W has been studied for different levels of W. We have thereby observed that the distributions became narrower and more Gaussian like the more accurate the normalization was made. As can be seen from Table VI the standard deviation will not become much smaller after having made the first step of the depth normalization.

10.2. Exclusion of disturbed readings

Some readings are for various reasons disturbed and are therefore not characteristic for the level of W of the track. The obviously disturbed readings, caused by dark spots, crossing tracks etc. are excluded by the operator and will not affect the level of W of a track in the further analysis.

A detailed qualitative discussion, treating different
Table VI

The standard deviation of the distribution of readings of W for four levels of W during the progress of the depth normalization.

<table>
<thead>
<tr>
<th>Mean value of track width</th>
<th>Standard deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U*-normalized</td>
</tr>
<tr>
<td>990</td>
<td>23.80</td>
</tr>
<tr>
<td>1640</td>
<td>17.01</td>
</tr>
<tr>
<td>2360</td>
<td>13.40</td>
</tr>
<tr>
<td>3090</td>
<td>11.65</td>
</tr>
</tbody>
</table>

kinds of disturbances which may appear along a track, has been given by Rosander (9). A statistical model describing the errors of W is presented in section 11. According to this model the distribution of readings of W is Gaussian. The most diverging readings in each plate were excluded as they may seriously distort the plate mean of W. The standard deviation of W in one plate is not known. Therefore we have used t-statistic to estimate a confidence interval for the mean. In this investigation we have put the confidence level at 90%. All readings beyond the 90% limits of the confidence interval were excluded.

10.3. Time normalization

The photometric measurements used in the analysis were made on three different occasions: the main measurements in the left half of the stack in Dec. 1969, the main measurements in the right half in May 1970 and the remeasurements...
in April 1971. The numbers of tracks measured on the different occasions are summarized in Table IV. Between the main measurements, no change was made on the photometer, while an optical adjustment was necessary before the remeasurements were started. In all 50 tracks were remeasured. Of these we could compare 41 with their levels of $W$ from the main measurements. 21 tracks were from the left and 20 from the right half of the stack. We found a slight increase in the mean level of $W$ when we compared the values from the remeasurements with those from the main measurements. The increase between the two measurements was found to be $0.2 \pm 0.2\%$ in the left half of the stack and $0.7 \pm 0.2\%$ in the right half of the stack.

For normalization purposes we remeasured 5 tracks from the main measurements in the left half during the main measurements in the right half.

All levels of $W$ were normalized to the level of $W$ of the main measurements in the left half. The normalizing factors were found to be $0.6\%$ for the right half and $0.1\%$ for the remeasurements. These figures are calculated as weighted means from the observed changes of the levels of $W$.

During the different measurements, each one amounting to about two weeks, some standard tracks were measured repeatedly. No systematic changes in their level of $W$ were observed.

The slight increase of the mean level of $W$ between the two main measurements may be caused by accumulation of dust on the optics or by small displacements of the optical system. The optics of the photometer was re-adjusted before the remeasurements and this certainly can explain the observed increase of the level of $W$ compared to the level of $W$ during the
main measurements in the right half of the stack.

10.4. **Operator normalization**

The measurements have been made by two operators. It is possible that the two operators adjust the tracks differently into the slit system. These differences may influence the level of W. Comparing a number of tracks which were measured by both operators, we have not been able to find any significant difference in W. Therefore no correction for different operators has been made.

10.5. **Plate normalization**

The stack was processed in three batches under as similar conditions as possible. In spite of this some differences exist both in the transparency and in the degree of development between the batches as well as between the different plates. To be able to use measurements from different parts of the stack all values of W must be normalized. In this normalization such tracks are generally used which do not change their track width. Because of the great number of plates in the stack it is difficult to obtain a perfect normalization. To obtain this it is required that many relativistic particles pass every plate and that at least a few particles pass so many plates that pellicles in one end of the stack can be linked to those in the opposite end of the stack. Furthermore it is required that no difference exists in the level of W in different parts of a plate.

We have done the normalization in two steps. In the first step we have assumed that the tracks used in the normalization procedure were produced by relativistic particles. This gave us a set of first order plate factors. In the se-
cond step we have payed greater attention to the fact that some tracks do not have constant track width. The resulting second order plate factors have been used for plate normalization.

10.5.1. First order plate factors

The plate mean values of \( W \) of all tracks were normalized to the corresponding value in a standard plate. We only used tracks produced by particles which from observations on the track width in different plates were considered as relativistic. For tracks passing through the standard plate we have calculated plate factors for every plate the track passes as the ratio between the plate mean of the standard and the actual plate. In this way we got a number of "individual" plate factors for each plate. Then a weighted mean of normalizing factors was calculated for every plate, using the number of readings as weights.

Since there are only a few tracks which pass through the standard plate as well as through plates in a large distance from it, we have set up a number of secondary standards. These are strongly linked to the standard plate with several tracks.

Thus the plate factors at a large distance from the standard plate have been calculated as weighted mean values of ratios directly to the standard plate as well as to the secondary standards.

10.5.2. Second order plate factors

After having got a set of first order plate factors tracks with measurable lengths exceeding 30 mm were studied closer. 44 tracks without visible change in the track width were in-
cluded in this analysis. If a typical particle does not lose any quantum of charge during its passage through the emulsion, its track width will increase along the track. This means that the derivative of $W$ with respect to the length coordinate $R$, $\frac{\delta W}{\delta R}$, will be larger than zero. Nevertheless we found that 27 tracks had derivatives less than zero. All these tracks were examined under a microscope with a 100x oil immersion objective in order to reveal small charge losses and to determine the direction of the particles unless this was known from collisions. We found that in two cases there had been a small charge loss and in one case the particle which produced the track had entered the stack from the lower hemisphere. All remaining long tracks, having $\frac{\delta W}{\delta R} < 0$, and a sample of shorter tracks were used for the determination of a new set of plate factors. The shorter tracks, chosen from the available tracks, had no visible derivative. Also we require all plates to be passed by as many tracks as possible. Thus we used a total of 64 tracks in an iterative procedure, where all tracks were assumed not to change their level of $W$ along the track.

In the iterative procedure all tracks were first normalized using the first order plate factors. The mean value of $W$ for a track was divided by the plate mean values of $W$ for this track, giving "individual" plate factors for each track. This was repeated for all 64 tracks. A new set of plate factors was determined as weighted means of individual plate factors for each plate using the reciprocals of the variances of the track means as weights. Then the tracks were normalized using the new plate factors and the total sum of squares of deviations of each plate mean from the corresponding track mean was calculated for all tracks. This quantity
is expected to decrease as the correctness of the plate factors increases. The procedure which determines the plate factors was repeated until the total sum of squares of deviations for all tracks was minimized. The final plate factors can be seen in Figure 6.

10.6. Dip normalization

The influence of the dip angle on the measured track width has been carefully examined by Kristiansson, Mathiesen and Waldeskog (2). They found that the dip factors chiefly depend on

1. overlapping of grains and $\delta$-rays
2. parts of the track being out of focus
3. defects of the instrument.

10.6.1. The effect of overlapping

A track produced by a multiply-charged particle is of a complicated nature. The detailed structure of the track is not completely known. In a first approximation it can be considered to be built up of a dense cylinder which is surrounded by completely separated grains. Following Kristiansson, Mathiesen and Waldeskog (2) the projected area of the track $S(\alpha)$ as seen by the photometer can be written

$$S(\alpha) = S(0) \left( a + b \sqrt{1 + \tan^2 \alpha} \right)$$  \hspace{1cm} (10.1)$$

where $\alpha$ is the dip angle in the unprocessed emulsion and $a$ and $b$ are positive constants such that $a + b = 1$. The constant $a$ represents the solid cylindrical part of the track and $b$ the free grains. The constants $a$ and $b$ depend on $W$.

In an attempt to obtain the dip normalizing factors we have estimated the constants $a$ and $b$ from absorption profiles of a number of tracks having different widths. The va-
The value of the constant b is shown in Table VII.

Table VII

The constant b used in the dip normalization for five levels of \( W \)

<table>
<thead>
<tr>
<th>( W )</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.34</td>
</tr>
<tr>
<td>1330</td>
<td>0.43</td>
</tr>
<tr>
<td>2000</td>
<td>0.62</td>
</tr>
<tr>
<td>2300</td>
<td>0.70</td>
</tr>
<tr>
<td>3000</td>
<td>0.90</td>
</tr>
</tbody>
</table>

10.6.2. Defocusing and instrumental effects

When dipping tracks are measured, the tracks are focused in the middle of the slit with both ends being out of focus. A defocused track gives a lower value of \( W \) than a track in focus. This decrease of \( W \) is called the defocusing effect. The defocusing effect depends on the dip angle, on the depth in the emulsion and on the width of the track. The defocusing is not very sensitive to the track width. In many cases, i.e. when only tracks which pass through the emulsion are measured, the defocusing effect can, in a first approximation, be assumed to depend only on the dip angle.

The instrumental effect depends on the difference in the spectral response of a human eye and the photomultiplier and on chromatic aberration of the optical system. In the present investigation an interference filter has been placed in front of the photomultiplier. Due to this the instrumental effect can be assumed to be relatively unimportant in our study. It has been treated together with the effect of defocusing. The magnitude of the compound effect has been es-
timated from measurements of W on flat tracks deliberately defocused. Its dependence on $\alpha$ is shown in Figure 7.

10.6.3. Dip factors

With the aid of data presented in Table VII and Figure 7 the dip normalizing factors have been calculated for different levels of W. The normalizing factors are shown in Figure 8. The correctness of the factors can be checked if well-separated groups of tracks exist in the material. In this investigation we have an admixture of non-relativistic particles and therefore it is hard to identify the groups and therefore also to check the dip normalization.

11. A statistical model for the analysis of track width readings

11.1. Theory

In order to study different sources of error involved in the method of photometric track width measurements we will discuss a statistical model which describes the data taken by our photometer.

In the normalization procedure the different effects were taken into consideration by applying factors to the raw readings of W. A statistical model can also be made as a factorial model. However, it can also be made as an additive one. The latter can easily be obtained from the first by a logarithmic transformation. In this report we have used an additive model.

Consider a reading of W taken in plate i from track j which was produced by a particle with charge number Z. Let $x$ denote the coordinate along the track within one plate.
Assume that the particle which produced the track was relativistic. Then a single reading of \( W \) can be expressed by

\[
W_{ij\ell}(Z) = W_0(Z) + \alpha_i(Z) + \beta_{ij}(Z) + \epsilon_{ij\ell}(Z) \quad (11.1)
\]

where \( W_{ij\ell}(Z) \) is a random variable. \( W_0(Z) \) is the true level of \( W \) of a track produced by a particle with charge number \( Z \). The quantity \( \alpha_i(Z) \) depends only on \( Z \). The stochastic variables \( \beta_{ij}(Z) \) and \( \epsilon_{ij\ell}(Z) \) are in a first order approximation assumed to be independent of each other and \( \epsilon_{ij\ell}(Z) \) is assumed to follow a normal distribution \( N(0, \sigma_0(Z)) \). No assumption is made for the distribution of \( \beta_{ij}(Z) \). When all corrections and normalizations are applied \( \alpha_i(Z) \) accounts for variations between the plates, \( \beta_{ij}(Z) \) accounts for variations within the plates and \( \epsilon_{ij\ell}(Z) \) accounts for variations between the successive readings. Their expected values \( E \) and variances \( V \) can be written

\[
E(\alpha_i(Z)) = \alpha_i(Z) \quad V(\alpha_i(Z)) = 0 \quad (11.2 \ a,b)
\]

\[
E(\beta_{ij}(Z)) = 0 \quad V(\beta_{ij}(Z)) = \sigma^2(\beta) \quad (11.3 \ a,b)
\]

\[
E(\epsilon_{ij\ell}(Z)) = 0 \quad V(\epsilon_{ij\ell}(Z)) = \sigma^2(\epsilon) \quad (11.4 \ a,b)
\]

The expected value of a single reading of \( W \) can be written

\[
E(W_{ij\ell}(Z)) = W_0(Z) + \alpha_i(Z) \quad (11.5)
\]

Plate normalizing means that all plates are normalized to the same level. The quotients \( \frac{W_0(Z)}{W_0(Z) + \alpha_i(Z)} \) will be the normalizing factors. The normalizing factors are assumed to be independent of the charge of the ion which produced the track, and thus \( \alpha_i(Z) \) must be assumed to be charge dependent. The expected value and the variance of a completely normalized reading \( W'_{ij\ell}(Z) \) can be written

\[
E(W'_{ij\ell}(Z)) = W_0(Z) \quad (11.6)
\]

\[
V(W'_{ij\ell}(Z)) = \left\{ \sigma^2(\beta) + \sigma^2(\epsilon) \right\} \left( \frac{W_0(Z)}{W_0(Z) + \alpha_i(Z)} \right)^2 \quad (11.7)
\]
As the normalizing factors approximatively equal unity the
effect of normalization on the variance will be negligible.
Thus the variance of a single reading can be written

\[ \text{var}(W_{ij}(Z)) \approx \sigma^2_\beta(Z) + \sigma^2_\alpha(Z) \] (11.8)

In order to decrease the standard error of a track mean the
number of used plates, \( N_j \), is increased. This yields a grea-
ter number of readings, \( n_j = \sum_{i=1}^{N_j} n_{ij} \), where \( n_{ij} \) is the number
of readings in plate \( i \), and we can write the variance of the
track mean as

\[ \text{var}(W_j(Z)) = \left\{ \sum_{i=1}^{N_j} n_{ij} \right\} \frac{\sigma^2_\beta(Z)}{n_j} + \frac{\sigma^2_\alpha(Z)}{n_j} \] (11.9)

The track mean \( W_j(Z) \) is defined by

\[ W_j(Z) = \left\{ \sum_{i=1}^{N_j} \sum_{j=1}^{n_{ij}} W_{ij}(Z) \right\} / n_j \] (11.10)

When the numbers \( n_{ij} \) are equal, equation 11.9 can be written

\[ \text{var}(W_j(Z)) = \frac{\sigma^2_\beta(Z)}{N_j} + \frac{\sigma^2_\alpha(Z)}{n_j} \] (11.11)

The appearance of \( N_j \) in the denominator in the first term
on the right is motivated by the assumption that the irregu-
larities are randomly distributed. If there is a systematic
unevenness in the stack the reduction of this term by the
factor \( N_j \) will be incorrect.

A discussion similar to this one has been carried out
by Kristiansson, Mathiesen and Waldeskog (2) using a diffe-
rent terminology.
27.

Next we consider a track produced by a non-relativistic particle. Letting \( x \) denote the coordinate along the track, a single reading of \( W \) can be written

\[
W_j(v(x),Z) = W^*(v(0),Z) + f^*(v(x),Z) \tag{11.12}
\]

where \( f^*(v(x),Z) \) is a function which accounts for the increase of \( W \) when the velocity \( v \) of the ion decreases. The origin of the coordinate \( x \) is assumed to be at the point where the first reading is taken. \( W^*_j(v(0),Z) \) is the function which describes the level of \( W \) at this point. In this investigation we have approximated \( f^*(v(x),Z) \) with linear functions yielding

\[
W_j(v(x),Z) = W^*_j(v(0),Z) + k_j(v(0),Z) x \tag{11.13}
\]

where \( k_j(v(x),Z) \) is the slope of the straight line.

Now let us consider the corresponding stochastic quantities. A single reading of \( W \) taken from a track produced by a non-relativistic particle can be written

\[
W_{ij\ell}(Z) = W_j(v(x_{ij\ell}),Z) + \alpha_i(Z) + \beta_{ij}(Z) + \epsilon_{ij\ell}(Z) \tag{11.14}
\]

or when using equation 11.13

\[
W_{ij\ell}(Z) = W^*_j(v(0),Z) + k_j(v(0),Z) x_{ij\ell} + \alpha_i(Z) + \beta_{ij}(Z) + \epsilon_{ij\ell}(Z) \tag{11.15}
\]

The identification of non-relativistic particles relies on the determinations of the track mean \( \overline{W}_j(Z) \) and of the slope \( k_j \). Therefore our interest now focuses on the expected value and the variance of \( k_j \). These can be written

\[
E(k_j) = f_1(\beta_{ij}(Z),v(0),Z) \tag{11.16}
\]

\[
V(k_j) = f_1(\beta_{ij}(Z),v(0),Z) \tag{11.17}
\]
where \( v(0) \) is the velocity of the particle at \( x=0 \) and \( x_{ijl} \) is the coordinate of the reading of \( W_j \). If the variations between and extended irregularities within the emulsions are put equal to zero we have

\[
E(\kappa_j) = f_3(v(0), Z) \tag{11.18}
\]

\[
V(\kappa_j) = \frac{\sigma_o^2(Z)}{\sum_{i=1}^{N_j} \sum_{l=1}^{nij} (x_{ijl} - \bar{x}_j)^2} \tag{11.18}
\]

where \( \bar{x}_j \) is the mean value of the length coordinate. This means that using a constant slit length the standard error of \( \kappa_j \) can be written as

\[
e_{\kappa} \approx k n^{-3/2} \tag{11.19}
\]

where \( n \) is the total number of readings of a track and \( k \) is a constant including \( \sigma_o(Z) \). In this first order theory \( \sigma_o(Z) \) is assumed to be independent of the velocity of the ion. In reality \( \sigma_o(Z) \) increases slightly when the velocity decreases.

The standard error of the slope which has been discussed above is valid when the readings are taken consecutively along the tracks. In order to fasten the measurements the innermost parts of the available track length can be skipped. As the number of readings then decreases, the accuracy also will decrease. The problems concerning how to estimate the amount of information lost using this method is discussed by Jensen (14).

11.2. Estimation of parameters

The accuracy of an investigation on cosmic-ray particles...
based on track width measurements in nuclear emulsions depends on the dispersions of the readings of \( W \) between and within plates, and on the numbers of plates and readings. In order to describe the dispersions we have introduced the variances \( \sigma^2_\beta(Z) \) and \( \sigma^2_o(Z) \). A straight ahead analysis of variance requires a group of tracks produced by nuclei of the same kind, having the same dip angle and passing a number of plates without changing their track width. These conditions are usually not fulfilled in a stack exposed to the cosmic radiation. This means that in the present study it is complicated to estimate the parameters. It seems possible to find consistent estimates, but it is more difficult to find unbiased ones. We have started with the estimate of \( \sigma_o(Z) \).

It can be shown that when all corrections and normalizations are applied the mean of the sampling distribution of

\[
s^2(Z) = \frac{\sum_{j=1}^{N_j} \sum_{i=1}^{N_i} \sum_{l=1}^{n_{ij}} (W_{ijl}(Z) - \overline{W}_{ij}(Z))^2}{n_{ij} - 1} (11.20)
\]

is equal to \( \sigma^2_o(Z) \) if there is no correlation between consecutive readings. \( N_Z \) is the number of tracks with charge \( Z \), \( N_j \) is the number of plates for track \( j \), \( n_{ij} \) is the number of readings in plate \( i \) and \( \overline{W}_{ij}(Z) \) is the plate mean of \( W \). The covariance between readings has been studied and no correlation could be proved. The distribution of readings of \( W \) was studied in order to estimate \( \sigma_o(Z) \). This has been made for four levels of \( W \). In Figure 9 \( \sigma_o(Z)/\overline{W}_o(Z) \) is shown as a function of \( Z \).

To verify the assumption that \( \sigma_o(Z) \) is normally distributed we have made \( \chi^2 \)-tests for goodness of fit. In the
tests we have compared the observed sample distribution with a theoretical frequency distribution with the same mean and the same standard deviation. This was repeated for the four levels of \( W \) studied. The hypothesis that the samples were taken from normal distributions could not be rejected. In addition we have compared observed values of skewness and excess with the theoretical ones for normal distributions. Also these results support our hypothesis.

Because of the lack of homogeneous groups of tracks the estimates of \( \sigma_g(Z) \) cannot be unbiased. We have studied the distributions of the plate means for all tracks passing each plate and the distributions of the plate means for each track in order to estimate \( \sigma_g(Z) \). The average value of this estimate is equal to or less than \( 3.5 \pm 0.9 \% \). However, this figure should be looked upon with care. At any rate it means that in the present stack the variations within the emulsions may be a serious source of error.

In the plate normalization the plates have been normalized using tracks from differently distributed regions for each plate. This means that each plate has been normalized as an average, but still almost every single track will be wrongly normalized owing to the irregularities within the plates. Thus in spite of the plate normalization, for a given track residual variations exist in the plate mean values of \( W \). These variations are in general different for different tracks.

The slope \( \kappa \) has been estimated from a linear regression analysis using the range in the emulsion stack as the independent variable. The best estimate of the slope should be obtained as the weighted mean of the slopes estimated se-
parately for all plates passed by the ion which produced
the track. However, this requires perfect depth normaliza-
tion for each single track and this cannot possibly be ob-
tained. Therefore, in this investigation we were compelled
to make one single estimate of the slope using all readings
taken on the track.

12. **The existence of non-relativistic particles**

From cutoff reasons it can be expected that at the site
of exposure there will be a non-negligible number of fast
but non-relativistic primary particles entering the earth's
atmosphere. The calculations by Shea and Smart (6) on the
cutoffs for Palestine, Texas, the site of launch, show that
particles with rigidities as low as 3.5 GV can be expected
within the zenith angle $\phi \leq 65^0$ accepted here. Furthermore
on account of the form of the rigidity spectrum the flux of
particles of species $i$ with rigidities greater than the cut-
off $P_0$ can be written $J_i(P>P_0) = J_{i0}P_0^{-\gamma}$ where $J_{i0}$ is a
constant for species $i$ and the exponent $\gamma$ is reasonably
constant $\approx 1.5$ at energies that include the bulk of the cos-
mic rays.

These two facts mean that the chance of non-relativis-
tic nuclei being registered is obvious. In addition, the
loss of energy during the passage of the overlying material
down to the point in the emulsion where the track is measured
has to be taken into account. Consequently there will be
particles whose charges will be wrongly determined owing to
the low velocities giving the particles too high levels of
$W$. However, these non-relativistic particles can be correct-
ly identified if the rate of change of $W$ along the tracks can
be determined with satisfactory accuracy. In section 14 the conditions for this will be discussed.

13. **The relation between track width and velocity**

When a charged particle passes through an absorbing material the electric field of the particle interacts with the bound electrons of the absorber. The charged particle loses energy and its velocity decreases. The average loss of energy per unit path length of a charged particle, \( \frac{dE}{dx} \), is expressed by the well-known formula of Bethe and Bloch:

\[
\frac{dE}{dx} = k \frac{Z^2}{\beta^2} \left\{ \ln \left( \frac{2 m_e c^2 \beta^2}{I(1 - \beta^2)} \right) - \beta^2 - C(\beta) \right\}
\]

where

- \( k \) is a constant which includes the average charge number of the absorbing material
- \( Z \) is the charge number of the particle
- \( \beta \) is the velocity of the ion relative to the velocity of the light
- \( m_e \) is the rest mass of the electron
- \( I \) is the mean excitation potential of the atoms of the absorber
- \( C(\beta) \) is a correction term which accounts for the density effect at high energies and for the shell correction at low energies.

The graph of \( - \frac{dE}{dx} \) versus \( E \) goes for low energies as \( \beta^{-2} \) and passes through a wide minimum at \( \beta \approx 0.93 \). For singly charged particles there will be a relativistic rise at high energies.

In investigations on the charge spectrum of primary cosmic ray particles using the nuclear emulsion technique, two
energy regions have essentially been payed attention to. The first of these is the low energy region where the charged particles come to rest in the detector and the second is the energy region where the loss of energy can be assumed to be nearly constant at a minimum level along all of the track. In these two cases it is relatively easy to identify the particles. In the first case the residual range and some other parameter of ionization can be measured and in the second case it is only necessary to know the level of some parameter of ionization.

If the energy of the particle is in the intermediate region the particle cannot be identified, unless the ionization and its rate of change along the track can be measured.

In the photometric method employed by us the track width is used as a measure of ionization. In a separate paper (15) we have studied the dependence of the track width on the velocity of the ion using the same photometer as in the present investigation. In that study we found an approximative relationship

$$W(\beta, Z) = W_0(Z) \beta^{-\alpha(Z)}$$

(13.2)

where $W(\beta, Z)$ is the level of $W$ of an ion with charge number $Z$ and velocity $\beta$ and $W_0(Z)$ is the hypothetical level of $W$ of an ion with charge number $Z$ at $\beta=1$. The exponent $\alpha(Z)$ is assumed to depend only on $Z$. The numerical value of $\alpha(Z)$ was found to be about one. This figure is rather different from the exponent of $\beta$ appearing in the formula of Bethe and Bloch. This discrepancy has been discussed in the above-mentioned study (15). We have assumed that the above relation is valid also for the emulsions used in the present study. From this expression we have obtained the relationship be-
the range of the particle. This makes it possible for us to identify non-relativistic particles by measuring their track width and its derivative. The measurements will be presented in the next section.

14. **Identification of non-relativistic particles**

Whether a particle can be identified from the method of combined measurements of \( W \) and \( \frac{\delta W}{\delta R} \) depends on the accuracy with which the derivative can be determined. In an attempt to find the non-relativistic particles we have calculated \( \frac{\delta W}{\delta R} \) for all particles in the stack which fulfil all criteria concerning track length, number of pellicles and dip angle stated in sections 5 and 7.

In this calculation it has been assumed that the relation between \( W \) and \( R \) can be approximated by a straight line. This means that the derivative \( \frac{\delta W}{\delta R} \) has been approximated by the slope \( \frac{\Delta W}{\Delta R} \). A straight line approximation is always satisfactory and does not give rise to systematic errors. This relation between \( W \) and \( R \) has been calculated by the method of least squares.

A total number of 197 tracks fulfilled the above-mentioned conditions. Of these tracks 16 were excluded from further analysis owing to nuclear interactions or their entering the stack from the wrong direction. Approximately 60 of the accepted tracks correspond to charges below \( Z \leq 16 \). They are included in the analysis for normalization and calibration purposes. Figure 10 shows the derivative \( \frac{\delta W}{\delta R} \) as a function of \( Z \) for different energies of the ion. The values of \( \frac{\delta W}{\delta R} \) for the iron group nuclei should be compared with the stan-
standard errors of $\frac{\Delta W}{\Delta R}$ shown in Figure 11. As it can be seen from Figure 11 the experimental points are distributed around the errors computed from equation 11.18. In addition we show in Figure 11 the expected standard errors assuming that internal variations exist in 20% of the used plates. The standard deviation of these disturbances has been assumed to be $\sigma_\beta = 5\%$. It can be seen that the standard error $e_\beta$ is not noticeably affected by disturbed emulsions. However, the expected value of the slope can be very seriously distorted by the disturbances, giving rise to a spurious derivative. Consequently, a particle can be wrongly identified owing to the false derivative. In Figure 12 we show the maximum spurious derivative for the iron group nuclei as a function of the relative number of disturbed emulsions. These emulsions are assumed to have irregularities which amount to 3% of $W$. Taking these results into account it can be concluded that severe conditions must be applied for identification of the particles from measurements of $W$ and $\frac{\Delta W}{\Delta R}$. In this investigation we have required that besides the conditions stated previously the tracks must have at least 180 accepted readings, be measured in at least 7 emulsions and have a derivative which exceeds its standard error. The two first conditions were fulfilled by 43 tracks and all three conditions by 20 tracks.

The relation between $W$ and $\frac{\Delta W}{\Delta R}$ has been calculated from equation 13.2 for particles with $16 \leq Z \leq 28$. The values needed for $W_0(Z)$ have been taken from the calibration line discussed in the next section. The relation between energy and range for heavy ions has been derived from the corresponding relation for protons given by Barkas (16). The result
of the calculations is shown in Figure 13.

In Figure 13 we also have plotted the results of the measurements of \( W \) and \( \frac{\Delta W}{\Delta R} \) of the 20 tracks which fulfilled the extended criteria stated above. Figure 13 has been used to identify the non-relativistic particles which fulfilled our measurement criteria for \( \frac{\Delta W}{\Delta R} \). The correctness of the identification is somewhat ambiguous. As it was pointed out earlier, false derivatives can be expected and these are not necessarily accompanied by large standard errors which would make it possible to reveal the spurious derivatives. In addition there are other sources of error which affect the identification. The discussion of the accuracy of this identification is found in section 17. Here we wish to mention the existence of two non-relativistic iron-group nuclei, having great derivatives, which support our calculations. Before dip normalization and before their \( \frac{\Delta W}{\Delta R} \) were taken into account these two tracks were identified as \( Z = 35 \). After dip normalization they were lowered to \( Z = 32 \). Finally, when their derivatives were taken into account, they appeared as non-relativistic iron-group nuclei.

15. Identification of relativistic particles

At the top of the atmosphere and at the geomagnetic latitude where the emulsions were exposed to the cosmic radiation, all particles are commonly supposed to be relativistic. Thus it is assumed that they can be correctly identified from measurements of one parameter of ionization only. However, in the present investigation we have found that there exists a non-negligible number of non-relativistic particles in the spectrum. These particles were revealed by measurements of
We have assumed that the particles with derivatives not significantly different from zero are relativistic. To obtain the relativistic levels of $W$ some kind of calibration is necessary. For this different methods can be used, such as $\delta$-ray counting, some track formation theory or studies nuclear interactions. The most reliable way is to use the theoretical dependence of some measurable quantity on the charge number. Such a dependence exists e.g. for the delta-ray density and can at least in principle be obtained for the width of particle tracks from existing theories. However, the delta-ray method is difficult to apply for $Z > 16$ and the track formation theories cannot at present be regarded sufficiently accurate for our purposes. Thus in the present investigation we had to rely on information about the general shape of the charge spectrum as determined in earlier investigations, e.g. as summarized by Shapiro and Silberberg (1). This information has been combined by observations of differences in $W$ found for particles which undergo small nuclear interactions with known losses of charge. The resultant calibration curve is shown in Figure 14. This calibration has been used for all particles which could not be identified by measurements of $W$ and $\Delta W / \Delta R$.

16. The charge distribution

16.1. Experimental results

The emulsion stack was exposed to the cosmic radiation by a high altitude balloon and the overlying matter, including packing material, corresponds to $2.8 \text{ g cm}^{-2}$. The distance along the track between the entering point of the par-
particle in the stack and the starting point for the measurements where the first reading was taken can vary. This is due to the criteria for scanning and selection of the tracks as discussed in sections 5 and 7. Further both the starting point and the measured lengths of the tracks can be essentially different from each other for different tracks. As a consequence the track mean values for different particles are usually obtained at different amounts of overlying matter. The mode of the distribution of the amount of overlying emulsion is 22.5 mm and the mean is 37.2 mm. Thus the measured level of W can be different for particles with the same charge number and the same velocity at the top of the stack. This effect has been taken into account only for these tracks which are identified by the quantities W and $\frac{\Delta W}{\Delta R}$.

Figure 15 a,b and c show the distribution of track mean values of W of all particles in the charge interval $Z > 16$ whose tracks fulfilled the conditions for measurement. In addition the histogram includes a number of particles with lower charges. These particles were included in the investigation for calibration and normalization purposes. In Figure 15 we have also marked off some charge numbers corresponding to the value of W as given by the calibration curve. It should be pointed out that the relative abundances of nuclei with charge number $Z < 16$ are not representative for the primary cosmic abundances at the site of exposure.

The spectrum presented in Figure 15 includes both particles identified from measurements of W only and non-relativistic particles identified from measurements of W and $\frac{\Delta W}{\Delta R}$. The charge spectrum is presented in three steps. In Figure 15a all normalizations but dip normalization are applied.
The spectrum shows a pattern, which is difficult to interpret correctly. The iron group peak is broad and the level of the relativistic iron is hard to fix. In Figure 15b dip normalization is added. This narrows in particular the iron peak. Furthermore the mode of the iron peak has decreased in comparison with Figure 15a. However, the spectrum is still distorted and the resolution between consecutive charges is not obvious everywhere in the examined charge interval. To improve the separation between consecutive charges the level of \( W \) of non-relativistic particles has been normalized to the corresponding relativistic level. The resulting spectrum is shown in Figure 15c. In extreme cases this normalization corresponds to a decrease amounting to 5-6 units of charge. The iron peak is drastically changed and it has obtained a more distinct structure. However, there are still some disturbances in the spectrum and completely resolved peaks are not obtained. The remaining disturbances are probably caused by the admixture of non-relativistic particles which could not be normalized to the relativistic levels of \( W \). Furthermore, there may exist particles which are identified wrongly according to the \( W - \frac{\Delta W}{\Delta R} \) method because the derivatives are sensitive to disturbances in the level of \( W \). A non-relativistic particle, whose low energy has not been corrected for, can be situated between two peaks representing consecutive charges. In an extreme case it can be included in a peak containing particles with higher charge number. Most peaks are distorted in this way and the effect is especially well pronounced in the iron group. Thus in Figure 15c the region of \( W \) having peaks at \( W = 3030 \) and \( W = 3210 \) contains a number of non-relativistic iron nuclei.
In Figure 16, b and c is shown the distribution of 20 non-relativistic particles identified by measurements of $W$ and $\frac{\Delta W}{\Delta R}$. These particles are included in Figure 15 as well. The normalizations are applied in the same order as in Figure 15. The dip normalization does not improve the resolution essentially. The normalization to relativistic levels moves the distributions drastically. It should be pointed out that the 20 particles in Figure 16 do not necessarily constitute a representative sample from the totality of non-relativistic very heavy nuclei in the primary cosmic radiation. Therefore the relative abundances of different elements in Figure 16 should not be expected to reflect the cosmic ray abundances. The imperfect resolution between consecutive charges seems to give an appropriate picture of the precision of the charge identification method attainable with the present data.

16.2. Analysis of sources of error

The sources of error which may give rise to incorrect identifications are somewhat different for the relativistic and the non-relativistic particles. Sources of error being common to both methods are those due to uncertainties in:
1. the dip normalization
2. the calibration curve
3. caused by the random spread $\sigma(Z)$ of the track width.
In addition to these sources the method of identification using combined measurements of $W$ and $\frac{\Delta W}{\Delta R}$ is sensitive to uncertainties in:
4. the determination of $\frac{\Delta W}{\Delta R}$
5. the validity of equation 13.2
6. the assumed $A/Z^2$-ratios.
The identification method using only the level of $W$ requires that all particles are relativistic or at least have approximately the same energy per nucleon. The identification according to this method will be affected if there exist

7. non-relativistic particles whose derivative of $W$ along the track cannot be determined with sufficient accuracy.

The above-mentioned sources of error will be briefly discussed.

1. The tracks which can be suspected to be wrongly identified have rather different dip angles, and therefore the error caused by the dip normalization cannot be considered to be systematic. However, it is reasonable to assume that dipping tracks with very high values of $W$ ($W \geq 3500$) will more likely be wrongly normalized because of the lack of satisfactory checking possibilities. The error introduced by a wrong dip normalization may in worst case amount to one unit of charge.

2. The uncertainty of the calibration curve is hard to estimate. If the calibration curve is wrongly determined, this can reasonably be done in only one direction because the abundance of trans-iron nuclei is very low. Assuming the calibration curve to be displaced some unit of charge, the peak identified as corresponding to iron nuclei actually contains particles of a lower charge, i.e. chromium or manganese. However, it seems unlikely to have only a few non-relativistic iron nuclei compared to the great number of non-relativistic nuclei having charge numbers between 20 and 24.

3. In the charge determination of relativistic particles
the random errors caused by variations due to $\varepsilon_{ij}(Z)$ are diminutive. The standard error of the track mean of $W$ is usually less than 1 %. This means that the corresponding error in the charge identification amounts to 0.2 - 0.3 units of charge in the charge interval studied. In the charge determination of non-relativistic particles the variations due to $\varepsilon_{ij}(Z)$ are more important. The magnitude of the random error of $\frac{\Delta W}{\Delta R}$ depends on the charge number of the particle and the measured track length. In Table VIII we show the corresponding errors in $Z$ for iron group nuclei as a function of the measured track length.

Table VIII

The maximum and standard error in the charge identification of a non-relativistic iron nucleus ($W=3200$) caused by
1) a spurious derivative ($\sigma_\beta=3$% and 20% disturbed plates)
2) an accidental derivative ($\sigma_\beta=0$)

<table>
<thead>
<tr>
<th>Track length (mm)</th>
<th>Spurious $\Delta Z$</th>
<th>Accidental $\Delta Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>2.0</td>
<td>1.2</td>
</tr>
<tr>
<td>40</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>50</td>
<td>1.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4. The effects of extended irregularities within the emulsions may give rise to false derivatives $\frac{\Delta W}{\Delta R}$, and therefore the particles can be wrongly identified. The magnitude of this error depends on the charge of the ion which produced the track, on the number of plates in which the track has been measured and also on the fraction of the disturbed plates. This problem has been examined in section 14. In Table VIII
we show the errors caused by spurious derivatives for iron
group nuclei. As can be seen from the Table the error in-
troduced by false derivatives may easily amount to one unit
of charge or more.

5. The relation 13.2 between an ion's value of W and its
velocity has been discussed in section 13. The great prob-
lems are whether this simple formula is valid in the same
form in this stack as in the stack where it was determined
and if it is valid in the velocity region considered here.
The same photometric equipment and the same type of nuclear
emulsions have been used in the examinations of the two
stacks, but unfortunately the correctness of the relation
cannot be completely verified because no ending tracks can
be found in the present stack. In order to study the sensi-
tivity of the charge identification procedure on the numeri-
cal value of \( \alpha(Z) \) in equation 13.2 we have repeated the
identification procedure using \( \alpha(Z) = 0.7 \) and \( \alpha(Z) = 1.2 \) for
all charge numbers. The value of \( \alpha(Z) \) effects the calcula-
ted value of \( \frac{\delta W}{\delta N} \) and thus the assigned charge numbers. In
Table IX we show the charge numbers obtained with the experi-
mentally found linear function \( \alpha(Z) \) and with the above-men-
tioned charge independent values of \( \alpha(Z) \).

6. As the masses of the ions registered in the emulsion
stack are not known we have been obliged to make assumptions
about the \( A/Z^2 \)-ratios. We have used for each charge number
the weighted means of \( A/Z^2 \) of the isotopes present at the
earth. These mean ratios may differ from the actual ratios,
but the identification of the particles will not be notice-
ably affected.

Table X shows a summary of errors caused by the previ-
Table IX

Charge numbers assigned to non-relativistic particles for different assumptions about $\alpha(Z)$.

<table>
<thead>
<tr>
<th>Particle number</th>
<th>$\alpha(Z) = a + bZ$</th>
<th>$\alpha(Z) = 0.7$</th>
<th>$\alpha(Z) = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>28.2</td>
<td>28.3</td>
<td>27.9</td>
</tr>
<tr>
<td>45</td>
<td>26.5</td>
<td>26.8</td>
<td>25.0</td>
</tr>
<tr>
<td>83</td>
<td>24.3</td>
<td>24.5</td>
<td>23.7</td>
</tr>
<tr>
<td>103</td>
<td>22.6</td>
<td>22.8</td>
<td>22.2</td>
</tr>
<tr>
<td>116</td>
<td>23.6</td>
<td>23.7</td>
<td>23.1</td>
</tr>
<tr>
<td>173</td>
<td>22.8</td>
<td>23.0</td>
<td>22.4</td>
</tr>
<tr>
<td>175</td>
<td>23.5*</td>
<td>23.7</td>
<td>23.0</td>
</tr>
<tr>
<td>236</td>
<td>18.5</td>
<td>18.8</td>
<td>18.0</td>
</tr>
<tr>
<td>255</td>
<td>25.7</td>
<td>26.1</td>
<td>24.3</td>
</tr>
<tr>
<td>327</td>
<td>23.2</td>
<td>23.6</td>
<td>22.4</td>
</tr>
<tr>
<td>831</td>
<td>23.8</td>
<td>23.8</td>
<td>23.0</td>
</tr>
<tr>
<td>845</td>
<td>21.9</td>
<td>22.1</td>
<td>21.3</td>
</tr>
<tr>
<td>846</td>
<td>24.5</td>
<td>24.6</td>
<td>24.1</td>
</tr>
<tr>
<td>848</td>
<td>19.1</td>
<td>19.7</td>
<td>17.9</td>
</tr>
<tr>
<td>849</td>
<td>23.6</td>
<td>23.8</td>
<td>22.9</td>
</tr>
<tr>
<td>858</td>
<td>21.2</td>
<td>21.6</td>
<td>20.5</td>
</tr>
<tr>
<td>862</td>
<td>25.8</td>
<td>26.0</td>
<td>25.0</td>
</tr>
<tr>
<td>863</td>
<td>23.8</td>
<td>23.9</td>
<td>23.2</td>
</tr>
<tr>
<td>870</td>
<td>26.4</td>
<td>26.6</td>
<td>26.1</td>
</tr>
<tr>
<td>872</td>
<td>25.1</td>
<td>25.2</td>
<td>24.6</td>
</tr>
</tbody>
</table>

ously discussed sources. From this Table it can be concluded that the conceivable error may easily amount to one unit of charge. The experimentally found errors for the non-rela-
tivistic particles are shown in Table XI.

Table X

Summary of the conceivable errors in the charge determination according to the method of combined measurements of W and \( \Delta W/\Delta R \) as discussed in the text.

<table>
<thead>
<tr>
<th>Source of error</th>
<th>( \Delta Z )</th>
<th>Type of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Dip normalization</td>
<td>( \approx 1 )</td>
<td>maximum error</td>
</tr>
<tr>
<td>2. Calibration curve</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>3. ( e(\bar{W}) = 1% ) W=2000</td>
<td>0.2</td>
<td>standard error</td>
</tr>
<tr>
<td>3. ( e(\bar{W}) = 1% ) W=2900</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>3. ( e(\Delta W/\Delta R) ) W=3200</td>
<td>0.5 - 1.2</td>
<td>-</td>
</tr>
<tr>
<td>4. Spurious ( \Delta W/\Delta R ) W=3200</td>
<td>1.3 - 2.0</td>
<td>maximum error</td>
</tr>
<tr>
<td>5. Wrong ( \alpha(Z) ), 0.7&lt;( \alpha(Z) )&lt;1.2</td>
<td>0.1 - 1.2</td>
<td>-</td>
</tr>
<tr>
<td>6. Wrong A/Z²</td>
<td>&lt; 0.01</td>
<td>-</td>
</tr>
</tbody>
</table>

7. The most dominant source of error in the charge identification by the use of W only, is that caused by the admixture of non-relativistic nuclei. The existence of particles with rather low energies may have different causes such as the low rigidity cutoff, particles being registered during ascent and incorrect assumptions about the zenith direction. All these effects decrease the low energy limit of the particles registered in the emulsion stack and thus increase the average loss of energy for all charges. This increase may lead to an incorrect charge identification. Of 43 particles whose tracks fulfilled the criteria for measurements according to the \( W \Delta W/\Delta R \) - method, 20 had observable derivatives and could be normalized to the relativistic levels. This means
Table XI

Data for the non-relativistic particles. $e = $ standard error

<table>
<thead>
<tr>
<th>Particle number</th>
<th>Dip angle</th>
<th>$W$</th>
<th>$e_\text{e}$</th>
<th>$K = \frac{\Delta W}{\text{mm}^{-1}}$</th>
<th>$e_\text{K}$</th>
<th>$N$</th>
<th>$Z$</th>
<th>$\Delta Z$</th>
<th>$E_{\text{kin}}$ (GeV/n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>0.16</td>
<td>3348</td>
<td>13.6</td>
<td>1.44</td>
<td>0.60</td>
<td>346</td>
<td>28.2</td>
<td>0.69</td>
<td>1.96</td>
</tr>
<tr>
<td>45</td>
<td>0.39</td>
<td>3568</td>
<td>20.6</td>
<td>9.04</td>
<td>0.62</td>
<td>313</td>
<td>26.5</td>
<td>0.34</td>
<td>1.07</td>
</tr>
<tr>
<td>83</td>
<td>0.39</td>
<td>3069</td>
<td>17.1</td>
<td>2.68</td>
<td>0.71</td>
<td>260</td>
<td>24.3</td>
<td>0.68</td>
<td>1.49</td>
</tr>
<tr>
<td>103</td>
<td>0.12</td>
<td>2833</td>
<td>11.9</td>
<td>1.69</td>
<td>0.53</td>
<td>411</td>
<td>22.6</td>
<td>0.56</td>
<td>1.49</td>
</tr>
<tr>
<td>116</td>
<td>0.27</td>
<td>2931</td>
<td>15.0</td>
<td>1.79</td>
<td>0.43</td>
<td>393</td>
<td>23.6</td>
<td>0.50</td>
<td>1.65</td>
</tr>
<tr>
<td>173</td>
<td>0.46</td>
<td>2877</td>
<td>16.7</td>
<td>2.00</td>
<td>1.72</td>
<td>211</td>
<td>22.8</td>
<td>1.68</td>
<td>1.29</td>
</tr>
<tr>
<td>175</td>
<td>0.38</td>
<td>2903</td>
<td>17.5</td>
<td>1.53</td>
<td>0.83</td>
<td>226</td>
<td>23.5</td>
<td>0.95</td>
<td>1.70</td>
</tr>
<tr>
<td>236</td>
<td>0.40</td>
<td>2404</td>
<td>18.1</td>
<td>2.03</td>
<td>1.05</td>
<td>224</td>
<td>18.5</td>
<td>1.08</td>
<td>1.20</td>
</tr>
<tr>
<td>255</td>
<td>0.38</td>
<td>3527</td>
<td>25.4</td>
<td>9.97</td>
<td>0.79</td>
<td>202</td>
<td>25.7</td>
<td>0.47</td>
<td>1.05</td>
</tr>
<tr>
<td>327</td>
<td>0.18</td>
<td>3087</td>
<td>19.0</td>
<td>4.65</td>
<td>2.42</td>
<td>182</td>
<td>23.2</td>
<td>1.47</td>
<td>0.98</td>
</tr>
<tr>
<td>831</td>
<td>0.34</td>
<td>3019</td>
<td>16.0</td>
<td>2.89</td>
<td>1.47</td>
<td>241</td>
<td>23.8</td>
<td>1.17</td>
<td>1.36</td>
</tr>
<tr>
<td>845</td>
<td>0.10</td>
<td>2861</td>
<td>13.2</td>
<td>3.18</td>
<td>0.86</td>
<td>362</td>
<td>21.9</td>
<td>0.68</td>
<td>1.22</td>
</tr>
<tr>
<td>846</td>
<td>0.17</td>
<td>2987</td>
<td>12.2</td>
<td>1.42</td>
<td>0.44</td>
<td>411</td>
<td>24.5</td>
<td>0.53</td>
<td>1.82</td>
</tr>
<tr>
<td>848</td>
<td>0.54</td>
<td>2724</td>
<td>21.7</td>
<td>7.11</td>
<td>2.10</td>
<td>210</td>
<td>19.1</td>
<td>0.83</td>
<td>0.75</td>
</tr>
<tr>
<td>849</td>
<td>0.12</td>
<td>3053</td>
<td>13.1</td>
<td>3.52</td>
<td>0.87</td>
<td>336</td>
<td>23.6</td>
<td>0.69</td>
<td>1.25</td>
</tr>
<tr>
<td>858</td>
<td>0.41</td>
<td>2947</td>
<td>18.0</td>
<td>6.22</td>
<td>2.02</td>
<td>192</td>
<td>21.2</td>
<td>1.02</td>
<td>0.85</td>
</tr>
<tr>
<td>862</td>
<td>0.09</td>
<td>3286</td>
<td>14.4</td>
<td>3.84</td>
<td>1.00</td>
<td>337</td>
<td>25.8</td>
<td>0.83</td>
<td>1.28</td>
</tr>
<tr>
<td>863</td>
<td>0.22</td>
<td>2979</td>
<td>14.0</td>
<td>2.16</td>
<td>0.89</td>
<td>369</td>
<td>23.8</td>
<td>0.88</td>
<td>1.49</td>
</tr>
<tr>
<td>870</td>
<td>0.11</td>
<td>3190</td>
<td>11.6</td>
<td>1.55</td>
<td>0.33</td>
<td>445</td>
<td>26.4</td>
<td>0.43</td>
<td>1.92</td>
</tr>
<tr>
<td>872</td>
<td>0.35</td>
<td>3066</td>
<td>11.8</td>
<td>1.68</td>
<td>0.52</td>
<td>506</td>
<td>25.1</td>
<td>0.56</td>
<td>1.77</td>
</tr>
</tbody>
</table>

that of the particles whose tracks did not fulfil the criteria about 50% may be wrongly identified according to their low energies.

16.3. **Comparison with other emulsion experiments**

The results of the present work will be compared with
some other investigations on the primary cosmic ray spectrum recorded in Texas, U.S.

In the work by Kristiansson, Mathiesen and Stenman (3) the charge interval $Z \geq 16$ was completely resolved but the statistical weight of their work is considerably lower than that of the present work. They used the same type of emulsions, Ilford G5, as in this work and the width of the slit of their photometer corresponded to 4.2 $\mu$m in the emulsion plane.

Waddington, Freier and Long (17) have also studied the composition of the primary cosmic radiation at Texas by means of photometric measurements in Ilford G5 nuclear emulsions. The stack used by them and the one used in the present study were exposed in the same balloon flight. The slit width of their photometer corresponded to 14.7 $\mu$m in the emulsion plane according to Long (18). The statistical weight of their investigation exceeds that of ours by a factor of two. The shape of the charge distribution presented by them is very similar to the shape of the distribution of $W$ shown in Figure 15b. In neither case the resolution between consecutive charges is brought to a satisfactory level.

One of the major causes for this seems to be the admixture of non-relativistic particles. Particles having low energies, $\approx 1$ GeV/nucleon, will be wrongly identified unless the low energy can be corrected for e.g. by the use of $\frac{dW}{dR}$. In a study on the relation between the track width of an ion and its velocity we found that the relation is sensitive to the width of the slit (15). A wider slit gives a stronger dependence of the track width on the velocity. Therefore using a narrow slit the admixture of non-relativistic particles will
not distort the spectrum as much as when using a wide slit. Furthermore, the placement of the background slits will affect the energy sensitivity of the photometer. Unfortunately, in the present work the placement of the background slits was not optimized with respect to the energy insensitivity.

17. The intensity of MH and VH nuclei

The data presented in Figure 15 c is not sufficiently detailed to admit a calculation of intensity of individual species in the radiation. However, the data seems to be appropriate for a calculation of the intensity of groups of particles. We have made such a calculation for two charge groups, $16 \leq Z \leq 19$ (MH nuclei) and $20 \leq Z \leq 28$ (VH nuclei). As input data in the calculation the total number of accepted $j$-type particles $n_j$ ($j = 1 \neq VH, j = 2 \neq MH$) is needed. The numbers $n_j$ are shown in Table XII. As the first step in obtaining the intensity of $j$-type particles the expected number of such particles at the entrance into the emulsion stack was calculated from

$$N_j = \sum_{i=1}^{n_j} \eta(Z)^{-1} \exp \left\{ \left( \lambda_i + y_i \right)/ \lambda(Z) \right\}$$

(17.1)

Here

- $\eta(Z)$ = the scanning efficiency for particles with charge number $Z$
- $\lambda_i$ = the distance in emulsion traversed by particle $i$ from the entering point to the point where the first reading was taken
- $y_i$ = the required minimum track length for particle $i$
- $\lambda(Z)$ = the interaction mean free path in emulsion for particles with charge number $Z$. 
Approximative charge numbers were assigned to each particle using the calibration curve shown in Figure 14. The interaction mean free paths in emulsion were taken from the work by Cleghorn, Freier and Waddington (19). The resulting numbers \( N_j \) at the top of the stack are shown in Table XII.

Table XII
Data used in and the results of the calculation of the intensity of MH and VH nuclei.

<table>
<thead>
<tr>
<th>Particle type ( j )</th>
<th>( 1 \text{ VH (20} \leq Z \leq 28) )</th>
<th>( 2 \text{ MH (16} \leq Z \leq 19) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of accepted</td>
<td>94</td>
<td>28</td>
</tr>
<tr>
<td>particles, ( n_j )</td>
<td>110.6</td>
<td>32.9</td>
</tr>
<tr>
<td>The numbers ( n_j ) corrected for scanning efficiency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected number of</td>
<td>172.1</td>
<td>48.5</td>
</tr>
<tr>
<td>particles entering the stack, ( N_j )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated number of particles entering the stack</td>
<td>( N_{j1} )</td>
<td>( N_{j2} )</td>
</tr>
<tr>
<td>before ( (N_{j1}) ) and after ( (N_{j2}) ) flip-over</td>
<td>19.1</td>
<td>6.1</td>
</tr>
<tr>
<td></td>
<td>153.0</td>
<td>42.4</td>
</tr>
<tr>
<td>Intensity ( J_j (0) ) ( (m^2 , sr , s)^{-1} )</td>
<td>Present work ( 0.38 \pm 0.04 )</td>
<td>( 0.092 \pm 0.012 )</td>
</tr>
<tr>
<td></td>
<td>Freier and Waddington (20)</td>
<td>( 0.405 \pm 0.023 )</td>
</tr>
</tbody>
</table>
through the overlying atmosphere. The amount of atmosphere traversed by the particles is given by \( x = h / \cos \phi \), where \( h \) is the vertical depth in the atmosphere and \( \phi \) the zenith angle.

It is customary to calculate the intensity \( J_j(x) \) of j-type particles using known flight data and geometrical acceptance criteria, and then to obtain the extrapolated intensity \( J_j(0) \) from the solution of the one-dimensional diffusion equations. The part of the solution which is relevant for our purposes is given by Kaplon, Noon and Racette (20) as

\[
J_1(x) = J_1(0) e^{-x/\Lambda_1} \\
J_2(x) = J_2(0) e^{-x/\Lambda_2} + C_{12} J_1(0) (e^{-x/\Lambda_2} - e^{-x/\Lambda_1})
\]

Here \( P_{ij} \) is the average number of j-type particles emerging from the collision of an i-type nucleus. The absorption mean free path of j-type particles, \( \lambda_j \), is defined by \( \lambda_j = \lambda_j / (1 - P_{jj}) \) where \( \lambda_j \) is the interaction mean free path in air for j-type particles.

In such an extrapolation procedure one single mean value of \( \phi \) has to be used. If large zenith angles are involved this procedure is inadequate. In our case particles approaching \( \phi = 90^\circ \) can enter the stack before flip-over and still be accepted for measurement. Also, on account of the small number of accepted particles we cannot subdivide the data into zenith angle classes and apply the equations 17.2 for each class separately. As a further complication the particles entering the stack before flip-over are in general indistinguishable from those registered during the main exposure if they fulfil the formal acceptance criteria. These
complications led us to develop the idea used by Mathiesen and Stenman (21) in extrapolating the particle flux from large zenith angles. Thus we combine the solution 17.2 of the diffusion problem with the definition equation for intensity \( J_j \) of \( j \)-type particles,

\[
dN_j = J_j \cdot t \cdot \cos \phi \cdot dS \cdot d\Omega , \quad j = 1, 2 \quad (17.3)
\]

Here \( dN_j \) denotes the number of \( j \)-type particles in the solid angle element \( d\Omega \) passing through the plane element of area \( dS \) during time \( t \) if the particle direction makes an angle \( \phi \) with the normal to the plane \( S \). Inserting \( J_j(x) \) from equations 17.2 and integrating over the solid angle of acceptance \( \Omega \) and the scan area \( S \) yields a set of two equations each for the numbers \( N_{jk} \) of \( j \)-type particles entering the stack. Here \( k=1 \) refers to particles gathered before and \( k=2 \) after the flip-over.

\[
\begin{align*}
N_{1k} &= J_1(0) \cdot Y_{1k} \\
N_{2k} &= J_2(0) \cdot Y_{2k} + J_1(0) \cdot C_{12} \cdot (Y_{2k} - Y_{1k}) \\
&= \begin{cases} 
J_1(0) \cdot Y_{1k}, & \text{if } k=1 \\
J_2(0) \cdot Y_{2k} + J_1(0) \cdot C_{12} \cdot (Y_{2k} - Y_{1k}), & \text{if } k=2
\end{cases} \\
&= \begin{cases} 
J_1(0) \cdot Y_{1k}, & \text{if } k=1 \\
J_2(0) \cdot Y_{2k} + J_1(0) \cdot C_{12} \cdot (Y_{2k} - Y_{1k}), & \text{if } k=2
\end{cases} \\
&= \begin{cases} 
J_1(0) \cdot Y_{1k}, & \text{if } k=1 \\
J_2(0) \cdot Y_{2k} + J_1(0) \cdot C_{12} \cdot (Y_{2k} - Y_{1k}), & \text{if } k=2
\end{cases}
\end{align*} \quad (17.4)
\]

Here it has been assumed that the particle intensity is isotropic in the upper hemisphere. The functions \( Y_{jk} \) are defined by

\[
Y_{jk} = t_k \int_S \int_\Omega \cos \phi \cdot \exp \left\{ -h/(\Lambda_j \cos \phi_j K) \right\} \cdot dS \cdot d\Omega \quad (17.5)
\]

with

\[
\phi_1 = \arccos (\cos \alpha \sin \theta) \\
\phi_2 = \phi = \arccos (\cos \alpha \cos \theta)
\]
\( a \) = the dip angle
\( \theta \) = the angle between the projection of the particle trajectory onto the emulsion plane and that edge of emulsion which was assumed to be parallel to the zenith direction during the main exposure.

The flight geometry is illustrated in Figure 17.

In equation 17.5 \( dS \) has been left within the integral over \( \Omega \). The reason for this is that the limits of integration over \( \Omega \) depend on the position of \( dS \) on the scan area \( S \). This dependence is due to the acceptance criteria discussed in sections 5 and 7. The importance of this for the calculation of the solid angle of acceptance is elucidated by the following argumentation. The geometrical factor for the stack is

\[
F = \int_{\Omega} \int_{S} dS \, d\Omega = 1.89 \cdot 10^{-2} \text{ m}^2 \text{ sr},
\]

yielding an effective solid angle \( \Omega_{\text{eff}} = F/S = 1.95 \text{ sr} \), whereas the integral of \( d\Omega \) over the maximum acceptance angles \( \alpha_{\text{max}} \) and \( \theta_{\text{max}} \) is

\[
\Omega_{\alpha} = \int_{-\alpha_{\text{max}}}^{+\alpha_{\text{max}}} d\alpha \int_{-\theta_{\text{max}}}^{+\theta_{\text{max}}} d\theta (\sin \alpha) = 2.32 \text{ sr}.
\]

The functions \( Y_{jk} \) depend on acceptance criteria, exposure details and \( \Lambda_{j} \) but are independent of the unknown intensities \( J_{j}(0) \). Thus they can be computed. By using the relations

\[
N_{j} = \sum_{k=1}^{2} N_{jk}, \quad j = 1, 2 \quad (17.6)
\]

where \( N_{j} \) is obtained from equations 17.1, it is possible to
solve equations 17.4 for \( J_j(0) \).

In our calculation of \( J_j(0) \) the diffusor parameters were given the same numerical values as those used by Freier and Waddington (22), i.e. \( A_1 = 16.2 \text{ g cm}^{-2} \), \( \gamma = 19.1 \text{ g cm}^{-2} \), \( P_{11} = 0.25 \), \( P_{12} = 0.15 \) and \( P_{22} = 0.15 \). The flight parameters were taken to be \( h = 2.8 \text{ g cm}^{-2} \), \( t_1 = 0.54 \cdot 10^4 \text{ s} \), \( t_2 = 3.18 \cdot 10^4 \text{ s} \). The result of the calculation is shown in Table XII. The errors shown for \( J_j(0) \) are based only on the statistical uncertainty in the number of registered particles. For comparison we have also included in Table XII the intensities found by Freier and Waddington (22). The agreement with this data is very good. It should be noted that the two sets of data were obtained in the same balloon flight.

18. Theoretical calculation of energy of arriving particles

In view of the appreciable number of particles registered before the flip-over, \( N_{j1} \) in Table XII, we deemed it necessary to obtain a theoretical estimate of the kinetic energy of all arriving particles to reassure that the observed non-relativistic particles were not accumulated before the flip-over of the stack. For ease in calculation we assume that the differential rigidity spectrum

\[
\frac{dJ_j}{dP} = J_{j0} P^{-\gamma}
\]

(18.1)

holds for all rigidities \( P \geq 3 \text{ GV} \) with a charge-independent exponent \( \gamma = 2.5 \) and that the intensity \( J_j \) of \( j \)-type particles at the top of the atmosphere is isotropic.

The integrals \( Y_{jk} \), equation 17.5, were decomposed into a number of zenith angle intervals and for each of these number densities \( N_{jk} \) were calculated according to equation 17.4.
Equation 1.8.1 was used to obtain the number densities at each $\phi$ for particles with different rigidities. For each value of $\phi$ and $P$ the degradation of the kinetic energy in the atmosphere was calculated, using the relation between kinetic energy and range of protons in emulsion as given by Barkas (16). It was assumed that the kinetic energy per nucleon and the direction of motion of the particles are not changed in nuclear interactions with surviving secondaries. The difference in range between VH primaries and MH secondaries has been taken into consideration. The calculations were repeated for a number of cutoff rigidities $P_0$.

The results of the calculations are shown in Figure 18 for particles entering the stack before flip-over and in Figure 19 for particles entering the stack after flip-over. The graphs show the total number of $j$-type nuclei entering the stack with kinetic energy less than a given value as a function of the kinetic energy. From these calculations it is evident that some 20 particles with energy less than say 1.2 GeV/nucleon impossibly can have been registered before flip-over if one single cutoff rigidity of about 4.5 GV is assumed for the site of exposure. A closer inspection of Figures 18 and 19 shows that the assumption of forbidden and allowed bands of rigidity down to about 3.5 GV in accordance with the calculations by Shea and Smart (6) indeed is capable of explaining our findings of non-relativistic particles in the spectrum. As a comforting fact we also note that even in this case the contribution of particles entering the stack before flip-over is relatively unimportant.
Some remarks on the histogram showing the charge spectrum

When all corrections and normalizations are applied the resultant track mean values of $W$ together with their standard errors can simplest be shown in a table. However, to make the data more accessible it is common to show the spectrum in a histogram.

A histogram is a picture of a number of observations. The observations are represented by equal units of area above a scale marked off in e.g. units of track width. It is important to represent the observations to the degree of precision achieved in the study. To provide a picture of data in general it is important to leave a sufficiently large number of intervals to discern the shape of the distribution, but it is also important not to have so many intervals that irrelevant detail obscures the picture. The precision of a track mean value of $W$ is given by its standard error and therefore the width of the representation should reflect the standard error.

Usually each particle is represented by a rectangle. The placement of the rectangle on the horizontal axis can be made differently. The rectangle can be placed with its middle exactly over the value of the track mean, or it can be placed so that the track mean falls within some predetermined interval. In the latter case the width of the interval can be smaller than the width of the rectangle. As a result the spectrum will show a "fine structure". Such a structure is also obtained when the rectangle is placed exactly over the track mean. However, in most cases the
width of the rectangle and the width of the interval on the horizontal axis are chosen to be equal, and very often the widths do not at all reflect the precision of the observations. This type of representation gives an incorrect picture of the charge spectrum. By a suitable choice of the widths it is possible to get an illusory resolution between consecutive charges, and this illusory resolution is especially well pronounced if the spectrum contains only a few particles.

A track mean is an average over a large number of observations. Each reading of $W$ follows a normal distribution and therefore the track mean value of $W$ also follows a normal distribution. Furthermore, owing to the central limit theorem, the track mean follows a normal distribution whatever the distribution of the single reading of $W$ is. In order to maintain the properties of the readings of $W$ lying behind the track mean, we have chosen to represent the particles by stepwise constant distributions which resemble normal distributions as much as possible. The distributions have equal units of area. To let the precision of the track mean be reflected in the histogram we use three distributions with different standard deviations. The standard error of the track mean decides which distribution has to be used so that the standard error and the standard deviation coincide as much as possible. The distributions are shown in Figure 20. In Figure 21 we show a plot of the distributions on a normal distribution/paper. As can be seen from Figure 21 the distributions look very much like normal distributions.

The histogram which shows the charge spectrum, Figure 15, contains all track mean values of $W$ obtained in the investi
gation. Each mean value has been represented by a small distribution. The histogram was put together by the use of a computer.

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FIGURE CAPTIONS.

Figure 1 Curve of flight for the balloon which carried the emulsion stack. The Waddington data (4) is determined from the ranges of α-particles. The National Center for Atmospheric Research (NCAR) data is an average of their photobarograph and Beacon data.

Figure 2 Sketch of the optical part of the photometric apparatus.

Figure 3 The wavelength intervals used in the photometric apparatus as defined by the limiting components. \( \lambda \) is the wavelength of light.
   a) Transmission curve of the filter in front of the reading microscope.
   b) Transmission curve of the interference filter.
   c) The spectral sensitivity of a human eye adopted for daylight, solid curve, and for darkness, dashed curve.
   d) The spectral response of the photomultiplier.

Figure 4 Principal diagram of the measuring unit and its connection to the data recording system.

Figure 5 The distribution of the slopes of the regression lines obtained in the first step of the depth normalization.
Figure 6 The inverse plate factors plotted as a function of the plate number. The limits of error indicate the standard error.

Figure 7 The defocusing effect as a function of the dip angle.

Figure 8 The dip normalizing factors as a function of the dip angle. The graph corresponding to $W=3600$ is obtained by extrapolation in the data.

Figure 9 The standard deviation of a single reading of $W$ plotted as a function of $Z$.

Figure 10 The expected value of $\frac{\delta W}{\delta R}$ plotted as a function of the charge number, with the kinetic energy as a parameter.

Figure 11 The standard deviation of $\frac{\Delta W}{\Delta R}$ for the iron group nuclei plotted as a function of the number of accepted readings. The open circles indicate experimental data. The solid line shows the computed standard deviation of $\frac{\Delta W}{\Delta R}$ according to equation 11.18 and the dashed line shows the computed standard deviation of $\frac{\Delta W}{\Delta R}$ assuming that 20% of the plates have internal variations amounting to 5% of $W$.

Figure 12 The maximum spurious error of $\frac{\Delta W}{\Delta R}$ for the iron group nuclei as a function of the relative number of dis-
turbed plates. The track length is used as the parameter. The irregularities within the emulsions are assumed to amount to 3% of W.

Figure 13 The track width as a function of its derivative along the track, with the charge number as the parameter. The open circles correspond to non-relativistic particles and the limits of error indicate the standard errors.

Figure 14 The calibration curve.

Figure 15 Distribution of track mean values of W of all used particles in the analysis. Each particle is represented by a stepwise constant distribution which resembles a normal distribution as much as possible. The standard error of the track mean decides the width of the used distribution. Some charge numbers corresponding to the value of W are marked off. The data is shown in three histograms: a) All normalizations but dip normalization are applied. b) All normalizations are applied. c) The level of W of the 20 non-relativistic particles has been normalized to the corresponding relativistic level.

Figure 16 The same as Figure 15 for the 20 non-relativistic particles.
Figure 17 One octant of the unit sphere showing the angles $\alpha$, $\theta$, $\phi$ and $\phi_1$. The angles are defined in the text. The emulsion plane is parallel to the plane AOB. The scan area lies on the plane AOC. Before flip-over the zenith direction is assumed to be OA, after flip-over it is assumed to be OB. The arrival direction of the particle is PO. With $\alpha = \alpha_{\text{max}}$ and $\theta = \theta_{\text{max}}$ the solid angle of acceptance is in each quadrant of the upper hemisphere equal to the spherical surface PEBD.

Figure 18 The calculated number of MH and VH particles with kinetic energy less than $E_{\text{kin}}$, $N(<E_{\text{kin}})$, arriving into the solid angle of acceptance before the flip-over of the stack. The number is given as a function of $E_{\text{kin}}$ with the cutoff rigidity $P_0$ as a parameter.

Figure 19 The same as Figure 18 for particles arriving after the flip-over of the stack.

Figure 20 The distributions used for representing the track mean values of $W$. Each particle is represented by one distribution.

Figure 21 The distributions used for representing the track mean values of $W$ plotted on a normal distribution paper.
Fig. 3
Fig. 8

DIP FACTOR

\[ \tan \alpha \]

W = 1600

2000

2600

3600

0.2

0.4

0.6
Fig. 12
Fig. 19

$N(<E_{\text{kin}})$ vs. $E_{\text{kin}}$ (GeV/nucleon) for different $P_0$ (GV) values:
- VH
- MH

$P_0$ (GV): 3.0, 3.5, 4.2, 5.6
STANDARD DEVIATION

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>10.3</td>
<td>16.3</td>
<td>21.6</td>
</tr>
</tbody>
</table>