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MAGNETIC ISLAND FORMATION IN A TOKAMAK PLASMA FROM HELICAL PERTURBATIONS OF THE PLASMA CURRENT

Paul Chrisman
John Clarke
James Rome

MASTER

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1. Introduction

In this report we discuss the formation, structure, and some of the consequences of magnetic islands in a tokamak plasma. These may be produced from helical current perturbations in the plasma. The existence, structure, and magnitude of the current perturbations causing the island formation is deduced from experimental measurements of the time rate of change of the poloidal magnetic field in the ORMAK device.

Figures 1, 2, and 3 illustrate typical experimental results. The measurement is carried out by poloidally distributing several pickup coils slightly inside the conducting aluminum shell of approximately 29 cm minor radius. The poloidal variation of the poloidal field is then deduced from the frequency and relative phases of the signals.

The exact distribution of the current perturbations in the plasma causing these poloidal magnetic field variations is not known, but based on the experimental results and theoretical models of the resistive tearing mode instabilities,[1] we chose what we believe to be a satisfactory model for the perturbations. In the following we will give physical justification for our model and show that it leads to the formation of magnetic islands of $\sim$3 cm width in the plasma. We will also give some physical insight as to the structure of the islands and how they break up into so called ergodic regions as the perturbation strength increases. Finally we will comment on the relevance of magnetic islands to tokamak phenomena.

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II. Model of the Current Perturbations

The experimental evidence, coupled with basic knowledge of the toroidal helicity of the tokamak magnetic field structure points to the existence of an effective helical perturbation of the plasma current, distributed poloidally according to the measured m number of the relative phases of the signals and toroidally such as to close on itself after an integral number of times around the torus. The radial distribution of the current perturbations cannot be measured but it is reasonable to assume that the perturbing currents lie on or in the vicinity of unperturbed rational magnetic surfaces[2] since on such surfaces the field lines close on themselves and thus current perturbations, following the field lines to first order, could conceivably close and be maintained i.e., if the perturbation didn't close on itself it would spread evenly over the surface in a few transits of the torus and there would be no poloidal variation in the current. (See Figure 4). It is easy to see that such a perturbation of the current in a rotating plasma (poloidally or toroidally or both) would produce a time rate of change in the poloidal field such as is observed.

Thus our model for the current perturbations was chosen to be a helical sheet current following the field lines on a rational magnetic surface of q equal to the measured value of m of the signals and radius determined by an assumed parabolic current profile. The helical perturbation is chosen to vary sinusoidally in the poloidal direction according to m and such as to close after m transits in the toroidal direction:

$$\underset{\sim}{\vec{J}} = \mathcal{J}_o\, \delta(r - r_o)\, \sin(m\theta - kz)\, [\hat{\theta}\cos\nu + \hat{z}\sin\nu]$$

where

r_o = radius of rational surface q = m

k = (major radius)$^{-1}$

ν = pitch angle of field lines on unperturbed rational surface.

III. Calculations

The magnetic field produced by such a perturbation is needed to calculate magnetic surfaces and drift orbits. All calculations are carried out in cylindrical coordinates with the toroidal effects modeled by:

(1) Allowing the unperturbed B toroidal to contain the normal 1/R dependence.

(2) In computing the drift orbits, an effective gravity term was included to account for toroidal curvature.

An analytical expression for the vector potential of the perturbed currents is obtainable by straightforeward integration:

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int d^3\vec{r}\,' \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \quad .$$

The components of $\vec{A}$ are:

$$A_r = (1/2)\, \mu_o\, \mathcal{J}_o\, r_o \cos\nu \cos(m\theta - kz)\, [K_{m-1}(Z) I_{m-1}(z) - K_{m+1}(Z) I_{m+1}(z)]$$

$$A_\theta = -(1/2)\, \mu_o\, \mathcal{J}_o\, r_o \sin\nu \sin(m\theta - kz)\, [K_{m+1}(Z) I_{m-1}(z) + K_{m+1}(Z) I_{m+1}(z)]$$

$$A_z = \mu_o \mathfrak{J}_o r_o \sin\nu \sin (kz - m\theta) K_m(Z) I_m(z)$$

where

for $r > r_o$, $Z = kr$, $z = kr_o$,

for $r < r_o$, $Z = kr_o$, $z = kr$

and $\mathfrak{J}_o$ is a constant depending on the amplitude of the perturbed current.

The above expressions for the vector potential can be checked by using the condition

$$\nabla^2 \vec{A} = \mu_o \vec{J}$$

The perturbing magnetic field is found by taking the curl of the vector potential.

All magnitudes were adjusted to fit ORMAK data. Including the effects of image currents in the conducting aluminum shell it was found that the magnitude of the perturbed currents necessary to produce the measured signal was approximately .13% of the total plasma current or approximately 80 kA. By the magnitude of the perturbed currents we mean the amount of perturbed current flowing in one sector, $(\frac{\pi}{m})$, of the helical sheet current. We shall speak of the ratio of this amount to the total plasma current as the perturbation strength.

Having analytic expressions for the total magnetic field we plot magnetic surfaces by numerical integration of the drift equation for a constant velocity zero mass particle which follows a field line exactly:

$$\dot{\vec{r}} = v \frac{\vec{B}}{|\vec{B}|}$$

Points of successive intersections of a field line with a given cross section of the torus map out a cross section of a magnetic surface. In the cylindrical model, periodic intersections are equivalent to toroidal transits. Figure 5 illustrates the features of typical sets of surface cross sections for m = 2 and m = 3 perturbations, however, this figure was constructed for illustrative purposes and in actuality the islands (cross sections) are slightly wider and more symmetric than the actual case.

IV. Physical Explanations and Results

Physically the islands are a result of having a harmonic component of the radial component of the perturbed magnetic field in resonance with the unperturbed rational surface field lines, in the presence of shear. Due to the small radial component of the perturbing field, an unperturbed field line would for example leave its unperturbed rational surface and move outward into a region of higher rotational transform in going around the device. Being now in a region of higher rotational transform it moves poloidally faster in going around the device than its original unperturbed position. It continues poloidally into the region where the small radial component of the perturbed field points inward thus moving the field line back to and past the rational surface whereupon it is in a region of lower rotational transform than the rational surface and moves poloidally slower. If the radial component has high enough harmonic content in resonance with the unperturbed rational field lines, then in many times around the torus the field line won't deviate significantly from the "crescent moon" shaped motion about the rational surface, and the indicated surface cross sections will be traced out by successive intersections. Figures 6 and 7 are actual computer plots of the magnetic surface cross sections (islands) generated as explained above by successive intersection of a field line with a cross section. Notice that as the perturbation strength is increased the islands break up, they are no longer well formed. We attempt a physical explanation of this.

We know the exact harmonic content of our radial component of the perturbed field is sin $(m\theta - kz)$. Since the toroidal field has a poloidal θ dependence, the unperturbed field lines are not of constant helical pitch on the rational surface, thus our constant pitch helical perturbation is not in exact resonance. When the amplitude becomes larger the off resonance destroys the nearly cyclic motion of the field line about the unperturbed rational surface, breaking the islands.

This condition would clearly be deleterious for plasma containment. Particles could move practically freely throughout the broken region by just following the field lines. A diffusion step size would be on the order of the region width $\sim$5 cm. But, it was found that to reach this highly unstructured condition would require approximately an order of magnitude higher perturbation amplitude than that value consistent with the experimental magnitudes. Furthermore, in our case, the condition of not having resonance is a result of our model of superposing a helically symmetric perturbation on a nonsymmetric unperturbed field. Based on the initial assumption that the perturbed currents follow the field lines, they would do so even if the field lines on an unperturbed surface were not helically symmetric. The off resonant effect mentioned above would then be absent. Thus in the actual tokamak plasma, we might expect to find well formed island structures even for larger perturbation strengths. However, the inherent nonsymmetry of a toroidal helical perturbation could also cause an off resonant condition. Also the presence of two or more perturbations lying at different rational surfaces could possibly destroy the resonance conditions because being of different pitch angles, each is off resonance with the other and if their amplitudes are large enough each destroys the other's island structures about their separate rational surfaces. We investigated this case briefly because in some of the experimental results there was evidence of mode mixing. However, for the interactions to be strong enough to destroy the island structure would again require inconsistently high values of the perturbation strengths at distances of separation investigated for the parabolic profile. The multiple perturbation cases for different current profiles are under further investigation at present at ORNL.

We present a few more comments on our model. We chose a sheet current model. One might ask what a finite width model or a more general Fourier distribution poloidally might modify. The answer is, very little, at least as far as island widths are concerned, because the size of the islands depends mainly on the magnitude of the harmonic component of $\underset{\sim}{B}$ radial in resonance with the unperturbed structure, and this will depend only very weakly on the distribution. A theoretical expression for the island width can be derived using simple arguments involving the shear and resonant component of $\underset{\sim}{B}$:

$$\delta = \frac{4\, r_o}{m^{1/2}} \sqrt{\frac{|\tilde{B}_r|}{B_o(r_o)}} \sqrt{\left|\frac{\iota_{r_o}}{r_o \iota'_{r_o}}\right|}$$

where ι is the rotational transform.

This expression is due to Matsuda and Yoshikawa in an unpublished report. The numerical results follow the theoretical curve until the islands break up. (See Figure 8.)

Concluding Remarks

A reasonable model, consistent with experimental observations in ORMAK points to the existence of well structured magnetic islands of 2-3 cm. width.

(1) The original purpose in attacking this problem was to see if in the presence of such perturbations the relativistic runaways might drift out of the device. We did drift orbit calculations for approximately zero magnetic moment relativistic electrons and found that the drift surfaces of these particles look almost exactly like the corresponding magnetic surfaces except they are slightly deformed and displaced. If these drift islands ever intersect the limiter, then we expect all the electrons on the whole island to be dumped onto the limiter producing a spontaneous burst of hard Xrays such as is seen in the experiment. We note that such bursts are often correlated with the MHD signals. This is by no means intended as conclusive explanation of the observed runaway escape, but it is clearly a possible mechanism.

In general transport processes are enhanced in toroidal devices due to magnetic perturbations.[3] However, a quantitative description of transport in the presence of the island structure is not complete. Such questions as the trapping of particles on the moving islands, the motion of the islands themselves, the possibility of leakage through the island separatricies, and others are not completely settled. As an example of the subtleties involved one might think that the effective step size for diffusion in the island structure would be approximately the island width. Note, however, that if a drift island has width of 2 or 3 cm, its effective width is reduced because particles in the outer regions of the islands are subject to pass through the separatrix where B radial is approximately zero (on the corresponding magnetic surface). Thus in traversal of the island, these particles tend to remain in the vicinity of the separatrix thereby reducing the effective width of the island for diffusion.

In further regard to the effects on transport we conjecture that perhaps the largest enhancement in cross field flow is of the convective type which will arise due to the electric fields produced by the time rate of change of $\vec{B}$. Thus the nature of the time dependence of the perturbations need to be investigated in detail.

Along with these rather physical arguments, if the plasma is unstable with respect to helical current perturbations of the form assumed, the above mentioned clumping of particles in the vicinity of the separatrix is seen to effectively cancel the current perturbation leading to a stabilization mechanism. This could account for the long steady observation of such modes in ORMAK.

We intend further detailed investigations of several of the above qualitative statements.

Acknowledgments

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2. T. Stix; *Phys. Rev. Letters*, 30(833).

3. T. Ohkawa; Gulf Gen. Atomic Report, GA-10630.

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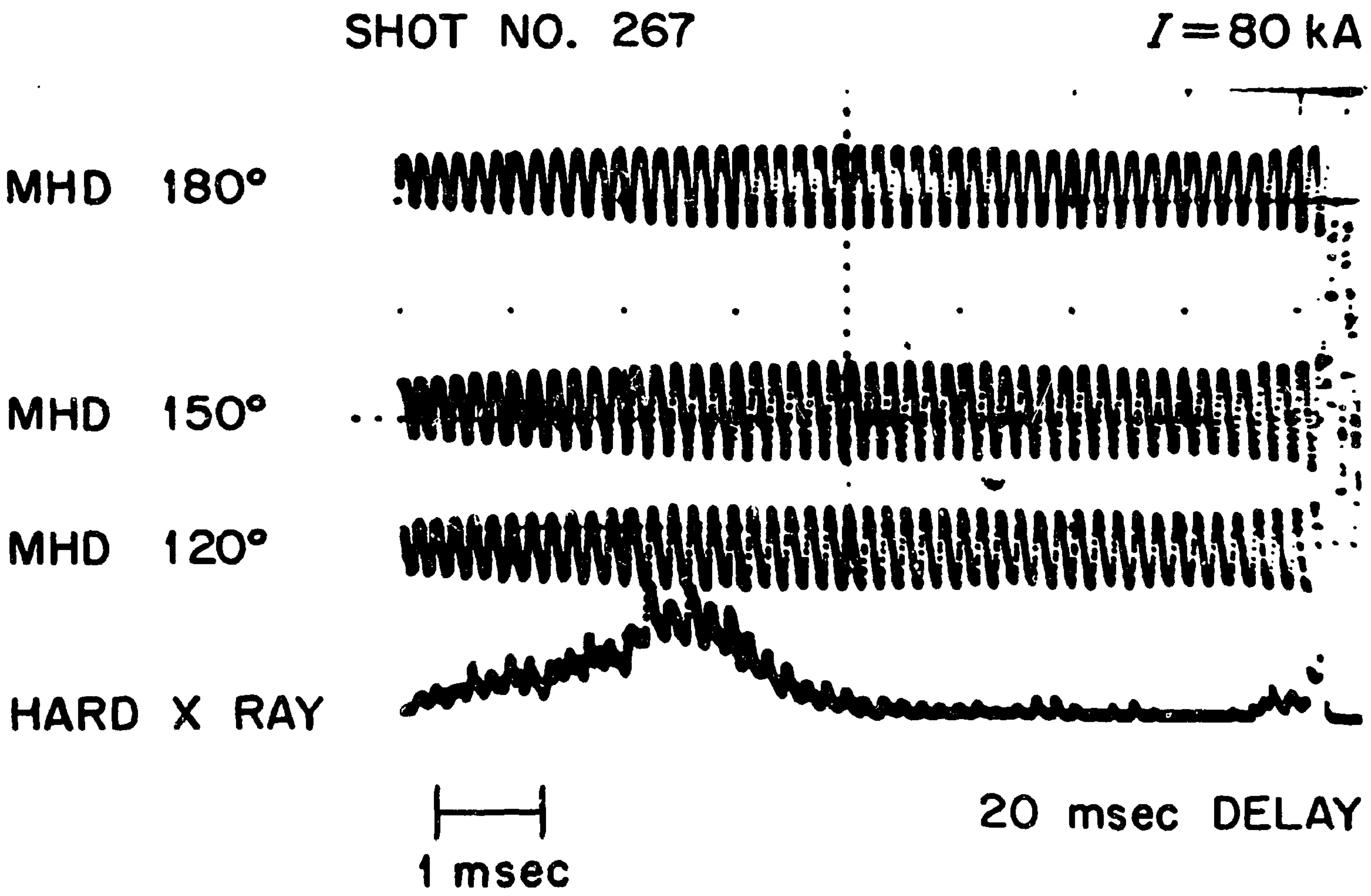

Fig. 1. $\frac{dB_{pol}}{dt}$ signals in ORMAK. The angles refer to the poloidal positioning of the pickup coils. Also included is the hard X-ray trace.

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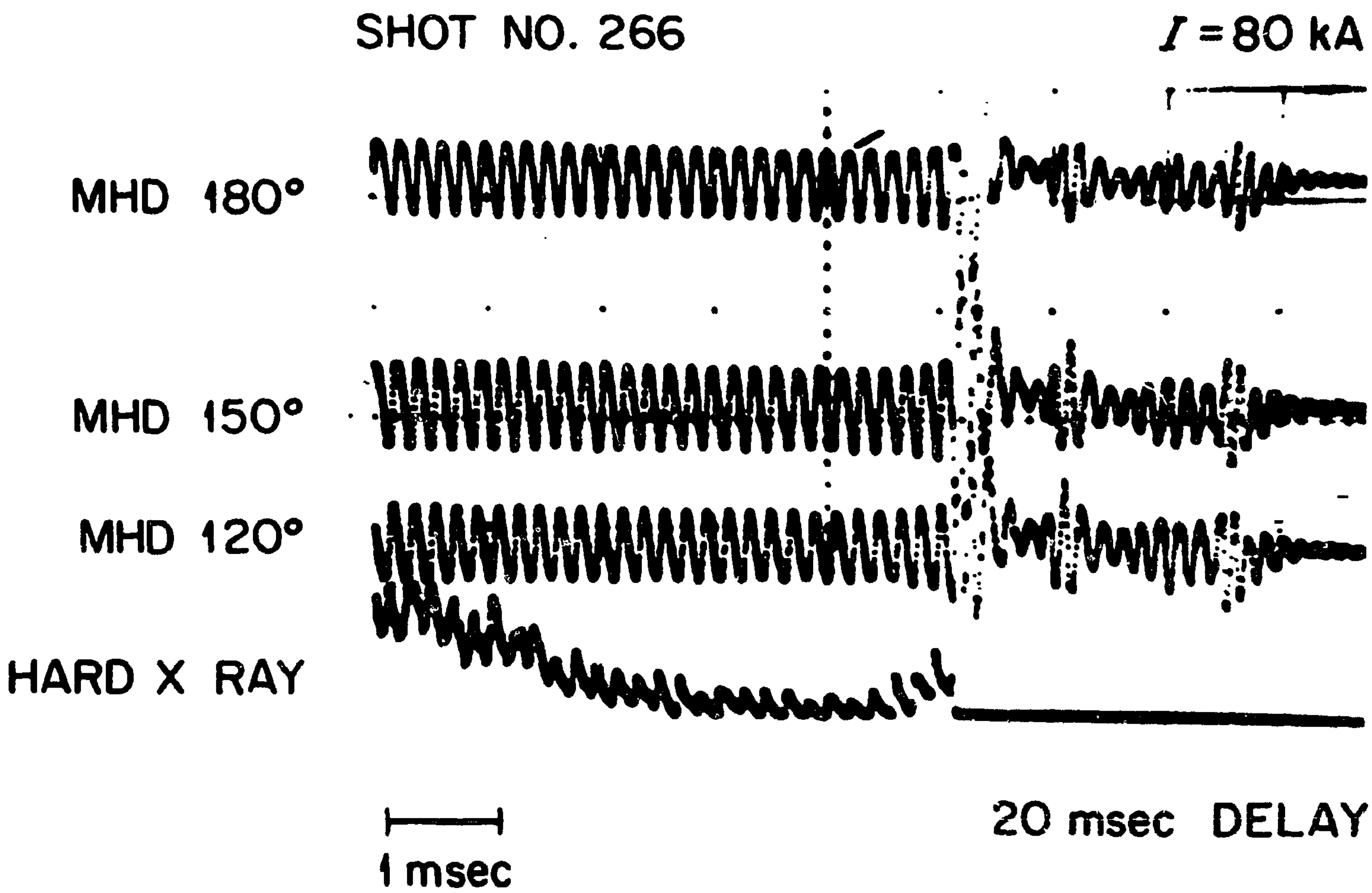
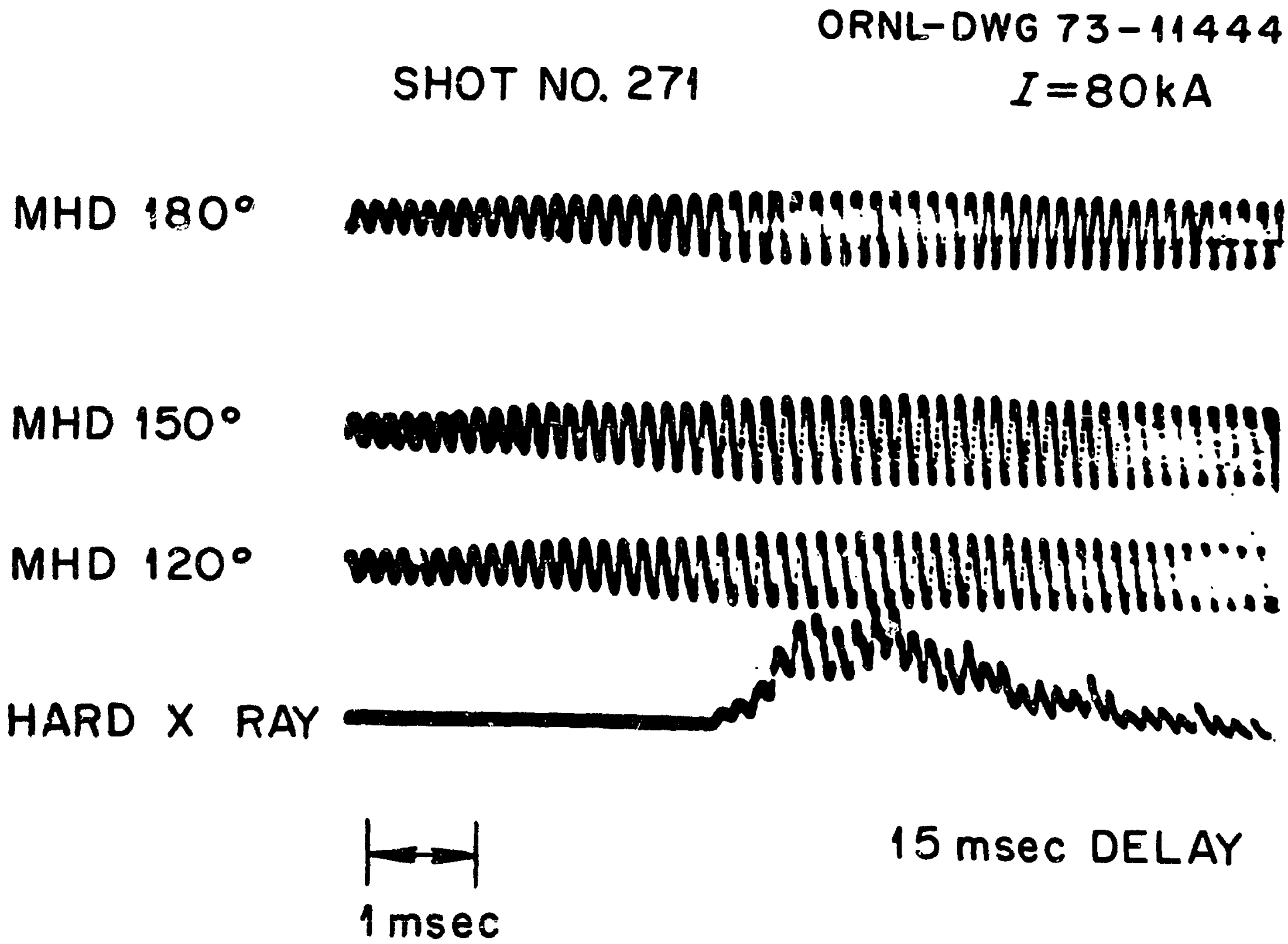


Figure 2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MHD 150°
MHD 120°
HARD X RAY
1 msec
15 msec DELAY

Figure 3

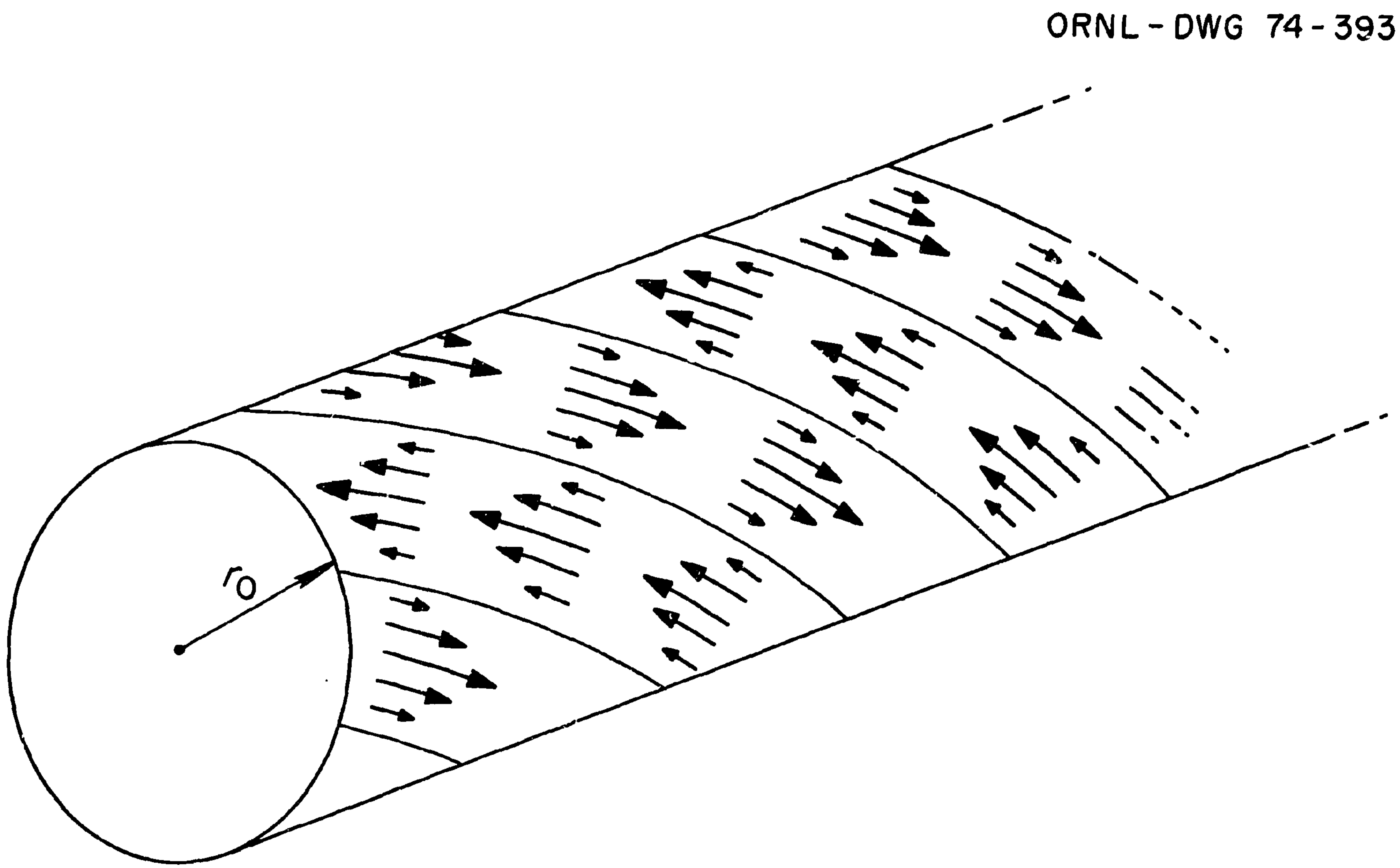

Fig. 4. Illustration of a helical sheet current distribution at a rational surface of radius r_o.

$$\underset{\sim}{\vec{J}} = \mathcal{J}_o \, \delta(r - r_o) \sin(m\theta - kz) \, [\hat{\theta} \cos \nu + \hat{z} \sin \nu]$$

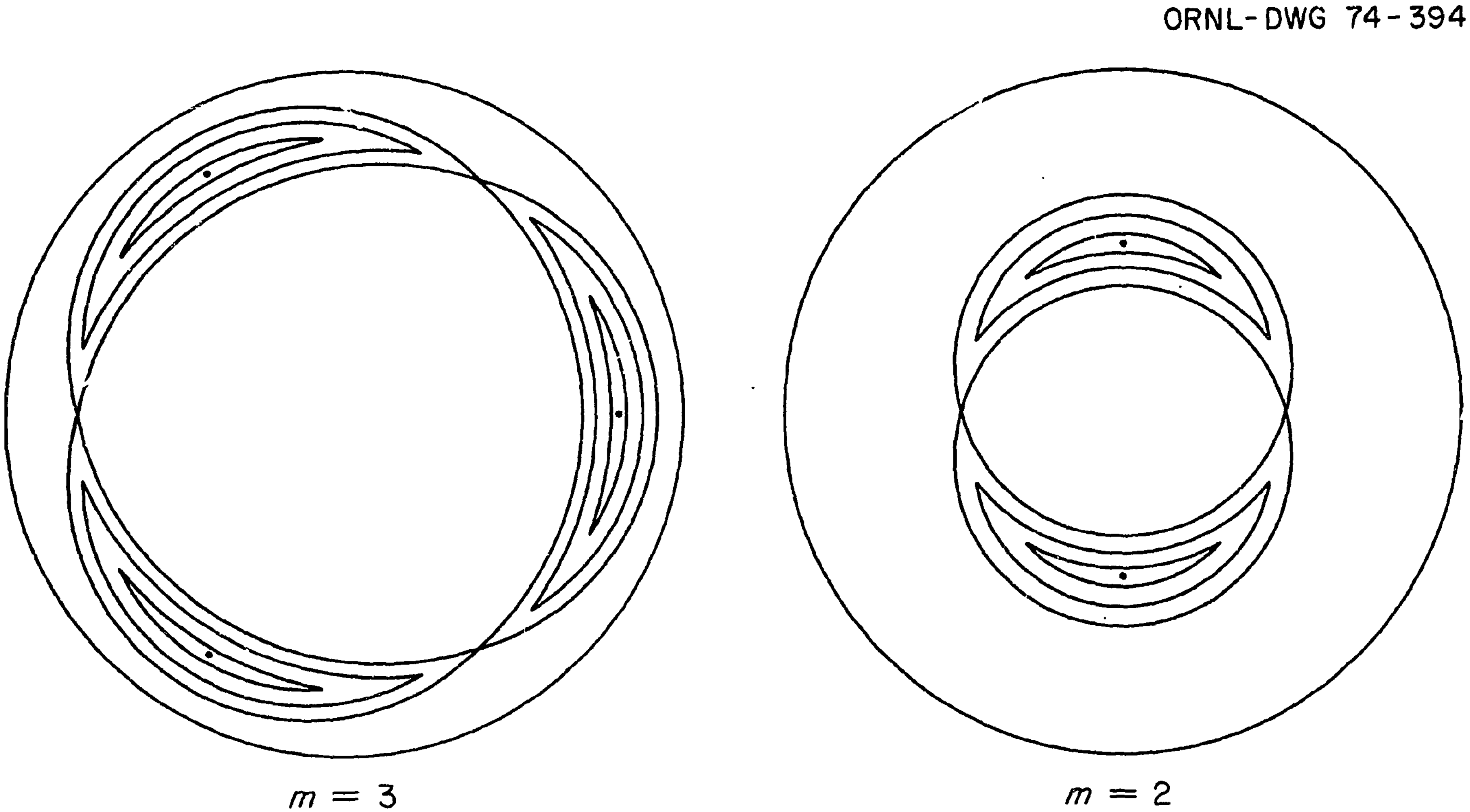

Fig. 5. Constructed cross sections of sets of nested island surfaces for m = 2 and m = 3 perturbations.

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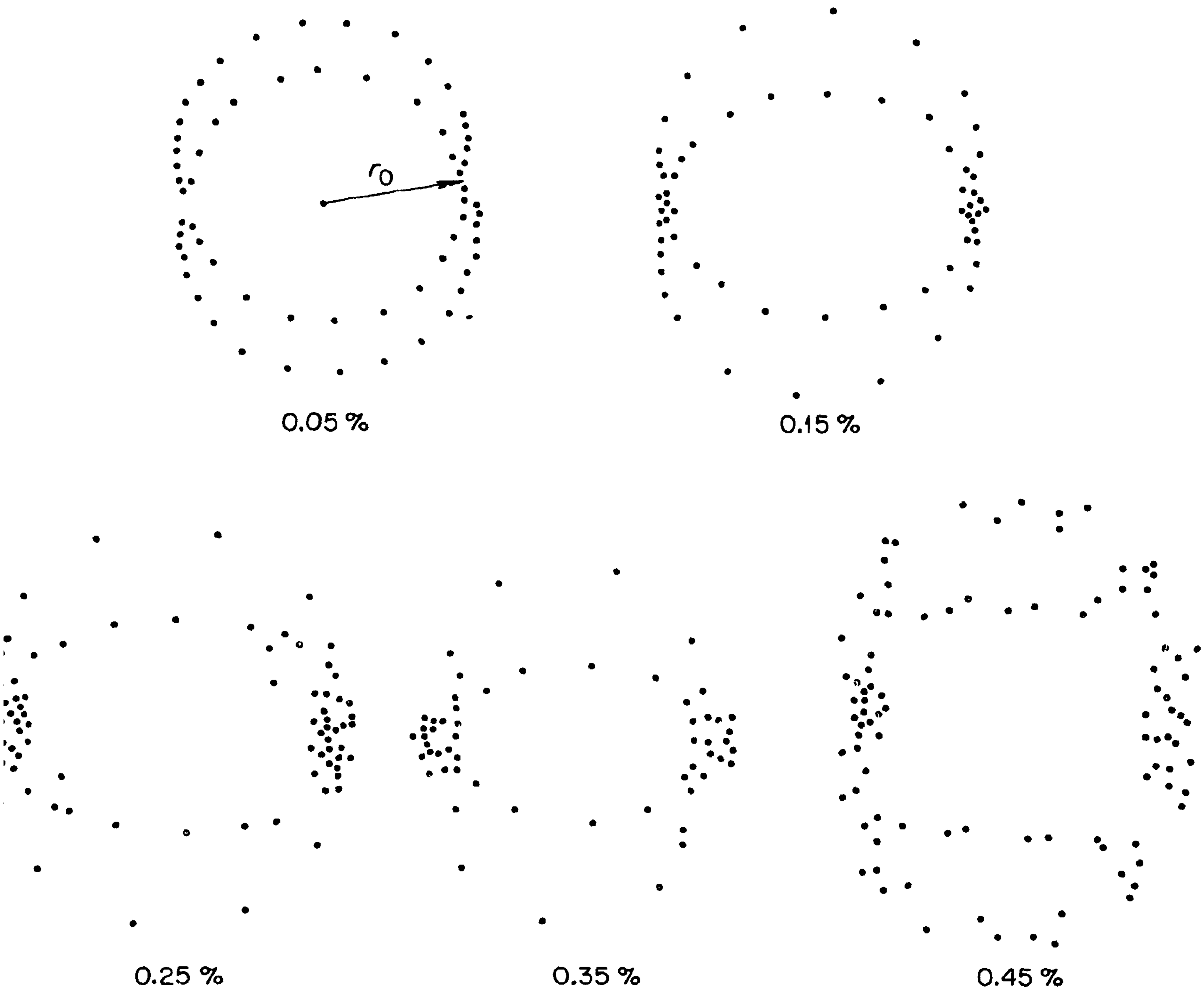

Fig. 6. Computer plots of m = 2 magnetic surface for values of perturbation strength ranging from .05% to .45%. The radius of the q = 2 surface is $r_o \approx 8$ cm. The plasma radius is 20 cm.

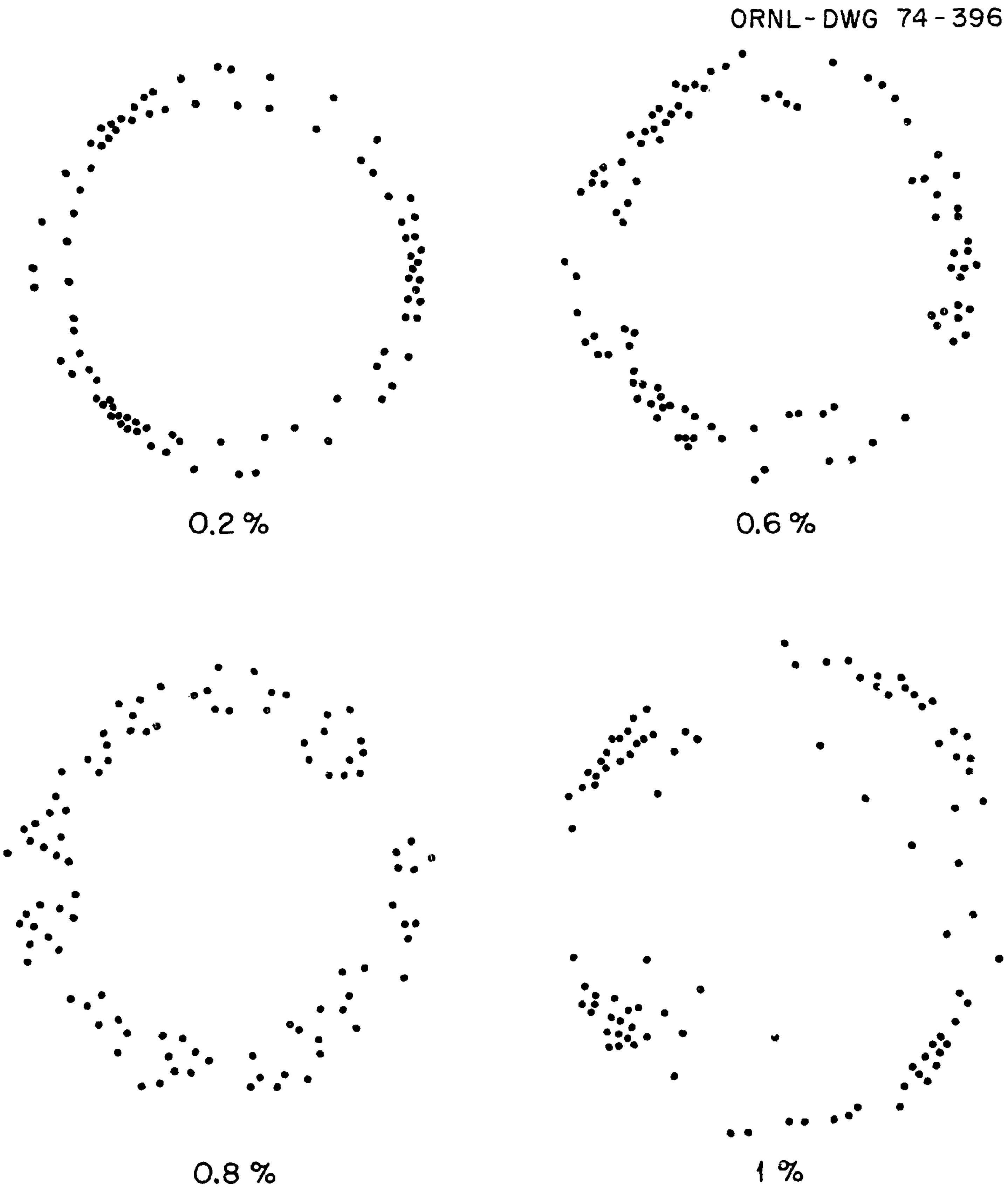

Fig. 7. Computer plots of m = 3 magnetic surfaces for perturbation strengths from .2% to 1%. The radius of the q = 3 surface is $r_o \approx 18$ cm. The plasma radius is 20 cm.

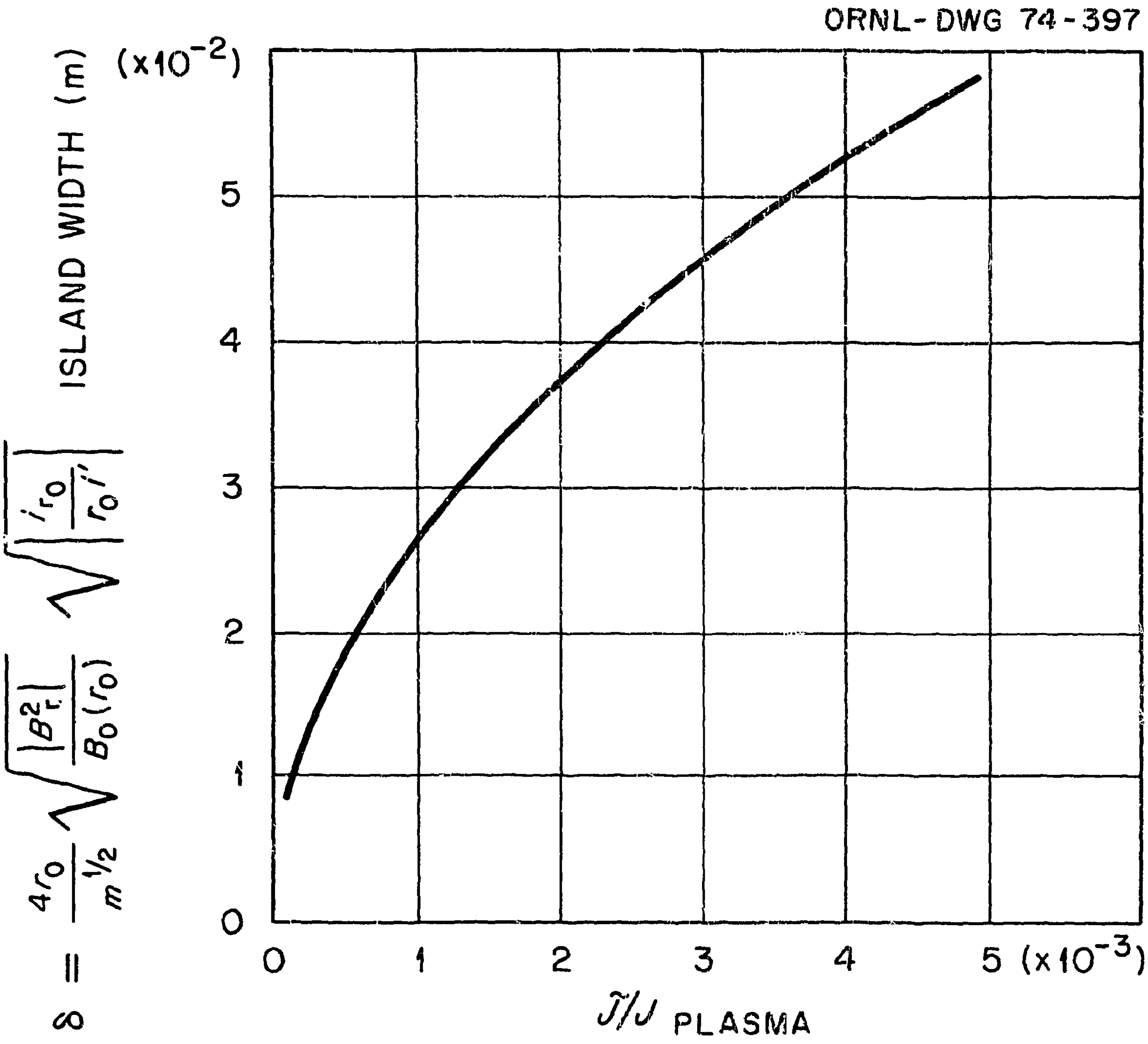

Fig. 8. Theoretical island width versus perturbation strength.