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IC/74/36

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SUPER-SYMMETRY AND NON-ABELIAN GAUGES

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1974 MIRAMARE-TRIESTE

IC/74/36

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

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SUPER-SYMMETRY AND NON-ABELIAN GAUGES *

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ABSTRACT

We show that the conventional Yang-Mills Lagrangian with the Yang-Mills field interacting with Majorana fermions belonging to the adjoint representation of an internal symmetry group like SU(n) is super-gauge invariant.

MIRAMARE - TRIESTE
May 1974

* To be submitted for publication.

The feasibility of combining fermions with bosons in irreducible multiplets of a new kind of global symmetry (super-gauge symmetry) has been established by Wess and Zumino *). These authors have given two examples [3),4)] of renormalizable Lagrangians which possess this symmetry. The question naturally arises as to whether and to what extent the new symmetry can be compatible with internal symmetries. Internal symmetries of the global type are easily incorporated. More interesting and more difficult is the question of compatibility with internal symmetries of the local type. What is clearly implied in such cases is that the vector mesons of the gauge system must be associated with new gauge particles which are fermions. In the absence of spontaneous symmetry breaking, the gauge fermions would have zero mass, like the gauge vector mesons.

Wess and Zumino have constructed a super-gauge invariant extension of quantum electrodynamics, i.e. of an Abelian local symmetry [4)]. The gauge multiplet consists of a photon together with a neutral (Majorana) zero-mass spinor, while the matter multiplet consists of an electron and two charged spin-zero particles. The resulting model is renormalizable.

In this paper we construct a super-symmetric Lagrangian which at the same time exhibits a non-Abelian gauge symmetry. In particular, we find the somewhat remarkable result that the Lagrangian for a conventional Yang-Mills system of fields interacting with a multiplet of Majorana fermions is super-invariant, provided that the Fermi fields (like the Yang-Mills fields themselves) belong to the adjoint representation of the internal symmetry group (e.g. SU(n)).

To construct the Lagrangian, we use the method of super-fields [5)]. A typical super-field, $\Phi(x,\theta)$, defined over the space of anticommuting Majorana spinors θ_α, is equivalent to a set of sixteen ordinary fields (half of them fermionic and the other half bosonic) defined over space-time. These ordinary fields are simply the coefficients in the expansion of $\Phi(x,\theta)$ in powers of θ (which expansion must terminate in the fourth order because of the anticommutativity of the variable θ). Differentiation with respect to

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*) These authors (Ref.1), following the usage in dual model theory where the concept originated (Ref.2), designate this Fermi-Bose symmetry by the expression "super-gauge". Since the word "gauge" has come to be associated more commonly with "gauges of the second kind" or local symmetries, it is confusing to use super-gauge to describe what is indeed a global symmetry of fermions and bosons. We suggest therefore that the expression "super-symmetry" might be more appropriate for the global concept and reserve the word "gauge" for local symmetries.

θ is easily defined and enjoys the usual properties except that attention must be paid to the order of factors. A particularly useful operator is the "covariant derivative" D_α defined by

$$D_\alpha\, \Phi(x,\theta) = \frac{\partial\Phi}{\partial\bar{\theta}^\alpha} - \frac{i}{2}\,(\gamma_\mu\theta)_\alpha\,\frac{\partial\Phi}{\partial x_\mu} \quad . \tag{1}$$

This derivative is a Dirac spinor under Lorentz transformations and an invariant under super-transformations,

$$\delta\Phi = -\,\bar{\varepsilon}^\alpha\left(\frac{\partial\Phi}{\partial\bar{\theta}^\alpha} + \frac{i}{2}\,(\gamma_\mu\theta)_\alpha\,\frac{\partial\Phi}{\partial x_\mu}\right) \quad . \tag{2}$$

The fundamental property of the operators D_α is that they generate a 16-dimensional Clifford algebra,

$$\{D_\alpha, D_\beta\} = -(\gamma_\mu C)_{\alpha\beta}\; i\,\frac{\partial}{\partial x_\mu} \quad . \tag{3}$$

This means, for example, that any product of five or more covariant derivatives can always be reduced. *)

A special kind of super-field can be expressed in terms of only four independent components, <u>viz.</u>

$$\Phi_\pm(x,\theta) = \exp\left[\mp\frac{1}{4}\bar{\theta}\gamma_\mu\gamma_5\theta\,\frac{\partial}{\partial x_\mu}\right]\left(A_\pm(x) + \bar{\theta}\psi_\pm(x) + \frac{1}{4}\,\bar{\theta}(1 \pm i\gamma_5)\theta\; F_\pm(x)\right) \;, \tag{4}$$

where $A_\pm$ and $F_\pm$ are complex, and $\psi_\pm$ are chiral projections $(i\gamma_5\psi_\pm = \pm\psi_\pm)$. Clearly, space reflections must carry Φ_+ into Φ_- and vice-versa. These super-fields, which are <u>irreducible</u> under the combined super-symmetry and (proper) Poincaré groups, identically satisfy

$$D_{R,L\alpha}\;\Phi_\pm \equiv \frac{1}{2}(1 \mp i\gamma_5)_\alpha^{\;\beta}\; D_\beta\;\Phi_\pm = 0 \quad , \tag{5}$$

which can be verified directly using (1) and (4). A fundamental property of the irreducible fields is that the ordinary product of two "left-handed" fields is again left-handed,

$$\Phi'_+(x,\theta)\;\Phi''_+(x,\theta) = \Phi'''_+(x,\theta) \quad , \tag{6}$$

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*) For example, $(\bar{D}D)^2 D_\alpha = -2i\bar{D}D(\gamma_\mu D)_\alpha\;\partial/\partial x_\mu$.

and similarly for the right-handed fields. The product of right with left fields, however, is a general 16-component super-field.

Now consider the action of a local symmetry transformation on a set of super-fields $\Phi(x,\theta)$ which belong to a representation of an internal symmetry group. It is natural to require that the parameters of such a transformation should themselves comprise a super-field,

$$\Phi(x,\theta) \to \Omega(x,\theta)\ \Phi(x,\theta)$$

The group property

$$\Omega_1(x,\theta)\ \Omega_2(x,\theta) = \Omega_3(x,\theta)$$

requires that the elements of Ω be super-fields either of the general 16-component type or else of one of the types Ω_+ or Ω_-. In this note we consider the more restricted type of gauge symmetry

$$\Phi_\pm \to \exp[i\Lambda_\pm]\ \Phi_\pm \tag{7}$$

where Λ_+ and Λ_- are matrices related by

$$\Lambda_+^\dagger = \Lambda_- \quad . \tag{8}$$

It may be instructive to see the rule (7) expressed in component form with

$$\delta\Lambda_+ = \exp[-\tfrac{1}{4}\,\bar\theta\partial\!\!\!/\gamma_5\theta]\left(\delta\omega_+ + \bar\theta\delta\chi_+ + \tfrac{1}{4}\,\bar\theta(1+i\gamma_5)\theta\ \delta\phi_+\right) \quad . \tag{9}$$

One finds

$$\begin{aligned}
\frac{1}{i}\,\delta A_+ &= \delta\omega_+\ A_+ \\
\frac{1}{i}\,\delta\psi_+ &= \delta\omega_+\ \psi_+ + \delta\chi_+\ A_+ \\
\frac{1}{i}\,\delta F_+ &= \delta\omega_+\ F_+ - \delta\bar\chi_-\ \psi_+ + \delta\phi_+\ A_+ \quad ,
\end{aligned} \tag{10}$$

with similar formulae for the right-handed components. The purely internal symmetry transformations of the fields $A_\pm$, $\psi_\pm$ and $F_\pm$, parametrized by $\delta\omega_\pm$, are here seen to be supplemented by parameters $\delta\chi_\pm$ and $\delta\phi_\pm$ to achieve compatibility with super-symmetry.

The gauge fields are to be contained in a hermitian matrix super-field Ψ which transforms according to

$$e^{\Psi} \to e^{i\Lambda_-} e^{\Psi} e^{-i\Lambda_+} \quad . \tag{11}$$

The Ψ fields must not be of the Ψ_+ and Ψ_- variety but are general super-fields. The main problem is to construct a gauge-invariant Lagrangian for them. The interaction of Ψ with the matter fields is controlled by the symmetry requirements. The coupling is contained in the invariant expression *)

$$\mathrm{Tr}\ (\bar{D}D)^2 \left[\Phi_+^\dagger\ e^{\Psi}\ \Phi_+ + \Phi_-^\dagger\ e^{-\Psi}\ \Phi_-\right] \quad . \tag{12}$$

Parity conservation requires that Ψ transform as a pseudoscalar super-field.

To construct a gauge-invariant Lagrangian for Ψ it is useful to define first the vector super-field

$$V_\mu = \frac{1}{2}\left(C^{-1}\ \gamma_\mu \frac{1 + i\gamma_5}{2}\right)^{\alpha\beta} D_\alpha\left(e^{-\Psi}\ D_\beta\ e^{\Psi}\right) \quad . \tag{13}$$

Corresponding to (11), one finds

$$V_\mu \to e^{i\Lambda_+}\ V_\mu\ e^{-i\Lambda_+} + \frac{1}{i}\ e^{i\Lambda_+}\ \partial_\mu\ e^{-i\Lambda_+} \quad , \tag{14}$$

i.e. a Yang-Mills type of transformation law. However, the inhomogeneous term here is an irreducible left-type field and so is annihilated by the right-handed covariant derivative (5), i.e.

$$D_{R\alpha}\ V_\mu \to e^{i\Lambda_+}\ D_{R\alpha}\ V_\mu\ e^{-i\Lambda_+} \quad . \tag{15}$$

Also note that

$$D_{R\alpha}\ D_{R\beta}\ V_\mu \equiv 0 \quad . \tag{16}$$

Using (15) and (16) it is easy to see that a suitable Lagrangian for the gauge field is given by

$$\mathrm{Tr}\ \bar{D}D(C^{-1})^{\alpha\beta}\left[(D_{R\alpha}\ V_\mu)(D_{R\beta}\ V_\mu) + (D_{L\alpha}\ V_\mu^\dagger)(D_{L\beta}\ V_\mu^\dagger)\right] =$$

$$= \frac{1}{2}\ \mathrm{Tr}\ (\bar{D}D)^2(V_\mu\ V_\mu + V_\mu^\dagger\ V_\mu^\dagger) \quad , \tag{17}$$

when a surface term is discarded.

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*) A detailed discussion of the properties of super-fields and their use in constructing Lagrangians is in preparation.

In a general gauge the Lagrangian (17) with V_μ defined by (13) is not a polynomial in the field variables and therefore the S matrix is not manifestly renormalizable. However, there does exist a remarkable gauge (due to Wess and Zumino [4]) in which the Lagrangian assumes a manifestly renormalizable form. To lowest order in Λ and Ψ the transformation rule (11) reads

$$\Psi \to \Psi - i\Lambda_+ + i\Lambda_- + \cdots \tag{18}$$

This indicates that from among the sixteen components in Ψ we can gauge away eight and leave Ψ in the special form:

$$\Psi = \frac{1}{4}\,\bar\theta i\gamma_\nu\gamma_5\theta\, A_\nu + \frac{1}{4}\,\bar\theta\theta\bar\theta\gamma_5\lambda + \frac{1}{32}\,(\bar\theta\theta)^2\, D \quad , \tag{19}$$

where A_ν is a transverse vector $(\partial_\nu A_\nu = 0)$. In this gauge, V_μ reduces to a polynomial in Ψ ,

$$V_\mu = \bar D\gamma_\mu\,\frac{1+i\gamma_5}{2}\,D\Psi - \tfrac{1}{2}\bar D\Psi\gamma_\mu D\Psi - \tfrac{1}{2}\left[\Psi,\bar D\gamma_\mu\,\frac{1+i\gamma_5}{2}\,D\Psi\right] \tag{20}$$

and the Lagrangian (17) reduces to

$$\mathrm{Tr}\left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu,A_\nu]\right)^2 + \bar\lambda\, i\gamma_\mu\left(\partial_\mu\lambda + i[A_\mu,\lambda]\right) + \tfrac{1}{2}D^2\right] \tag{21}$$

Apart from the $\frac{1}{2}D^2$ term (which on variation gives the field equation $D = 0$ in the absence of matter), this is the Yang-Mills Lagrangian for the gauge fields A_μ and the Majorana spinors λ . The super-symmetry implies that each of the fields A_μ and λ acts as a gauge particle for the other. It would be interesting to investigate if this system (with a Majorana λ) shows the same diminution of infinities as has been noticed by Illiopoulos and Zumino [6] for other super-symmetric Lagrangians. For example, it is amusing that charge renormalization in this theory is finite in the lowest order provided there are in the theory three distinct matter super-multiplets (each containing two spin-zero bosons and a Majorana fermion) belonging to the adjoint representation of the internal symmetry group. Whether on account of super-invariance of the theory such a result persists in higher orders is an open question.

We wish to conclude by pointing out that the involvement of Majorana fermions in super-symmetric schemes such as the one discussed here is not inevitable. One quite simple way to avoid them is by introducing a multiplet of massless matter super-fields $\Phi_\pm$ in the adjoint representation. The fermionic part of the total Lagrangian is given by

$$i\,\bar{\lambda}^k\,\gamma_\mu\left(\partial_\mu\,\lambda^k + \varepsilon^{k\ell m}\,A^\ell_\mu\,\lambda^m\right) + i\,\bar{\psi}^k\,\gamma_\mu\left(\partial_\mu\,\psi^k + \varepsilon^{k\ell m}\,A^\ell_\mu\,\psi^m\right) + $$

$$+\,2\,\varepsilon^{k\ell m}\,\bar{\lambda}^k\left(A^\ell + \gamma_5\,B^\ell\right)\,\psi^m \quad , \qquad (22)$$

where A^ℓ and B^ℓ denote, respectively, the real and imaginary parts of A^ℓ_+. If the Majorana spinors ψ^k and λ^k are combined into the complex form

$$\chi^k \;=\; \psi^k + i\,\lambda^k \;,$$

then the terms (22) can be arranged in the form

$$i\,\bar{\chi}^k\,\gamma_\mu\left(\partial_\mu\,\chi^k + \varepsilon^{k\ell m}\,A^\ell_\mu\,\chi^m\right) + i\,\varepsilon^{k\ell m}\,\bar{\chi}^k\left(A^\ell + \gamma_5\,B^\ell\right)\,\chi^m \qquad (23)$$

after discarding a gradient. This Lagrangian admits the phase transformations

$$\chi^k \to e^{i\alpha}\,\chi^k \quad , \quad \bar{\chi}^k \to e^{-i\alpha}\,\chi^k \qquad (24)$$

(with the boson components A^k, A^k_μ, etc., real and invariant).

More elaborate schemes for complexifying the fermions are possible. One which we have examined employs a "colour" triplet of matter fields, ${}^{(a)}\Phi_+$, each of which transforms under the local symmetry as a triplet. If the self-interaction term

$$\bar{D}D\ \mathrm{Tr}\left({}^{(1)}\Phi_+\ {}^{(2)}\Phi_+\ {}^{(3)}\Phi_+ + {}^{(1)}\Phi_-\ {}^{(2)}\Phi_-\ {}^{(3)}\Phi_-\right)$$

is added to the Lagrangian, then it is possible to rearrange the four Majorana triplets, λ^k, ${}^{(a)}\psi^k$, into two complex triplets in such a way that phase transformations are admitted.

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