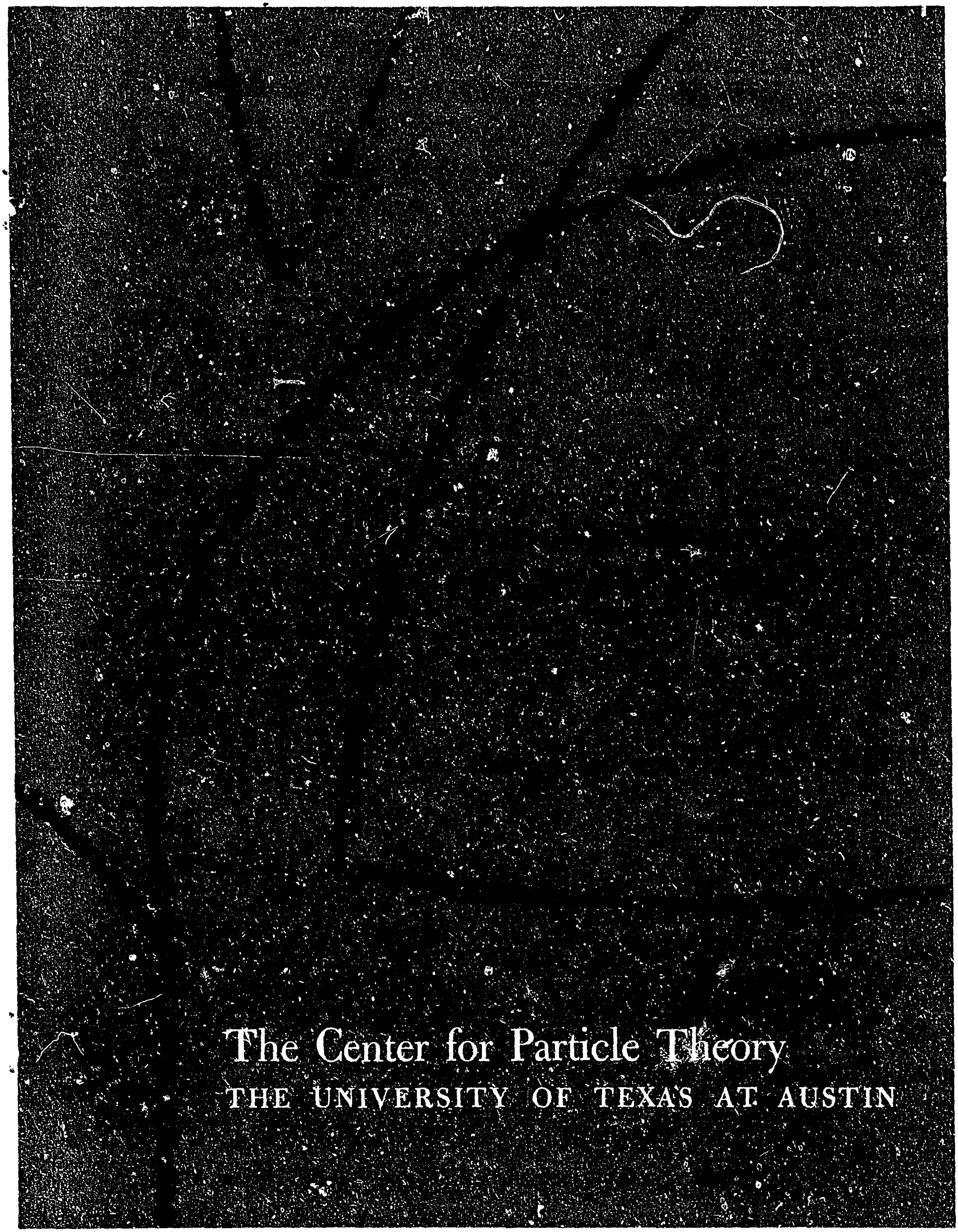

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Validity of Second Order Analysis of Superdense Matter

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ABSTRACT

The limitations of relativistic calculations of the properties of superdense matter obtained from strictly second order terms is discussed. Extension of the model to overcome these limitations leads to serious complications which can only be overcome by a fully self-consistent treatment.

A relativistic treatment of infinite fermion matter at zero temperature which retains interactions strictly to order g^2 in perturbation theory has recently been presented.[1] The analysis, which utilizes relativistic finite density Green's functions, treats the strong interactions through the exchange of scalar, pseudoscalar and vector mesons. In this note we discuss two major limitations of this approach: (1) For densities at which the method is viable, relativistic effects are not necessary; (2) Strict adherence to Born approximations which retain no terms of order greater than g^2 is too restrictive an approach to a many-body theory applicable to superdense matter. An extension of the analysis to properly include unitarity by retaining second order self-energies summed to all orders leads to serious fundamental difficulties involving the appearance of anomalous states in the physical spectrum.[2] Our arguments are based on detailed formal and numerical analysis of a model similar to the one discussed by Bolsterli.

It is tempting to base a model of a dense system of strongly interacting relativistic fermions on Green's functions obtained from solution of an effective lagrangian which contains baryon fields interacting via exchange of the observed mesons whose coupling strengths have been fitted to nucleon-nucleon scattering data. The analysis of such a model obviously requires approximations at several levels if numerical results are desired. The simplest possible approach would involve retention of second order self-energies.[3] For a finite density Green's function approach based on interacting relativistic fields, the second order self-energies consist of two terms: (1) density dependent pieces which are finite; (2) terms formally identical with divergent zero density self-energies common to relativistic field theory. The latter may be regularized, but will contribute off mass shell terms to the fermion propagators. The second order self-energies, denoted simply by $\Sigma(p,q_F)$, then may be used to construct the interacting fermion Green's function $G(p,q_F)$ through, for example, Dyson's equation

$$G(p,q_F) = S(p,q_F) + S(p,q_F)\ \Sigma(p,q_F)\ G(p,q_F). \tag{1}$$

The boundary conditions reflecting the system's finite density enter through the non-interacting function $S(p,q_F)$. Evidently the solution for G contains Σ and therefore g^2 to all orders. Before addressing ourselves to the second order implications of (1), we emphasize that although the inverse Green's function for spin ½ fermions satisfying the Dirac equation

$$G^{-1}(p,q_F) = \not{p}-m-\Sigma(p,q_F) \tag{2}$$

contains explicit dependence on Σ to second order only, the off physical mass shell behavior of G is essential in determining the collective properties of the system.

If attention is restricted only to order g^2 then as demonstrated by Bolsterli[1] the off mass shell effects are of order g^4, and within the spirit of the approximation scheme are dropped.

There are at least two serious objections to this procedure, the first involving unitarity, and the second the asymptotic behavior of the Green's functions.

One of the principle reasons motivating second order calculations is that they represent the lowest order at which unitarity may be verified. This is especially important for many-body theory since unitarity determines, among other things, the stability of the Fermi sea. To better understand this, we may consider the fermion inverse Green's function to order g^2:

$$G^{-1}(p,q_F) = \not{p}-m-\Sigma_1^E(p)-\Sigma_1^n(p,q_F)$$

$$- i[\Sigma_2^E(p) + \Sigma_2^n(p,q_F)] \qquad (3)$$

where the term containing explicit density dependence has been denoted by superscript n and real and imaginary parts have been separated. In order for the Fermi sea to be stable, the total imaginary part of the self-energy must vanish at the Fermi surface. Although this is found to occur, $\Sigma_2^E(p)$ and $\Sigma_2^M(p,q_F)$ do not vanish individually when $\vec{p} = q_F$. Since the Fermi surface corresponds to excitation energies off physical mass shell neglecting $\Sigma_2^E(p)$ results in a finite imaginary part at $\vec{p} = q_F$ <u>even</u> <u>in</u> <u>second</u> <u>order</u>. Therefore $\Sigma_2^E(p)$ must be retained if unitarity is to be preserved.

A more serious drawback to the possible applications of this approach to superdense matter is the fact that corrections to the fermion propagators restricted to order g^2 contain a density dependence proportional to (q_F/m). This leads to anomalous asymptotic behavior, and would indicate possible causality violations.

In order to obtain any physically significant information pertinent to relativistic systems from such a model the approximations must be improved. The most obvious step in this direction would be to sum the second order regularized self-energies to all orders by using Dyson's equation for the Green's functions, as indicated by eqn. (1). This eliminates objections based on unitarity, incorporates off physical mass shell effects contained in Σ_1^E and Σ_2^E, and eliminates possible acausal behavior at asymptotic densities. When the approximations are thus relaxed, it is no longer true that off mass shell effects are ignorable. Instead they enter to all orders in g^2. Furthermore, detailed numerical analysis shows that they are driven well off mass shell by density dependence, and that they play a major role in determining such physical quantities as excitation energies, chemical potentials and the equation of state.

In an attempt to construct a model of superdense fermions at zero temperature we considered a model like the one discussed above.[4] The strong interactions were described by SU(3) symmetric Yukawa couplings to the low mass mesons needed to fit nucleon-nucleon scattering data. Second order bubble diagrams were retained to all orders through Dyson's equations, and the fermion chemical potentials were found numerically for a wide range of coupling constants. Results were obtained for a single baryon in four cases. These corresponded to the interactions mediated by: (1) a single scalar exchange (a sigma meson); (2) pseudoscalar exchange (pions); (3) a single vector exchange (rho); (4) the complete set $\{\pi,\rho,\eta,\phi,\delta,\omega\}$ needed to fit scattering data.[5] In all cases the results were disappointing, and led to the appearance of anomalous states at densities near or <u>below</u> nuclear density for any physically reasonable choice of the coupling constants. The anomalies which develop can be identified with indefinite metric states, such as one found in the Lee model, which manifest themselves here as negative baryon number densities. This should not come as a surprise since we are working in a renormalized formalism. If the couplings are restricted to sufficiently small values to eliminate these states ($g_\pi^2/4\pi<5.$, $g_\rho^2/4\pi < 2.$ for example) solutions for the chemical potentials may be obtained. However, in all instances they had the opposite behavior to that expected on physical grounds. For example, the weak exchange of pions, which is expected to produce attraction, implies an equation of state harder than a free gas, while vector exchange leads to a softening of the equation of state. In all cases examined we found that physically reasonable behavior resulted in solutions which developed anomalous states at non-relativistic energies.

Recent work has shown that a self-consistent treatment of proper baryon number density must be incorporated in any serious treatment of a system of relativistic fermions at zero temperature.[6,7] These effects are not necessarily small at low densities where one may be tempted to ignore terms of order g^4 or higher. In fact the correct incorporation of the self-consistency constraints would eliminate anomalous indefinite metric states in the physical spectrum. The requirement of self-consistency leads to substantial complications, even in a numerical approach to the problem. Self-consistency relations involve non-linear integral equations for the Green's functions which contain off mass shell effects even to lowest order in the self-energy. A numerical algorithm for the correct evaluation of these conditions has not yet been developed.

In conclusion we emphasize that any calculation strictly to order g^2 is inadequate on physical as well as mathematical grounds. The situation is somewhat improved by incorporating vacuum self-energies which would contribute to the physical spectrum in fourth order, since these inclusions guarantee the stability of the Fermi sea. This advantage is more than offset because it introduces anomalous states which can only be eliminated by a fully self-consistent treatment.

REFERENCES

1. M. Bolsterli, Phys. Rev. D11, 312 (1975).

2. R. Bowers, D. Pedigo, A. Gleeson, and R. L. Zimmerman, To appear in the Astrophysical Journal; R. L. Bowers, A. Gleeson and D. Pedigo, to be published.

3. R. L. Bowers, J. A. Campbell, and R. L. Zimmerman, Phys. Rev. D7, 2278 (1973); and D7, 2289 (1973).

4. A brief discussion of this attempt is given in the second paper of ref. 2.

5. K. Erkelenz, Physics Reports 13, 191 (1974).

6. J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974).

7. G. Kalman, Phys. Rev. D9, 1656 (1974).