

Accuracy of Single Photoelectron Time Spread  
Measurement of Fast Photomultipliers

Branko Leskovar  
Lawrence Berkeley Laboratory  
University of California  
Berkeley, California, U.S.A.

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Abstract

The accuracy of time spread measurements of fast photomultipliers has been investigated, using single photoelectrons. The effect of the finite light pulse width on the measurement accuracy is determined and discussed. Experimental data have been obtained on a special measuring system for light pulse widths ranging from 200 psec to 10 nsec, using fast photomultipliers 8850 and C31024 with optimized operating conditions for minimum transit time spread. A modified exponential function expression and curve-fitting parameters are given, which fit closely the experimentally obtained data over a wide dynamic range of light pulse widths.

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1. Introduction - The investigation of time-resolution capabilities of fast photomultipliers has been the subject of both experimental and theoretical investigations for a long time, and a comprehensive survey of the literature has been recently given by the author.<sup>1)</sup> It was pointed out that the photomultiplier transit time spread investigations are becoming increasingly important in a multitude of research areas, such as atomic and molecular subnanosecond fluorescence decay time measurements,<sup>2)</sup> nuclear research instrumentation,<sup>3)</sup> optical ranging experiments,<sup>4)</sup> optical communication,<sup>5)</sup> and photon statistics experiments.<sup>6)</sup> The time resolution capabilities of fast standard and microchannel type photomultipliers are essentially determined by the random deviations in transit time of electrons traveling from photocathode to collector. The electron transit time spread is mainly caused by fluctuations of the individual time of flight of photoelectrons and secondary electrons, due to their different trajectories and their initial velocity differences. Other causes are differences in trajectory length and electrical field strength in different portions of the photocathode-first-dynode region, and between various dynode sections. The best characterization of the photomultiplier electron transit time spread is given by the single photoelectron time spread performance, because it is particularly useful in the determination of photomultiplier optimum operating conditions for minimum time spread, and in the evaluation, selection and comparison of photomultipliers. The single photoelectron time spread measurements have previously been done for fast photomultipliers having dynodes with conventional secondary emitting surfaces,<sup>7)</sup> and for photomultipliers having dynodes with cesium-activated gallium-phosphide secondary emitting surfaces.<sup>8-9)</sup> Our measurements and considerations, made on a special measuring system,<sup>10)</sup> have shown that the measurement accuracy is essentially determined by a finite width of the light pulse of the subnanosecond light pulse generator and by the time walk and resolution characteristics of the timing discriminator used during measurement. This is particularly true for measurements made on contemporary photomultipliers which, under optimized operating conditions, have a relatively small transit time spread. The influence of the discriminator timing error on the measurement accuracy has recently been practically eliminated in the measuring system, which incorporates a specially designed constant fraction discriminator with a time walk of  $\pm 15$  psec or less over a dynamic range of input pulse amplitudes from 200 mV to 5 V.<sup>11)</sup> However, the accuracy of the time spread measurements is significantly influenced by the finite width of the light pulse. The light pulse width contributes directly to the amount of the electron time spread, since photoelectrons can be emitted at any time during the existence of the light pulse. Therefore, it is important to

determine the dependence of the accuracy of the photomultiplier time spread measurements upon the light pulse width by experiment and by an approximation function fit of experimental data. The approximation function fit enables one to obtain by extrapolation a true value of the device time spread from experimental data which include the measuring error.

2. Photomultiplier Total Transit Time Spread - The total electron transit time spread of an electrostatically focused photomultiplier consists of the photoelectron transit time spread between the photocathode and the first dynode of the electron multiplier, the transit time spread in the electron multiplier, and that between the electron multiplier and the anode. The initial stage of a photomultiplier contribute with the greatest weight to the total transit time spread. In the latter stages, the larger number of electrons in the pulse provide many samples of transit time through the stage and reduce the transit time spread of that stage in the manner of the standard error of mean value. The total value of the photomultiplier single photoelectron transit time spread, expressed by the full width at half maximum, is approximately given by the equation:<sup>1)</sup>

$$\tau_{3\text{FWHM}} \approx 2.30 \left[ \tau_{CD1}^2 + \frac{\tau_{D1D2}^2}{g_1} (1 + g_{\sigma 1}^2) + \frac{\tau_{DD}^2}{g_1(g-1)} (1 + g_{\sigma}^2) \right]^{1/2}, \quad (1)$$

where  $\tau_{CD1}^2$  is the variance of the photoelectron transit time between the photocathode and the first dynode,  $\tau_{D1D2}^2$  is the variance of the electron transit time between the first and the second dynode,  $\tau_{DD}^2$  is the variance of the electron transit time between two successive dynodes,  $g_1$  is the gain of the first dynode,  $g$  is the gain of all dynodes,  $g_{\sigma 1}^2$  is the variance of the gain  $g_1$  and  $g_{\sigma}^2$  is the variance of the gain  $g$ . It can be seen from the equation that the total device time spread is considerably reduced using dynodes with emitting surfaces that have a high and uniform secondary emission yield. The ultimate limitation on time spread is determined by initial velocity effects of photoelectrons and secondary electrons. The transit time spread resulting from the initial velocity distribution is decreased by increasing the voltages between the photocathode and the first dynode and between various dynodes in the electron multiplier.

5. Dependence of the Accuracy of the Time Spread Measurement Upon the Light Pulse Width - The measuring system shown in Fig. 1 of reference<sup>1)</sup> was used to determine the dependence of the accuracy of the photomultiplier time spread measurement upon the light pulse width. The system consists of a subnanosecond light pulse generator, a very low time-walk current discriminator, leading edge discriminators, delay cables, a time-to-pulse-amplitude converter, a multichannel analyzer, and a pulse shaper. The light level of the subnanosecond light pulse generator, with variable light pulse width capability, was adjusted to a low intensity necessary to cause the emission of predominantly single photoelectrons. Typical single photoelectron pulses from an RCA C51024 operated at 3.5 kV, using a 200 psec impulse excitation from the reversed-biased electroluminescent diode, Ferranti type XP-23, are shown in Fig. 1.

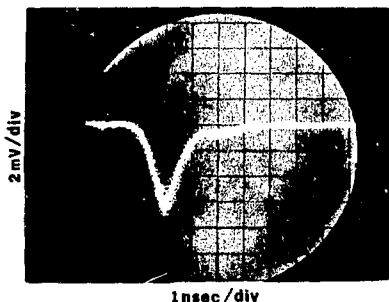


Fig. 1. Typical single photoelectron pulses from an RCA C31024 operated at 3.5kV using a 200 psec impulse excitation from the reverse-biased electroluminescent diode, Ferranti type XP-23.

Measurements were done on several 8850's and C31024's with a small 1.6 mm diameter photocathode area and also with full photocathode illumination. Using a photomultiplier with optimized operating conditions for minimum single photoelectron time spread, the measurements were made as a function of width of the light pulse, with the supply voltage between the anode and cathode as a parameter.

The results of measurements for the type 8850 are given in Figs. 2 and 3 for a 1.6 mm diameter area and for full photocathode illumination, respectively. The experimental results are shown by open triangles. The experimental data points represent the typical time spread, FWHM, obtained by averaging the data from four photomultipliers. For the purpose of interpolation and extrapolation, a modified exponential function expression was used to represent the log-log domain of experimental data. The relationship between measured single photoelectron time spread,  $t_{0M}$ , and light pulse width,  $u$ , in nanoseconds, for supply voltages between anode and cathode as the parameters, can be well fitted with a function of the type

$$t_{0M} = \exp \left\{ c_1 + c_2 [\ln u + (\alpha_1 + \alpha_2 \ln u + (\ln u)^2)^{1/2}] \right\}, \quad (2)$$

where  $c_1$ ,  $c_2$ ,  $\alpha_1$ , and  $\alpha_2$  are the curve-fitting parameters. They depend on the single electron time spread characteristics and operating conditions of the particular photomultiplier. The curve-fitting parameters were determined from averaged experimental data using the MIGRAD version of the MINUIT computer program.<sup>11)</sup> The MIGRAD is a minimization subroutine based on a variable metric method by Dividon.<sup>12)</sup> The variable metric

method is an algorithm for minimizing real-valued differentiable functions, and has been found to compare favorably with all other gradient methods. The essential feature of the algorithm is that it proceeds toward the minimum by making successive approximations to the covariance matrix. It uses a technique to converge simultaneously toward minimum and toward the true covariance matrix. Using initial estimates for the curve-fitting parameters from the averaged experimental data, the MIGRAD minimization subroutine gives results of calculations as shown in Table 1.

Table 1. Final Calculated Values of the Curve-Fitting Parameters for RCA 8850 Photomultiplier

Curve-Fitting Parameters	1.6 mm-Diameter Area Photocathode Illumination			Full Photocathode Illumination		
	Supply Voltage Between Anode and Cathode			Supply Voltage Between Anode and Cathode		
	2000 V	2500 V	3000 V	2000 V	2500 V	3000 V
$c_1$	-0.617136	-0.705811	-0.800014	-0.612580	-0.607638	-0.607776
$c_2$	.545081	.548509	.547538	.545247	.552985	.552148
$a_1$	.993405	1.081292	1.434351	1.796370	1.495494	1.226228
$a_2$	1.137837	1.454467	1.783053	.757145	1.143816	1.313148

Furthermore, using equation (2) and the final values of the curve-fitting parameters from Table 1, the single electron time spread, as a function of the light pulse width, is calculated and plotted as a solid line in Figs. 2-3, over a wide dynamic range of light pulse width values. Also, the error between the experimental data and calculated values was investigated. The error is larger than 1% in absolute amount only in isolated cases, for light pulse widths between 200 psec and 10 nsec. This is better than the accuracy of the measurement from which experimental data were obtained. Any extension of the single photoelectron time spread representation by equation (2) for  $n < 200$  psec and  $n > 10$  nsec is an extrapolation, and all of the uncertainties inherent in such extensions of experimental data must apply. However, extension of the data for 200 psec  $> n > 10$  nsec, and  $n \geq 10$  psec, is more acceptable from physical interpretations of the measurements, since the constraint of asymptotic behavior of equation (2) for  $n \ll 1$  nsec and  $n \gg 1$  nsec was used in the original fit.

It can be seen from Figs. 2 and 3 that the single electron time spread is a well-behaved monotonically increasing function of the light pulse width. In the case where the light pulse width is considerably shorter than the photomultiplier time spread, the time spread curves show very small dependence upon the light pulse width. Consequently, the

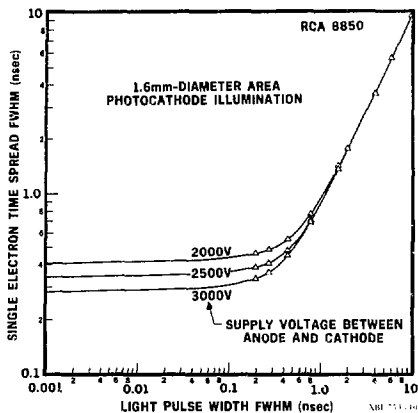


Fig. 2. Single electron time spread of RCA 8850 as a function of the light pulse width for 1.6 mm diameter area of photocathode illumination, with supply voltage between the anode and cathode of the parameter.

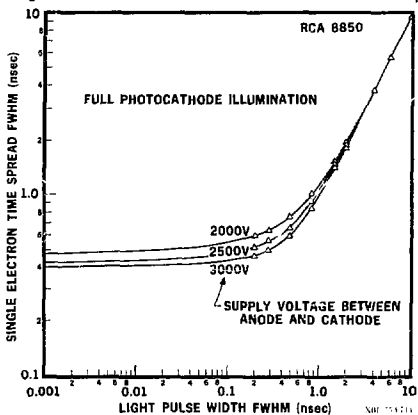


Fig. 3. Single electron time spread of RCA 8850 as a function of the width of the light pulse for full photocathode illumination, with supply voltage between anode and cathode as the parameter.

measured value of the photomultiplier time spread is practically equal to the true value of the time spread. Extrapolation shows that the error between the measured and true value, taking the true value at the 10 psec pulse width point, is approximately +10.8% for a light pulse width of 200 psec, for a supply voltage of 2000 V. If the light pulse width equals 600 psec, a typical value used extensively in previous measuring systems, the error between the measured time value of the time spread is +54%. When the light pulse width is 1 nsec, the error is +124%. Furthermore, when the light pulse width is considerably larger than the single electron time spread, the measured value of the time spread is closely equal to the width of the light pulse, and the difference between the measured and true values of the transit time spread is very large. Under these conditions, the measured emission probability of photoelectrons is proportional to the light intensity. Also, the accuracy of the measurement decreases as the photomultiplier time spread decreases, for a constant value of the light pulse width, as can be seen from Figs. 2 and 3 for supply voltages of 2500 V and 3000 V. For example, it can be seen from Fig. 2 that, for a supply voltage of 3000 V, the extrapolation taken at the 10 psec light impulse width shows an error between the measured and true time spread values of approximately +16%, at a pulse width of 200 psec. In such cases, the accuracy of the extrapolation to obtain the true value of time spread is also decreased.

Similar measurements and calculations of the curve-fitting parameters were performed for photomultiplier C31024. The results are shown in Figs. 4 and 5, for a 1.6 mm diameter area and for full photocathode illumination, respectively. The experimental data, shown by open triangles, represent the typical time spread, FWHM, obtained by averaging the data from four photomultipliers.

In Table 2 the results of the calculations are shown, using the MIGRAD minimization subroutine for the curve-fitting parameters.

**Table 2.** Final Calculated Values of the Curve-Fitting Parameters for RCA C31024 Photomultipliers

Curve-Fitting Parameters	1.6 mm Diameter Area Photocathode Illumination			Full Photocathode Illumination		
	Supply Voltage Between Anode and Cathode			Supply Voltage Between Anode and Cathode		
	3000 V	3500 V	4000 V	3000 V	3500 V	4000 V
$c_1$	-0.651142	-0.699446	-0.707063	-0.640176	-0.608411	-0.618533
$c_2$	.524279	.525219	.517914	.535736	.527851	.524908
$a_1$	1.159440	1.303110	1.333039	1.474335	1.218926	1.162727
$a_2$	1.766629	1.951619	2.195640	1.244283	1.412855	1.582473

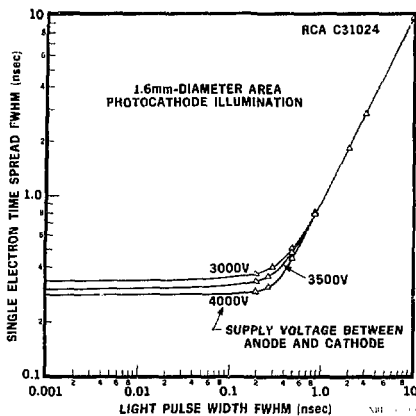


Fig. 4. Single electron time spread of the RCA C31024 as a function of the light pulse width for 1.6 mm diameter area of photocathode illumination, with supply voltage between anode and cathode as the parameter.

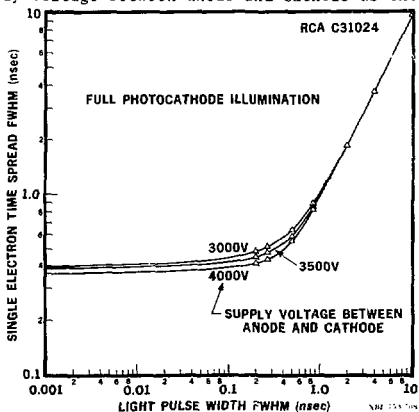


Fig. 5. Single electron time spread of RCA C31024 as a function of the width of the light pulse for full photocathode illumination, with supply voltage between anode and cathode as the parameter.



Using equation (2) and the final values of the curve-fitting parameters from Table 2, the transit time spread, as a function of the light pulse width, is calculated and plotted as a solid line in Figs. 4-5, over a wide dynamic range of light pulse width values.

It can be seen from the Figs. 2, 3, 4 and 5, from equation (2), and from comparison with the data obtained for the 8850 photomultiplier, that the finite width of the light pulse has a larger influence on the C31024 true value than for type 8850. This is caused by the fact that the C31024 photomultiplier, with optimized operating conditions, has a smaller value of the single photoelectron transit time spread than the 8850.

4. **Conclusions** - It is apparent from the above considerations that, in applications where a true value of the single photoelectron time spread is essential, the accuracy of the time spread measurement should be determined for a particular photomultiplier and its operating conditions. This can be accomplished with reasonable accuracy by interpolation and the extrapolation of the experimental data, since the transit time spread is a well-behaved monotonically increasing function of the light pulse width, with asymptotic values for  $u \gg 1$  nsec and  $u \ll 1$  nsec. A high accuracy interpolation and an acceptable extrapolation of the experimental data can be performed, using equation (2) and a suitable minimization procedure for obtaining the curve-fitting parameters from averaged experimental data for a particular photomultiplier.

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