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LOW VOLTAGE INITIATION OF DAMAGING ARCS BETWEEN ELECTRICAL CONTACTS

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ABSTRACT

Metallic arcs were found to precede the firm contacting of electrical contacts which were closed without bounce. When the open-circuit voltages were below the ionization potential, the initiation of these arcs was found to depend on the presence of asperities on the surfaces and on asperity contacting, melting, and pinching off by magnetic forces. The arc is thought to be initiated inductively when the molten metallic asperity contact is pinched off, and the electrode damage is similar to that produced by the arcing of opening contacts. Arcing could not be produced for exceptionally smooth surfaces, or, for rough surfaces when the open-circuit potential was below the melting voltage of the electrode metals. In order to prevent damage to contact surfaces by melting or arcing, it is suggested that test potentials be limited to below the melting voltages, that the current be limited, the test circuits be designed to prevent inductively generated high voltage transients, and the contact surfaces be very smooth. In order to facilitate arc initiation in arc welding applications, it is suggested that the surfaces of electrodes and work pieces be roughened.

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INTRODUCTION

It is well known that metallic arcs may be initiated between opening electrical contacts [1] when the molten metallic bridge is vaporized. The bridge is drawn between the contacts as a result of Joule heating of the last contacting areas. When the open circuit potential is below the "minimum arcing voltage" (11-15 V) [2], the arcing potential may be generated inductively as a result of a momentary decrease in current which occurs when the bridge is destroyed. Similarly, an arc may be formed between closing contacts when contact bounce occurs [3].

In this paper it is proposed that a metallic arc may also be initiated between closing contacts, in the absence of contact bounce, when the first asperities to touch are melted and then vaporized or pinched off by magnetic forces. The melting voltages for the electrode metals were found to be lower limits for the initiation of this process. (The melting voltage is a consequence of the correlation between the temperature and the potential drop across a current constriction such as an asperity contact. Since heat flows in the same path as the electric current, isothermal surfaces are also equipotential [4].)

The importance of this work is primarily in the testing of electrical contacts where it may be desirable to avoid damage to contact surfaces.

EXPERIMENTAL

The essential features of the contacting experiments reported here are (1) a very sensitive piezoelectric micrometer for bringing the electrical contacts together, (2) very rigid and compact fixtures for contact mounting to avoid uncontrolled relative motions, and (3) effective vibration isolation which has been described previously [5].

A calibration curve (Fig. 1) was generated for the piezoelectric micrometer (Spectra Physics Model 415A) using a laser interference fringe microscope [6] to measure the displacement of one optical flat relative to another. The flats were pivoted at one end to form an air wedge of variable thickness between them. Fringe motions corresponding to displacements of less than 161 Å (5 V d.c. across the piezoelectric crystal) were detected easily by viewing the laser-illuminated wedge through the microscope. No evidence of vibration could be detected in the integrally mounted wedge/micrometer at this optical sensitivity. Figure 1 is presented to show that the displacement is a highly linear function of the applied potential and that the deviations of individual points from the best-fit-line are very small in any limited range chosen for operation. The micrometer calibration was checked by two independent means: (1) A He-Ne laser beam was reflected from a mirror, which was rotated using the micrometer, and the displacement of the beam was measured at the end of a 20 m light path, and (2) the displacement of the core of a high sensitivity linear variable differential transformer was measured as a function of the micrometer crystal voltage. The results of the three calibration techniques were in agreement.

The contacting fixture is shown in Fig. 2. The upper contact was mounted on a stiff cantilever arm which was spring loaded to the open position against either

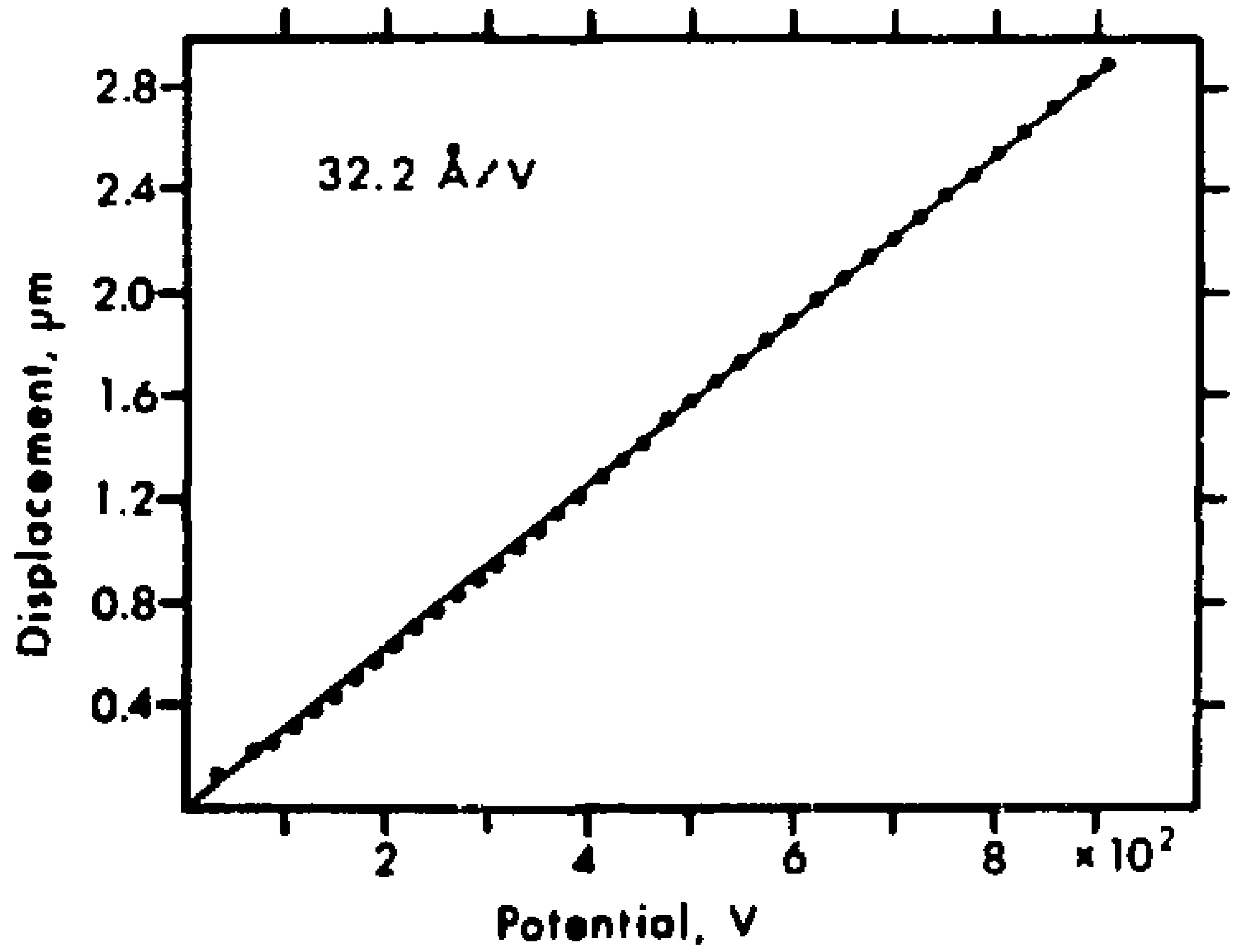

FIGURE 1

Piezoelectric micrometer calibration curve obtained using a laser interference fringe microscope.

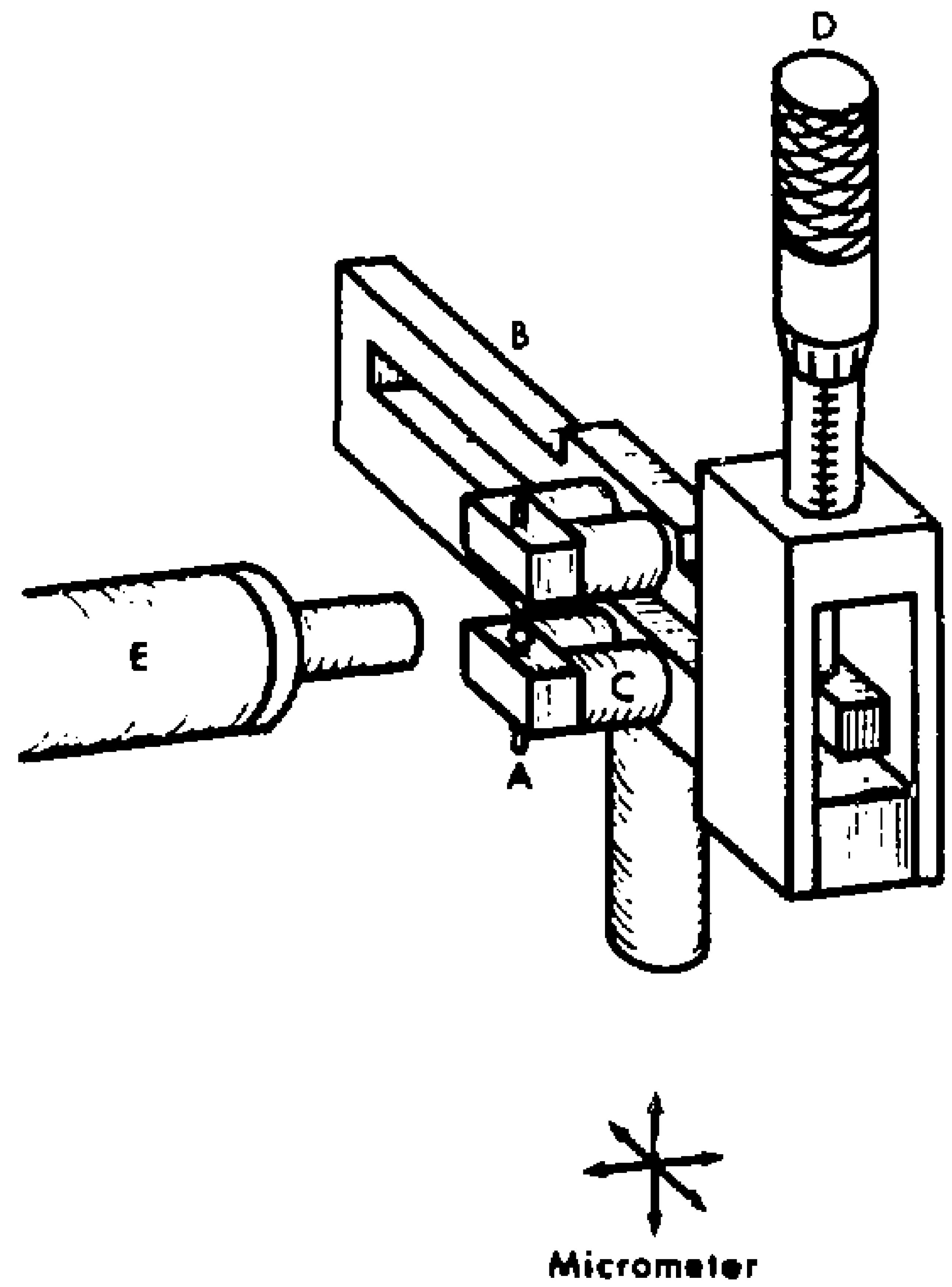

FIGURE 2

Apparatus for contacting the ends of sample rods A which are electrically isolated from cantilever arm B by ceramic spacers C. The rods are contacted by bending the arm using either a standard micrometer D or a piezo-electric micrometer and arcs across the gap are observed through a microscope E.

a standard micrometer (Fig. 2) or the piezoelectric micrometer. The cantilever arm and lower contact mount were machined from a single solid brass bar in order to provide greater rigidity and freedom from vibration or relative motions of the contacts. The closure arcs were observed through a horizontally mounted microscope (Fig. 2).

Arcs were produced between metallic contacts using the circuit shown in Fig. 3. A regulated d.c. variable power supply (Kepco Model KS8-15M, 0-8V, 0-15A) was connected to the contacts by a 1 m cable, the inductance of which was varied to simulate typical contact testing conditions. Potentials were measured across the contacts using a Kiethley Model 160 digital multimeter for open circuit potentials and a Tektronix Model R7704 oscilloscope with high frequency probes for arcing potentials. This oscilloscope contained a [illegible]0 nsec internal delay and was triggered by potential excursions which preceded arcing. Simultaneously, discharge currents were measured using a 50 mΩ current viewing resistor in the circuit or a Tektronix Current Probe Model P6042 (d.c. to 50 MHz, 7 nsec risetime). This probe was tested at sweep times down to 20 nsec/division using a high frequency signal generator and was found to display a true current waveform. High currents were measured with the probe and an auxiliary clip-on current transformer (Tektronix Model CT-5, 20 mA div. to 1000 A div., 17.5 nsec risetime). The current probe output was monitored using a Tektronix Model 7904 oscilloscope which was triggered by the +Gate signal from the R7704 (potential viewing oscilloscope).

Extremely smooth gold contact surfaces were prepared by quenching from the melt. No structure could be detected on these surfaces (prior to arcing) using a scanning electron microscope (SEM) at a magnification of 30,000 X. Gold surfaces of controlled roughness were prepared by repetitively arcing opening contacts. Fresh surfaces of various other metals were produced by cutting off the ends of rods. Base metal samples were arced very soon after preparation, and only once

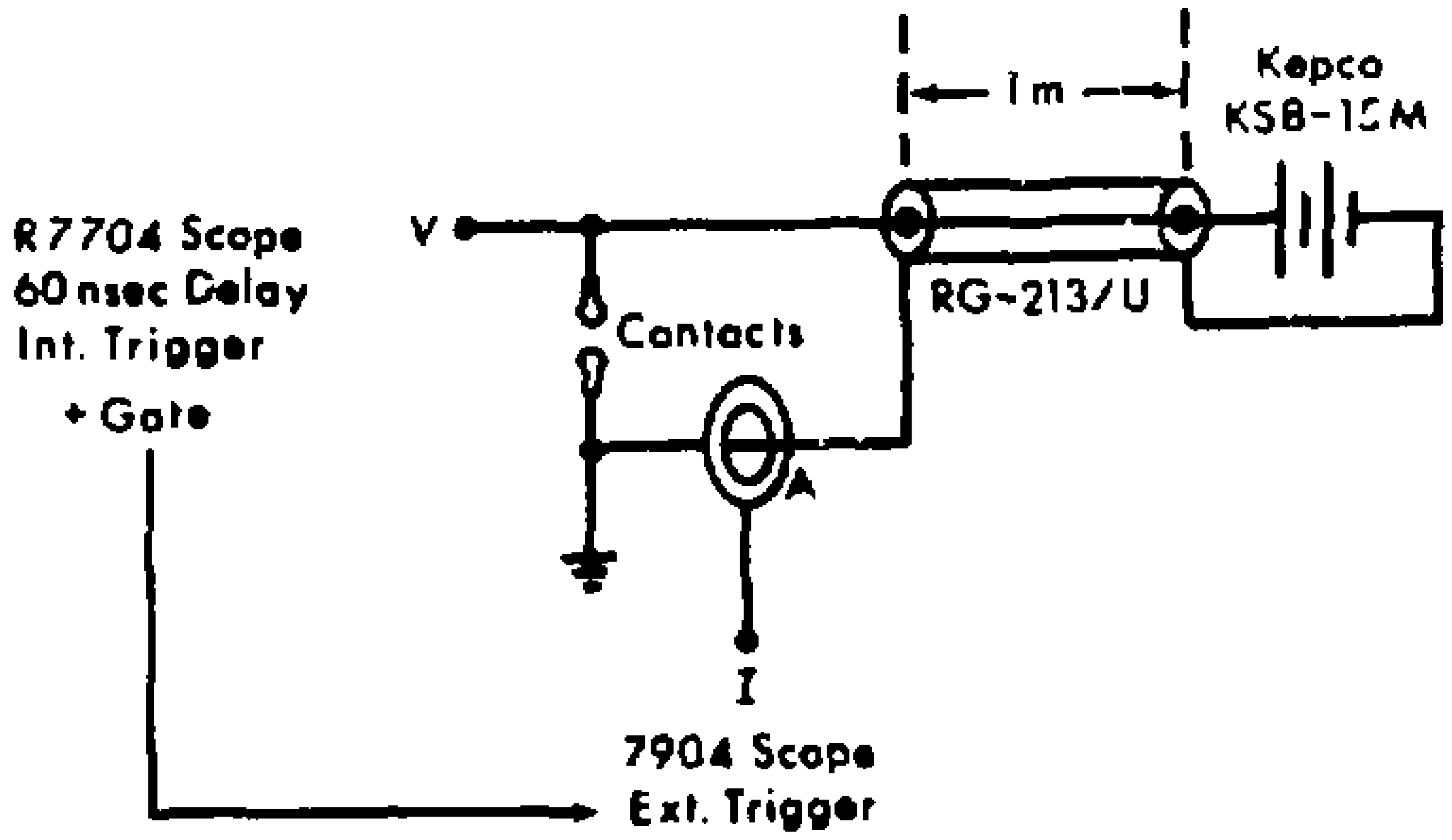

FIGURE 3

Circuit for generating closure arcs and measuring arcing potentials and currents.

each, since oxidation in air rapidly increased their voltage stand-off capability.

RESULTS AND DISCUSSION

Electrical contact testing is often performed at potentials below the "minimum arcing voltage" (11-15 V) in order to avoid the damage produced by arcing between closing contacts. However, it was found in the present investigation that more drastic potential limitations are required to prevent arcing.

Gold contacts were mounted on the cantilever beam assembly shown in Fig. 2 and were closed by bending the beam using either a standard or a piezoelectric micrometer. Figure 4 shows typical oscilloscope traces of the potentials across the contacts during closure. The upper photograph shows a trace which was triggered by an event which was too fast to record at this slow sweep speed and which occurred about 29 msec prior to firm contact closure (indicated by the fall in potential from 6 V open-circuit to below 1 V closed-circuit). There are no indications of contact bounce in this trace, and, as will be shown later, bounce was indeed prevented. When the triggering transitions were viewed using faster sweep speeds, indications of intermediate short duration contacting followed by arcing were found (Fig. 4, center and lower). The initial drop in potential is interpreted as a result of asperity contacting and melting. The subsequent slow potential rise to about 2 V is interpreted as a result of heating of the molten asperity and necking down under electromagnetic forces. The rapid rise to the arcing voltage (about 12 V for gold), which was sustained for several hundred nanoseconds, may have been inductively generated as a result of a momentary decrease in current when the molten asperity bridge was either pinched off by magnetic forces or vaporized as a result of the high current density. Note that either an open-circuit or another short duration closed-circuit may follow the arc. Simultaneous current

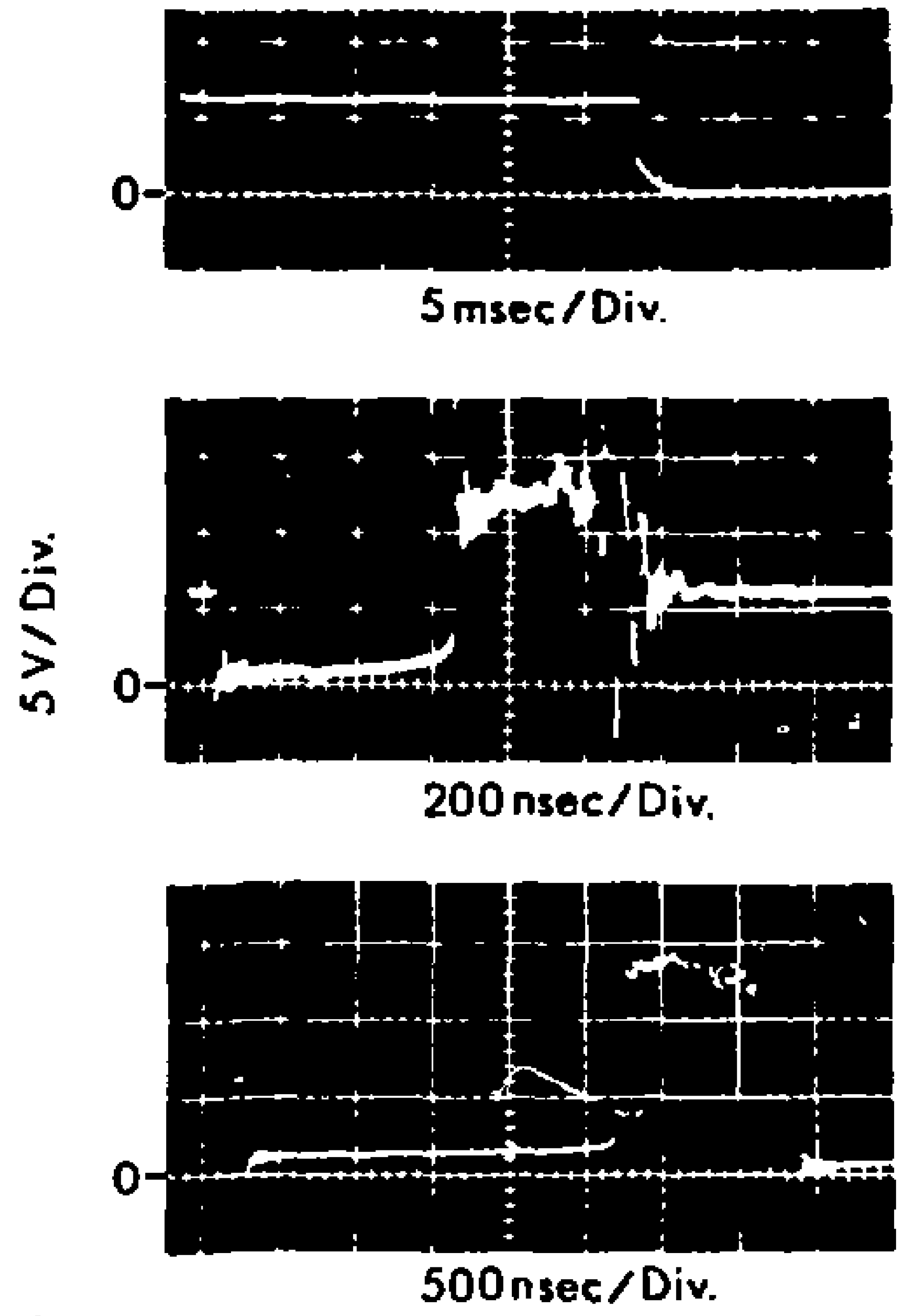

FIGURE 4

Oscilloscope traces of the potential across closing gold contacts at 6 V open-circuit potentials. The upper trace, which was triggered by events such as those shown in the lower two photographs, shows the potential drop at the firm closure of the solid contacts with no indications of contact bounce. The two lower traces show the drop in potential associated with asperity contacting, the slow rise in potential as Joule heating occurs, and the rapid increase to the arcing voltage (11-13 V) which may occur when the asperity contact is melted and pinched off by magnetic forces. Note that the post arc condition may be an open-circuit (center) or another short duration closed-circuit (lower).

and potential traces are shown in Fig. 5. For the eventual firm contacting (left), the current approached 90 A prior to dropping to a 6 A steady state closed-circuit current. Asperity contacts would not be expected to sustain currents of this magnitude, and, as shown in Fig. 5 (center and right), they do not.

Some variation in the asperity contact period, the arc duration, and the time between asperity contacting and eventual firm contact closure is shown in Figs. 4 and 5. Since the contact closure rate which was obtained with the micrometers was approximately constant (~5×10^{-5} m/sec), an estimate of the asperity sizes may be obtained from the product of the closure velocity and the time interval between arcing and eventual firm closure (the times between triggering and potential drop shown in Figs. 4 upper and 5 left). These sizes (0.15 to 1.5 μm) are in agreement with the dimensions of features shown by SEM on the surfaces of repetitively arced gold contacts (Fig. 6). The variations mentioned above may be interpreted in terms of variations in asperity size.

The above mechanism for arc initiation, which involves the contacting, melting, necking down, and pinching off of asperities prior to firm contact closure, is supported by the absence of arcs on any of the first closures of gold contacts which were rendered exceptionally smooth by quenching from the melt. These data also support the conclusion that bounce was prevented in these closures. Arcs were observed consistently for the closure of gold contacts which were purposely roughened by arcing on opening. The conclusion that metallic arcs were indeed observed is supported by Figs. 4 and 5 which show that the discharges were characterized by low potentials (~12 V), high currents (2-6 A), and were sustained over several hundred nanoseconds at the arcing potential. In addition, the discharges were highly luminous (Fig. 7) and of a brilliant blue-white color. Current densities of 10^{12} to 10^{14} A/m^2 (considerably greater than the minimum requirement of 10^{10} A/m^2 [2]) were calculated from the measured currents at arc initiation (Fig. 5)

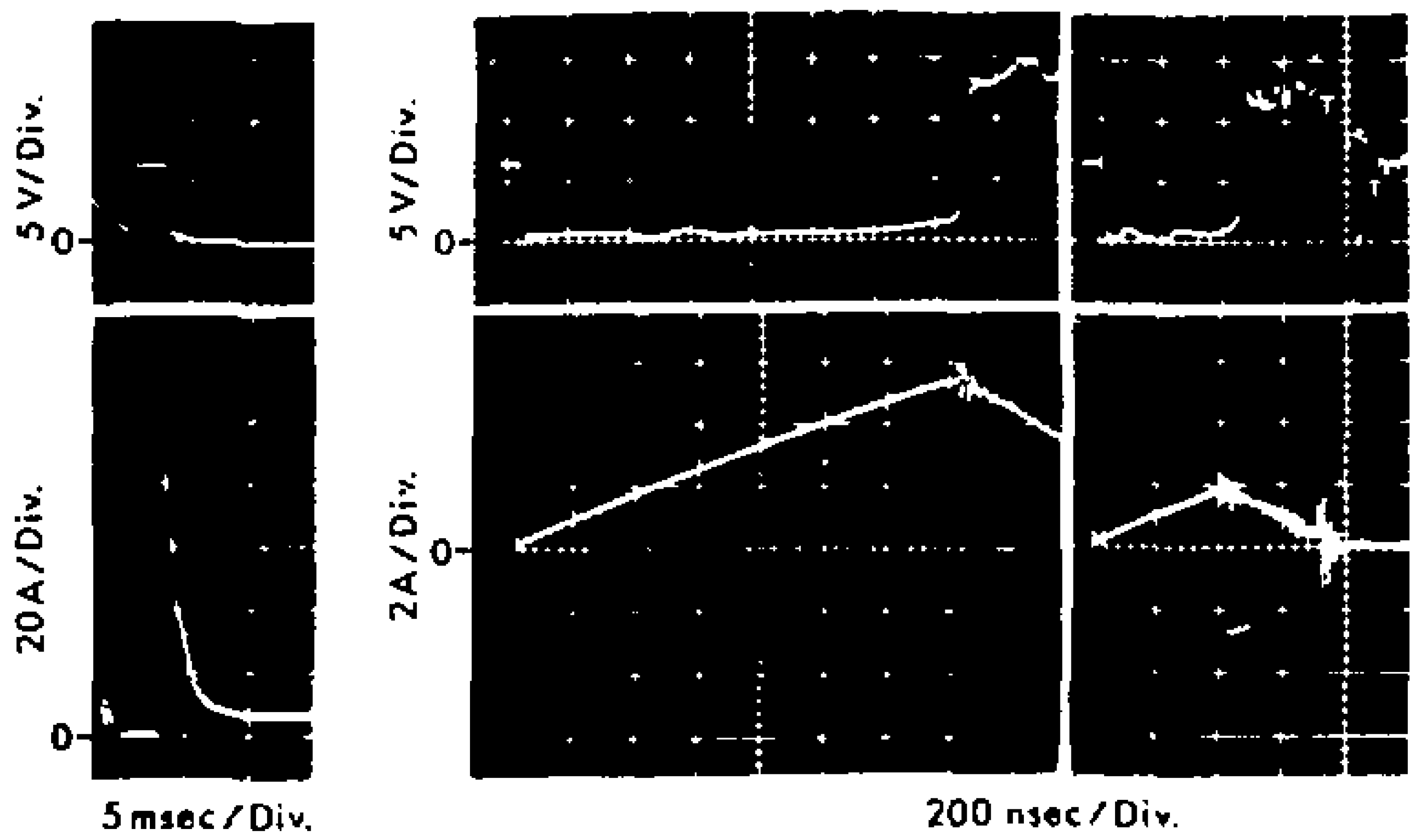

FIGURE 5

Potential and current traces for firm closure (left) and for intermittent asperity contact (center and right). Note that the current approaches 90 A for firm closure while for asperity contacts the current is limited and arcs are formed.

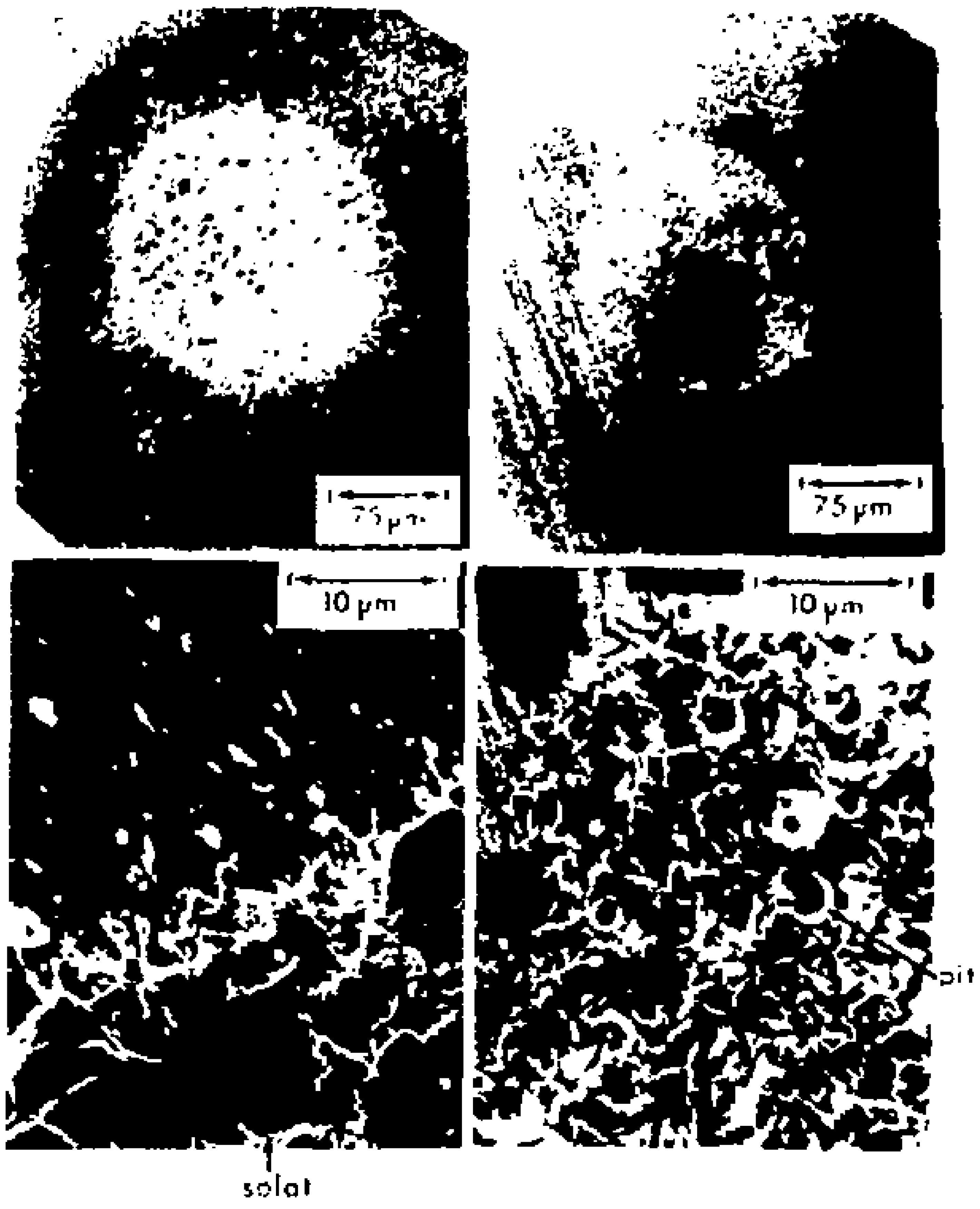

FIGURE 6

Gold splats on a gold anode (left) and pits on a gold cathode (right) produced by multiple arcs initiated at open-circuit potentials of 0.45 V. The damage is essentially equivalent for opening and closing contacts without bounce.

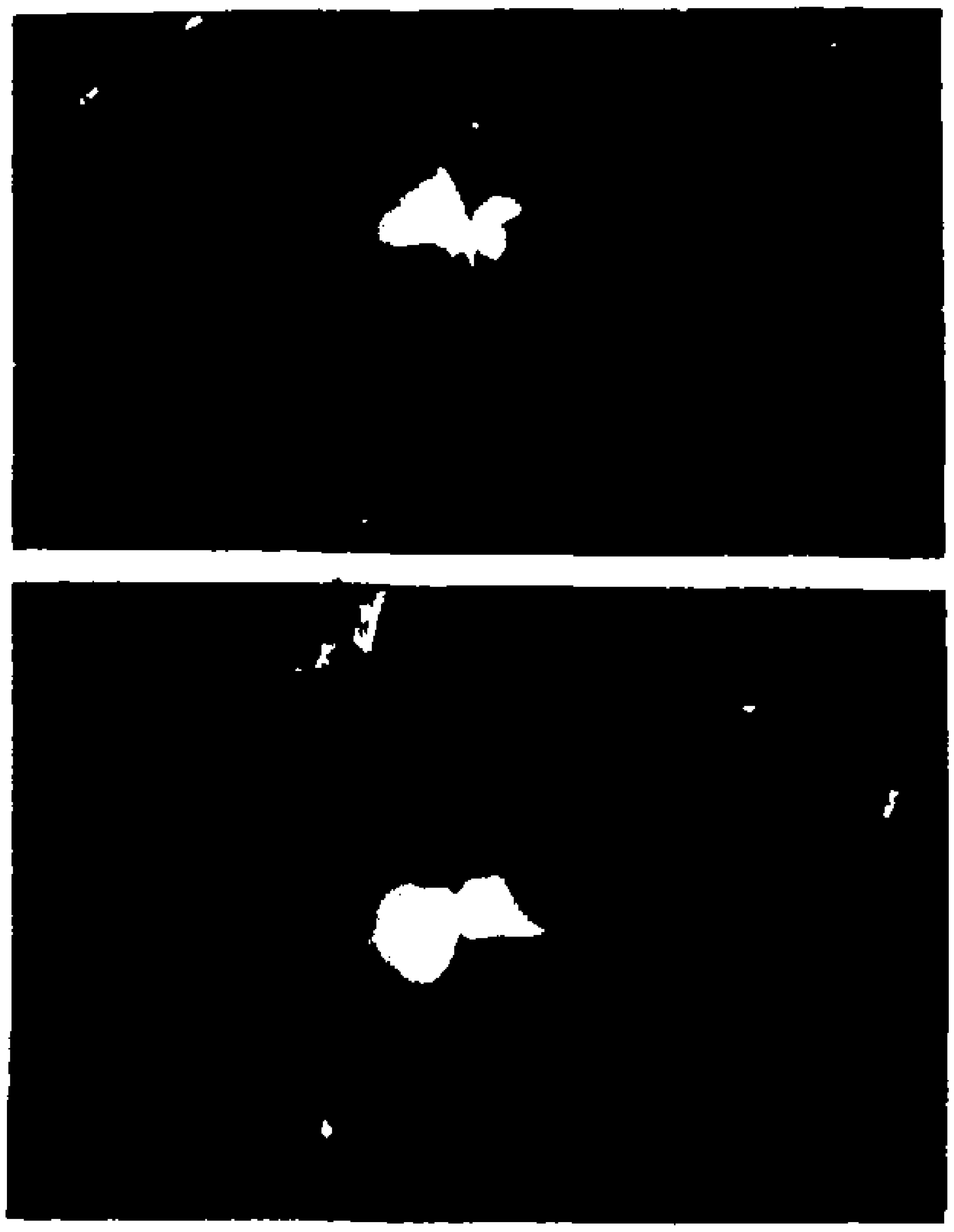

FIGURE 7

Luminous arcs initiated at 0.45 V between gold electrodes (upper) and at 0.46 V between copper electrodes (lower) for contact closures without bounce.

and the estimated asperity dimensions (Fig. 6).

The above mechanism for arc initiation depends on the melting of the asperities. It was found in this investigation that, for a variety of metals, closure arcs could not be produced when the open-circuit potentials were below the melting voltages for the electrode metals. The minimum arc initiation potentials are shown in Table I for these metals along with the softening, melting, and boiling potentials reported by Holm [7].

TABLE I. Softening, Melting, Boiling [7], and Minimum Arc Initiation Voltages for Electrical Contact Materials

Material	Softening Voltage	Melting Voltage	Boiling Voltage	Minimum Arc Initiation Voltage*
Ag	0.09	0.37	0.75	0.40
Al	0.1	0.3		0.40
Au	0.08	0.43	0.9	0.45
Cu	0.12	0.43	0.8	0.40
Fe	0.21	0.[illegible]		0.65
Mo	0.25	0.75	1.1	0.75
Ni	0.22	0.65		0.68
Pt	0.25	0.71	1.3	0.75
W	0.6	1.1	2.1	1.1

*This work.

The data presented in this paper were obtained using a circuit (Fig. 3) which included a 1 m length of RG-213/U coaxial cable and two parallel leads (between the cable and the contacts): each lead was 0.106 m long, of 1.27×10^{-3} m conductor radius, and separated by [illegible].35 $\times 10^{-2}$ m between axes. Static field inductances of 2.24×10^{-7} H for the cable and 1.77×10^{-7} H for the leads, or a total of 4.01×10^{-7} H, were estimated geometrically using expressions shown elsewhere [8]. Similar inductances (within less than one order of magnitude) were calculated using the data shown in Fig. 5 center, where the expression for an L-R circuit was assumed to apply after asperity contacting [9]:

$$V = V_o - L\, di/dt \quad , \qquad (1)$$

where V is the measured potential (1.34 V just prior to arcing, and 11.04 V just after the arc was initiated), V_o is the open circuit or supply potential (6 V), L is the inductance, and di/dt is the slope of the current-time trace (2.91×10^6 A/sec just prior to arcing, and -2.68×10^6 A/sec just after the arc was initiated). Inductances of <u>1.60×10^{-6} H</u> and <u>1.88×10^{-6} H</u> were estimated for the pre-arc and the arc periods, respectively.

In order to estimate when melting of the gold asperities occurred for Figs. 4 and 5, potential traces were obtained for an electrode metal which was melted prior to contacting. The oscilloscope traces for arcs between a liquid gallium cathode and a tungsten anode are shown in Fig. 8. Based on the similarity of the potential traces (Figs. 4, 5 and 8), it is concluded that the gold asperities were melted very soon after contacting, possible during the initial drop from the open-circuit potential. The slight decrease in slope of the current-time trace (Fig. 5 center) over the interval between contacting and arcing is interpreted as a result of an increase in specific resistance caused by Joule heating and as a result of necking down of the melt under magnetic pinch forces.

For frequencies between 10^6 Hz and 10^9 Hz, the surface resistance for the portions of the gold rods between the potential measuring probes is extremely small ($R_s = 2.96 \times 10^{-7} A \sqrt{f}$, where A is the surface area in m^2 and f is the frequency in Hz [10]. The combined rod lengths were 3.3×10^{-2} m, the rod diameters were 1.52×10^{-3} m, and surface resistances of $4.66 \times 10^{-8}\ \Omega$ for 10^6 Hz and $1.47 \times 10^{-6}\ \Omega$ for 10^9 Hz were calculated). Also, the conductive skin depth ($\delta = .075 \sqrt{f}$, [10] is large (75 µm at 10^6 Hz and 2.37 µm at 10^9 Hz) compared to the asperity dimensions estimated by the two earlier methods (0.15 to 1.4 µm). Therefore, the measured potential drop across the asperity contact and the current just after asperity

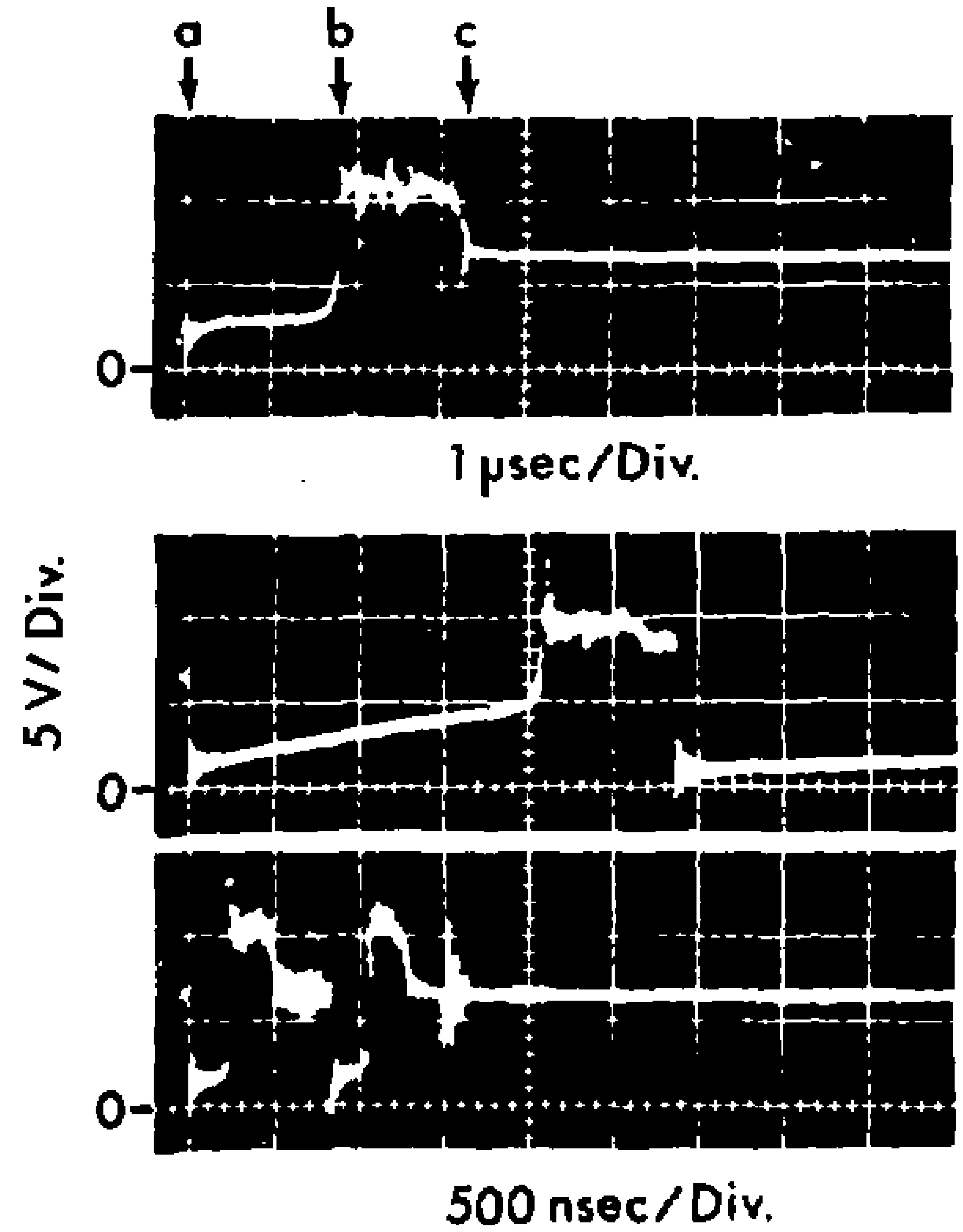

FIGURE 8

Potential traces for the arcing which occurs during the closure of liquid gallium and tungsten contacts. Contacting occurs at a, necking down of the liquid between a and b, and arcing between b and c. Note that the arc may be followed by another short duration contact (center) or by an open-circuit (upper and lower). The lower trace shows two successive arcing sequences.

contacting (0.13 V and 0.75 A) may be used in conjuction with Holm's equation for the constriction resistance [11] to provide a third estimate of the asperity size ($R = \rho/d$, where ρ is the specific resistance and d is the diameter of the conductive spot). This value (0.21 μm) falls within the range of the earlier estimates (.15 to 1.5 μm). For these calculations, the temperature of the liquid asperity was assumed to be near the melting point ($\rho = 3.70 \times 10^{-8}$ Ω-m [12]).

Since the relative motion of the contacts as a result of vibration was eliminated (within an optical detection capability of ~ 160 Å using the interference fringe microscope), other possibilities for premature closure or bounce will now be examined. It has been shown [13] that the electrostatic attraction forces between the hemispherical contacts may be estimated using an expression which was derived using the bispherical coordinate system,

$$F = \frac{\pi}{4} \epsilon_o V^2 \sum_{n=0}^{\infty} G_n \left[(2n+1)\, G_n - (2n+2)\, G_{n+1}\right] \quad , \qquad (2)$$

where, $\epsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$, V is the potential difference, and,

$$G_n = e^{-(n+1/2)\alpha_o}/\text{Sinh}\ (n+1/2)\alpha_o \qquad (3)$$

or,

$$G_n = 2/(e^{(2n+1)\alpha_o} - 1) \quad , \qquad (4)$$

and,

$$d/2R = \cosh \alpha_o \quad , \qquad (5)$$

for spheres of radii R and center-to-center separation d. Equation (2) was tested experimentally by measuring the force between steel spheres as a function of the sphere radii (between 3.175×10^{-3} m and 1.095×10^{-2} m), the potential difference (between 100 V and 2000 V), and the surface-to-surface separation (between 25 µm and 400 µm). Figure 9 shows that the correlation between the measured and the calculated values is very nearly one-to-one (the solid line).

Using Eq. (2), the electrostatic force of attraction between the hemispherical gold surfaces was calculated for a radius of curvature of 1.59×10^{-3} m and a potential difference of 6 V. The electrostatic forces were found to be too small by about 7 orders of magnitude (at 1 µm separation) or by about 3 orders of magnitude (at 100 Å separation) to bend the cantilever bar (spring loaded to the open position) and cause premature closure or bounce. The electrostatic forces were found to be too small by about 10 orders of magnitude to elastically elongate the gold rods and cause closure over a 1 µm separation. Electrostatic forces are also too small to cause closure in several other less probable situations such as (1) the concentration of the total force to elongate a single asperity, (2) the elongation of two opposed asperities, and (3) the elongation of a liquid metal asperity, opposed by surface tension, in the event melting occurs prior to contacting.

When Holm's expression [15] for the magnetic force component opposing firm closure, $F = 10^{-7} i^2 \ln (B/a)$, (where i is the current, B is the radius of the gold rods, and a is the radius of the asperity contact) was used to calculate the force which may elastically compress the gold rods (after asperity contact occurs), it was found that, in the contact opening directions, a metallic compression of only 10^{-3} Å would be produced and opening of the contacts under these forces is not probable. However, the magnetic pinch forces which arise as a result of the current through the asperity may be comparable to the surface tension and may pinch off the asperity after melting occurs. The magnetic flux density, $B \approx \mu_0 i/2\pi r$ [16],

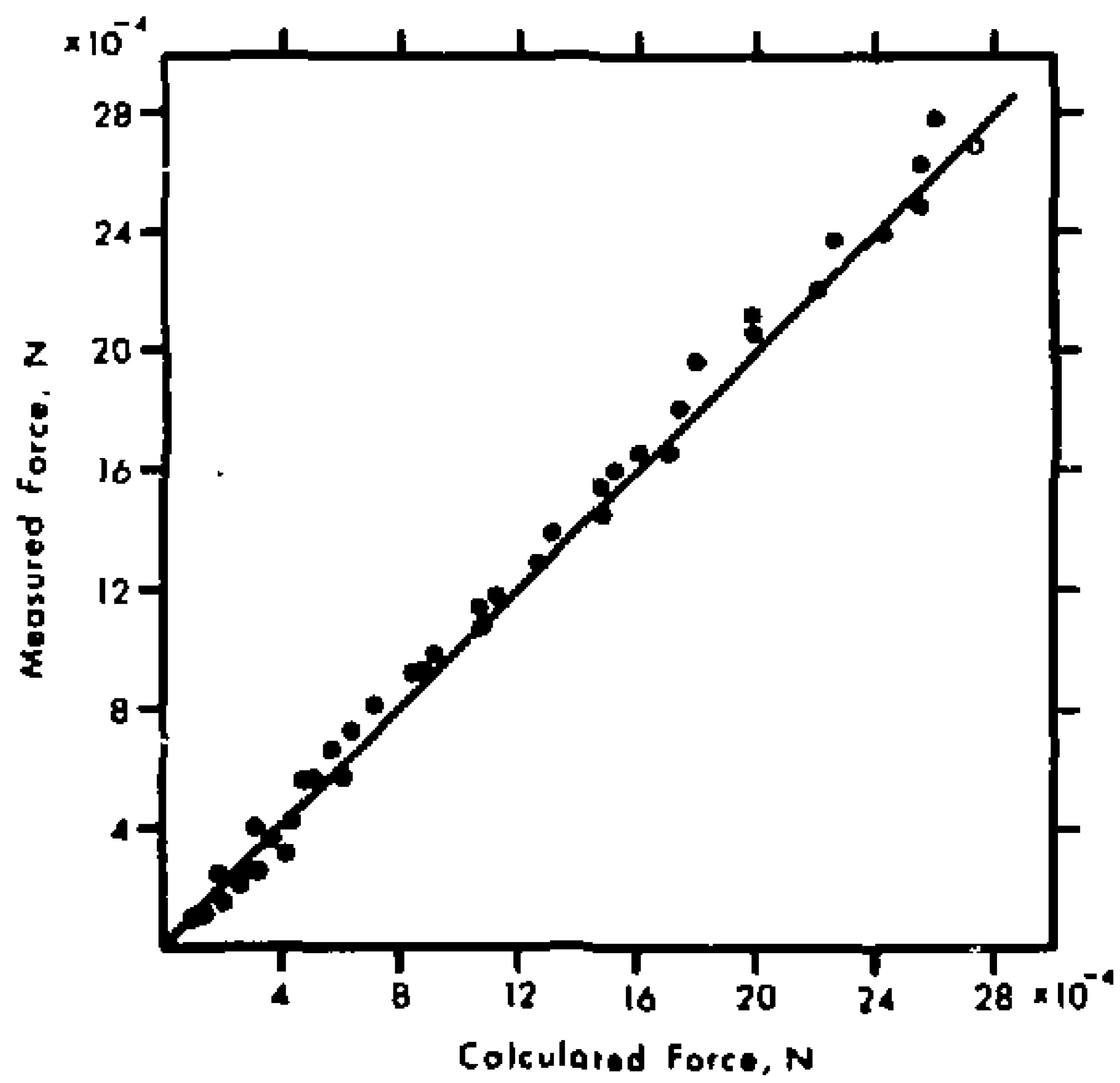

FIGURE 9

The agreement of measured with calculated electrostatic forces between oppositely charged steel spheres. The solid curve represents a one-to-one correlation.

and the pinch pressure, $P = B^2/2\mu_o$ [17], may be estimated from the current i and the asperity radius r, where $\mu_o = 1.257 \times 10^{-6}$ H/m. By combining these equations and multiplying the result by the asperity surface area, $A = 2\pi rl$, where the asperity length l is assumed to be about 2r, an expression for the pinch force was obtained.

$$F_p \approx \mu_o i^2/2\pi \quad . \tag{6}$$

The surface tension force opposing the pinch may be of the order of $2\pi r\gamma$. For a current of 5.34 A, an asperity diameter of 0.53 μm, and a surface tension (γ) for liquid gold of about 1000 dyn/cm [18], the pinch force is about 5.7×10^{-6} N and the opposing surface tension force is about 1.7×10^{-6} N. Therefore, magnetic pinch forces may destroy the molten asperity bridge and the arc may be initiated inductively as a result of the associated momentary decrease in the current. A fourth estimate of the asperity dimensions may be obtained by equating the expressions for the magnetic pinch force and the surface tension force. This estimate (0.91 μm) is within the range of the earlier estimates (0.15 to 1.5 μm).

CONCLUSIONS

Low voltage, high current, sustained, luminous arcs were found to precede firm contacting on the closure of electrical contacts without bounce. Simultaneous oscilloscope traces of the potential and the current during the closures were interpreted in terms of asperity contacting, melting, magnetic pinch-off of the molten metal bridge, and the inductive generation of the "minimum arcing voltage." The melting voltages of the contact metals were found to be the lower open-circuit limits for the initiation of this process which apparently occurs only for rough contact surfaces. Arcs were not observed on the closure of exceptionally smooth gold contacts. The arc damage to the closing contact surfaces is similar to that observed on the arcing of opening contacts.

This work indicates (1) that test potentials should be limited to below the melting voltages (a 50 mV limit is suggested), (2) that currents should also be limited (a 50 mA limit is suggested), (3) that cables between contacts and test equipment or power supplies should be of high quality, coaxial, low inductance, and properly terminated to prevent inductively generated high voltage transients, and (4) that contact surfaces should be as smooth as possible. In addition, the results indicate that electrode tips and work surfaces which are used in arc welding should be roughened to about a 40 microinch finish (or rougher) in order to facilitate arc initiation.

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