COARSE-MESH REBALANCING ACCELERATION FOR EIGENVALUE PROBLEMS

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ABSTRACT

The coarse-mesh rebalance method is adopted for Monte Carlo schemes for aiming at accelerating the convergence of a source iteration process. At every completion of the Monte Carlo game for one batch of neutron histories, the scaling factor for the neutron flux is calculated to achieve the neutron balance in each coarse-mesh zone into which the total system is divided. This rebalance factor is multiplied to the weight of each fission source neutron in the coarse-mesh zone for playing the next Monte Carlo game.

The numerical examples have shown that the coarse-mesh rebalance Monte Carlo calculation gives a good estimate of the eigenvalue already after several batches with a negligible extra computer time compared to the standard Monte Carlo.

INTRODUCTION

The Monte Carlo method has recently become more and more to play an important role in a wide field of theoretical studies of reactor physics because of the development of the computer softwares and hardwares. In addition, it gives a benchmark to check other mathematical models as well as the physics data attached to them. This tool is, however, not yet handled as skilfully as one might do, mainly due to a large calculation time required for obtaining the result within a reasonably small statistical error. To improve this situation, an effort will be required for developing further new biasing methods and their theoretical foundations, as well as practical methods for obtaining usable sampling functions [1].

In eigenvalue problems, also the acceleration of the convergence rate of iterative processes is essential for enlarging the range of applicability

of the method. This situation is the same as for deterministic methods for which the outer or source iterations are required for solving problems. An acceleration algorithm of iteration prccesses having been developed for deterministic methods can therefore be applied in principle also to Monte Carlo calculations with iterative batch processes.

CONVERGENCE ACCELERATION ALGORITHMS

Many acceleration techniques have been suggested for use with iterative deterministic methods for solving the steady state transport equation. The Chebychev acceleration is used most often in one-dimensional problems and is usually quite effective. In two-dimensional geometries, however, the technique is less satisfactory by reasons which are not throughly understood. The most effective acceleration method is generally the coarse-mesh rebalance method though there are problems for which the use of this technique leads to an unstable algorithm. The synthetic method as an acceleration technique has similar characteristics and these two methods are in fact equivalent, provided that the balance equation is used as a low-order equation for the synthetic algorithm [2].

For applying the acceleration algorithm of the outer iterations to the Monte Carlo method, consideration should be taken not to consume an extra time for executing the algorithm in computer codes. This suggests that the coarse-mesh acceleration technique is most suited to the Monte Carlo calculation, if the homogeneous regions of the reactor system are chosen as coarse-mesh zones.

The source iteration method can be written in the form:

$$L\varphi_{n+1} = S\varphi_n , \tag{1}$$

where φ_n stands for the angular flux at the n-th outer iteration. The low-order equation to it is a balance equation written as

$$\tilde{L}_n \tilde{\varphi}_{n+1} = \tilde{S}_n \tilde{\varphi}_n . \tag{2}$$

For practical applications, the source operator S can be assumed to be isotropic and hence $S\varphi_n$ depends only on the scalar flux ϕ_n. The geometrical system is divided into the coarse-mesh zones and the transport equation is integrated over all directions and over a coarse-mesh zone. The resulting equations, one for each coarse-mesh zone, involve the scalar fluxes within the coarse-mesh zone and the currents across the coarse-mesh boundaries:

$$I(r) = \int d\underline{\Omega}\, (\underline{\Omega}\cdot\underline{n})\, \varphi(r, \underline{\Omega}). \tag{3}$$

Therefore, a low-order approximation to the equation for the scalar flux ϕ is sought instead of Eq.(1) for φ. Upon assuming that an approximation φ_n (and therefore ϕ_n) is available and the flux shape within a coarse-mesh zone is that of ϕ_n, then the additional assumption that the current obtained from a relationship of $\tilde{\phi}_n$ must match $I(r)$ from $\tilde{\varphi}$ leads to a set of equations

for the flux amplitude or a set of balance equations.

The iteration proceeds as follows. Given Φ_n (therefore $S\Psi_n$), $\tilde{\Psi}_n$ is then obtained from

$$L\tilde{\Psi}_n = S\Psi_n . \tag{4}$$

The currents $I(r)$ are computed from appropriate integrals of $\tilde{\Psi}_n$. A set of balance equations are formulated for these fluxes and currents, and the scalar flux Φ_{n+1} is obtained by solving these equations.

APPLICATION OF COARSE-MESH REBALANCE METHOD

Since in Monte Carlo games neutrons crossing the boundaries between different media are sampled for the determination of their flight distances, the homogeneous regions can conveniently be chosen as the coarse-mesh zones. The balance equation for obtaining the scale factor f_ℓ, by which all fluxes in the region ℓ are multiplied to insure the neutron balance, may then be written as

$$f_\ell \left[\sum_{\ell'} (CUR)_{\ell\to\ell'} + (AB)_\ell + (SD)_\ell \right] = (QQ)_\ell + \sum_k f_k (CUR)_{k\to\ell} , \tag{5}$$

where, upon denoting the weight of the h-th history neutron at the j-th collision point by W_{hj},

$(CUR)_{\ell\to\ell'} = \sum_{h=1}^{H} \sum_j W_{hj}^{\ell\to\ell'}$; the total neutron current across the boundary from the region ℓ to ℓ',

$(AB)_\ell = \sum_h \sum_j (\Sigma_a/\Sigma_t)_\ell W_{hj}$; the total number of neutrons absorbed in the region ℓ,

$(SD)_\ell = \sum_h W_{hJ}^{SD}$; the total number of neutrons slowed down below the cut-off energy in the region ℓ (J stands for the last collision number for the h-th history neutron),

$(QQ)_\ell$ = the total number of fission source neutrons for the region ℓ, which is normalized as

$\sum_\ell (QQ)_\ell$ = (the total number of histories for the present batch, H) × (initial weight of each source neutron, W_0) = 1.

The source term can be rewritten as

$$(QQ)_\ell = f_\ell (FS)_\ell / k_{eff} , \tag{6}$$

where $(FS)_\ell = \sum_k \sum_j (\nu\Sigma_f/\Sigma_t)_\ell W_{kj}$; the total number of fission neutrons produced in the region ℓ.

In this case, a source iteration is required for determining the rebalance factors and it gives a better estimate of the eigenvalue k_{eff}.

Equation (5) is solved by using a forward elimination and backward substitution [3]. In one-dimensional geometries, for example, by assuming

$$\sum_{\ell'} (CUR)_{\ell\to\ell'} = (FL)_\ell + (FR)_{\ell+1},$$
$$\sum_k (CUR)_{k\to\ell} = (FR)_\ell + (FL)_{\ell+1}, \qquad (7)$$

where $(FL)_\ell$ is the total neutron current crossing leftwards the left boundary of the region ℓ from the ℓ to $(\ell-1)$-th region, and $(FR)_{\ell+1}$ is the current crossing rightwards the right boundary of ℓ from ℓ to $\ell+1$, the set of Eq.(5) can be rewritten as

$$\begin{bmatrix} a_1 & -b_1 & & & 0 \\ -c_2 & a_2 & -b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & & -b_{L-1} & \\ 0 & & & -c_L & a_L \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_L \end{bmatrix} = \begin{bmatrix} QQ_1 \\ QQ_2 \\ \vdots \\ QQ_L \end{bmatrix}, \qquad (8)$$

where

$$a_\ell = (FL)_\ell + (FR)_{\ell+1} + (AB)_\ell + (SD)_\ell,$$
$$b_\ell = (FL)_{\ell+1}, \qquad c_\ell = (FR)_\ell.$$

The forward elimination is therefore performed as (with $H_0 = G_0 = 0$)

$$T_\ell = a_\ell - c_\ell H_{\ell-1}, \quad G_\ell = (QQ_\ell + c_\ell G_{\ell-1})/T_\ell,$$
$$H_\ell = b_\ell/T_\ell, \quad \ell = 1, 2, \cdots, L, \qquad (9)$$

and then the backward substitution is carried out as follows:

$$f_L = G_L,$$
$$f_\ell = G_\ell + H_\ell f_{\ell+1}, \quad \ell = L-1, \cdots, 2, 1. \qquad (10)$$

If the source iteration for Eq. (5) with (6) fails to converge, if any factor f_ℓ becomes negative; or, if any denominator T_ℓ becomes zero, all f_ℓ's are set equal to the single-system factor f given by

$$f = \sum_\ell (QQ)_\ell \Big/ \left[TCR + \sum_\ell (AB)_\ell + (SD)_\ell \right], \qquad (11)$$

where TCR stands for the total neutron flow leaking out of the whole system.

The rebalancing based on Eq. (5) with (6) will be required only for the first few outer iterations (batches) of the Monte Carlo game and thereafter Eq. (5) with the fixed source $(QQ)_\ell = (FS)_\ell / k_{eff}$ will be enough for obtaining the rebalance factors. As the number of batches increases, the factors all usually approach unity and the deviations from unity will become much smaller than the standard deviations of the quantities of Eq. (5), for example, the standard deviation of k_{eff} as a typical representative of the quantities:

$$(\overline{k_{eff}^2} - \overline{k_{eff}}^2)^{1/2} / \sqrt{N} \gg \max_\ell |1 - f_\ell|$$

where N stands for the total number of batches. In this case, Eq. (11) will also be applied for achieving the whole system rebalance or the rebalance technique will not be necessary any more.

NUMERICAL EXAMPLES AND DISCUSSIONS

The present coarse-mesh rebalance technique has been incorporated in the MORSE computer code [4] by attaching two new subroutines. One is the COARSE subroutine for evaluating the quantities of Eq. (5) and the other is the REBAL for computing the rebalance factors according to Eqs. (9) and (10), or (11) if required. After obtained the converged values of f_ℓ's as a result of i iterations in the REBAL, the weight of each fission neutron produced in the region ℓ, stored in the fission bank in the MORSE code, is multiplied by $\prod f_\ell^{(k)} \cdot k_{eff}^{(0)} / k_{eff}^{(1)}$, where $k_{eff}^{(0)}$ and $k_{eff}^{(1)}$ are respectively the k_{eff} values obtained in the MORSE prior to the rebalance calculation and in the REBAL after the i iterations.

In order to test the effectiveness of the coarse-mesh rebalance acceleration applied to Monte Carlo eigenvalue calculations, the k_{eff} of a homogeneous spherical fast reactor of 55 cm radius was calculated with a 5-energy-group model by dividing the system into two coarse-mesh zones at a radius of 35 cm.

The results for the average values of k_{eff} and their standard deviations are shown in Fig. 1 as a function of the total number of neutron histories, where the first batch is excluded from calculating the averages. As seen from Fig. 1, after several batches the values of $k_{eff}^{(1)}$ obtained from the coarse-mesh rebalance Monte Carlo (white circles in Fig. 1) are already very close to the S_{12} value calculated by the ANISN computer code [5], while the standard Monte Carlo by using the original MORSE (black circles) gives still a slightly too low value after about 4,000 histories. It should be noted that the coarse-mesh rebalance Monte Carlo requires only a negligible extra computer time (only 2 % more per collision) compared to the standard

calculation.

In Table I are shown the final results for the total number of collisions suffered in each coarse-mesh zone and the number of fission neutrons produced in each zone. The Monte Carlo results obtained from about 4,000 histories agree well with the S_{i2} values.

As another example, the calculations were performed on a two-region spherical fast reactor composed of the core of 35 cm radius and the blanket of 55 cm outer radius. The coarse-mesh zones are chosen as the same as these two regions.

It is seen from Fig. 2 that in this case the coarse-mesh rebalance version is much better than the standard version, though neither of two calculations did not converge yet to the S_{i2} value. The difference between the Monte Carlo results for k_{eff} and the S_{i2} value comes mainly from the underestimation of the fourth group contribution in the Monte Carlo calculations (see Table II). This fact indicates that it is desirable to sample more neutron histories per batch than used here. It may be worthwhile to mention here again that the coarse-mesh rebalance Monte Carlo calculation takes more computer time by only 2 % per collision than that of the standard calculation.

CONCLUSIONS

It has been shown in this article that the coarse-mesh rebalance method can successfully be applied to Monte Carlo calculations for accelerating the convergence of a source iterative process. Even for the present numerical examples on spherical systems with only two coarse-mesh zones, the eigenvalue obtained from the coarse-mesh rebalance Monte Carlo calculation approaches quickly to the correct value. The computer time required for the extra rebalance calculation is negligibly small compared to that for the standard Monte Carlo calculation.

It is therefore expected that the coarse-mesh rebalance technique is certainly more effective for complex systems composed of different media. In addition, the technique will be useful also for estimating the neutron flux at a point detector position.

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TABLE I

The Total Number of Collisions and the Number of Fission Neutrons Produced in Each Coarse-Mesh Zone of a Homogeneous Spherical Fast Reactor with a Unit Neutron Source

Calculation Method	Energy-Group Index	$\int dV\ \Sigma_t\phi$		$\int dV\ \nu\Sigma_f\phi$	
		First Zone	Second Zone	First Zone	Second Zone
Standard Monte Carlo	1	0.043	0.049	0.0073	0.0083
	2	1.234	1.210	0.1148	0.1128
	3	5.256	5.111	0.1938	0.1880
	4	6.075	5.738	0.2199	0.2068
	5	0.733	0.574	0.0309	0.0242
Coarse-Mesh Rebalance Monte Carlo	1	0.046	0.050	0.0079	0.0086
	2	1.284	1.251	0.1193	0.1160
	3	5.298	5.250	0.1957	0.1938
	4	5.871	0.537	0.2114	0.1995
	5	0.759	0.611	0.0320	0.0257
S_{12}	1	0.049	0.052	0.0085	0.0089
	2	1.181	1.230	0.1099	0.1144
	3	5.172	5.243	0.1905	0.1931
	4	5.867	5.723	0.2120	0.2068
	5	0.710	0.679	0.0299	0.0286

TABLE II

The Total Number of Collisions and the Number of Fission Neutrons Produced in Each Region of a Two-Region Spherical Fast Reactor with a Unit Neutron Source

Calculation Method	Energy-Group Index	$\int dV\ \Sigma_t\phi$		$\int dV\ \nu\Sigma_f\phi$	
		Core	Blanket	Core	Blanket
Standard Monte Carlo	1	0.057	0.035	0.0097	0.0111
	2	1.634	0.759	0.1510	0.0929
	3	7.276	7.976	0.2677	0.0188
	4	8.100	10.104	0.2924	0.0240
	5	1.015	0.090	0.0427	0.0004
Coarse-Mesh Rebalance Monte Carlo	1	0.060	0.032	0.0103	0.0103
	2	1.666	0.696	0.1553	0.0851
	3	7.675	7.940	0.2827	0.0187
	4	8.432	10.048	0.3055	0.0239
	5	0.996	0.083	0.0419	0.0004
S_{12}	1	0.078	0.032	0.0133	0.0103
	2	1.778	0.610	0.1653	0.0747
	3	8.021	7.303	0.2955	0.0172
	4	9.032	9.728	0.3264	0.0231
	5	1.003	0.110	0.0422	0.0005

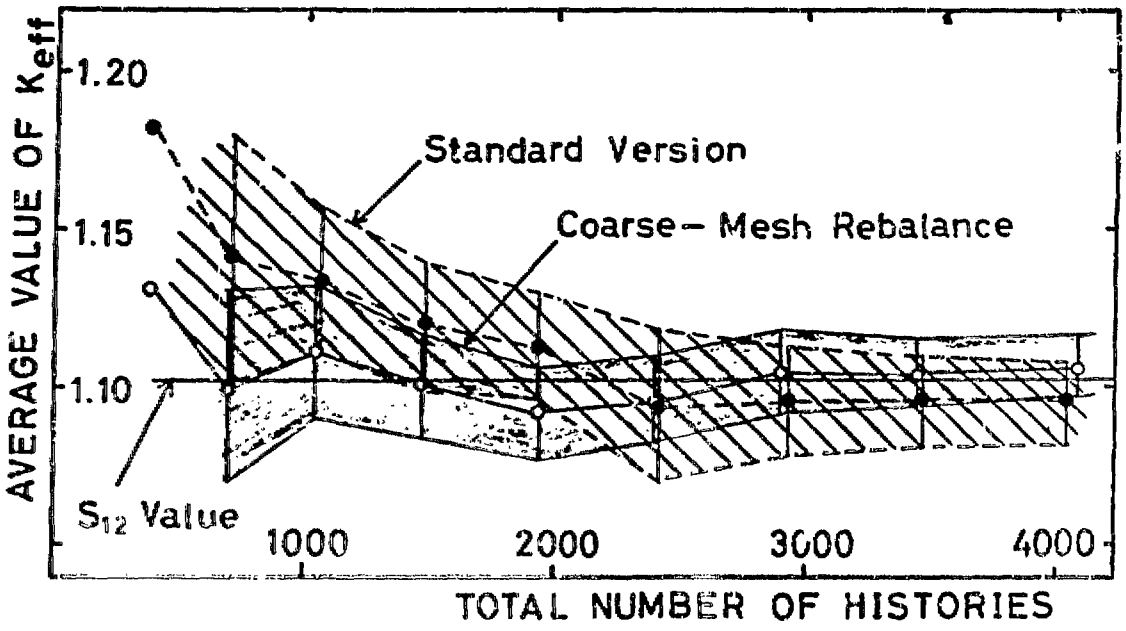

Fig. 1. The average value of k_{eff} and its standard deviation of a homogeneous spherical fast reactor.

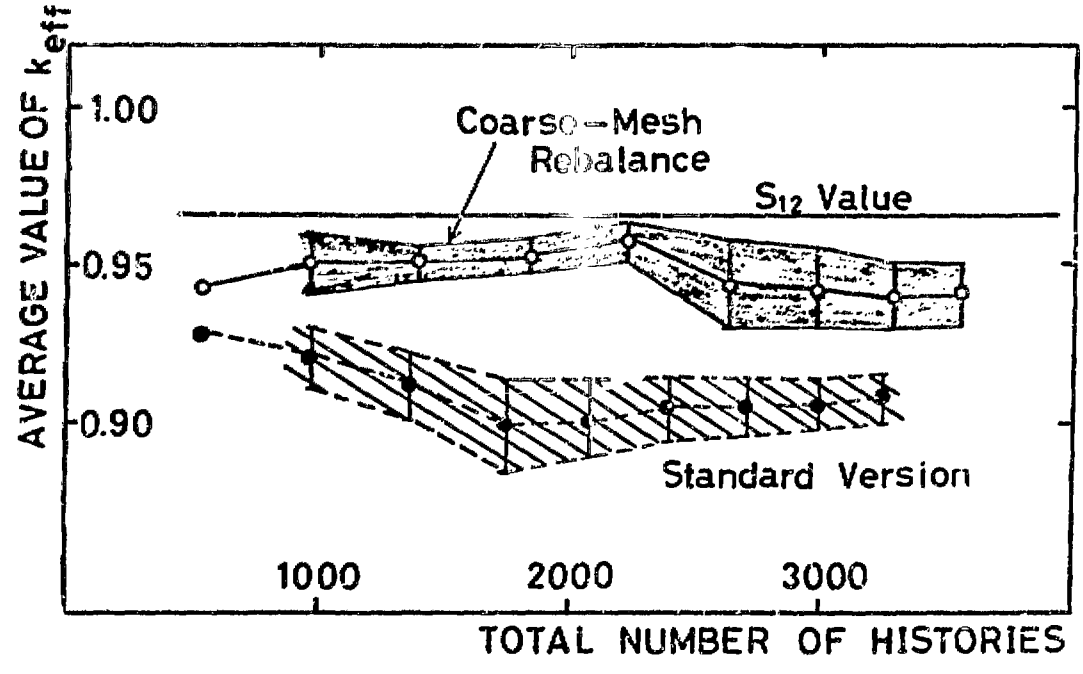

Fig. 2. The average value of k_{eff} and its standard deviation of a two-region spherical fast reactor.