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LOCAL ALGEBRAS IN EUCLIDEAN QUANTUM FIELD THEORY *

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I. INTRODUCTION

Euclidean quantum field theory is based on the idea of replacing the usual description of quantum field theory in Minkowski space by a description in Euclidean space. This proposal was made in a very lucid way by Schwinger some years ago [42,44] and was considered a "possible avenue for the future development of quantum field theory" [1]. The correspondence between the two formulations can be obtained considering the vacuum expectation values of products of quantum fields and making the analytic continuation from the points of physical space-time to the points of complex space-time where the spatial coordinates are real and the time coordinate is purely imaginary (Schwinger points). The possibility of this analytic continuation is based on firm physical grounds since the work of Wightman and followers [51,27] on the general structure of quantum field theory.

The resulting Euclidean theory is described in terms of the so-called Schwinger functions, which are essentially the Wightman functions at the Schwinger points. The basic covariance group of the theory is given by the inhomogeneous Euclidean group. The obvious structural simplifications obtained in this way are related to the fact that the indefinite metric of the Lorentz group is replaced by the positive definite metric of the orthogonal group and the difficult hyperbolic problem is replaced by a more manageable elliptic problem.

Further decisive progress was made by Symanzik [53,54] who stressed the fact that the Euclidean theory can be described through a field structure (Euclidean fields) with peculiar dynamical properties. In the Boson case the Euclidean fields are commutative random variables and their dynamical structure has strong analogy with classical statistical mechanics, as was shown by Symanzik [54] in some particular cases.

The recent work of Nelson [32, 33, 34] has put Euclidean quantum field theory on a solid physical and mathematical basis. Nelson [illegible] the crucial Markov property as peculiar locality property of Euclidean fields and consequently established a rigorous Euclidean covariant Feynman-Kac path integral formula. Moreover he solved the main problem left open by Schwinger and Symanzik, i.e. the possibility to reconstruct a Wightman field theory starting from Euclidean fields.

As a result it has been possible to use the methods of Euclidean field theory for the advancement of the constructivo quantum field theory program of Glimm and Jaffe and their followers [13,14,15, 20, 25,48,55,56] . One of the first new results using Nelson Euclidean covariant scheme was the proof of the convergence of the energy density in the infinite volume limit for $P(\varphi)_2$ [17,19].

All recent contributions to constructive quantum field theory exploit, in one way or another, ideas and methods of the Euclidean-Markov theory. In particular the analogy with classical statistical mechanics [26,54] has proven to be very useful. It is the core of a general program advocated by Guerra, Rosen and Simon [20] aiming at the exploitation of the rigorous methods of modern statistical mechanics [43] for the study of constructive quantum field theory. In particular correlation inequalities have been proved for the Schwinger functions [20], and consequently their monotone convergence in the infinite volume limit has been established in some cases [35,20]. It has been also possible to prove the Lee-Yang theorem in Euclidean field theory [50] and to derive important consequences about the infinite volume limit [47,49]. We refer to [20] and [48] for a general account of methods and results of the statistical mechanics program and to [18, 21, 22,45,49] for more recent progress. See also [8,14,25,28,36].

In its structure Euclidean quantum field theory can be seen as a spectacular synthesis of many different ideas and methods, [illegible] Feynman path integral [illegible] statistical mechanics [illegible] random processes [11] and stochastic mechanics [31 [illegible]].

In the last few year it has been proven to be very useful in constructive quantum field theory [55], on the other hand the availability of reconstruction theorems (how to go from the Euclidean to the Minkowski formulation), based on the properties of Euclidean fields [33] or of the Euclidean-Schwinger functions [38,48], makes clear that the Euclidean methods can have large applications in many problems of relativistic quantum mechanics. Moreover the general structure of Euclidean quantum field theory is very rich and poses interesting problems in its own sake, which could be solved using general methods of structural theory, like those advocated by Haag and Kastler [24] and Doplicher, Haag and Roberts [5]

This talk is mainly expository, we will try to describe the general structure of the local observable algebras of Euclidean quantum field theory, considering the very simple examples of the free scalar field [34], the vector meson field [16,3] and the electromagnetic field [3]. In particular we will stress the role of Markov property and the relations between the Euclidean theory and the Hamiltonian theory in Minkowski space-time. The considerations on the vector meson field and the electromagnetic field are based on joint work with S. De Siena and P. Ruggiero [3].
The Euclidean formulation of half-integer spin fields introduces interesting problems. For a recent account of the many possibilities we refer to the very authoritative paper by Fröhlich and Osterwalder [9]. An analysis of the problem, with emphasis on the Euclidean-Markov field structure, is the subject of a forthcoming paper by De Falco and the author [2]. The main idea of [2] is to consider the Euclidean-Fermion fields as generators of an infinite dimensional Grassmann algebra, on which the free and interacting Schwinger functions define a linear functional. It is then possible to define a Markov property very similar to the Boson case, and to embed the Hamiltonian theory in the Euclidan

theory, provided proper care is taken toward the lack of reflexivity of the theory.

Finally we refer to [45,40,30] for the construction of the finite volume renormalized Euclidean Schwinger function of the interacting Yukawa theory in two dimensional space-time.
For the recent spectacular results of Feldman-Osterwalder and Magnen-Seneor on the infinite volume limit of the weakly coupled φ^4 in three dimensions, see [6] and [29].

2. LOCAL ALGEBRAS OF THE FREE EUCLIDEAN-MARKOV FIELD

We review here the local structure of the free Euclidean-Markov scalar field. For more information we refer to the original Nelson paper [34] and to [20].

We work in the four-dimensional Euclidean space-time $\mathbb{R}^4$, and define the Euclidean-Markov scalar field φ as the real Gaussian random process indexed by $\mathcal{D}(\mathbb{R}^4)$ with mean zero and covariance given by

$$E\left(\varphi(f)\varphi(g)\right) = \langle f, (-\Delta + m^2)^{-1} g \rangle ,$$

where $f, g \in \mathcal{D}(\mathbb{R}^4)$ (i.e. infinitely differentiable functions on $\mathbb{R}^4$ with compact support), Δ is the Laplacian in four dimensions, m^2 is a constant, $m^2 \neq 0$, and $\langle \ , \ \rangle$ is the scalar product on $L^2(dx)$, with dx the Lebesgue measure on $\mathbb{R}^4$.

It is clear that $\varphi(f)$ can be extended to smearing functions f belonging to the real Sobolev-Hilbert space N, with scalar product $\langle f, g \rangle_N = \langle f, (-\Delta + m^2)^{-1} g \rangle$.

We call (Q, Σ, μ) the underlying probability space, then the fields $\varphi(f)$ will be represented by $L^p(Q, \Sigma, \mu)$ functions, $1 \leq p < \infty$, and the expectations E will be represented as integrals on Q space

$$E\left(\varphi(f_1) \ldots \varphi(f_n)\right) = \int_Q \varphi(f_1) \ldots \varphi(f_n)\, d\mu .$$

There are many possible concrete realizations of Q space, see [48] for a complete discussion. One possible way is to identify Q with the space of distributions $\mathcal{D}'(\mathbb{R}^4)$, then the field is represented by the coordinate function $\cdot$ $\mathcal{D}'(\mathbb{R}^4)$

$$\varphi(f) = \langle \varphi, f \rangle \quad , \quad \varphi \in \mathcal{D}'(\mathbb{R}^4) \quad , \quad f \in \mathcal{D}(\mathbb{R}^4) \; ,$$

and μ is the cylinder measure on $\mathcal{D}'(\mathbb{R}^4)$ uniquely defined by

$$E\left(e^{i\varphi(f)}\right) = e^{-\frac{1}{2}\langle f, (-\Delta + m^2)^{-1} f \rangle} = \int_{\mathcal{D}'(\mathbb{R}^4)} e^{i\varphi(f)} \, d\mu \; ,$$

using Minlos theorem, see [10]. But in all applications made so far in field theory it is not necessary to specify the particular representation.

Without loss of generality we assume that Σ is the smallest σ-algebra with respect to which all fields $\varphi(f)$ are measurable. Roughly speaking this means that any physical observable (= measurable function) is a function of the fields $\varphi(f)$.

Due to the Euclidean invariance of the scalar product in N , the full Euclidean group, E O(4) can be represented in the natural way as a group of measure preserving automorphisms of the σ-algebra Σ . The minimality of Σ has a consequence that the translation subgroup acts ergodically on Σ , that means that the only translation invariant measurable functions on Q are the constants.

The following local structure plays a considerable role in the development of the theory. To each open region Λ of $\mathbb{R}^4$ let us associate the sub-σ-algebra Σ_Λ of Σ generated by the fields $\varphi(f)$ with $\operatorname{supp} f \subset \Lambda$. For any region Λ of $\mathbb{R}^4$ we can define $\Sigma_\Lambda = \bigcap_{\Lambda' \supseteq \Lambda} \Sigma_{\Lambda'}$, where the intersection is taken with respect to all open regions Λ' containing Λ . If $\varphi(f)$ is indexed by N then it is convenient to define Σ_Λ as the sub-σ-algebra generated by the fields $\varphi(f)$ with $\operatorname{supp} f \subseteq \Lambda$, for any closed Λ . We remark explicitely here that we mean always equality between σ-algebras as equality modulo subsets of Q of zero measure.

It is clear that this local structure has the natural covariance properties with respect to the Euclidean group. We can consider the spaces $L^p(Q, \Sigma_\Lambda, \mu)$, $1 \le p \le \infty$, as spaces of observables associated to the region Λ . We call E_Λ the conditional expectation with respect to the σ-algebra Σ_Λ .

Now, following Nelson [34], we introduce the Markov property in the following form :

Proposition 1

Let Λ be an open region of $\mathbb{R}^4$, Λ' its complement and $\partial\Lambda$ its boundary, then for any observable u , which is $\Sigma_{\Lambda'}$ measurable, we have

$$E_{\partial\Lambda}\, u = E_\Lambda\, u \, .$$

In some cases it is convenient the following equivalent form.

Proposition 2

Consider a closed three-dimensional manifold σ which divides $\mathbb{R}^4$ in two closed regions Λ_1 and Λ_2 having σ in common. Then we have $E_{\Lambda_1} E_{\Lambda_2} = E_{\Lambda_2} E_{\Lambda_1} = E_\sigma$.

Markov property is a simple consequence of the fact that the scalar product on N is expressed through the inverse of a local differential operator. It is a generalization of the analogous property of stochastic processes [11]. Roughly speaking it tells us that all informations exchanged between Λ_1 and Λ_2 are "felt" by the separating boundary. Therefore the observables in Λ_1 and Λ_2 are conditionally independent given the observables at the separating boundary σ . This kind of generalized Markov property has been considered also in classical statistical mechanics of lattice systems, see for example [4].

In order to describe the connection with the Hamiltonian theory first of all let us remark that if we consider hyperplanes π then all $L^2(Q,\Sigma_\pi,\mu)$ spaces are isometrically isomorphic. Moreover we have

Proposition 3

Each $L^2(Q,\Sigma_\pi,\mu)$ can be identified with the physical Hilbert space $\mathcal{H}$, the fields supported on the hyperplane π coincide, from a stochastic point of view, with the zero time free Wightman field of the Hamiltonian theory.

The following reflexivity property is also very important.

Proposition 4

Let π be an hyperplane, E_π the conditional expectation on Σ_π and R_π the representative of the reflection in $\mathbb{R}^4$ with respect to the hyperplane π, then we have

$$R_\pi E_\pi = E_\pi ,$$

i.e. the observables connected to the hyperplane π are left point-wise invariant by the transformation R_π.

Let us now consider a family of parallel hyperplanes π_t, let us say $x_4 = t$, $t \in \mathbb{R}$, call Σ_t the related sub-σ-algebras and E_t the conditional expectations. Let J_t be the isometric embedding of $\mathcal{H}$ into $L^2(Q,\Sigma,\mu)$ with image $L^2(Q,\Sigma_t,\mu)$ then we have

Proposition 5

The free Hamiltonian H_0 in physical Hilbert $\mathcal{H}$ space can be expressed in the form

$$e^{-|t-s|H_0} = J_s^* J_t .$$

We call this the free Feynman-Kac formula, it can be generalized to the interacting case, see [illegible] and [20]. We remark here that the semi-group property of $J_0^* J_t$ is related to Markov property, and the selfadjointness to reflexivity.
If we recall all definitions and properties introduced we see immediately that the fields $\varphi(f)$ have played the following role. First of all they have been used to define the local structure, then to define the representation of the Euclidean group. Therefore by postulating the local structure and the existence of the representation of the Euclidean group, we can abstract a general structure of local observables in Euclidean quantum field theory in the following way.

A) (Local structure on the probability space) :
There is a probability space (Q,Σ,μ) whose measurable functions are the observables of the theory. To each bounded region Λ of $\mathbb{R}^4$ we associate a sub-σ-algebra Σ_Λ of Σ. The Σ_Λ measurable functions will be interpreted as local observables associated to the region Λ. We make the obvious isotony requirement : $\Sigma_\Lambda \subseteq \Sigma_{\Lambda'}$, if $\Lambda \subseteq \Lambda'$. Moreover we can assume that Σ is the smallest σ-algebra containing all Σ_Λ, so that Σ may be interpreted as quasi-local σ-algebra. The averages of the observables are calculated through the measure μ. Therefore μ represents a state on the algebra of observables.

B) (Euclidean covariance) :
There is a representation of the inhomogeneous Euclidean group as a group of measure preserving automorphisms of the σ-algebra Σ, compatible with the locality structure introduced in A).
We may assume that the translation subgroup acts ergodically on Σ, in this case μ is a pure state. Otherwise μ will represent a mixture and will be decomposable in extremal states.

C) (Markov property) :

Consider a smooth three-dimensional manifold σ dividing $\mathbb{R}^4$ in two closed regions Λ_1 and Λ_2 then

$E_{\Lambda_1} E_{\Lambda_2} = E_{\Lambda_2} E_{\Lambda_1} = E_{\sigma}$, where E_{Λ} in general denote the conditional expectation with respect to the σ - algebra Σ_{Λ} introduced in A) .

D) (Reflexivity) :

Let R_{π} be the representative of the reflexion in $\mathbb{R}^4$ with respect to the hyperplane π , then

$$R_{\pi} E_{\pi} = E_{\pi}$$

where E_{π} is the conditional expectation with respect to Σ_{π} .

The free Euclidean-Markov scalar field satisfies all properties A-B-C-D. In the next Sections we introduce the vector meson field and the electromagnetic field in order to provide some more examples.

We conclude this Section with the warning that properties A-B-C-D are probably the bare minimum to require for a sensible structure of Euclidean observables and that more structure will be requested in particular cases.

3. THE VECTOR MESON FIELD

Higher spin fields have been considered previously by many authors [9,12,16,37,39,40,42]. Here we will consider their structure especially in relation to the Markov and reflexivity properties. We follow mainly [3].

For the vector meson field our starting point is given by the two point Wightman function

$$W_{\mu\nu}(x-y) = \langle \Omega_0, B_\mu(x)\, B_\nu(y)\, \Omega_0 \rangle =$$

$$= (2\pi)^{-3} \int e^{-i\omega(\vec{k})(x_0-y_0)}\, e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \left(m^{-2} k_\mu k_\nu - g_{\mu\nu}\right) \frac{d\vec{k}}{2\omega(\vec{k})} ,$$

where $\omega(\vec{k}) = (\vec{k}^2 + m^2)^{\frac{1}{2}}$, $\vec{k} \in R^3$, $d\vec{k}$ is the Lebesgue measure on $\mathbb{R}^3$. Notice that the condition of transversality, $\partial^\mu W_{\mu\nu} = 0$, is satisfied.

By analytic continuation to the Schwinger points, using also a matrix transformation, which changes the Minkowski indexes into the Euclidean ones, we get the free Schwinger function

$$S_{ij}(x-y) = \frac{1}{(2\pi)^4} \int \frac{e^{i p(x-y)}}{p^2 + m^2} \left(\delta_{ij} + m^{-2} p_i p_j\right) dp .$$

For real test functions $f_i(x) \in \mathcal{D}(\mathbb{R}^4)$, $x \in \mathbb{R}^4$, $i = 1,\ldots,4$, let us introduce the scalar product

$$\langle f, g \rangle_{\mu} = \sum_{ij} \iint f_i(x)\, S_{ij}(x-y)\, g_j(y)\, dx\, dy =$$

$$= \langle f, T^{-1} g \rangle ,$$

where T is recognized as the differential operator

$$(Tf)_i(x) = (-\Delta + m^2) f_i(x) + \partial_i \partial_j f_j(x) \ .$$

Then we have :

<u>Definition 6</u>

The Euclidean vector meson field $B(f)$ is the real Gaussian process indexed by $f = \{f_i(x)\} \in \mathcal{D}''(\mathbb{R}^4)$ with mean zero and covariance given by

$$E(B(f)B(g)) = \langle f,g \rangle_N \ .$$

Since T is a local operator it is trivial to verify [3] Markov property. The reflexivity property is also satisfied [16,3], as are the analogs of Propositions 3 and 5 . Thus we see that the structure of the Euclidean vector meson field is perfectly analogous to the well-known one of the scalar field.

At present we are studying the lattice approximation [20] of the Euclidean vector meson field in order to see whether it is possible to prove correlation inequalities analogous to that established in the scalar case [20].

It is also possible to relate the Euclidean-Markov theory of the vector meson with stochastic mechanics as was done in [23] for the scalar field.

4. THE ELECTROMAGNETIC FIELD

A more complex structure is offered by the electromagnetic field. Here we have to distinguish between a description based on observable fields and one based on potentials, which carry additional unphysical gauge degrees of freedom but allow a simple and direct introduction of the interaction.

It is very well-known that in Minkowski case the introduction of local covariant potentials enforces the existence of an indefinite metric in the theory. We refer to the work of Strocchi and Wightman [52] and references cited there for an extensive discussion of the resulting indefinite metric theory.

Here we will see [3] that in the Euclidean case no indefinite metric is necessary. Local (in the sense of Markov), covariant potentials can be introduced inside a purely stochastic framework with positive definite metric.

First of all let us consider the case of the observable electromagnetic field.
As usual our starting point is given by the Wightman function

$$W_{\mu\nu;\mu'\nu'}(x-x') = \langle \Omega_0, F_{\mu\nu}(x)\, F_{\mu'\nu'}(x')\, \Omega_0 \rangle =$$

$$= \left[\partial_\mu \partial_{\mu'} g_{\nu\nu'} + \dots \right] \frac{1}{(2\pi)^3} \int e^{i|\vec{k}|(x_0 - x'_0)}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \frac{d\vec{k}}{2|\vec{k}|} \ ,$$

where $+\dots$ contains the other permutations of the indexes $(\mu\nu)$, $(\mu'\nu')$ in such a way that the resulting expression is antisymmetric for the exchange $\mu \leftrightarrow \nu$ and $\mu' \leftrightarrow \nu'$. By analytic continuation to the Schwinger points and matrix transformation we get the Euclidean two-point function

$$W_{ij,i'j'}(x-x') = \frac{1}{(2\pi)^4}\int \left[p_i p_{i'} \delta_{jj'} + \dots \right] e^{ip(x-x')} \frac{dp}{p^2} .$$

Then for real test functions $f_{ij}(x) \in \mathcal{D}(R^4)$, antisymmetric in ij, we can define the scalar product

$$\langle f, f' \rangle_W = \sum_{i<j,\, i'<j'} \iint f_{ij}(x)\, W_{ij,i'j'}(x-x')\, f'_{i'j'}(x')\, dx\, dx' ,$$

where $f \equiv \{ f_{ij}(x) \}$.

Definition 7

The free Euclidean electromagnetic field is the real Gaussian process $F(f)$ indexed by $f = \{f_{ij}(x)\} \in \mathcal{D}^6(R^4)$ with mean zero and covariance

$$E(F(f)F(f')) = \langle f, f' \rangle_W .$$

We can introduce the probability space (Q, Σ, μ) and the local structure $\Lambda \to \Sigma_\Lambda$ induced by the fields $F(f)$, i.e. Σ_Λ will be the smallest σ-algebra with respect to which all fields $F(f)$, with $\mathrm{supp}\, f_{ij} \subseteq \Lambda$, are measurable. Then it is simple, but not trivial, to verify [3]

Theorem 8

The localization $\Lambda \to \Sigma_\Lambda$ satisfies the Markov property.

The proof is slightly more complicated than in the scalar or vector meson case, because the covariance of the electromagnetic field is not given by the inverse of a local differential operator.

We have also :

Proposition 9

The analogs of Propositions 3, 4, 5 are satisfied for the electromagnetic field.

We have therefore a completely satisfactory solution for the free electromagnetic field in the Euclidean region. According to usual quantum field theory in order to introduce electromagnetic interactions it is necessary to consider also the potentials, from which the fields can be derived. It is expected that the potentials will describe additional unphysical gauge degrees of freedom.

Given a measurable function $d(k^2) \geq 0$ we introduce the two point function

$$S_{ij}(x-y) = \frac{1}{(2\pi)^4}\int e^{ip(x-y)} \left[\delta_{ij} - \frac{k_i k_j}{k^2} + d(k^2)\frac{k_i k_j}{k^2} \right] \frac{dk}{k^2} .$$

Definition 10

The electromagnetic potential, with gauge function $d(k^2)$, $A(g)$, is the real Gaussian random field indexed by $g = \{g_i(x)\} \in \mathcal{D}^4(R^4)$ with mean zero and covariance given by

$$E(A(g)A(g')) = \sum_{ij} \iint g_i(x)\, S_{ij}(x-y)\, g'_j(y)\, dx\, dy .$$

We call $(\bar{Q}, \bar{\Sigma}, \bar{\mu})$ the underlying probability space, then it is immediate to verify that the electromagnetic field F of definition 7 can be expressed through the potentials of Definition 10

$$F_{ij} = \partial_i A_j - \partial_j A_i .$$

In general the probability space $(\bar{Q}, \bar{\Sigma}, \bar{\mu})$ is larger than (Q, Σ, μ).

The electromagnetic potentials of Definition 10 satisfy obviously the covariance property and positive definiteness. It is simple to verify that also Markov property can be enforced [3].

Proposition 11

If $d^{-1}(k^2)$ is a polynomial in k^2 then the electromagnetic potentials of Definition 10 satisfy the Markov property.

The proof is very simple and is based again on the fact that the covariance is expressed through the inverse of a local differential operator.

It is also interesting to consider the case $d = 0$ (transverse gauge). First of all we have [3]

Proposition 12

The transverse electromagnetic potentials A^T $(d = 0)$ satisfy the Markov property.

Then we can immediately verify that the probability space of the transverse potentials A^T is the same as that of the electromagnetic field F. Therefore on the same probability space (Q, Σ, μ) we have two different local structures with Markov property, one generated by A^T, $\Lambda \to \Sigma'_\Lambda$, the other by F, $\Lambda \to \Sigma_\Lambda$. We have $\Sigma_\Lambda \subseteq \Sigma'_\Lambda$, in general Σ'_Λ is larger than Σ_Λ and contains also measurable functions which do not correspood to physical observables. We have also :

Proposition 13

The localization $\Lambda \to \Sigma'_\Lambda$ does not satisfies the reflexivity property of Proposition 4 , therefore the Hamiltonian introduced by Proposition 5 on $\mathcal{H} = L^2(Q, \Sigma'_t, \mu)$ is not selfadjoint. By introducing a suitable indefinite metric on $\mathcal{H}$ we can have a selfadjoint Hamiltonian. Obviously the Hamiltonian defined on the subspace $\mathcal{H}_{phys.} \equiv L^2(Q, \Sigma_t, \mu)$ is selfadjoint and coincides with the physical Hamiltonian of the free electromagnetic field.

In this way we see that there is a relation between the need of indefinite metric [51] in Minkowski space and the lack of reflexivity of the electromagnetic potentials in Euclidean space.

On the other hand in the Euclidean space no conflict arises between covariance (in the Euclidean sense) and locality (in the Markov sense) on one hand and positive definiteness of the metric on the other hand. This fact could be useful for the study of quantum electrodynamics or other gauge theories in the Euclidean framework.

It would be also interesting to reconsider systematically all cases in which spaces with indefinite metric have been introduced (for example in non-abelian gauge theories) in order to see whether Euclidean methods can be helpful.

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- REFERENCES -

[1] S. ALBEVERIO and R. HOEGH-KROHN
Mathematical Theory of Feynman Path Integrals
Preprint, University of Oslo, 1974.

[2] D. De FALCO and F. GUERRA
Euclidean Fermion Fields,
in preparation.

[3] S. De SIENA, F. GUERRA and P. RUGGIERO
On the Markov Property of the Free Euclidean Vector
Meson and Photon Fields,
in preparation.

[4] R.L. DOBRUSHIN
Gibbsian Random Fields for Lattice Systems with Pairwise
Interactions,
Funct. Anal. Applic. 2, 292 (1968).

[5] S. DOPLICHER, R. HAAG and J.E. ROBERTS
Local Observables and Particle Statistics, I
Commun. math. Phys. 23, 199 (1971).

[6] J. FELDMAN and K. OSTERWALDER
The Wightman Axioms and the Mass Gap for weakly coupled
$(\varphi^4)_3$ Quantum Field Theories,
Preprint, Harvard University, February 1975.

[7] R.P. FEYNMAN and A.R. HIBBS
Quantum Mechanics and Path Integrals,
Mac Graw Hill, New York (1965).

[8] J. FRÖHLICH
The Quantized "Sine-Gordon" Equation with a Non-Vanishing
Mass Term in Two Space-Time Dimensions,
Phys. Rev. Lett. 34, 833 (1975).

[9] J. FRÖHLICH and K. OSTERWALDER
Is There a Euclidean Field Theory for Fermions.
Preprint, Harvard University, 1974.

[10] I.M. GEL'FAND and N. Ya. VILENKIN
Generalized Functions, Vol.IV, Applications of Harmonic
Analysis
Academic Press, New York (1964).

[11] I.I. GIHMAN and A.V. SKOROHOD
Introduction to the Theory of Random Processes,
W.B. Saunders Co., Philadelphia 1969.

[12] J. GINIBRE and G. VELO
The Free Euclidean Massive Vector Field in the Stuckelberg
Gauge,
Preprint, Université de Paris-Sud (1974).

[13] J. GLIMM and A. JAFFE
Quantum Field Theory Models,
in Statistical Mechanics and Quantum Field Theory,
Les Houches 1970
C. De Witt, R. Stora, Editors, Gordon and Breach,
New York (1971).

[14] J. GLIMM and A. JAFFE
Particles and Bound States and Progress toward Unitarity
and Scaling
Report presented at the International Symposium on *Mathematical Problems in Theoretical Physics, Kyoto*,
January 23-29, 1975 ; and references quoted there.

[15] J. GLIMM, A. JAFFE and T. SPENCER
The Wightman Axioms and Particle Structure in the $P(\varphi)_2$
Quantum Field Model,
Ann. Math. 100, 585 (1974).

[16] L. GROSS
The Free Euclidean Proca and Electromagnetic Fields,
Report to the Cumberland Lodge Conference on Functional
Integration and its Applications, April 1-4, 1974.

[17] F. GUERRA
Uniqueness of the Vacuum Energy Density and Van Hove
Phenomenon in the Infinite Volume Limit for Two-Dimensional
Self-Coupled Bose Fields,
Phys. Rev. Lett. 28, 1213 (1972).

[18] F. GUERRA
Some Remarks on the Spontaneous Magnetization and the
Long Range Order in Quantum Field Theory,
in preparation.

[19] F. GUERRA, L. ROSEN and B. SIMON
Nelson's Symmetry and the Infinite Volume Behavior of the
Vacuum in $P(\varphi)_2$,
Commun. Math. Phys. 27, 10 (1972).

[20] F. GUERRA, L. ROSEN and B. SIMON
The $P(\varphi)_2$ Euclidean Quantum Field Theory as Classical
Statistical Mechanics,
Ann. of Math., 101, 111 (1975).

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[21] F. GUERRA, L. ROSEN and B. SIMON
Correlation inequalities and the Mass Gap in $P(\varphi)_2$.
III. Mass Gap for a Class of Strongly Coupled Theories with Non-Zero External Field,
Commun. math. Phys. 41, 19 (1975).

[22] F. GUERRA, L. ROSEN and B. SIMON
Boundary Conditions in the $P(\varphi)_2$ Euclidean Quantum Field Theory,
Preprint in preparation.

[23] F. GUERRA and P. RUGGIERO
New Interpretation of the Euclidean-Markov Field in the Framework of Physical Minkowski Space-Time,
Phys. Rev. Lett. 31, 1022 (1973).

[24] R. HAAG and D. KASTLER
An Algebraic Approach to Quantum Field Theory,
J. Math. Phys. 5, 848 (1964).

[25] A. JAFFE
Status of Constructive Field Theory.
Report at the "International Conference on High Energy Physics",
London, July, 1974.

[26] G. JONA-LASINIO
Relativistic Field Theory with Symmetry-Breaking Solution,
Nuov. Cim. 34, 1790 (1964).

[27] R. JOST
The General Theory of Quantized Fields
Am. Math. Soc. Publ. Providence, R.I. (1965).

[28] A. KLEIN and L. LANDAU
An Upper Bound on the Energy Gap in the $(\lambda\phi^4 + \sigma\phi^2)_2$ Model,
Preprint, University of California at Irvine, (1975).

[29] J. MAGNEN and R. SENEOR
The Infinite Volume Limit of the φ_3^4 Model,
Preprint, Ecole Polytechnique, February 1975.

[30] O.A. McBRYAN
Volume Dependence of Schwinger Functions in the $Yukawa_2$ Quantum Field Theory,
Preprint, Rockefeller University (1975).

[31] E. NELSON
Dynamical Theories of Brownian Motion,
Princeton University Press (1967).

[32] E. NELSON
Quantum Fields and Markoff Fields,
in Proceedings of Summer Institute of Partial Differential Equations, Berkeley 1971,
Amer. Math. Soc., Providence 1973.

[33] E. NELSON
Construction of Quantum Fields for Markoff Fields,
J. Funct. Anal. 12, 97 (1973).

[34] E. NELSON
The Free Markoff Field,
J. Funct. Anal. 12, 211 (1973)

[35] E. NELSON
Probability Theory and Euclidean Field Theory,
Contribution to [55].

[36] C.M. NEWMAN
Moment Inequalities for Ferromagnetic Gibbs Distributions,
J. Math. Phys., to appear.

[37] K. OSTERWALDER and R. SCHRADER
Euclidean Fermi Fields and a Feynman-Kac Formula for Boson-Fermion Models,
Helvetica Physica Acta 46, 277 (1973).

[38] K. OSTERWALDER and R. SCHRADER
Axioms for Euclidean Green's Functions,
Commun. math. Phys. 31, 83 (1973), and
Harvard University Preprint, 1974.

[39] H. ÖZKAYNAK
Euclidean Fields for Massive Particles of Arbitrary Spin,
Preprint, Harvard University (1974).

[40] A.L. REBENKO
Euclidean Fermi Fields and Green Functions without Momentum Cutoff in the $Yukawa_2$ Field Theory,
Preprint, ITP-74-54E, Kiev (1974).

[41] D. RUELLE
Statistical Mechanics,
Benjamin, New York (1969).

[42] J. SCHWINGER
On the Euclidean Structure of Relativistic Field Theory,
Proc. Nat. Acad. Sc. U.S. 44, 956 (1958).

[43] J. SCHWINGER
Four-Dimensional Euclidean Formulation of Quantum Field Theory, 1958 Annual International Conference on High-Energy Physics at CERN, ed. by B. Ferretti, CERN, Genève (1958).

[55] G. VELO and A. WIGHTMAN
Constructive Quantum Field Theory,
Lecture Notes in Physics, Vol. 25, Springer-Verlag,
Berlin (1973).

[56] A.S. WIGHTMAN
Constructive Quantum Field Theory : Introduction to
the Problems, in
in Fundamental Interactions in Physics and Astrophysics,
ed. B. Kursunoglu et al., Plenum, 1973.

[44] J. SCHWINGER
Euclidean Quantum Electrodynamics,
Phys. Rev. 115, 721 (1959).

[45] E. SEILER
Schwinger Functions for the Yukawa Model in Two-Dimensions
with Space-Time Cutoff,
I.A.S. Preprint, 1974.

[46] B. SIMON
Correlation Inequalities and the Mass Gap in $P(\varphi)_2$.
I. Domination by the Two Point Function,
Commun. math. Phys. 31, 127 (1973).

[47] B. SIMON
Correlation Inequalities and the Mass Gap in $P(\varphi)_2$.
II. Uniqueness of the Vacuum for a Class of Strongly
Coupled Theories,
Ann. of Math., 101, 260 (1975).

[48] B. SIMON
The $P(\varphi)_2$ Euclidean (Quantum) Field Theory
Princeton University Press, 1974.

[49] B. SIMON
Bose Quantum Field Theory as an Ising Ferromagnet :
Recent Developments
Preprint, Princeton University, (1975).

[50] B. SIMON and R.B. GRIFFITHS
The $(\phi^4)_2$ Field Theory as a Classical Ising Model,
Commun. math. Phys. 33, 145 (1973).

[51] R. STREATER and A.S. WIGHTMAN
PCT, Spin and Statistics, and All That,
Benjamin (1964), New York

[52] F. STROCCHI and A.S. WIGHTMAN
Proof of the Charge Superselection Rule in Local Relativistic Quantum Field Theory,
J. Math. Phys. 15, 2198 (1974).

[53] K. SYMANZIK
Euclidean Quantum Field Theory. I. Equations for a
Scalar Model,
J. Math. Phys. 7, 510 (1966).

[54] K. SYMANZIK
Euclidean Quantum Field Theory,
in Local Quantum Theory,
R. Jost, Editor, Academic Press, New York (1969).