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NON-STATIONARY ANALYSIS OF OHMIC AND VISCOUS HEATING OF A CYLINDRICAL PLASMA SHELL

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ABSTRACT

Possible heating mechanisms for a cylindrical plasma shell confined in an axial magnetic field are investigated. Two different cases are studied, namely ohmic and viscous heating of a rotating plasma and ohmic heating of a static plasma. The nonstationary plasma fluid equations are solved by using a numerical code. Of primary interest have been the maximum temperatures obtained by ohmic heating of a static plasma. It is shown that, in the first classical approximation, it is possible to reach temperatures around \( T \approx 2-4 \times 10^5 \) K at plasma densities, magnetic field strengths and induced current of the order \( n = 10^{21} \text{ m}^{-3} \), \( B = 1 \) tesla and \( J = 100 \) kA. The particle, momentum, and heat losses along the magnetic field lines have been neglected.
1. Introduction

Recently the idea has been put forward of ohmic heating of an impermeable plasma by induced currents directed along a poloidal magnetic field [1]. Experiments have already been performed in order to study this heating mechanism [2].

This report describes a study of the effect of the induced currents on the particle, momentum and energy balances by using a numerical code in solving the time-dependent plasma-fluid equations. A simplified cylindrical shell model has been used. Two cases have been considered, namely viscous and ohmic heating of a rotating plasma and ohmic heating of a static plasma. Of particular interest is the maximum temperatures obtained at a given ohmic heating current.

2. Basic Equations and Assumptions

Neglecting the effect of neutral gas the fundamental macroscopic equations in the plasma fluid equations can be written [3]

\[ \frac{3n}{\partial t} + \text{div}(nv) = 0 \]  \hspace{1cm} (1)

\[ nm \frac{dv}{dt} + \frac{m_i m_e}{e^2 m} (\mathbf{j} \times \mathbf{E}) (\mathbf{j}/n) = \mathbf{j} \times \mathbf{\dot{\mathbf{E}}} - \text{div} \mathbf{\dot{\tau}} \]  \hspace{1cm} (2)
Here \( n_i = \text{ion plasma density} \), \( n_e = \text{electron plasma density} \), \( n = n_i = n_e \), \( \mathbf{v} = \text{centre-of-mass velocity} \), \( m_i = \text{ion mass} \), \( m_e = \text{electron mass} \), \( m = m_i + m_e \), \( e = \text{electron charge} \), \( \mathbf{j} = \text{current density} \), \( \mathbf{B} = \text{magnetic field} \), \( \pi_e = \text{electron pressure tensor} \), \( \pi_i = \text{ion pressure tensor} \), \( \pi = \pi_i + \pi_e \), \( \mathbf{E} = \text{electric field} \), \( T_e = \text{electron temperature} \), \( T_i = \text{ion temperature} \), \( T = T_i = T_e \), \( p_e = \text{electron scalar pressure} \), \( p_i = \text{ion scalar pressure} \), \( p = p_i + p_e \), \( U_e = 3p_e/2n \), \( U_i = 3p_i/2n \), \( U = 3p/2n \), \( \nu_{ei} = \text{electron-ion collision frequency} \), \( q = \text{heat flux} \), \( R = \text{radiation losses} \). Eqs. (1)-(4) are now applied to an infinitely long cylindrical plasma shell. All derivatives in the azimuthal and axial directions are neglected as a first approximation. The magnetic field is parallel to the axis of symmetry. A cylindrical coordinate system is introduced with the z-axis along the axis of symmetry. For plasmas with densities around
n = 10^{21} \text{m}^{-3} \text{ and magnetic fields around } B = 1 \text{ tesla} \ [3],

Eqs. (1)-(4) reduce to

\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (n rv_r) = 0 \tag{5}

\frac{\partial \psi}{\partial t} + \frac{v_r}{r} \frac{\partial \psi}{\partial r} = \frac{1}{nm} \frac{1}{r} \frac{\partial}{\partial z} \int \left[ \frac{r}{3} \frac{\partial}{\partial r} (rv_r) \right] \tag{6}

v_r = -2kn_1 \frac{T}{B} \frac{\partial n}{\partial r} - \frac{1}{2} kn_2 \frac{n}{B^2} \frac{\partial T}{\partial r} + \frac{nmv^2}{r} \frac{\partial \psi}{\partial r} + \frac{1}{2} \frac{\partial^2 \psi}{\partial r^2} \tag{7}

\frac{\partial T}{\partial t} + \frac{v_r}{r} \frac{\partial T}{\partial r} = \frac{\nu_r}{3nk} \left[ \frac{1}{r} \frac{\partial}{\partial r} (rv_r) \right]^2 + \frac{1}{3nk} \frac{1}{r} \frac{\partial}{\partial r} (kr \frac{\partial T}{\partial r}) -

- \frac{2}{3} T \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{n_1}{3nk} \frac{\partial \psi}{\partial r}^2 + \frac{\eta}{3nk} \frac{\partial \psi}{\partial z}^2 \tag{8}

j_r = \frac{\Psi}{B} - \frac{nmv^2}{Br} \tag{9}

Here \( r \) = distance from axis of symmetry, \( k \) = Boltzmann's constant, \( v_\psi \) = azimuthal velocity, \( v_r \) = radial diffusion velocity, \( j_r \) = radial current density, \( \eta_r \) = resistivity parallel to \( \vec{B} \), \( \eta_\perp \) = resistivity perpendicular to \( \vec{B} \), \( \mu \) = coefficient of viscosity perpendicular to \( \vec{B} \), \( \kappa \) = coefficient of heat conductivity perpendicular to \( \vec{B} \). According to Spitzer [4]
\[ n_i = 65.3 \frac{Z(\ln A)}{T_e^{3/2}} \quad (10) \]

\[ n_e = 129 \frac{Z(\ln A)}{T_e^{3/2}} \quad (11) \]

Here, \( \Lambda = \) ratio between Debye distance and impact parameter, \( Z = \) charge number. According to Braginskii [5],

\[ \mu = \frac{nkT}{\nu_{ii}} b_1(2x) \quad (12) \]

\[ \kappa = \frac{nk^2T}{m_i \nu_{ii}} b_2(x) \quad (13) \]

\[ b_1(x) = \frac{1.2x^2 + 2.23}{x^4 + 4.03x^2 + 2.33} \quad (14) \]

\[ b_2(x) = \frac{2x^2 + 2.645}{x^4 + 2.70x^2 + 0.677} \quad (15) \]

\[ x = \frac{\omega_{ki}}{\nu_{ii}} \quad (16) \]

\[ \nu_{ii} = 6 \times 10^{-8} \frac{n_{ii}^{4}(\ln A)}{A_{1/2}T_i^{3/2}} \quad (17) \]
Here $v_{ii}$ = ion-ion collision frequency, $\omega_{gi}$ = ion gyration frequency, $Z$ = charge number, $A$ = mass number. Non-stationary solutions of Eqs. (5)-(9) have been obtained by using a numerical code. Stationary solutions for the case of pure viscous heating have earlier been investigated by Hellsten [6]. The boundary values of the plasma density $n_b(r_1)$ and $n_b(r_0)$ are assumed to be determined by the external neutral gas pressure and can therefore be chosen arbitrarily within certain limits. The boundary value of temperature $T_b$ is approximated by a characteristic temperature in the transition region between a partially and fully ionized plasma. In the density regions considered the variation of $T_b$ with density is neglected as a first approximation. The boundary value of the rotational velocity $\nu_{\varphi b} = 0 \text{ m s}^{-1}$. This is due to the friction between the neutral gas and plasma in the boundary layers. The initial value of the density distribution has been chosen equal to a constant, for the sake of simplicity. The rotational velocity and temperature are assumed initially to have parabolic shapes with maximum values corresponding to approximately stationary conditions. The results obtained are not particular sensitive to the initial profiles choosen.
3. Numerical Code

A combined explicit and implicit scheme has been used in forming the finite difference approximations of Eqs. (5)-(9). The convective terms have been approximated by an explicit upstream-downstream method \[7\], while the diffusion terms have been treated by using an implicit method by Laasonen (1947) \[7\]. The stability of the numerical code has been tested and is considered satisfactory.

4. Viscous and Ohmic Heating of a Rotating Plasma

Eqs. (5)-(9) have been solved in the case with viscous and ohmic heating under the following conditions:

\[
\begin{align*}
    r_i &= 0.24 \text{ m} \\
    r_o &= 0.30 \text{ m} \\
    n_b(r_i) &= n_b(r_o) = 2 \times 10^{21} \text{ m}^{-3} \\
    v_b &= 0 \text{ m s}^{-1} \\
    T_b &= 3 \times 10^4 \text{ K} \\
    B &= 0.5 \text{ T} \\
    v_b \text{ max} &= 4 \times 10^4 \text{ m s}^{-1} \\
    T_{\text{max}} &= 4 \times 10^4 \text{ K}
\end{align*}
\]

Here \(v_b \text{ max}\) = initial maximum rotational velocity, \(T_{\text{max}}\) = initial maximum temperature. The radial driving current \(I_r(t)\) has the time dependence shown in Fig. 1. The radial current density \(j_r(r, t)\) has been obtained from
Here $dS$ = differential area element. The value of $I_r(t=0 \mu s)$ is chosen such that approximately stationary conditions prevail for the particular initial choices of density and rotational velocity. Further, the value of $I_r(t=200 \mu s)$ has been chosen such as to assure a maximum rotational velocity not exceeding $v_{\text{max}}$. Here $v_{\text{max}} = c \cdot v_{\text{crit}}$. In this specific example $c = 1.7$ and $v_{\text{crit}}$ is the critical velocity [8]. The ohmic heating current $I_z(t)$ has the time dependence shown in Fig.2. A maximum induced current of about 100 kA is assumed possible to realize. The ohmic current density $j_z(r,t)$ has been obtained from the following relations

$$j_z = \frac{T^{3/2}}{65.3(\ln \Lambda) \cdot E_z} \quad (20)$$

$$I_z = \int j_z dS \quad (21)$$

The induced electric field is approximated by a linear function of radius. This relation holds exact for the case of a primary circuit consisting of an infinitely long solenoid and the secondary circuit being an internal plasma cylinder. It is also necessary for the plasma to be mainly resistive.

The solutions of Eqs. (5)-(8) are shown in Figs. 3-6. The density profiles show a characteristic local maximum close to
the center of the shell during the first 500 µs. This is of course due to the outward radial diffusion caused by the centrifugal force. This effect disappears after about 500 µs. This is due to the fact that after about 300 µs the rotational velocity decreases (see Fig.4) while the temperature is still increasing (see Fig.6) and the radial diffusion due to the centrifugal force therefore becomes strongly reduced. The density after this moment approaches a value being consistent with $v_r = 0$ as can be seen from Fig.5. This implies according to Ohm's law that the density and temperature approximately satisfies a $n^2 T = \text{const.}$ relation. The characteristic maximum values of the plasma density at the boundaries is typical for an impermeable plasma surrounded by a neutral gas blanket as pointed out by several authors [9,10,11]. The temperature profile is very flat in the central part during the first 200 µs (see Fig.6). This is due to the fact that during this time the viscous heating effect is dominating. The viscous power is being absorbed close to the outer and inner plasma boundaries where there are substantial velocity gradients. It might even be anticipated that for a suitable choice of plasma parameters the temperature profile could have a non-stationary local minimum in the central plasma parts. The maximum temperature corresponds to $T = 1.1 \times 10^5$ K. This temperature could of course be increased by choosing lower plasma density and a larger magnetic field strength.
5. Ohmic Heating of a Static Plasma

This investigation follows the same scheme as outlined in the previous section. The difference is that the rotation of the plasma is suddenly stopped. The time when this occurs has somewhat arbitrarily been chosen at $t = 200 \mu s$. The results obtained do not crucially depend on this choice of time. The situation after this time corresponds to ohmic heating of a static plasma. Typical solutions to Eqs. (5)-(8) using the same set of parameters as in the previous chapter are shown in Figs. 7-9. The local maximum in the density distribution (see Fig.7) disappears rather rapidly and a minimum in the central parts of the plasma is established after a few hundred microseconds for reasons explained in section 4.

For a particular choice of parameters such as the magnetic field strength it might be anticipated that the local density maximum will not have time to disappear. That means that the possibility exists to a certain extent to control the density profile by external means. The radial diffusion velocity, (see Fig.8), shows after a few hundred microseconds an almost sinusoidal shape. In the outer plasma parts the radial velocity is dominated by the resistive diffusion due to the temperature gradient. Closer to the axis of symmetry the diffusion is dominated by the resistive diffusion due to the density gradient. The maximum plasma temperature, (see Fig. 9), is shifted slightly towards the outer plasma radius. This is caused by the induced electric field which is directly proportional to the radius.
The maximum temperature as a function of the magnetic field strength $B$ for $n_b(r_i) = n_b(r_o) = 2 \times 10^{21} \text{m}^{-3}$ and $I_z = 100 \text{kA}$ is shown in Fig.10. The initial values have been chosen as described in section 5. The increase in temperature for increasing magnetic field strength is mainly due to the decrease in heat conduction losses. These losses scale as $B^{-2}$ for $\omega_{gi}/v_i >> 1$. The maximum temperature dependence on plasma boundary density is shown in Fig.11 for $I_z = 100 \text{kA}$ and $B = 1.0 \text{ tesla}$. The initial and boundary values are chosen in the same way as before. As can be seen, a maximum temperature $T_{\text{max}} = 2 \times 10^5 \text{ K}$ is reached for a plasma boundary density $n_b = 1.0 \times 10^{21} \text{ m}^{-3}$. The inverse dependence of the maximum temperature versus density is explained by the fact that the dominating losses by heat conduction scales as $n^2$ when $\omega_{gi}v_i >> 1$. The general dependence of maximum temperature on magnetic field strength and plasma density remains unchanged for another wide range parameter value. We should therefore expect a temperature around $4 \times 10^5 \text{ K}$ for $n = 1 \times 10^{21} \text{ m}^{-3}$ and $B = 2 \text{ tesla}$. The effect of different ohmic heating currents on the temperature is shown in Fig.12, for $B = 1.0 \text{ tesla}$ and $n_b(r_i) = n_b(r_o) = 2 \times 10^{21} \text{ m}^{-3}$. There is an almost linear relationship. From this investigation it therefore seems possible that a plasma temperature of between $2 \times 10^5 - 4 \times 10^5 \text{ K}$ can be reached for possible realizable values of density, magnetic field strength and ohmic current.
Discussion and Conclusions

Pure viscous heating of a plasma shell with the dimensions as given in Section 4 results according to theory in a maximum temperature around $5 \times 10^4$ K for densities and magnetic field strengths in the range $n = 10^{21}$ m$^{-3}$ and $B = 0.5$ T [12]. Ohmic heating of a static plasma with induced currents of the order 100 kA results on the other hand in approximately three times larger maximum temperature for the same density and magnetic field strength. The maximum temperature could even further be increased to at least $4 \times 10^5$ K for physically realizable values of the external parameters.

The maximum temperature in the case of ohmic heating of a static plasma is strongly dependent on the plasma density and the magnetic field strength. Further, the density profile can to a certain extent be controlled by external means. This implies that, for certain choices of magnetic field strengths, maximum rotational velocity and duration of preheating by viscous forces, the density profile can have either a local maximum or local minimum at central plasma parts during the ohmic heating phase of a static plasma.

In this investigation it has been assumed that there are no heat losses along the magnetic field lines. For increasing plasma temperatures these losses will become important.

Throughout the calculations, the classical values of the resistivity have been used. There is of course the possibility of an increased ion-electron collision frequency and therefore also an increased resistivity due to anomalous effects. An increase in the resistivity due to impurity effects is also possible.
In a more detailed investigation of ohmic plasma heating by induced currents directed along the magnetic field lines, particular attention therefore has to be paid to heat losses along the magnetic field lines, anomalous effects, impurity effects, and eventually to effects resulting from the slightly deformed magnetic field.

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References


Captions to the Figures

Fig. 1. Typical time dependence of relative radial current driving the plasma rotation.

Fig. 2. Typical time dependence of the induced currents along the magnetic field lines.

Figs. 3-6. Density, rotational velocity, radial diffusion velocity and temperature profiles at different times for the case with ohmic heating of a rotating plasma. Here \( n_b(r_i) = n_b(r_o) = 2 \times 10^{21} \text{ m}^{-3} \), \( T_b(r_i) = T_b(r_o) = 3 \times 10^4 \text{ °K} \), B=0.5 T and \( J_z = 100 \text{ kA} \).

Figs. 7-9. Density, radial diffusion velocity and temperature profiles at different times for the case with ohmic heating of a static plasma. Here \( n_b(r_i) = n_b(r_c) = 2 \times 10^{21} \text{ m}^{-3} \), \( T_b(r_i) = T_b(r_o) = 3 \times 10^4 \text{ °K} \), B=0.5 T and \( J_z = 100 \text{ kA} \).

Fig. 10. Maximum temperature \( T_{\text{max}} \) as a function of magnetic field strength B for ohmic heating of a static plasma. Induced heating current \( J_z = 100 \text{ kA} \) and plasma density at the boundaries \( n_b = 2.0 \times 10^{21} \text{ m}^{-3} \).

Fig. 11. Maximum temperature \( T_{\text{max}} \) as a function of plasma density at the boundary \( n_b \) for ohmic heating of a static plasma. Magnetic field strength B = 1 tesla and induced heating current \( J_z = 100 \text{ kA} \).

Fig. 12. Maximum temperature \( T_{\text{max}} \) as a function induced current \( I_z \) for ohmic heating of a static plasma. Magnetic field strength B = 1 tesla and plasma density at the boundaries \( n_b = 2 \times 10^{21} \text{ m}^{-3} \).
Fig. 12

![Graph showing the relationship between \( T_{\text{max}} \) and \( J_z \).]
Possible heating mechanisms for a cylindrical plasma shell confined in an axial magnetic field are investigated. Two different cases are studied, namely ohmic and viscous heating of a rotating plasma and ohmic heating of a static plasma. The nonstationary plasma fluid equations are solved by using a numerical code. Of primary interest have been the maximum temperatures obtained by ohmic heating of a static plasma. It is shown that, in the first classical approximation, it is possible to reach temperatures around $T = 2-4 \times 10^5 \, \text{°K}$ at plasma densities, magnetic field strengths and induced current of the order $n = 10^{21} \, \text{m}^{-3}$, $B = 1 \, \text{tesla}$ and $J = 100 \, \text{kA}$. The particle, momentum, and heat losses along the magnetic field lines have been neglected.

**Key words** Viscous heating, ohmic heating, induced currents, rotating plasma, static plasma, maximum temperatures.