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THE SATURNIAN RINGS

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§ 1 Importance of cosmogonic effects and resonance effects

Like most of the structure of the Solar System the Saturnian ring system was formed by cosmogonic processes four or five billion years ago. However, because the Kepler motion of its particles is perturbed by the gravitational force of the satellites of Saturn, part of its structure - especially the fine structure - may be a result of forces acting today. The purpose of this paper is to analyse the relative importance of these effects in the light of new observational and theoretical results. Of these the new photometric curves by Coupinot (1) are important because together with Dollfus' curve (2)(2a) they clarify some of the structural details. Also the better understanding of the cosmogonic processes in general has now made it possible to discriminate between possible mechanisms with increased degree of certainty (3), (4).

§ 2 Comparison with the asteroid belt

Because in certain respects the asteroid belt is similar to the Saturnian ring system, a comparison is motivated. From a theoretical point of view we should expect that the (m, a) diagram is more relevant to the study of cosmogonic processes than the usual (n, a) diagram (m = total mass, n = number of asteroids per interval of semimajor axis a). This is confirmed by the fact that the (m, a) plot gives a much more regular picture of the structure (4)(5).

The (m, a) diagram (Fig. 1) (5) shows two conspicuous features:

(a) The belt has very sharp limits both at the inside and outside.

As the Kepler periods at these limits do not coincide with any low order commensurability (with Jupiter's period) they cannot be due to resonance effects. Moreover, there is no resonance effect known which can produce such limitations. Hence this feature must be due to cosmogonic effects. Indeed it can be understood as a result of a condensation from a partially corotating plasma (3)(4).

(b) There are a number of Kirkwood gaps.

As these are located exactly at the points of the Jupiter commensurabilities there can be no doubt that they are due to resonance effects. However, attempts to explain the gaps by celestial mechanics effects alone have not been successful. The pri cipal difficulty is to explain the absence of bodies at the Kirkwood gaps but at the same time the presence of bodies at the Hilda and Thule resonances. As shown by Ip (6) a theory of the gaps must include collisions between the bodies, and the resonance effects which formed the gaps must have been acting at cosmogonic times.

The only asteroids which are massive enough to be conspicuous in the (m, a) diagram are the Hilda asteroids located in the 2:3 resonance point. Also these can be understood as a product of cosmogonic processes. Ip (7).

The asteroidal results are partially applicable to the Saturnian rings. There are important similarities: Some of the limits viz the inner limit of the A ring and both the outer and inner limits of the B ring are due to cosmogonic shadow effects. On the other hand there are also important differences: the outer limit of the A ring caused by the Roche limit. Furthermore, there is no observable analogy to the Kirkwood gaps.

§ 3 Kirkwood gaps in the ring system

At a superficial comparison between a photograph of the Saturnian rings and a diagram of the asteroid belt it is tempting to identify the Cassini division as an analogy to the Kirkwood gaps. Furthermore, Cassini's division is located close to the 1:2 Mimas resonance, and as long as the observational accuracy was low, an agreement within the limits of error could be claimed. The higher accuracy of Dollfus' and more recently Coupinot's photometric curves make such an identification impossible. Moreover the small mass of Mimas means that its perturbations should be four orders of magnitude smaller than the Jupiter perturbations in the asteroid belt. This means that the breadth of the Kirkwood gaps in the ring system should have a relative size four orders of magnitude smaller than in the asteroidal belt. Hence the gaps must be far below the present detectability. It is out of the question that they can produce such a conspicuous feature as the Cassini's division. The paper by Franklin et al (8) showing a very poor agreement between a "resonance theory" and observation is supporting this conclusion (although the authors claim the contrary).

§ 4 Identification of the main ring features

(a) Outer border of A ring. This is traditionally identified with the Roche limit. There is no reason to doubt this identification although it is likely that we should rather speak of a "modified Roche limit" than of a Roche limit in the classical sense (4).

(b) Cassini's division. This is "the cosmogonic shadow" of Mimas (Fig. 2). The effect is similar to the effect which produces the outer limit to the asteroidal belt.

However, in the latter there is no real correspondance to the A ring (except the Hilda group and Thule). According to Coupinot the brightness minimum is located at 17.22" in the excellent agreement with Dollfus' value. As the orbital radius of Mimas is 27.1" the fall-down ratio is α = 1.57 which differs from the theoretical value by less than 5%. It is doubtful whether we should consider such a discrepancy as significant because there may have been changes in the structure during the 4 - 5 billion years since it was found. However, it is interesting to point out that the production of a shadow implies that the fall-down ratio must increase. According to (9) the value 1.57 is quite reasonable.

According to Coupinot the breadth of the Cassini division is 0.70" or 4%. If the shadow effect is produced by a circular jet stream in which Mimas is accreting, this is reconcilable for the order of magnitude with the thickness which has been tentatively estimated (it is smaller by about 40%). See (4).

(c) Limit between B and C rings. This should be the cosmogonic shadow of the outer border of the A ring. In the asteroidal belt it corresponds to the decrease in mass at 2.3 AU. In order to check this the photometric diagram is diminished by a factor 2:3 and reversed. As is seen in Fig. 2 its shadow agrees very well with the observed light curve. In fact the positions of the points of inflexion of the outer limit to the A ring is 19.8" and of the B-C ring border is 13.2", giving a fall-down ratio of 1.50. It should be observed that as the B-C border is inside the limit

$$R = R_{syn}(\tfrac{2}{3}\cos\lambda)^3 =$$

(r_{syn} = radius of synchronous satellite, λ = latitude) where partial corotation can be established, the theoretical value should be 3:2 = 1.50 without any correction. Hence the agreement between theory and observation is excellent.

(d) Guérin's division. There is a minimum in the light curve at 10.45" which has recently been discovered by Guérin and is called after him. According to Fig. 2 its position coincides with 2/3 of the highest point (at 16.1") in the photometric curve. Hence it may be a cosmogonic shadow effect. The fall-down ratio is $\alpha = 16.1/10.45 = 1.50$. Because we are far inside the synchronous orbit, there should be no correction to the theoretical value $\alpha = 3/2$.

(e) Inner border of D ring. There is a not very well defined minimum at 9.2" - 9.6", to the right of which the D ring is located. It should be observed that according to (4) the lowest possible distance at which matter from a condensing plasma in practical corotation could condense is

$$r = (2/3)^{4/3)} r_{syn} = 0.58\ r_{syn}$$

which is 9.4". It is possible that the inner border of the D ring is given by this relation. However, we are very close to the Saturnian upper atmosphere, and an effect due to the interaction with this cannot be ruled out.

As both (d) and (e) are phenomena very close to the planet and extremely difficult to observe, the discussion of these necessarily is uncertain.

§ 5 Possible resonance effect with the axial rotation of Saturn

Even if the perturbations by the satellites are too small to produce any noticable effects we cannot rule out other types of resonances. As pointed out by Allan [(10), see also (3) and (4)] a satellite with an orbital period which is an integer fraction of the spin period of a planet might be seriously perturbed under the condition that the gravity field of the planet has high order terms. The most important resonance is with bodies moving in synchronous satellite orbits. The location of such bodies in the Saturnian environment is at $1.10 \cdot 10^{10}$ cm or 16.1". In the Coupinot curves a light minimum appears just at this point. In Dollfus' curve it is slightly displaced to the left. If this identification is confirmed a calculation of the irregularities of the Saturnian gravitational field can be made and conclusions drawn about the inner structure of Saturn.

§ 6 Other markings on the rings

Except the mentioned features a large number of bright and dark markings have been reported. Artists' drawings of the rings often show a dozen such "divisions" or bright regions. Dollfus' photometric curve shows four pronounced minima on the A ring and an equal number on the B ring. Coupinot's two curves also exhibit some light minima but less conspicuous. The agreement between them and Dollfus' minima is not very good. The minimum at 15.2" which is well visible in Dollfus' curve, and which was tentatively identified with the shadow of Janus, is at most a slight irregularity in Coupinot's curve. In fact the only minimum which is obviously present on all the curves is the one we have discussed in § 5.

The very good agreement between features (a), (b), and (c) in § 4 make us confident about their reality and possible permanency. The difference between the curves may be due to the difference in the techniques by which they have been prepared. Perhaps they could also be due to "seasonal" differences produced by the difference in solar irradiation. In any case it is essential to try to discriminate between the permanent and the transient features.

§ 7 Inner limit to partial corotation

The equilibrium characterizing the state of partial corotation can be established only if the angular velocity of the spin of the central body exceeds the angular velocity given by the partial corotation criterion. If the radius of a synchronous satellite is r_{syn} the inner limit to the partially corotating plasma is $(2/3)^{1/3}R_{syn}$ [(3),(4)]. Hence above this limit the fall-down ratio should exceed 1.5 - as it does for Mimas-Cassini's division - whereas below it it should be exactly 1.50 (assuming an exact dipole field) - as we have found it for the shadow producing the border between the B and C rings and possibly also for some other features.

It is easily seen that in the region where the decrease in the fall-down takes place, there must be an increase in grain density. This region is located a little inside the synchronous satellite distance. In Coupinot's curves a maximum of about the expected type is seen. Indeed the maximum brightness of the whole ring system is located just at this place. However, it is not very pronounced in Dollfus' curve. Also the theory need to be further developed before any convincing conclusions can be drawn.

§ 8 Brightness of the A and B rings

The brightness of the A ring increases inwards. The brightness of the B ring is considerably larger than that of the A ring, but decreases inwards. There are at least three different effects which should contribute to the radial variations in the brightness.

(a) The radial variation of the density of the condensing plasma. From extrapolation of the mass sequence of the inner Saturnian satellite, we should expect the density to decrease rapidly inwards and at 9"4 it should become zero. We could check the density function by the increase in the fall-down ratio (9).

(b) The size of the grains may have a radial variation. As the disruptional effect of the tides of Saturn decreases rapidly outwards the grains may be bigger when the distance to the planet increases. At the Roche limit they should become "infinite". This effect should produce a decrease in brightness outwards.

(c) As the A ring is formed by grains which have condensed outside the orbit of Mimas they have to pass the orbit of Mimas before they reach their final destination. At the formation of the ring Mimas was also in statu nascendi which means that it was condensing from its jet stream. We have dereived the diameter of this from the breadth of the Cassini division. If the thickness of the jet stream perpendicular to the plane of motion is not very much smaller than the radial thickness, a considerable part of the condensing grains will hit the the jet stream and not pass it. This should produce a reduction in the intensity of the A ring compared to the B ring. For geometrical reasons this reduction will increase with distance. These effects may explain why the A ring is fainter than the B ring and perhaps also why A ring brightness decreases outwards.

From the combined action of (a), (b), and (c) it should be possible to calculate brightness variations theoretically. From such a study we may draw important conclusions about the cosmogonic processes. However, at present this is difficult for several reasons. The observational brightness variations are not known very well as shown by the difference between Dollfus' and Coupinot's curves. Furthermore we do not know the relation between the observed brightness and the density of the condensing plasma, among others because of the unknown radial dependence of the albedo and the size of the grains.

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Fig. 1. The (m, a) diagram of asteroids, showing the mass (m) per unit of semimajor axis a. A prominent feature is the Kirkwood gap caused by Jupiter resonance. Except the resonance-captured Hilda asteroids at a = 3.9, there are no appreciable mass outside Jupiter's "cosmogonic shadow". The inner limit is given by the "own shadow" of the belt which causes a decrease at 2.30 and a cut-off at 2.15. See ref. (4) and (5).

Fig. 2. Comparison between Dollfus' and Coupinot's photometric curves. The agreement between the main features is very good, and a comparison with theory can be based on them. The differences in some details (maxima and minima in A and B rings) have to be checked by further observations before they can be used theoretically.

Fig. 3. Coupinot's photometric curves. The curves diminished in the ratio 2:3 and turned upside down are shown in the upper left corner, demonstrating the theoretically expected shadow effect (analogous to the asteroid belt).

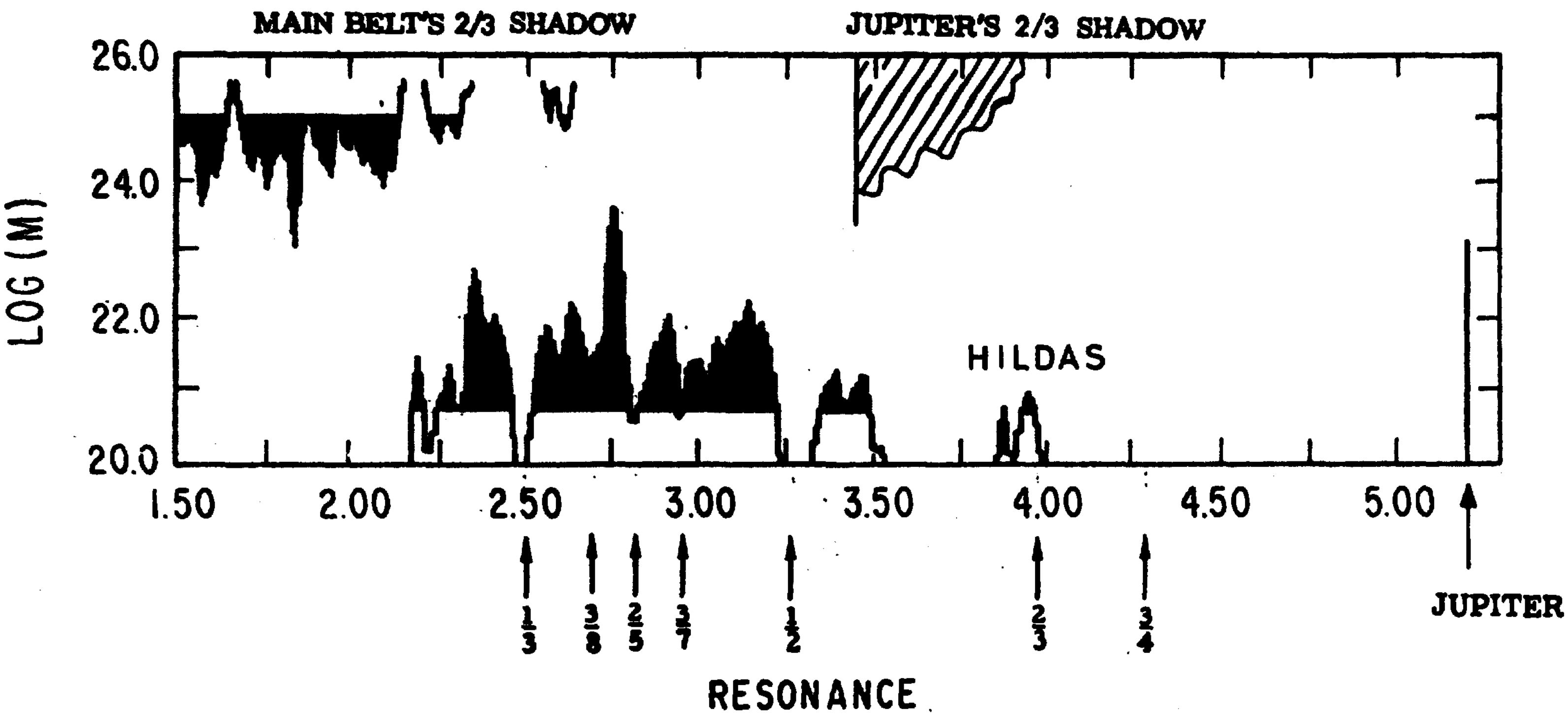
MAIN BELT'S 2/3 SHADOW
JUPITER'S 2/3 SHADOW
26.0
24.0
22.0
20.0
LOG (M)
HILDAS
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
1/3
3/8
2/5
3/7
1/2
2/3
3/4
JUPITER
RESONANCE

Fig. 1

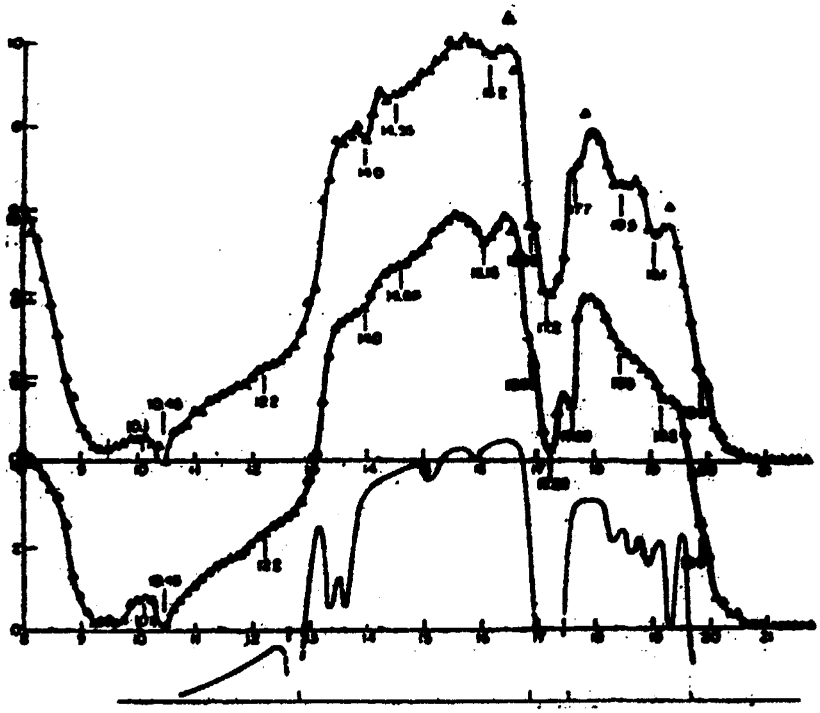

Fig. 2

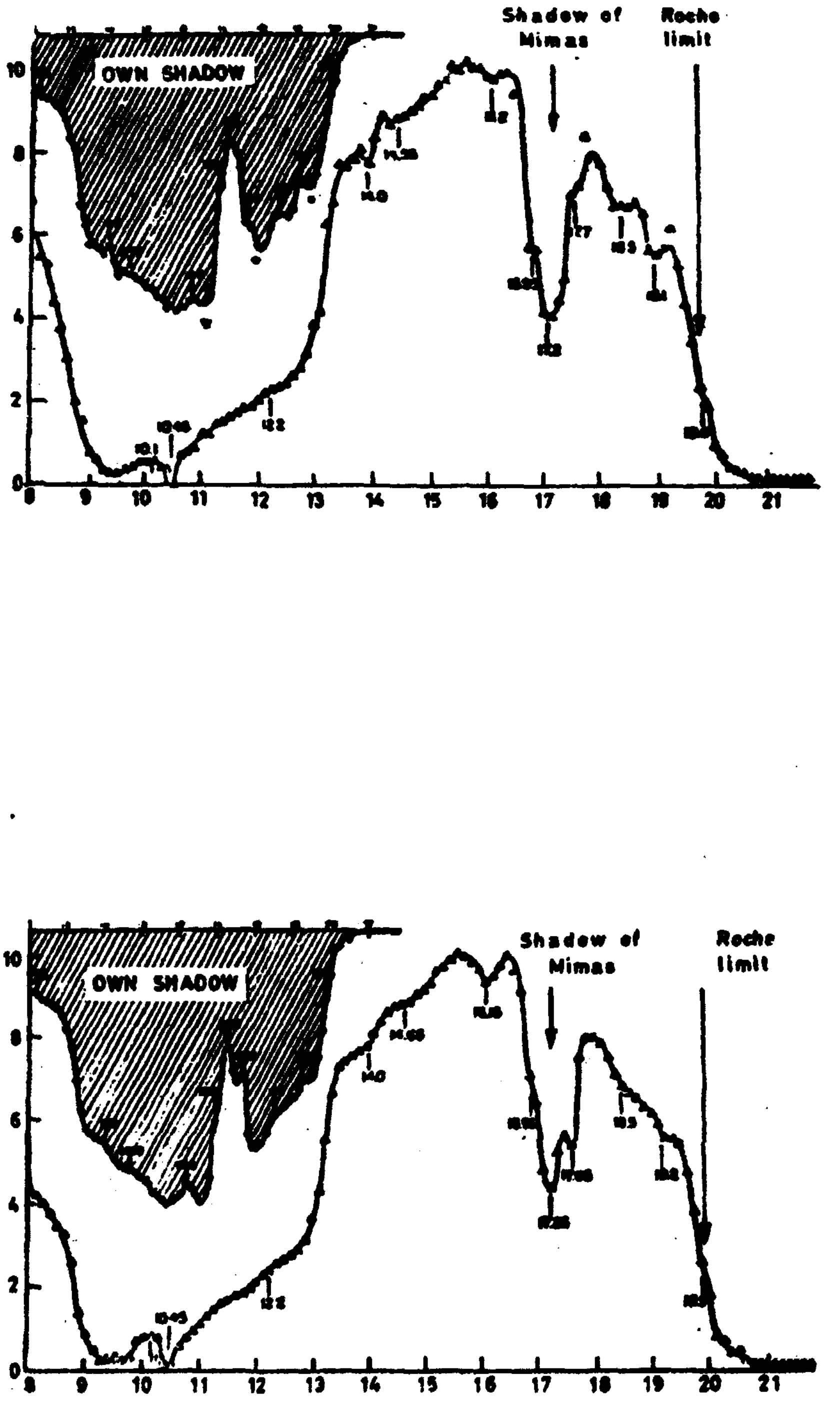
OWN SHADOW
Shadow of Mimas
Roche limit
OWN SHADOW
Shadow of Mimas
Roche limit

Fig. 3

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The structure of the Saturnian rings is traditionally believed to be due to resonances caused by Mimas (and possibly other satellites). It is shown that both theoretical and observational evidence rule out this interpretation.

The increased observational accuracy on one hand and the increased understanding of the cosmogonic processes on the other makes it possible to explain the structure of the ring system as a product of condensation from a partially corotating plasma. In certain respects the agreement between theory and observations is about 1%.

Key words Saturn, planets, solar system, cosmogony, cosmic plasma physics, partial corotation.