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NUCLEAR BETA DECAY AND THE WEAK INTERACTION

(NUCLEAR PHYSICS READING COURSE NOTES - 1973-74)

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NUCLEAR BETA DECAY AND THE WEAK INTERACTION.

These notes were used as the basis for six weekly 90-minute tutorial sessions which formed part of the 1973-74 Nuclear Physics reading course. Sessions took the form of a discussion by approximately five students (about the optimum number), the identity of the student to lead the discussion each week being determined by lottery at the beginning of the session.

The intention of the course was to give students (and the author of these notes) an introduction to nuclear beta decay and the weak interaction. There appears to be no textbook available which gives a suitable treatment of the latter, and so most of these notes are concerned with that section of the course.

D.C. KEAN.

OUTLINE OF COURSE

Section I. Nuclear Beta-Decay

I.1 Introduction, Instrumentation, Simple Fermi Theory, Kurie Plots (from Enge 11-1 to 11-4)

I.2 ft Values and Selection Rules (from Marmier and Sheldon 8.4, 8.4.1, and Cohen 11-3)

I.3 The 4-Fermion Interaction (from notes)
Matrix Elements (from notes and Cohen 11-4)

I.4 Electron Capture (from Enge 11-7)

I.5 Neutrinos (from Enge 11-9, Marmier & Sheldon 8.2.2, Enge 11-9, 11-10, and notes)

Section II. The Weak Interaction

II.1 Lee and Yang's experiment on parity violation (from Enge 11-8)

II.2 Scalars, Pseudo-scalars, Vectors, Pseudo-Vectors, and more discussion of Lee and Yang's experiment (from notes)

II.3 General Form of the Weak Interaction (from notes)

II.4 Why is the Weak Interaction called the Weak Interaction? Coupling Constants (from notes)

II.5 Four-Vectors and Four-Currents. Current-Current Formalism of the Weak Interaction. Cabibbo's Hypothesis. Leptonic, Semi-leptonic and Non-leptonic Processes (from notes)

II.6 The Intermediate Vector Boson, The Universal Fermi Interaction, Conserved Vector Current Theory, Second-Class Currents (from notes)

References: Harald Enge, Introduction to Nuclear Physics (Addison-Wesley)
Pierre Marmier and Eric Sheldon, Physics of Nuclei and Particles (Academic Press)
Bernard Cohen, Concepts of Nuclear Physics (McGraw-Hill)

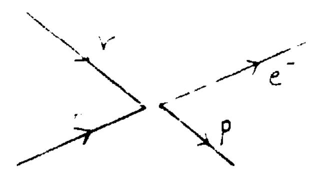
Section I. Nuclear Beta-Decay

I.1 Enge 11-1, Introduction to Beta Decay
11-2, Instruments of Beta Spectroscopy.
[The above sections were not discussed, except for specific questions, in the tutorial sessions.]

Enge 11-3, Simple Theory of Beta Decay
11-4, Kurie Plots.

I.2 Marmier and Sheldon 8.4, Classification of Beta Transitions,
8.4.1, Degrees of Forbiddenness.
Cohen 11-3, Selection Rules.

I.3 Note on Beta Decay in Terms of a 4-fermion Vertex interaction.



Beta decay (e.g. of a neutron, as pictured above) proceeds via the beta-neutrino field which consists of an infinite number of negative energy, unobservable electrons and neutrinos pervading all of space. A β^- decay is then an exchange of energy between a nucleus and that field in which the nucleus contributes energy by changing a neutron into a proton, and this energy excites the field by changing a neutrino into an electron; the missing neutrino behaves physically as an antineutrino. Conversely, in a β^+ decay the nucleus converts a proton into a neutron, and the energy given to the field converts an electron into a neutrino, with the missing electron behaving as a positron. It is often more convenient in discussions to ignore the difference between particles and antiparticles and think of an electron and a neutrino as being created in the process.
[The above note is useful for understanding the next reading section.]
Cohen 11-4, Matrix Elements in Beta Decay (to line 8, page 279, only).

Derivation of Fermi Matrix Element for Super-Allowed Transition

For single nucleon decay (n→p)

$$M_F = \int_{\text{volume of nucleus}} \psi_p^*(r)\, t_-\, \psi_n(r)\, d\tau$$

t_- is the operator which transforms a neutron into a proton.

For decay of whole nucleus

$$|M_F|^2 = \sum_{\text{all final substates } M_f} |M_F|^2 \text{ to substate } M_f$$

$$= \sum_{M_f} |\int \Psi_f^* \sum_i t_-^i \Psi_i\, d\tau|^2$$

$$= \sum_{M_f} |\int \Psi_f^* T_- \Psi_i\, d\tau|^2$$

where the isospin lowering operator T_- satisfies (see any text on isospin)

$$T_- \phi_T^{T_z} = \sqrt{T(T+1) - T_z(T_z-1)}\ \phi_T^{T_z-1}$$

Suppose we have a superallowed transition.

i.e. $\Psi_f = \Psi_i$ except that $T_{Zf} = T_{Zi} - 1$

$$|M_F|^2 = \sum_{M_f} |\int \Psi_f \sqrt{T(T+1) - T_z(T_z-1)}\ \Psi_f\, d\tau|^2$$

$$= T(T+1) - T_Z(T_Z-1) \quad \text{since } M_f = M_i$$

Derivation of Gamow-Teller Matrix Element for Super-Allowed Transition

For a single nucleon decay,

$$M_{GT} = \int \psi_p^*(r)\, \sigma\, t_-\, \psi_n(r) d\tau$$

where σ is the Pauli spin matrix. M_{GT} is a vector, with components corresponding to the components of σ.

For the decay of a whole nucleus,

$$|M_{GT}|^2 = \sum_{M_f} \sum_x |\int \Psi_f^* \sum_i t_-^i \sigma_x^i \Psi_i\, d\tau|^2$$

From the above, H_{GT} changes not only T_z but also spin, and some nuclear forces are not spin independent, there is little overlap between $H_{GT}\Psi_i$ and Ψ_f even for transitions between analogs. G.T. transitions are weaker than Fermi transitions.

Considering transitions between analogs, $j_f = j_i$ (these are the angular momenta of the neutron and the proton into which it decays within the nucleus; we are making the approximation that only a single nucleon is involved, this being an approximation which is not required for the Fermi transition treated above). Suppose also $j = \ell + 1/2$ (a similar result can be obtained for $j = \ell - 1/2$). Consider (arbitrarily) case where $m_j = j$ (e.g. $j = 1/2$) in initial state.

$$|M_{GT}|^2 = \sum_{M_f} \sum_x |\int \psi_i^{m_j *} \sum_i t_-^i \sigma_x^i \psi_i^{m_j = j}\, d\tau|^2 \qquad \text{(A)}$$

Now $\sigma_x \psi_j^j = \tfrac{1}{2}(\sigma_+ + \sigma_-)\psi_j^j = \tfrac{1}{2}\sigma_- \psi_j^j$ (see NOTE below)

since $\sigma_+ \psi_j^{m_j} = (\quad)\ \psi_j^{m_j+1} + (\quad)\ \psi_{j-1}^{m_j+1}$

And σ_- has the property

$$\sigma_- \psi_{\ell+1/2}^{m+1/2} = \frac{2}{2\ell+1}\left[\sqrt{(\ell+1)^2 - m^2}\ \psi_{\ell+1/2}^{m-1/2} - \sqrt{(\ell+m)(\ell+m+1)}\ \psi_{\ell-1/2}^{m-1/2}\right.$$

$$\tfrac{1}{2}\ \sigma_- \psi_j^j = \frac{\sqrt{2j}}{2j}\psi_j^{j-1} + (\quad)\ \psi_{j-1}^{j-1}, \quad \text{using } \ell = j - \tfrac{1}{2}$$

(the 2nd term will disappear in the matrix element since it is orthogonal to ψ_j, which is the final state.

NOTE $\sigma_+ = \sigma_x + i\sigma_y$ rather than the more common $\tfrac{1}{2}(\sigma_x + i\sigma_y)$

$\sigma_- = \sigma_x - i\sigma_y$ " " " " " $\tfrac{1}{2}(\sigma_x - i\sigma_y)$.

Similarly $$\sigma_y \psi_j^j = \frac{1}{2i} (\sigma_+ - \sigma_-) \psi_j^j$$

$$= -\frac{1}{i} \frac{\sqrt{2j}}{2j} \psi_j^{j-1} + (\quad) \psi_{j-1}^{j-1}$$

Also $$\sigma_z \psi_j^j = \frac{2}{2j} \times j \psi_j^j + (\quad) \psi_{j-1}^j$$

$$= \psi_j^j + (\quad) \psi_{j-1}^j$$

since $$\sigma_z \psi_{\ell+\frac{1}{2}}^{m+\frac{1}{2}} = \frac{2}{2\ell+1} \left[(m+\tfrac{1}{2}) \psi_{\ell+\frac{1}{2}}^{m+\frac{1}{2}} + \sqrt{\ell(\ell+1) - m(m+1)} \psi_{\ell-\frac{1}{2}}^{m+\frac{1}{2}} \right]$$

From eq (A)

from σ_z term from σ_x term

$$|M_{GT}|^2 = |\int \psi_j^{*j} t_- \psi_j^j d\tau|^2 + |\int \psi_j^{*j-1} t_- \frac{1}{\sqrt{2j}} \psi_j^{j-1} d\tau|^2$$

$$+ |\int \psi_j^{*j-1} t_- (-\tfrac{1}{i}) \frac{1}{\sqrt{2j}} \psi_j^{j-1} d$$

from σ_y term

$$= 1 + |\frac{1}{\sqrt{2j}}|^2 + |-\frac{1}{i} \frac{1}{\sqrt{2j}}|^2 = 1 + \frac{1}{j} = \frac{j+1}{j}$$

For $j = \ell - \frac{1}{2}$, one obtains $|M_{GT}|^2 = \frac{j}{j+1}$

Example: $^3He \rightleftarrows {}^3H$

Assume decay consists of an $s_{1/2}$ proton decaying to an $s_{1/2}$ neutron.
Thus $j_f = j_i = \frac{1}{2} = \ell + \frac{1}{2}$ since $\ell = 0$.
Initial substates $m_j = \pm \frac{1}{2}$ are equally probable, and matrix element for $m_i = j$ must be same as for $m_i = -j$.
Specialization to $m = j$ in above treatment doesn't matter.
Get $|M_{GT}|^2 = 3$. The experimental value is 3.8.
Only for $n \to p$ is the above single-particle treatment precisely correct.

Comparative Half-Lives in Mixed Transitions

Enge eq.(11-37) gives, for a transition via a single interaction characterized by a coupling constant G

$$f(Z,E_o)t_{1/2} = \frac{2\pi^3 \log_e 2 \hbar^7}{m_o^5 G^2 c^4 |M_{if}|^2} \qquad (1)$$

[This enables us to get G_V, the coupling constant for Fermi decays from the experimental f t for superallowed transitions, for which $|M_{if}|^2$ is calculable, e.g.

2 for $^{14}O \to {}^{14}N$, 1 for $^7Be \to {}^7Li$].

For a transition which proceeds by both F and G-T, we have instead of just $G^2 |M_{if}|^2$ in eq.(1)

$$G_V^2 |M_F|^2 + G_A^2 |M_{GT}|^2$$

Knowing G_V, M_F, M_{GT} we can obtain G_A from $ft_{1/2}$ for neutron decay. One finds

$$|G_A| \approx 1.2 |G_V| .$$

PROBLEMS.

1. a) In not more than three sentences, describe a practical method for the determination of the half-life of the neutron.

b) What is the half-life and Q-value for neutron decay?

c) What is the matrix element for the decay of the first excited state of Al^{26} to Mg^{26}?

d) The $ft_{1/2}$ value for the above Al^{26} decay is 3027 sec. Deduce a value for the Fermi coupling constant G_V (more commonly known as the vector coupling constant).

e) What is the nature (allowed, forbidden, Fermi, Gamow-Teller?) of the beta decay of He^6 ?

f) What is the theoretical value of the Gamow-Teller matrix element in neutron decay?

g) In one sentence, explain why you expect this theoretical G-T matrix element to be more accurate for neutron decay than, eg. for the G-T decay of tritium.

h) The $ft_{1/2}$ for neutron decay is (depending on who you believe) about 1080 sec. Using the value of G_V obtained in (d), deduce a value for G_A, the G-T coupling constant (more commonly known as the axial vector coupling constant), and hence of the ratio $\left|\frac{G_A}{G_V}\right|$.

I.4. Enge 11-7, Electron Capture.
[The above section was not discussed, except for specific questions, in the tutorial sessions].

I.5. Enge 11-9, Neutrinos.
(to end of 1st paragraph, page 335, only).

Marmier and Sheldon 8.2.2, Neutrino Hunting
(to end of 1st paragraph, page 341, only).

Enge 11-9, Muon- and Electron-Associated Neutrinos
(last 2 paragraphs only).

Enge 11-10, Measurement of Neutrino Helicity.

NOTE ON RECOIL EFFECTS IN NEUTRINO HELICITY EXPERIMENT
(Enge, page 337).

We saw before that on both emission and absorption (prior to resonant scattering) of the gamma ray, the nucleus recoils by amount $E_\gamma^2/2Mc^2$. Therefore the total amount of energy going into recoil is

$$\frac{2 \times E_\gamma^2}{2Mc^2} \quad . \qquad (1)$$

It appears that if the neutrino had energy E_ν, there is recoil (towards the scatterer for neutrinos upwards) of $\frac{E_\nu^2}{2Mc^2}$. This is given to the gamma increasing its energy, but is only enough to half-compensate for the recoil energy (1). This last step is incorrect. $\frac{E_\nu}{2Mc^2}$ is given to the recoiling nucleus following neutrino emission, but it is not the energy given to the gamma.

The (Doppler) energy shift of the gamma is $E_\gamma \frac{v_R}{c} \cos\theta$

v_R = recoil velocity of nucleus following ν emission

θ = angle between v_R and direction of emission of gamma.

v_R is given by $M v_R = \frac{E_\nu}{c}$

$\therefore$ Energy shift of gamma due to ν emission is ($\theta = 0°$)

$$E_\gamma \frac{E_\nu}{Mc^2} \qquad (2)$$

This corrects for (1) if $E_\gamma = E_\nu$.

$\therefore$ one gets maximum resonant scattering when $E_\gamma = E_\nu$.

SECTION II. The Weak Interaction

II.1 Enge 11-8, Parity Violation in Beta Decay.
(Lee and Yang's Experiment).

II.2 Vectors and Pseudo-Vectors, Scalars and Pseudo-Scalars

(Further discussion of Lee and Yang's Experiment)

In the parity operation, when the coords of each point (x,y,z) are changed to (-x,-y,-z).

Polar vectors (or vectors) change sign
Axial vectors (or pseudo-vectors) don't change sign
Scalars don't change sign
Pseudo-scalars change sign

It is found that the β-decay matrix element is a sum of scalar and pseudo-scalar interactions (see II.3). The distinction is that a scalar does not change when the parity operation is performed - that is, when the coordinates of each point (x,y,z) are changed to (-x,-y,-z) - whereas a pseudo-scalar changes sign in this operation. An example of a scalar is the Coulomb interaction potential, or any other interaction potential that can be written $V(r)$. As the argument r is the magnitude of a vector $\underset{\sim}{r}$, V(r) does not change in the parity operation of reflection r through the origin, even though $\underset{\sim}{r} \to -\underset{\sim}{r}$ in this operation. Since the matrix element of V(r) is a certain average of this quantity, the matrix element is a scalar. In fact, with the exception of the beta decay interaction potential matrix element, the matrix elements of all the known interaction potentials of classical, atomic, and nuclear physics are scalars. For instance, consider the spin-orbit interaction potential matrix element which is proportional to $\underset{\sim}{S}.\underset{\sim}{L}$. We investigate what happens to $\underset{\sim}{S}.\underset{\sim}{L}$. in the parity operation by writing $\underset{\sim}{L} = \underset{\sim}{r} \times \underset{\sim}{p}$ and realizing that in this operation both $\underset{\sim}{r}$ and $\underset{\sim}{p}$ change sign.* Thus $\underset{\sim}{L}$ does not change sign. Neither does $\underset{\sim}{S}$. To bring out the distinction between their behaviour in the parity operation, vectors which change sign, such as $\underset{\sim}{r}$ and p, are called polar vectors, and vectors which do not change sign, such as L, are called axial vectors. All angular momentum vectors, including $\underset{\sim}{S}$, are axial vectors. Therefore we see that the spin-orbit

* p changes sign since m dx/dt→m d(-x)/dt = -m dx/dt, and similarly for the other momentum components.

interaction potential matrix element is a scalar because S.L does not change sign in the parity operation. An example of a pseudo-scalar is the quantity P.A, (or $\sigma \cdot P$ of neutrino) where P is a polar vector and A is an axial vector. This is a pseudo-scalar since it changes sign in the parity operation because P changes sign and A does not. The beta decay interaction potential matrix element contains quantities of this type. As the parity operation is equivalent to leaving the point (x,y,z) fixed and reflecting the coordinate axes through the origin, the operation amounts to changing from a right-handed to a left-handed coordinate system and, as at least part of the beta decay interaction potential matrix element changes sign in this operation, the description of the beta decay processes which this matrix element produces will depend on whether a right-handed or a left-handed coordinate system is used. When this was first discovered, it was extremely surprising because it had been a fundamental assumption that the description of all physical processes must be independent of whether a right-handed or left-handed coordinate system is used.

This assumption was proved incorrect for beta decay in experiments performed by Wu and collaborators (1957), at the suggestion of Lee and Yang (1956). In these experiments, which we describe first in a right-handed coordinate system, the spin angular momentum vectors I of electron emitting $_{27}Co^{60}$ nuclei were aligned with respect to an external magnetic field by using the hyperfine structure interaction at temperatures so low that the thermal equilibrium energy kT is comparable to the hyperfine splitting energy. In these circumstances, a majority of atoms will be in the lowest energy state, and in this state I is essentially aligned with respect to the applied magnetic field. The angle between the direction of I and the direction of the momentum vector p was then measured for each emitted electron, and the distribution of these angles was plotted. It was found that this distribution is asymmetric, with about a 30 percent preference for the emission of electrons whose momentum vectors p are in the opposite hemisphere from the spin vector I. So in the average emission process $\underset{\sim}{p}.\underset{\sim}{I} < 0$. But $\underset{\sim}{p}.\underset{\sim}{I}$ is a pseudo-scalar since $\underset{\sim}{p}$ is a polar vector and $\underset{\sim}{I}$ is an axial vector. Thus in any left-handed coordinate system the sign of this pseudo-scalar will change, in the average emission process $\underset{\sim}{p}.\underset{\sim}{I} > 0$, and the description of this process therefore depends on whether a right-handed or a left-handed coordinate system is used. This proves the beta decay interaction potential matrix element must be a pseudo-scalar or at least contain a pseudo-scalar part. If it were purely scalar the

the electron distribution would necessarily be symmetric so that in the average emission process $\underset{\sim}{p}.\underset{\sim}{I} = 0$, and the description of this process would be independent of whether a right-handed or a left-handed coordinate system is used, even though the pseudo-scalar p.I changes sign, because -0 = +0.

Since the beta-decay interaction potential matrix element is not a scalar, it is not true that for the interaction potential $V(-x,-y,-z) = V(x,y,z)$.

II.3. PARITY VIOLATION AND THE GENERAL FORM OF THE WEAK INTERACTION

Theoretically, the weak interaction Hamiltonian can be of the general form

$$H_W = C_S H_S + C_V H_V + C_A H_A + C_T H_T + C_P H_P \qquad (1)$$

S = Scalar

V = Vector

A = Axial Vector

T = Tensor

P = Pseudo-scalar, where the names apply to the matrix elements constructed from the lepton and nuclear wave functions (remember the GT matrix element was an (axial) vector?)

Of the above terms, scalar and vector give Fermi selection rules, axial vector and tensor give G.T. selection rules, and pseudo-scalar gives $\Delta J = 0$, $\Delta\pi$ = yes (see below).

From electron polarisation and electron-neutrino correlations, it is found that $C_S = C_T = 0$ (or at least very small). As discussed in the next section, the shapes of forbidden spectra indicate $C_P \simeq 0$.

$$H_W = C_V H_V + C_V' H_V' + C_A H_A + C_A' H_A'$$

where now we have split H_V and H_A into scalar and pseudo-scalar parts. [For detailed discussion see Preston, Physics of the Nucleus, page 394.)

When we now evaluate a transition probability, say for a pure G-T transition,

$$P \sim \left|\langle\alpha|C_A H_A + C_A' H_A'|\beta\rangle\right|^2$$
$$\sim C_A\langle\alpha|H_A|\beta\rangle^2 + C_A'\langle\alpha|H_A'|\beta\rangle^2 + C_A C_A' \langle\alpha|H_A|\beta\rangle\langle\beta|H_A'|\alpha\rangle$$
$$+ C_A C_A' \langle\alpha|H_A'|\beta\rangle\langle\beta|H_A|\alpha\rangle$$

Now the last two terms change sign when $\underset{\sim}{r} \to -\underset{\sim}{r}$, so the last term gives rise to an asymmetry (such as detected in Wu's ^{60}Co experiment) so that $P(\theta) \neq P(-\theta)$
Therefore provided C_A and C_A' are both non-zero, one gets an asymmetry, and hence one gets parity violation (i.e. non-invariance with respect to spatial reflection) in the ^{60}Co experiment. Note that the selection rules for H_V and H_V' are the same, likewise for H_A and H_A'.

It is in fact found that $C_A = C_A'$, $C_V = C_V'$, a situation called maximum parity violation. It turns out that $\langle\alpha|H_V\ H_V'|\beta\rangle$ is zero, so there is no asymmetry in a pure Fermi decay, even although $C_V' \neq 0$. It is found that in general, $C_A = -C_V$, so that the interaction is of form V - A, where V is the vector interaction and A is the axial vector. For nuclear β-decay, $C_A \neq -C_V$, because of the non-conservation of axial-vector current in the presence of strong interactions, as described later.

The Pseudo-Scalar Interaction.

If account is taken of a possible pseudo-scalar term in the weak interaction, the rules become (Lee-Whiting, Can. J. Phys. 36 (1958) 1199)

$$\underset{\sim}{I}_f = \underset{\sim}{I}_i + \underset{\sim}{L} + \underset{\sim}{S} \qquad (1)$$

$$\Pi_i = (-1)^{L+\eta}\ \Pi_f \qquad (2)$$

where $L + \eta$ is the forbiddenness of the transition, and
$\eta = 1$ for pseudoscalar, $\eta = 0$ otherwise.
The term η is included in the definition of forbiddenness because the matrix elements for $\eta = 1$ are expected to be smaller than those for $\eta = 0$ by a factor of about $\frac{v}{c}$, where v/c is some mean value for the nuclear states involved.

It is clear that there can be no allowed (forbiddenness = 0) transitions proceeding via the pseudoscalar interaction. Also from (2)

the pseudoscalar interaction cannot induce transitions between spin zero states of the same parity.

As we saw in the earlier discussion of selection rules there is a complex hierarchy of forbidden transitions corresponding to increasing values of L and contributions from relativistic correction terms. Allowance for a pseudoscalar component in the weak Hamiltonian introduces another source of forbidden transitions, about a factor $(v/c)^2$ weaker than transitions proceeding via the vector or axial vector interactions with the same L.

The pseudoscalar component of the weak interaction is difficult to detect from its effect on selection rules and hence on spectrum shapes. The selection rules and magnitude of pseudoscalar matrix elements are such that this interaction would be noticeable only in $\Delta J = 0$, $\Delta\pi$ = yes transitions, and then only with great difficulty. The problem is that the pseudoscalar operator can contribute only in $\Delta J = 0$ first forbidden and in higher order transitions. In such transitions ambiguities in interpretation arise both from the need in the calculations to go to a non-relativistic form for a relativistic operator and also because other contributions can arise from the main axial vector interaction. Considerable effort has gone into the $0^- \to 0^+$ ^{144}Pr decay, without conclusive results (Krinpotic and Tadic, Phys. Rev. 178 (1969) 1804).

NOTE. The above discussion is concerned with a possible non-zero term $C_p H_p$ in eq. (1), where H_p is the pseudoscalar interaction. It is not concerned with the pseudo-scalar parts of the other interactions, e.g. H_V', H_A', which have no effect on selection rules.

II.4 INTERACTION STRENGTHS AND COUPLING CONSTANTS

(Supplementary material, not discussed in tutorials.)

Comparison of Strengths of Different Interactions

The strength of the nuclear interaction in a typical nucleus is ~10 MeV (separation energy of nucleon constant at ~8 MeV). The electrostatic energy of 2 protons at a similar internucleon distance (say 2 fm) is 0.7 MeV. The interaction of dipole moments gives a magnetic potential energy ~ 0.03 MeV. The gravitational potential energy is ~10^{-38} MeV. To make a comparison with the weak interaction (which is the absence of the intermediate vector boson is zero range, sometimes called a contact interaction) we can use a typical transition matrix element, e.g. $n \rightarrow p$.

Enge eq. 11-20, p. 315 gives, from simple Fermi theory,

$$|H_{if}| \sim \frac{G}{V} |M_{if}|$$

Setting the overlap integral equal to 1, and V ~ nuclear volume $(5 \times 10^{-13})^3$
$G = 10^{-49}$ erg cm^3

$$H_{if} = \frac{10^{-49}}{1.6 \times 10^{-6} \times (5 \times 10^{-3})^3} \doteq 10^{-5} \text{ MeV.}$$

Coupling Constants

The most well-known of the coupling constants in that for the electromagnetic interaction, commonly known as the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$$

This appears in the expression for the atomic spin-orbit interaction which produces fine-structure splitting,

$$\frac{\Delta E_{\underset{\sim}{S}\cdot\underset{\sim}{L}}}{E_{\text{unperturbed}}} = \alpha^2 f(\ell, \text{ quantum nos})$$

This expression involves α to the same power as the order of the process; ΔE must depend both on the magnitude of the electron magnetic moment and the magnitude of the atomic magnetic field. The Compton scattering cross-section also depends on α^2, since the photon is absorbed then re-emitted.

So does the mean life for 2-quanta electron-positron annihilation.

It is straight-forward to construct an analogous dimensionless coupling constant for the strong interaction by replacing e, the electronic charge, by g, the coupling constant of the pion exchange potential. This substitution is valid because e and g have the same dimensions

[Yukawa potential $V(r) = \frac{-g^2}{r} e^{-r/R}$

Coulomb potential $V(r) = \frac{e^2}{r}$]

The precise value of g obviously depends on the functional forms which it multiplies in the pion-exchange potential, ie. it depends on the specific form of the interaction. The result is

$$0.1 \leq \frac{g^2}{\hbar c} \leq 15$$

A compromise value is $g^2/_{\hbar c} \sim 1$.

The construction of a dimensionless coupling constant for the weak interaction is rendered less straightforward, and more arbitrary, by the fact that the numerical constant which is characteristic of the weak interaction, namely the Fermi coupling constant G_F, is dimensionally different from e and g

$$G_F = 1.4 \times 10^{-49} \text{erg cm}^3$$

$$\therefore \frac{G_F^2}{\hbar c} \text{ has dimensions } \frac{\text{erg}^2 \text{ cm}^6}{\text{erg cm}}$$

To get a dimensionless quantity, use

$$\frac{G_F^2}{(\hbar c)^2 (\text{length})^4}$$

A suitable quantity for the length might appear to be the range of the strong nuclear force, but this could be criticised on the grounds that it is not known that the weak interaction is of the same form or range as the strong force. The quantity taken as the fundamental length is the Compton wavelength of the pion

$$(\lambda_c)_\pi = \frac{\hbar}{m_\pi c} = 1.413 \text{ fm},$$

since this is of the same order of magnitude as the Compton wavelength of other fundamental particles. Insertion of this value gives a dimensionless constant

$$\frac{G_F^2}{(\hbar c)^2} \left(\frac{m_\pi c}{\hbar}\right)^4 \sim 5 \times 10^{-14}$$

The Newtonion gravitational constant $G = 6.67 \times 10^{-8}$ erg cm gm^{-2}
With (lack of) logic similar to that described above for the weak interaction, a dimensionless constant can be constructed

$$\frac{G\, m_N^2}{\hbar c} \simeq 2 \times 10^{-39}$$

where m_N is the mass of a nucleon.

Interaction	Coupling Constant
Nuclear(Strong)Interaction	1
E - M Interaction	1/137
Weak	10^{-14}
Gravitational	10^{-39}

II.5. FOUR-VECTORS

A vector with four components V_μ is a four-vector if its components satisfy the Lorentz transformation eg. an event $E(x,y,z,t)$ is linked to an event $E'(x',y',z',t')$ by the Lorentz transformation

$$s^2 = x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c^2 t'^2$$

The four-vector in this case is

$$x_\mu = (x_1, x_2, x_3, x_4)$$

where
$x_1 = x$
$x_2 = y$
$x_3 = z$
$x_4 = ict$

Other examples are $P_\mu = (P_x, P_y, P_z, \frac{iE}{c})$

$$A_\mu = (A_x, A_y, A_z, \frac{i\phi}{c})$$

In relativistic electromagnetic theory one uses the four-current density.

$$j_\mu = (j_1, j_2, j_3, j_4)$$

where $j_1 = j_x,\ j_2 = j_y,\ j_3 = j_z,\ j_4 = ic\rho$

and ρ = charge density.

$$\therefore \quad \frac{\partial j_\mu}{\partial x_\mu} \left(= \sum_\mu \frac{\partial j_\mu}{\partial x_\mu} \right) = \text{div}\, \underset{\sim}{j} + \frac{\partial j_4}{\partial x_4} = \text{div}\, \underset{\sim}{j} + \frac{\partial \rho}{\partial t}$$

$\therefore$ The equation for conservation of charge

$$\text{div}\, \underset{\sim}{j} + \frac{\partial \rho}{\partial t} = 0$$

becomes, in four-vector notation, conservation of four-current density

$$\frac{\partial j_\mu}{\partial x_\mu} = 0$$

Thus electromagnetic four-current is conserved.

Treatment of four-vectors in matrix notation

$$x_\mu^1 \xrightarrow{\text{L.T.}} x_\mu$$

$$\Longrightarrow x_\mu^1 = \sum_{r=1}^{4} a_{\mu r} x_r \qquad 1.$$

where $$(a_{\mu r}) = \begin{pmatrix} \gamma & 0 & 0 & \frac{iv}{c}\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{iv}{c}\gamma & 0 & 0 & \gamma \end{pmatrix}$$

This is the Lorentz Transformation matrix. It is unitary. From 1. a four-vector is a first-rank tensor in four-space.

CURRENT-CURRENT FORMALISM OF THE WEAK INTERACTION

Refs: Blin-Stoyle and Nair, Adv. in Physics 15 (1966) 493-545
Blin-Stoyle, Fundamental Interactions and the Nucleus (North-Holland, 1973)
Blin-Stoyle, Wilkinson, in Nature Sept. 18 1975.

The current-current formalism of the weak interaction is based on strong analogies with electromagnetism. Classically, the potential energy of a current element in the field of another is given by

$$W_{12} = (\text{const}) \frac{\underset{\sim}{j_1} \cdot \underset{\sim}{j_2}}{r} \, d\tau_1 \, d\tau_2$$

where j_1 and j_2 are the corresponding current densities.

The current-current formalism of the weak interaction sets up for all weak interaction processes, an energy of interaction in the form of a coupling constant times a product of two currents (or rather the product of current with itself), however these currents are four vectors, as discussed previously.

The general weak interaction is written

$$H_W = \frac{G}{2\sqrt{2}} \{\mathcal{J},\mathcal{J}^*\}_+ = \frac{G}{2\sqrt{2}} [\mathcal{J}\mathcal{J}^* + \mathcal{J}^*\mathcal{J}]$$

Note that there is no implication at this stage that G has the same value for all types of weak interaction processes.

The weak four-vector current $\mathcal{J}$ can be decomposed as follows -

$$\mathcal{J} = J + S + j \qquad (1)$$

(usually these currents have a subscript λ to denote the component λ of the four vector; it has been omitted here for the sake of clarity).
J is the strangeness-conserving hadronic current (a hadron is any particle which interacts via the strong interaction)
and can itself be decomposed into polar and axial vector parts

$$J = J^{(V)} + J^{(A)}$$

Because of the strong interactions experienced by hadrons, it is not possible in general to write down an explicit form for J with any confidence. However, in spite of this, it is possible go make considerable progress.

S is the strangeness-changing hadronic current, which can similarly be written as the sum of vector and pseudo-vector parts $S^{(V)}$, $S^{(A)}$.

j is the leptonic current (the leptons are $e^{\pm}$, $\mu^{\pm}$, ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$) which can be decomposed into electronic and muonic currents.

$$j = j_e + j_\mu$$

The form of the lepton current can be written down explicitly (Blin-Stoyle and Nair, eq (5)) from the 2-component theory of the neutrino. It consists of equally strong polar and axial vector parts.

In 1963 it was suggested by Cabibbo (P.R.L. 10, 531) that eq (1) for the weak current should be modified so that the hadronic currents J,S are not coupled equally -

$$\mathcal{J} = J\cos\theta + S\sin\theta + j \qquad (2)$$

where θ is called the Cabibbo angle. The consequences of this hypothesis will become apparent later.

Examples of Weak Interaction Processes

1) Pure leptonic processes eg. $\mu^- \rightarrow e^- + \tilde{\nu}_e + \nu_\mu$

$$H_W = -\frac{G_\mu}{2\sqrt{2}}\{j,j^*\}_+ \quad \text{where } j = j_e + j_\mu$$

Thus from muon decay it is possible to obtain a value for the coupling constant $G_\mu = 1.4354 \pm 0.0003 \times 10^{-49}$ ergcm³ (Nucl. Phys. B29 (1971) 296).

2) Semi-leptonic interactions.

On the basis of eq (2), two separate parts can be identified as responsible for semi-leptonic processes. These are

$$H_W^{(s.L,\Delta S=0)} = -\frac{G}{2\sqrt{2}}\{\cos\theta J + j,\ \cos\theta J^* + j^*\}_+$$

$$= -\frac{G}{2\sqrt{2}}[\cos\theta J j^* + \cos\theta J^* j + \cos\theta J^* j + \cos\theta J j^*]$$

keeping only leptonic-hadronic terms

$$= -\frac{G}{\sqrt{2}}\cos\theta\{J,j^*\}_+$$

and similarly $H_W^{(s.L,\Delta S=1)} = -\frac{G}{\sqrt{2}}\sin\theta\{S,j^*\}_+$

The former is responsible for strangeness conserving semi-leptonic processes (eg. nuclear β decay, muon capture), the latter for the decays of K, Λ, Σ muons.

If θ is a small angle, as turns out to be the case, ΔS = 1 decays are relatively weak. The β-decay Hamiltonian can be written

$$H_\beta = -\frac{G_\beta}{\sqrt{2}}\{J,j_e^*\}_+ = -\frac{G_\beta^{(V)}}{\sqrt{2}}\{J^{(V)},j_e^*\} - \frac{G_\beta^{(A)}}{\sqrt{2}}\{J^{(A)},j_e^*\}$$

Note that Cabibbo's hypothesis changes the relation between G and the coupling constant for β decay, so that $G_\beta = G\cos\theta$.

3) Non-leptonic interactions

As in the previous section these can be split into two parts depending on whether strangeness is conserved or not.

$$H_W = -\frac{G}{2\sqrt{2}}\{\cos\theta J + \sin\theta S,\ \cos\theta J^* + \sin\theta S^*\}$$

$$= -\frac{G}{2\sqrt{2}}[\cos^2\theta JJ^* + \sin^2\theta SS^* + \sin\theta\cos\theta SJ^* + \sin\theta\cos\theta JS^* + \cos^2\theta JJ^* + \sin^2\theta SS^* + \sin\theta\cos\theta SJ^* + \sin\theta\cos\theta JS^*]$$

$$= -\frac{G}{2\sqrt{2}}[\cos^2\theta\{J,J^*\}_+ + \sin^2\theta\{S,S^*\}_+]$$

$$-\frac{G}{2\sqrt{2}}[\sin\theta\cos\theta\{S,J^*\}_+ + \sin\theta\cos\theta\{J,S^*\}]$$

The first term is a strangeness-conserving interaction, the second part is strangeness-nonconserving. If θ is a small angle, $\Delta S = 1$ decays will be relatively weak.

Effects generated by the $\Delta S = 0$ part are virtually impossible to observe since they form a small ($\leq 10^{-6}$) background in what are overwhelmingly strong interaction processes. However its effect can be studied in nuclear phenomena where this part of the interaction manifests itself as a weak parity violating internucleon potential (eg. α-particle decay of the 2^- 8.88 MeV state in O^{16} to the ground state of C^{12} (Hattig, Munchen and Waffler, P.R.L. 25 (1970) 941).

The $\Delta S = \pm 1$ part is repsonsible for many weak non-leptonic decay processes of elementary particles (eg. $\Lambda \rightarrow N\pi$, $\Sigma \rightarrow N\pi$, $K \rightarrow 2\pi, 3\pi$).

II.6. Intermediate Boson or Weakon

It has been shown by Feynman and Gell-Mann (P.R. 109 (1958) 193) that the absence of the reactions

$$\mu^{\pm} \rightarrow e^{\pm} + e^{+} + e^{-} \qquad (1)$$

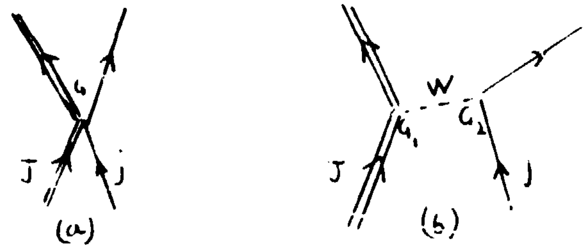
or the neutrinoless conversion of μ^- into e^- in the proximity of nuclei,

$$\mu^{-} + N \rightarrow e^{-} + N \qquad (2)$$

which cannot be explained by any known selection rules, could be explained if weak interactions were transmitted by a charge particle. This hypothetical particle has been named the intermediate boson, or weakon. If it exists, the weak interaction coupling of fermions would become similar to their electromagnetic coupling. The weakon would have the following properties: spin 1 and undefined parity
non-zero electric charge
mass $>$ mass of K-meson (500 MeV) to account for stability of K against $K^{\pm} \rightarrow W^{\pm} + \gamma$

[An alternative explanation for the absence of reactions (1) and (2) above is that muon-like leptons (meptons) must be conserved and electron-like leptons (eptons) must also be conserved
meptons $= \mu, \nu_{\mu}$
eptons $= e, \nu_e$]

With the condition that $m_W > m_K$, the two diagrams below give similar results for all reactions of interest



The matrix element from (b) contains an extra propagator factor $(k_W^2 - m_W^2)^{-1}$ in which k_W is the four-momentum transfer between the two fermion pairs. Thus as long as $k_W^2 << m_W^2$ this factor is practically constant and the effects of the "non-locality" of the interaction are very small. In nuclear β-decay $k_W \simeq$ MeV: somewhat larger effects can be expected for μ decay, where $k_W \sim 100$ MeV.

If the interaction at the vertex in fig (a) above has a coupling constant G, then the "weak charge" at each vertex of (b) is not simply $\sqrt{G}$ because of the possible difference between the coupling of the weakon to hadrons and leptons. If the couplings at the two vertices are G_1, G_2 then it can be shown that

$$G = \frac{\sqrt{2}\, G_1 G_2}{m_W^2} \qquad m_W = \text{mass of weakon.}$$

The Universal Fermi (Vector) Interaction

So far we have said nothing about the relative values obtained for the coupling constant G for the various types of weak interaction processes.

If values of the vector coupling constant G_V are extracted from the observed rates of weak processes, very similar values are obtained for purely leptonic processes such as muon decay, for semi-leptonic processes such as nuclear beta decay, muon capture and $\Sigma \rightarrow \Lambda$ decay and for the strangeness changing semi-leptonic and non-leptonic decays of K-mesons and hyperons. From these $\Delta S = 1$ decays it is found that $\sin\theta = 0.221 \pm 0.004$ where θ is the Cabibbo angle. (Historically it was of course the experimental values of the coupling constants which prompted Cabibbo to suggest on theoretical grounds the ($\cos\theta$, $\sin\theta$) weighting of the $\Delta S = 0$ and $\Delta S = 1$ hadronic currents J and S).

The experimental data is most precise for nuclear beta decay and muon decay. From the ft values of superallowed $0^+ \rightarrow 0^+$ pure Fermi beta-decays it is found that $G_\beta^{(V)}$ is constant for all such transitions and

$$G_\beta^{(V)} \left(1 + \frac{\alpha C}{4\pi}\right) = (1.4150 \pm 0.0011) \times 10^{-49} \text{erg cm}^3$$

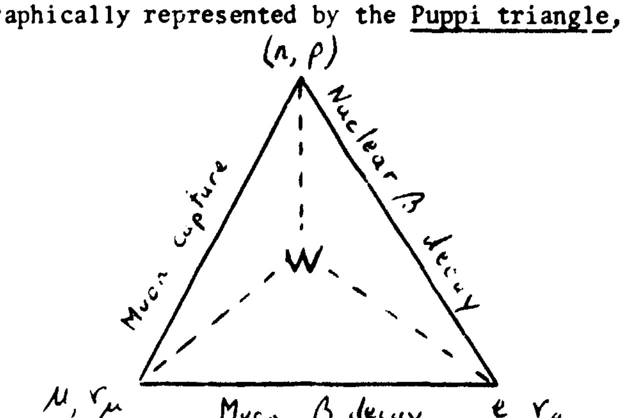
while from the half life of the muon,

$$G_\mu^{(V)} = (1.4343 \pm 0.0007) \times 10^{-49} \text{ erg cm}^3$$

$\frac{\alpha C}{4\pi}$ is a model-dependent, electromagnetic radiative correction term. The above values of $G_\beta^{(V)}$ and $G_\mu^{(V)}$ are consistent with $G_\beta^{(V)} = G_\mu^{(V)} \cos\theta$, provided $\frac{\alpha C}{4\pi} = 2.3 \pm 0.3\%$. This value of $\frac{\alpha C}{4\pi}$ is consistent with theoretical calculations which assume the existence of an intermediate boson (with mass $m_W > 20$ nucleon masses). The present weight of opinion (ie. there is some disagreement on the subject) is that the required value of $\frac{\alpha C}{4\pi}$ cannot be reconciled with theories which do not require an intermediate boson.

The need for small model-dependent corrections such as that described above (another obvious, although less important one is the coulomb correction to $|M|^2 = 2$ for $0^+ \rightarrow 0^+$ superallowed decays) limits the accuracy with which one can extract the coupling constants from the experimental data. However all the data are consistent with G_V being a constant for all weak interactions, supporting the theory of a universal Fermi interaction as first suggested by Feynman and Gell-Mann (Phys. Rev. 109 (1958) 193). Conversely, if the constancy of G_V is accepted then the experimental data is now sufficiently accurate and the correction terms other than $\frac{\alpha C}{4\pi}$ can now be calculated sufficiently accurately, that $\frac{\alpha C}{4\pi}$ can be evaluated with sufficient accuracy to distinguish between models of the weak interaction with and without an intermediate boson.

The universal Fermi interaction for leptonic and semi-leptonic processes can be graphically represented by the Puppi triangle, as shown below



Fermions at the vertices of the triangle interact via the univeral Fermi interaction. In the centre is the intermediate boson W presumed to act as the field particle of a weak-interaction exchange force. The triangle may be extended to a tetrahedron, with an extra vertex of positive strangeness, to include the decay of strange particles.

Conserved Vector Current (CVC) Theory

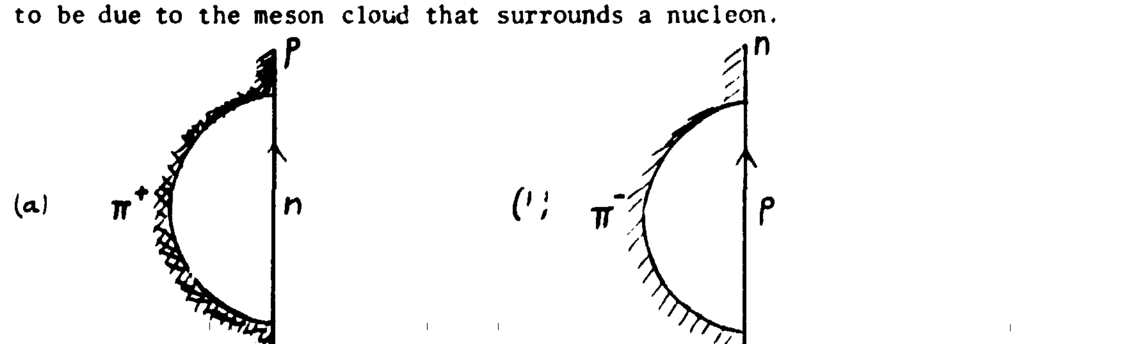
The equality of the vector coupling constants for different weak interaction processes seems to be in satisfactory and straight-forward agreement with the idea of a universal Fermi interaction between fermions. However, a little more thought indicates that there is more interesting physics in the situation than first appears.

It would be natural to assume that the idea of universal Fermi interaction applies to "bare" fermions, ie. those which, like the the muon and electron, satisfy the point-charge, no anomalous magnetic moment Dirac equation or, in other words, that are not surrounded by complicated meson "clouds" of strongly coupled fields.

It should be the bare nucleons and not the physical nucleons, that are vector-coupled with a constant equal to $G_V^{(\mu)}$. The observed coupling of the "physical" neutron should be smaller than for a bare neutron, since the "physical" neutron is "part time" decomposed into $p + \pi^-$, and the proton cannot emit β^-.

But it is found experimentally that it is the "physical" nucleon which is vector coupled like the muon. This is a new and significant fact, not necessarily included in the idea of univeral Fermi interaction.

We can observe in this connection that the vector coupling of the physical proton and the muon are identical for the electromagnetic field: this is because electromagnetic charge is conserved in the virtual reaction $p \leftrightarrows n + \pi^+$. and the pion carries the charge that the proton has lost. However, the tensor (anomalous magnetic moment) coupling of the physical proton to the electromagnetic field is different from that of the muon: this is known to be due to the meson cloud that surrounds a nucleon.



In the figure (a) above, the electric charge (cross-shading) remains constant in the virtual decomposition of the proton : the vector coupling constant does not depend on the presence of the meson cloud : (b) the weak interaction vector coupling (diagonal shading) also remains the same despite the virtual decomposition of the neutron (from De Benedetti, Nuclear Interactions). This implies that the pion can interact via the weak field · in fact it does β decay ($\pi \to e + \nu$, $\pi \to \mu + \nu$).

Thus the behaviour of the electromagnetic field and of the weak field in this respect are similar : in both there is vector current conservation. But the conservation does not extend to other modes of coupling. Presumably the interaction with mesons which produce an anomalous magnetic moment also accounts for the fact that it is found that the weak axial vector current, responsible for nuclear G.-T. transitions, the axial vector decay of the muon, etc. is not conserved.

Whereas muon decay data is, like nuclear beta decay, unambiguously described by the V-A interaction, it requires $G_A^{\mu} = - G_V^{\mu}$ to a high degree of accuracy, whereas $G_A^{\beta} = -(1.24 \pm 0.03)G_V^{\beta}$. In the jargon of the theorists, the effect of the virtual meson cloud is to "renormalise" the axial vector interaction. Thus G_A varies with the meson field, so that unlike G_V, it would be expected to vary from nuclide to nuclide in nuclear β-decay. However, since G_A for neutron decay is only 20% different from that found (in muon decay) in the complete absence of a pion field, we would not expect great changes in G_A from nuclide to nuclide. In any case clarification of the matter is inextricably bound up with accurate determination of G.T. matrix elements.

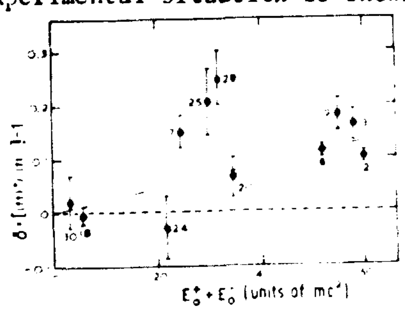
Second-Class Currents in Nuclear β-Decay.

In addition to "renormalising" the axial vector coupling constant G_A, strong interactions may also impose induced scalar (G_{IS}) pseudoscalar (G_{IP}) and tensor (G_{IT}) terms on the underlying presumably universal V-A weak interaction. It can be shown that CVC says $G_{IS} = 0$, but G_{IP} and G_{IT} are associated with G_A, about which CVC says nothing. But PCAC (Partially Conserved Axial-vector Current) theory specifies a magnitude for G_{IP} which is not in disagreement with experimental data on muon capture. The value of G_{IT} is an open question : it might be expected to have value zero or about $\frac{G_V}{2000}$ depending whether the weak interaction is invariant under the product of charge symmetry and charge conjugation (G-parity).

Because the interference between axial-vector and induced tensor has opposite sign for β^+ and β^- emission, it may be possible to investigate G_{IT} from the relative ft values of axial vector (G.T.) mirror transitions (eg. $^{25}Na \to Mg^{25} + \beta^- + \bar{\nu}$, $^{25}Si \to Mg^{25} + \beta^+ + \nu$). If it is possible to correct for other small effects (eg. coulomb), any difference between the values ft_- and ft_+ for such decays would be due to non-zero G_{IT}. Huffaker and Greuling (P.R. 132 (1963) 738) have shown that

$$\delta = \frac{ft_+}{ft_-} - 1 \approx \frac{4}{3} \left|\frac{G_V}{G_A}\right| \frac{G_{IT}}{G_V} (W_o^+ + W_o^-)$$

where W_o^+ and W_o^- are the end point energies (in units of electron masses). The experimental situation is shown below.



The asymmetry $\delta = [(ft)^+/(ft)^-] - 1$ in mirror axial vector β-decays. The sloping straight line corresponds to $f_T = 2 \times 10^{-3}$.

Studies of $\frac{ft_+}{ft_-}$ ratios and of the constancy of G_V for superallowed $0^+ \to 0^+$ decays, (see eg. P.R.L. 26 (1971) 1127, Nucl. Phys. A223 (1974) 157) are among the most interesting areas of experimental nuclear physics at the present time, primarily because there is an unusually direct connection between experimental results and the properties of one of the fundamental interactions.