INSTITUTE OF PLASMA PHYSICS
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I. DIRECT CURRENTS PRODUCED BY HF HEATING OF PLASMA

II. ON THE ANGULAR MOMENTUM TRANSFERRED DURING ELECTROMAGNETIC ENERGY ABSORPTION AND EMISSION

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I. DIRECT CURRENTS PRODUCED BY HF HEATING OF PLASMA

ABSTRACT

In addition to the well-known diffusion currents, toroidal direct currents arise in HF heated plasmas as a consequence of momentum transfer from HF field to plasma particles. In the present paper, estimates of steady state conditions are given for these currents. Particularly, the possibility of stationary operation of a Tokamak device is analyzed.

During high - frequency (HF) heating of plasma, direct currents [1, 2, 3] were observed in toroidal experiments and interpretted theoretically in some particular cases. Various utilisations of these currents were proposed by Thonemann et al. [1] and by Wort [4]. The recent analysis [5] indicated the general nature of these phenomena and their relevance for experiments. The direct currents in question arise from the fact that a HF field excited by external sources transfers not only energy but also momentum [5] to the plasma. In the present contribution, estimates of steady state conditions are given for these direct currents.

We consider an axially symmetric torus (major radius $R \gg a$ = minor radius) of plasma immersed in a toroidal magnetic field $H_z \gg H_\varphi$ = the poloidal field. The following set of hydrodynamic equations is adopted to describe the motion of ions, electrons which absorb the HF field momentum, and the remaining electrons (subscripts "i", "a" and "e") along the torus:

$$(1)\quad m_e \nu_{ai} n_a (V_i - V_a) + m_e \nu_{ei} n_e (V_i - V_e) = -n_i m_i V_i \tau_i^{-1}$$

$$(2)\quad m_e \nu_{ae} n_a (V_a - V_e) + m_e \nu_{ai} n_a (V_a - V_i) = f$$

$$(3)\quad m_e \nu_{ae} n_a (V_e - V_a) + m_e \nu_{ei} n_e (V_e - V_i) = 0$$

m are masses of particles, $\nu_{j,k}$ are effective (generally not classical) collision frequencies for the longitudinal momentum change, $n_i = n_a + n_e = n$ are concentrations, V are perturbations of hydrodynamic velocities arising due to the time-averaged dragging force density f from the HF field [5] and τ_i is a characteristic time of decelaration of ions due to ion viscosity and/or the finite containment time of ions. Electron viscosity, curvature of magnetic field lines, locally trapped particles and neutrals are omitted in Eqs. (1) - (3). An arbitrary wave propagating along the surface of the torus with phase velocity ω / k_z supplies power P and momentum $k_z P/\omega$ [5] to the plasma electrons. Since Eqs. (1) - (3) are, in fact, zero - dimensional, we may put $f = k_z P_1/\omega$, where $P_1 = P/(2\pi^2 a^2 R)$. The electric current density

$$j = e(n V_i - n_e V_e - n_a V_a)$$

then is

$$(4)\qquad j = \frac{-e k_z P_1 (\nu_{ae} n + \nu_{ei} n_e)}{\omega m_e (\nu_{ae} \nu_{ei} n_e + \nu_{ae} \nu_{ai} n_a + \nu_{ai} \nu_{ei} n_e)}$$

This expression is independent on the (not well defined) quantity m_a in three particular cases:

$$(5) \qquad j \simeq -e k_x P_1 / (m_e \omega \nu)$$

where $\nu = \nu_{ei}$ for $\nu_{ai} \simeq \nu_{ei}$, $\nu = \nu_{ei}(\nu_{ae}+\nu_{ai})/(\nu_{ae}+\nu_{ei})$ for $m_a \ll m_e$ and term $\sim m_a$ negligible, $\nu = \nu_{ai}$ for $m_a \gg m_e$ and terms $\sim m_e$ negligible. We introduce the factor $K_A = \nu/\nu_{coul}$ of the friction anomaly, where ν_{coul} is the frequency of classical (Coulomb) electron - ion collisions. If the toroidal currents (5) are intended to replace the discharge current in a Tokamak [4], the necessary HF power absorbed in plasma is given by the following obvious generalization of formula (5.7) in [5]:

$$(6) \qquad P \simeq \frac{8\pi^{3/2} K_A c a^2 \omega H_z \lambda e^3 n}{3 q T_e k_x v_e},$$

λ is the Coulomb logarithm, T_e is the electron temperature $v_e = (2T_e/m_e)^{1/2}$ and $q = aH_z/(RH_\varphi)$ is the safety factor [6]. The uncertainty of Eq. (6) arising from the a priori unknown factor K_A should disappear in a large-scale toroidal plasma, where the friction should be essentially classical [6]. For existing devices with negligible production of nuclear energy, this uncertainty is reduced by the following consideration. We naturally assume that plasma energy losses are mainly from thermal energy. Since, from (1) - (3), $V_i = \tau_i k_z P_1/(m_i n \omega)$, the last assumption is valid when the (sufficient) inequality

$$\tau_i^2 k_z^2 T_e \ll \tau_c m_i \omega^2 \tau_E$$

is satisfied; τ_c is the containment time of particles, τ_E is the energy containment time [6]. Then we have $T_e = P\tau_E/(2\pi^2 a^2 R n)$ and Eq. (6) gives

$$(7) \qquad P \simeq 4\pi^{7/4} a^2 n \left(\frac{K_A c \omega H_z \lambda e^3 R}{3 q \tau_E k_x v_e} \right)^{1/2}$$

The value of P is proportional to $K_A^{1/2}$ here and the experimentally observed values of resistivity ($K_A \simeq 3 \div 30$) [6] may be used for estimates. The steady state temperature is

$$(8) \qquad T_e \simeq \left(\frac{4 \tau_E K_A c \omega H_z \lambda e^3}{3 \pi^{1/2} R q k_x v_e} \right)^{1/2}$$

As it is well-known, the value of plasma pressure is limitted by the condition $\beta \leq \beta_{max}$ where $\beta \simeq 8\pi n T_e / H_\varphi^2$, $\beta_{max} \simeq R/a$ for equilibrium and $\beta_{max} \simeq 1$ for the suppression of flute instability. Consequently, the condition of plasma confinement reads

$$(9) \qquad \frac{16\pi n}{a^2}\left(\frac{aR}{H_z}\right)^{3/2}\left(\frac{\tau_E K_A c\omega \lambda e^3}{3\pi^{1/2} k_z v_e}\right)^{1/2} \leq \beta_{max} .$$

The toroidal motions of electrons imply a "hydrodynamic" criterion of the maximum HF power which can be absorbed at resonant (Čerenkov) heating of electrons by one travelling wave. Since $n_a \leq n$ and $V_a \lesssim \omega / k_z$, we obtain from (1) - (3) that

$$(10) \qquad P_1 \lesssim \frac{m_i n \omega^2 m_e \nu_{ai}}{k_z^2 (\tau_i m_e \nu_{ai} + m_i)}$$

If P_1 is larger than (10) with classical ν_{ai}, some additional mechanism of wave absorption must be present. At $\tau_i \to \infty$, the whole plasma would move with the wave phase velocity ω / k_z and with no resonant (in the sence mentioned above) absorption.

Consequencies of the ion motion with velocity V_i for equilibrium and stability have not yet been considered. However, the HF power necessary for stopping of the ions would be much less than P [4].

The results of the present approach may be supported by the fact that the basic formula (5) predicts currents of magnitude just observed in the experiments [2], see also [5]. A rigorous comparison with the experiments reported in [3] is, unfortunately, impossible because the HF power absorbed in plasma is not given there.

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II. ON THE ANGULAR MOMENTUM TRANSFERRED DURING ELECTROMAGNETIC ENERGY ABSORPTION AND EMISSION

ABSTRACT

In the framwork of classical electrodynamics, the angular momentum transported into (and/or from) a volume bounded by a rotational surface is derived. The dependences of electromagnetic field on space and time, and also the electromagnetic properties of the interacting masses are arbitrary. The results may be useful for the theory of toroidal currents induced by high-frequency heating of plasmas. A connection with the Manley-Rowe equations is given.

1. INTRODUCTION

The energy U and the angular momentum $\vec{M}$ of a partial spherical harmonic wave are related by the equation [1] $M_z = mU/\nu$, where m is the azimuthal number of the partial electromagnetic wave in question and ν is the (angular) frequency. Similar relations hold under considerably different circumstances, viz., in the theory of electric rotating machines. These relations are derived there either from the analysis of induced currents [2a], or from the generalized theory [2b] of Manley-Rowe equations [3]. Another example is the angular momentum of the eigen-modes of a plasma cylinder [4]. On the other hand, the angular momentum transferred at high-frequency heating of toroidal plasmas has not yet been analyzed. We recall that the consequencies of this transfer may be relevant either for plasma confinement [5, 6] or for experimental checking of diffusion currents [7] in high-frequency heated plasmas.

The purpose of the present paper is to deduce, on the basis of classical electrodynamics, relations between the angular momentum transfer and the power absorbed (emitted) by an arbitrary bounded object. The only essential assumption used is that a closed rotational surface exists which lies in vacuum and which surrounds the object in question. The final results derived below are just of the form which may be expected, at least intuitively, from the general laws of quantum electrodynamics. Therefore, rather than to obtain new physical results, this paper is intended to give simple formulas, which can be useful for the theory of nonlinear currents in high-frequency heated plasmas, as it has been mentioned above. Also certain connections of the present results with the Manley-Rowe equations could be of some interest.

2. THE z-COMPONENT OF ANGULAR MOMENTUM

Assume that an arbitrary object interacting with an electromagnetic field $\vec{E}, \vec{H}$ is placed inside a closed rotational surface $\mathcal{S}$ with the axis of symmetry z . The surface $\mathcal{S}$ itself lies in vacuum, further interacting objects may be present outside this surface. The changes of the angular momentum in the volume V bounded by $\mathcal{S}$ are given by the force density $\vec{f} = \nabla . T$, where T is the stress tensor. Since $T_{jk} = T_{kj}$ (angular momentum conservation), we have

$$(2.1) \qquad \frac{d\vec{M}}{dt} = \int_{\mathcal{S}} \vec{r} \times (T . \vec{e}_n)\, d\mathcal{S} \quad ,$$

where $\vec{M}$ is the total angular momentum in the volume V, $\vec{r}$ is the position vector with $\vec{r} = 0$ on the z-axis and $\vec{e}_n$ is a unit vector of the outer normal of the surface $\mathcal{S}$. In cylindrical coordinates r, ϑ, z, only the z-component M_z of the angular momentum $\vec{M}$ will be considered. Substituting the well-known Maxwell stress tensor for T into (2.1), one finds (in Gaussian units)

$$\text{(2.2)} \qquad \frac{dM_z}{dt} = \frac{1}{4\pi} \int_{\mathcal{S}} (E_\vartheta E_n + H_\vartheta H_n) r^2 d\vartheta \, ds ,$$

where $E_n = \vec{e}_n \cdot \vec{E}$, $H_n = \vec{e}_n \cdot \vec{H}$ and s is a curve arising by intersection of the surface $\mathcal{S}$ with a plane containing the z-axis. The curve s is either closed ($\mathcal{S}$ is topologically toroidal) or it has its boundary points on the z-axis ($\mathcal{S}$ is topologically spheroidal). We separate the time-independent and the oscillating components of the electromagnetic field, labelling the corresponding symbols by subscripts "0" and "1", respectively:

$$\text{(2.3)} \qquad \vec{E} = \vec{E}_0(\vec{r}) + \vec{E}_1(\vec{r},t) , \qquad \vec{H} = \vec{H}_0(\vec{r}) + \vec{H}_1(\vec{r},t) .$$

As we are interested in the time average of Eq. (2.2), the relevant expressions are

$$\text{(2.4)} \qquad T^0_{\vartheta n} = \frac{1}{4\pi} (E_{0\vartheta} E_{0n} + H_{0\vartheta} H_{0n})$$

and

$$\text{(2.5)} \qquad T^1_{\vartheta n} = \frac{1}{4\pi} (E_{1\vartheta} E_{1n} + H_{1\vartheta} H_{1n}) .$$

Using the Fourier transform of time dependences, we have

$$\text{(2.6)} \quad T^1_{\vartheta n} = \frac{1}{4\pi} \iint d\omega' d\omega'' e^{-i(\omega'+\omega'')t} \left[E_\vartheta(\omega') E_n(\omega'') + H_\vartheta(\omega'') H_n(\omega') \right],$$

where we use the notation

$$E_\vartheta(\omega) = \frac{1}{2\pi} \int E_{1\vartheta} e^{i\omega t} dt$$

etc., the integration limits ($-\infty$) and ($+\infty$) being omitted. The surface $\mathcal{S}$ is assumed to lie in vacuum; consequently, the field quantities in (2.2) and in (2.6) satisfy the free space Maxwell equations. Therefrom,

$$\text{(2.7)} \quad H_n(\omega) = -\frac{ic}{\omega} \left[\vec{e}_n \cdot \vec{e}_z \left(\frac{\partial E_z(\omega)}{r \partial \vartheta} - \frac{\partial E_\vartheta(\omega)}{\partial z} \right) + \vec{e}_n \cdot \vec{e}_r \left(\frac{\partial r E_\vartheta(\omega)}{r \partial r} - \frac{\partial E_r(\omega)}{r \partial \vartheta} \right) \right],$$

where $\vec{e}_r$, $\vec{e}_\vartheta$ and $\vec{e}_z$ form the local unit basis of cylindrical coordinates r, ϑ, z. A similar equation holds for $E_n(\omega)$. It is useful to introduce a new orthonormal basis $\vec{e}_n$, $\vec{e}_\vartheta$, $\vec{e}_s$ = the unit tangential vector of the curve s. Using the obvious relations

$$\vec{e}_n \cdot \vec{e}_r = \vec{e}_s \cdot \vec{e}_z \,, \qquad \vec{e}_n \cdot \vec{e}_z = -\vec{e}_s \cdot \vec{e}_r$$

in (2.7), we obtain the following expressions for the field components on $\mathcal{S}$:

(2.8a) $$H_n(\omega) = -\frac{ic}{\omega}\left(\frac{\partial E_s(\omega)}{r\partial\vartheta} - \frac{\partial r E_\vartheta(\omega)}{r\partial s}\right),$$

(2.8b) $$E_n(\omega) = \frac{ic}{\omega}\left(\frac{\partial H_s(\omega)}{r\partial\vartheta} - \frac{\partial r H_\vartheta(\omega)}{r\partial s}\right).$$

Equation (2.6) then is

(2.9) $$T^1_{\vartheta n} = \frac{ic}{4\pi r}\iint d\omega' d\omega'' e^{-i(\omega'+\omega'')t}\left[\frac{E_\vartheta(\omega')}{\omega''}\left(\frac{\partial H_s(\omega'')}{\partial\vartheta} - \frac{\partial r H_\vartheta(\omega'')}{\partial s}\right) - \frac{H_\vartheta(\omega'')}{\omega'}\left(\frac{\partial E_s(\omega')}{\partial\vartheta} - \frac{\partial r E_\vartheta(\omega')}{\partial s}\right)\right].$$

According to (2.2), (2.5) and (2.9), the angular momentum M^1_z transported by the oscillating field is given by the following equation:

(2.10) $$\frac{dM^1_z}{dt} = \frac{ic}{4\pi}\iiint_{\mathcal{S}} d\omega' d\omega'' d\vartheta\, ds\, e^{-i(\omega'+\omega'')t}\, r\left[\frac{E_\vartheta(\omega')}{\omega'}\left(\frac{\partial H_s(\omega'')}{\partial\vartheta} - \frac{\partial r H_\vartheta(\omega'')}{\partial s}\right) - \frac{H_\vartheta(\omega'')}{\omega'}\left(\frac{\partial E_s(\omega')}{\partial\vartheta} - \frac{\partial r E_\vartheta(\omega')}{\partial s}\right)\right].$$

The integration over time scale from ($-\infty$) to ($+\infty$) leads to the multiplier $\delta(\omega'+\omega'')$ on the right side. The integral of the left side is denoted as ΔM^1_z:

(2.11) $$\Delta M^1_z = \frac{c}{2i}\iint_{\mathcal{S}} d\omega\, d\vartheta\, ds\, \frac{r}{\omega}\left[E_\vartheta(\omega)\left(\frac{\partial H^*_s(\omega)}{\partial\vartheta} - \frac{\partial r H^*_\vartheta(\omega)}{\partial s}\right) + H^*_\vartheta(\omega)\left(\frac{\partial E_s(\omega)}{\partial\vartheta} - \frac{\partial r E_\vartheta(\omega)}{\partial s}\right)\right].$$

The relations $H_{s,\vartheta}(-\omega) = H^*_{s,\vartheta}(\omega)$ have been used, the asterisk labelling the complex conjugate values. Since the terms involving $\partial/\partial s$ cancel out at performing the integration along s, they can be ignored:

$$\Delta M_{\varkappa}^{1} = \frac{c}{2i} \int_{\mathscr{S}} \int d\omega \, d\vartheta \, ds \, \frac{r}{\omega} \left(E_{\vartheta}(\omega) \frac{\partial H_s^*(\omega)}{\partial \vartheta} + H_{\vartheta}^*(\omega) \frac{\partial E_s(\omega)}{\partial \vartheta} \right) \tag{2.12}$$

The ϑ -dependences of $E_{\vartheta}(\omega)$, $H_s(\omega)$ etc. are now expanded in a Fourier series, viz.,

$$E_{\vartheta}(\omega) = \sum_m E_{\vartheta}(\omega, m) e^{im\vartheta}, \tag{2.13a}$$

$$H_s(\omega) = \sum_m H_s(\omega, m) e^{im\vartheta} \tag{2.13b}$$

etc. Using the Parseval theorem, one finds from (2.12) that

$$\Delta M_{\varkappa}^{1} = \sum_m \int \frac{m}{\omega} W(\omega, m) \, d\omega, \tag{2.14}$$

where

$$W(\omega, m) = \frac{c}{2} \int_{\mathscr{S}} r \, d\vartheta \, ds \left[E_s(\omega, m) H_{\vartheta}^*(\omega, m) - E_{\vartheta}(\omega, m) H_s^*(\omega, m) \right]. \tag{2.15}$$

3. INTERPRETATION AND MODIFICATIONS

The physical interpretation of the value of $W(\omega, m)$ is readily established by determining the energy ΔW supplied across $\mathscr{S}$ into the volume V:

$$\Delta W = -\iint \vec{e}_n \cdot \vec{S} \, d\mathscr{S} \, dt, \tag{3.1}$$

where $\vec{S}$ is the Poynting vector. Since a simple algebra gives

$$\Delta W = \sum_m \int W(\omega, m) \, d\omega, \tag{3.2}$$

$W(\omega, m)$ is the (complex) spectral energy density of the mode with the azimuthal number m.

It is sometimes suitable to exclude the integrations over negative frequencies in Eqs. (2.14) and (3.2). For this purpose, we note that equations (2.13) imply relations of the following type:

$$E_{\vartheta}(-\omega, m) = E_{\vartheta}^*(\omega, -m). \tag{3.3}$$

According to (2.15) and (3.3),

(3.4) $$W(-\omega, m) = W^*(\omega, -m) .$$

Consequently, equations (3.2) and (2.14) are

(3.5) $$\Delta W = 2\sum_m \int_0^\infty \mathrm{Re}\, W(\omega, m)\, d\omega ,$$

(3.6) $$\Delta M_k^1 = 2\sum_m \int_0^\infty \frac{m}{\omega} \mathrm{Re}\, W(\omega, m)\, d\omega .$$

The summation with respect to m includes (similarly as in the previous equations) both positive and negative integers m, corresponding to the modes with positive and negative phase velocities along the azimuth ϑ. The quantum interpretation of Eqs. (3.5) and (3.6) is obvious: If a field quantum transfers the energy $\hbar\omega$ into the volume V, the angular momentum $m\hbar$ is transferred simultaneously.

Let us consider some modifications of the "symmetric" relation (2.14).

(I) If the oscillating field is non-zero only in a finite time interval $(-T/2) \leq t \leq (T/2)$, the average angular momentum flow into the volume V is

(3.7) $$K_k^1 = \frac{\Delta M_k^1}{T} = \frac{1}{T} \sum_m \int \frac{m}{\omega} W(\omega, m)\, d\omega .$$

The value of $W(\omega, m)/T$ may be treated as the spectral density of power transported into the volume in question. More usually, this quantity is introduced in case that

(II) the intensities of the oscillating field are stationary random functions of time. Then it is possible to introduce the spectral density $G(\omega, m)$ by using the method of "rectangular cuts" 8, viz.,

(3.8) $$G(\omega, m) = \lim_{T\to\infty} \frac{W(\omega, m)}{T}$$

Consequently, equation (3.7) gives

(3.9) $$K_k^1 = \sum_m \int \frac{m}{\omega} G(\omega, m)\, d\omega .$$

(III) If, moreover, the time dependences of the field components are ergodic, it is possible to take average over a statistic set in Eq. (2.5). The requirement of time independence then leads again to the multiplier $\delta(\omega' + \omega'')$ in the corresponding equation (2.6) etc., with the same result (3.9).

(IV) In the case of a discrete spectrum with frequencies $\Omega_1, \Omega_2, \ldots, \Omega_N$ we find

$$K_z^1 = \sum_{j=1}^{N} \sum_{m} \frac{m P_{mj}}{\Omega_j} , \tag{3.10}$$

where P_{mj} is the power transported into the volume V by the modes with the azimuthal number m and with the frequency Ω_j.

It is interesting to demonstrate certain connections of the present results with the Manley-Rowe equations [2b, 3]. If equation (3.10) is applied to a rigid body rotation, K_z^1 is a ratio of some mechanical power to some angular velocity of rotation. Equation (3.10) then acquires the well-known form [2b]. Another case is the nonlinear interaction of three modes with frequencies $\Omega_1, \Omega_2, \Omega_3$ and with azimuthal numbers m_1, m_2, m_3. Let us assume that the total power flowing into the volume V and the corresponding angular momentum transfer (3.10) are zero:

$$P_1 + P_2 + P_3 = 0 ,$$

$$\frac{m_1 P_1}{\Omega_1} + \frac{m_2 P_2}{\Omega_2} + \frac{m_3 P_3}{\Omega_3} = 0 ,$$

where $P_{1,2,3}$ are powers corresponding to the individual modes. Using the matching conditions $m_1 = m_2 + m_3$ and $\Omega_1 = \Omega_2 + \Omega_3$ with $m_1 \Omega_2 \neq m_2 \Omega_1$, we easily arrive at equations (26) and (27) of Ref. [3].

4. CONCLUSION

The results presented above determine the transfer of angular momentum in terms of spectral power density. If the contribution of the static field (2.4) is zero (e.g., at $E_{0\vartheta} = H_{0n} = 0$), the angular momentum transferred is given, by the oscillating field. If, moreover, the changes of the field angular momentum are negligible (e.g., a quasi-steady state of the field), then K_z^1 is the z-component of the torque acting on the object inside the surface $\mathcal{S}$. Naturally, the motions generated by this torque depend on the concrete mechanisms of absorption and/or emission. Some particular cases concerning the high-frequency dragging in toroidal plasmas have been discussed previously [5, 6], the toroidicity being neglected. The theory given above represents a first step to the analysis of specific toroidal effects. Another approach, also neglecting toroidicity, has been used recently by Midzuno [9]. His theory is apparently applicable for time intervals much less than the collisional relaxation time of velocity distribution.

Concluding these remarks we have to point out a further possible application of the present results. If the diffusion currents [7] perturb magnetic surfaces unfavourably, the high-frequency dragging can be used for an appropriate compensation.

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)1 A note on the text below Eq. (2.13) in Ref. [5]: Instead of "i.e." should be "proportional to ".

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