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ABSTRACT

This report describes two applications of optimal control of the reactor analogue model. Both cases deal with the control A-1 reactor neutron density. The control loops containing the real controlled process-the reactor of the first Czechoslovak nuclear power plant A-1-are simulated on an analogue computer. Two versions of optimal control algorithm are derived using Modern Control Theory (Pontrjagin's maximum principle, the calculus of variations, and Kalman's estimation theory), the minimum time performance index, and the quadratic performance index.

The results of the analysis of optimal control are compared with conventional control of the A-1 reactor.

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LIST OF MAIN SYMBOLS

n $[1/cm^3]$	neutron density
n_0 $[1/cm^3]$	neutron density of subcritical (initial) state of reactor
n_1 $[n/cm^3]$	estimated state variable of reactor model
N $[1]$	relative deviation of neutron density
N_1 $[1]$	estimated relative state variable of reactor model
$\Delta N[1]$	deviation of neutron density from set-point value
δk $[1]$	reactivity
$\delta k_0 [1]$	reactivity of subcritical (initial) state of reactor
ρ $[\$]$	reactivity in dollars
$\rho_0 [\$]$	reactivity in dollars of initial state of reactor
C_i $[1/cm^3]$	delayed neutron precursors of the i-th group
C $[1/cm^3]$	delayed neutron precursors of one equivalent group
λ_i $[1/s]$	precursor decay constant of the i-th group
λ $[1/s]$	precursor decay constant of one equivalent group
l $[s]$	neutron mean prompt generation time in critical reactor
β_i $[1]$	effective delayed neutron fractions of the i-th group
β $[1]$	sum of β_i
u	control variable
u_0	initial state of control variable
u^*	optimal control variable
$t[s]$	time
T_1, T_2, T_3, T $[s]$	time constants
x	state variable
y	output variable

w	noise driving the process
v	noise contaminating measurement
$s\,[cm]$	control rod position
$E\,[V]$	voltage of electronic amplifier
$f\,[Hz]$	frequency of generator
$\bar{A}, \bar{B}, \bar{C}, \bar{D}$, $\bar{G}, \bar{H}, \bar{\Phi}, \bar{\Psi}$, $\bar{\xi}, \bar{\eta}, \bar{R}, \bar{Q}, \bar{F}_x$	matrices
$\bar{K}_c$	Kalman's matrix of optimal control
$\bar{K}_R$	matrix of optimal control coefficients
$\bar{K}_f$	Kalman's matrix of extimation
$\bar{K}_e$	matrix of estimation coefficients
$M_{\bar{u}}$, $M_{\bar{q}}$	minors (subdeterminants)
h	rank of matrix
μ	switching curve
Ω_u	definitial domain of variable u
I	performance index
f_k	function of the terminal state
λ_k	Lagrangian coefficients
$\wedge$	symbol for estimation
$*$	symbol for optimality
$-$	written above capital letters this symbol denotes matrices; above lower letters it denotes vectors
T	as superscript it denotes transposition of a matrix

INTRODUCTION

In 1972 the first Czechoslovak nuclear power plant in Jaslovské Bohunice was put into operation. This nuclear power plant has produced the electrical energie and simultaneously it has been widely used as a research and experimental facility. In 1974 the control computer Tesla RPP-16 was installed in the nuclear power plant A-1 for research and experimental purposes. After putting the computer into operation in the open loop (as an information system for data acquisition and processing) it will also be possible to verify gradually some theoretical studies concerning reactor optimal control in closed loop on a real process with a concrete computer. The results of these experiments can be made full use of later, mainly in further nuclear power plants with VVER type reactor.

Some studies concerning reactor model optimal control have been carried out in Nuclear Power Construction Department, ŠKODA WORKS Plzeň, as the first step towards the goal mentioned above. As a model of the controlled process the reactor A-1 model was chosen and the neutron density was defined as a controlled variable. Two problems of optimal control were simulated on the analogue computer Tesla AP3-M. The time optimal control (with the application of Pontrjagin's maximum principle) was the first problem that has been solved. The other one was the problem of the optimal control with quadratic performance index and with the application of the calculus of variations and Kalman's theory of estimation. It is necessary to state beforhand that the influence of stochastic disturbances in both cases and the control and state variables constraints in the second case were neglected.

The examples of control are compared with the behaviour of conventional control loop SUZ (system of protection and control) of reactor A-1.

The determination of the controlled process (reactor A-1) model, the derivation of the control laws and optimal control loops (simulated on analogue computer) analysis are introduced in the following chapters.

1. DERIVATION OF THE CONTROLLED PROCESS A-1 REACTOR MODELS

The real reactor A-1 is approximated by the point model of the reactor kinetics with six groups of delayed neutron precursors. From the mathematical point of view the real reactor is described by the following non-linear system of differential equations whose order is 7 (reactor in subcritical state), /1/:

$$\frac{dn}{dt} = \frac{\delta k - \beta}{l} n + \sum_{i=1}^{6} \lambda_i c_i + \frac{\delta k_0 n_0}{l} \qquad (1.1)$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{l} n - \lambda_i c_i \qquad (1.2)$$

On the presumption that 6 groups of delayed neutron precursors are approximated by one equivalent group, /2/, /3/, then:

$$\frac{dn}{dt} = \frac{\delta k - \beta}{l} n + \lambda c + \frac{\delta k_0 n_0}{l} \qquad (1.3)$$

$$\frac{dc}{dt} = \frac{\beta}{l} n - \lambda c \qquad (1.4)$$

1.1. DERIVATION OF MODEL 1

The term $\frac{\delta k_0 n_0}{l}$ (the source term representing the source of neutrons) is negelcted for critical state of reactor. The product λc is eliminated from equation (1.3) and by the differentiation of the obtained expression the term $\frac{dc}{dt}$ is obtained. The terms λc and $\frac{dc}{dt}$ are substituted into equation (1.4) and multiplied by λ . Then the following equation is obtained:

$$\frac{d^2n}{dt^2} + \frac{dn}{dt}\left(\lambda - \frac{\delta k}{l} + \frac{\beta}{l}\right) - n\left(\frac{\lambda}{l}\delta k - \frac{1}{l}\frac{d\delta k}{dt}\right) = 0 \qquad (1.5)$$

When considering the small reactivity changes around the steady state, the relations (1.6) to (1.8) are valid:

$$\lambda - \frac{\delta k}{l} \ll \frac{\beta}{l} \tag{1.6}$$

$$\frac{d^2 n}{dt^2} \ll \frac{dn}{dt} \cdot \frac{\beta - d\delta k}{l} \tag{1.7}$$

$$\frac{d^2 n}{dt^2} \ll - \frac{\lambda}{l} n \delta k \tag{1.8}$$

Substituting
$$\rho = \frac{\delta k}{\beta} \tag{1.9}$$

into eq. (1.5) and with respect to (1.6), (1.7) and (1.8), eq. (1.5) assumes the shape as follows:

$$\frac{dn}{dt} - n\left(\lambda\rho + \frac{d\rho}{dt}\right) = 0 \tag{1.10}$$

When
$$N = \frac{n - n_\infty}{n_\infty} \tag{1.11}$$

is the relative deviation of neutron density around the steady state and when $p = \frac{d}{dt}$ is the Laplace operator, then

$$\frac{dN}{dt} = \frac{1}{n_\infty} \cdot \frac{dn}{dt} = \frac{1}{n} \cdot \frac{dn}{dt} \tag{1.12}$$

and
$$pN = \lambda\rho + p\rho \tag{1.13}$$

The control variable u is defined as a speed of reactivity insertion $\frac{d\rho}{dt}$.

Then the system of differential equations has the form:

$$pN = \lambda\varrho + p\varrho$$
$$p\varrho = u \tag{1.14}$$

and the transfer function of reactor with actuating unit is:

$$F(p) = \frac{N}{u} = \frac{Tp+1}{Tp^2} \tag{1.15}$$

where $$T = \frac{1}{\lambda} \tag{1.16}$$

The eq. (1.13) describes the reactor model as the linear system of the 2nd order (MODEL 1) which will be used in the case of time optimal control /2/.

1.2. DERIVATION OF MODEL 2

The relation $$\delta k \cdot n = u \tag{1.17}$$
is substituted into eq. (1.3) in order to linearize the system, /3/. Then:

$$\frac{dn}{dt} = \frac{1}{l}u - \frac{\beta}{l}n + \lambda c + \frac{1}{l}u_o \tag{1.18}$$

$$\frac{dc}{dt} = \frac{\beta}{l}n - \lambda c \tag{1.19}$$

The transfer function of subcritical reactor is derived from eqs. (1.18) and (1.19) (the Laplace operator $p = \frac{d}{dt}$ is introduced):

$$F(p) = \frac{n}{u+u_o} = \frac{T_1p+1}{T_2p(T_3p+1)}, \tag{1.20}$$

where: $$T_1 = \frac{1}{\lambda} \tag{1.21}$$

$$T_2 = \frac{\lambda l + \beta}{\lambda} \qquad (1.22)$$

$$T_3 = \frac{l}{\lambda l + \beta} \qquad (1.23)$$

The differential eq. of the 2nd order is derived from the transfer function (1.20):

$$T_2 T_3 p^2 n + T_2 p n = T_1 p (u + u_0) + u + u_0 \qquad (1.24)$$

When the substitutions:

$$pn = n_2$$
$$p^2 n = p n_2 = n_3$$

are introduced into eq. (1.24), the system of differential eqs. of the 1st order is obtained:

$$T_2 T_3 p n_2 + T_2 n_2 = (T_1 p + 1)(u + u_0) \qquad (1.25)$$

$$pn = n_2 \qquad (1.26)$$

From the eq. (1.25) it is obtained:

$$p \left[T_2 T_3 n_2 - T_1 (u + u_0) \right] = -T_2 n_2 + u + u_0 \qquad (1.27)$$

When denoting the expression in square brackets as n_1 then:

$$p n_1 = -T_2 n_2 + u + w_0 \qquad (1.28)$$

Substituting eq. (1.28) into (1.27) and by application of step integration method, the eq. (1.29) is obtained

$$T_2 T_3 n_2 - T_1 (u + w_0) = n_1 \qquad (1.29)$$

n_2 is eliminated from eq. (1.29) and substituted into eqs. (1.28) and (1.26).

Since the simulation of the reactor control loop is considered with neutron deviation N_1, the substitutions N = n and $N_1 = n_1$ are also respected. Then:

$$pN_1 = \frac{-1}{T_3} N_1 - \frac{T_1 - T_2}{T_3}\left(u + u_0\right) \qquad (1.30)$$

$$pN = \frac{1}{T_2 T_3} N_1 + \frac{T_1}{T_2 T_3}\left(u + u_0\right) \qquad (1.31)$$

After substituting the constants of A-1 reactor

$\beta = 6.274\text{x}10^{-3}$
$l = 10^{-3}$ s

$\lambda_1 = 1.24\text{x}10^{-2}$ 1/s	$\beta_1 = 2.07\text{x}10^{-4}$
$\lambda_2 = 3.05\text{x}10^{-2}$ 1/s	$\beta_2 = 1.37\text{x}10^{-3}$
$\lambda_3 = 1.11\text{x}10^{-1}$ 1/s	$\beta_3 = 1.23\text{x}10^{-3}$
$\lambda_4 = 3.0099\text{x}10^{-1}$ 1/s	$\beta_4 = 2.48\text{x}10^{-3}$
$\lambda_5 = 1.13$ 1/s	$\beta_5 = 7.2299\text{x}10^{-4}$
$\lambda_6 = 3.40$ 1/s	$\beta_6 = 2.6399\text{x}10^{-4}$
$\lambda = 7.727\text{x}10^{-2}$ 1/s	

into eq. (1.30) and (1.31) it is obtained:

$$pN_1 = -6.372\, N_1 - 82.454\left(u + u_0\right) \qquad (1.32)$$

$$pN = 77.519\, N_1 + 1003.101\left(u + u_0\right) \qquad (1.33)$$

The eqs. (1.30), (1.31) resp. (1.32), (1.33) describe the reactor model as the linear system of the 2nd order (MODEL 2) which will be used for optimal control with quadratic performance index.

2. THE PROBLEM OF OPTIMAL CONTROL

The process to be controlled is described in a general form by the following equations:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u} \qquad (2.1)$$

$$\bar{y} = \bar{C}\bar{x} + \bar{D}\bar{u}\ , \qquad (2.2)$$

where $\bar{x}$ is the n-dimensional state vector of the process, $\bar{u}$ is the m-dimensional (in eq. (2.1.)) or r-dimensional (in eq. (2.2)) control vector and $\bar{y}$ is the p-dimensional output vector; $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ are time invariant matrices of the order nxn, nxm, pxn and pxr.

Two states, $\bar{X}_0$ and $\bar{X}_1$, of the controlled process are defined. It is necessary to look for such a control variable n, which transfers the control process from the initial state $\bar{X}_0$ to the new state $\bar{X}_1$ in such a way that the performance index I is minimal. The condition

$$|u| \leq \Omega_u \tag{2.3}$$

must be fulfilled; Ω_u defines the domain of variable u (the constrains of u).

Before the synthesis of the optimal control loops the observability and the controllability of the process are tested.

According to /5/ the following criteria of observability and controllability are defined:

The process is observable (controlable) when the rank of a matrix:

$$\begin{aligned} &\bar{H} = [\bar{C}^T \ \bar{A}^T\bar{C}^T \ (\bar{A}^T)^2\bar{C}^T \ldots (\bar{A}^T)^{n-1}\bar{C}^T] \\ &(\bar{G} = [\bar{B} \ \bar{A}\bar{B} \ \bar{A}^2B \ldots A^{n-1}\bar{B}] \ ) \end{aligned} \tag{2.4}$$

is equal h , the value of which must be the same as the order of the differential equation describing the process. In other words det $\bar{H}$ (det $\bar{G}$) must be $\neq 0$. The real value $\hat{h}$ is determined from the values of minors (subdeterminants) $M_{\bar{H}}$ and $M_{\bar{G}}$.

When the process is observable and controlable, then the optimal control laws are derived as a next step of the work.

2.1. THE TIME OPTIMAL CONTROL OF THE REACTOR MODEL (MODEL 1)

The time optimal control of reactor is derived with the use of Pontrjagin's maximum principle /4/. As a model of controlled process the MODEL 1 is prepared. With respect to eqs. (2.1), (2.2) the system of differential equations is set:

$$
\begin{aligned}
pN &= \lambda\varrho + u \\
p\varrho &= u
\end{aligned}
\qquad (2.5)
$$

$$N_e = N \qquad (2.6)$$

where N_e is denotation for the observable (estimated) value of N. In matrix form:

$$p\begin{bmatrix} N \\ \varrho \end{bmatrix} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} N \\ \varrho \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot u \qquad (2.7)$$

$$N_e = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} N \\ \varrho \end{bmatrix} \qquad (2.8)$$

From eqs. (2.7) and (2.8) follows:

$$\bar{A} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \bar{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \bar{D} = 0$$

$$\bar{x} = \begin{bmatrix} N \\ \varrho \end{bmatrix} \qquad \bar{u} = u$$

2.1.1. The test of observability

$$\bar{A}^T\bar{C}^T = \begin{bmatrix} 0 & 0 \\ \lambda & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \end{bmatrix}$$

$$\bar{H} = \begin{bmatrix} \bar{C}^T & \bar{A}^T\bar{C}^T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\det \bar{H} = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix} = \lambda \neq 0$$

The rank of matrix $\bar{H}$ is

h = 2

because the minors of matrix $\bar{H}$

$$M_{\bar{H}_1} = |\lambda| \neq 0 \qquad \text{and} \qquad M_{\bar{H}_2} = \begin{vmatrix} 1 & 0 \\ 0 & \lambda \end{vmatrix} \neq 0$$

The system (2.5) is observable.

2.1.2. The test of controllability

$$\bar{A}\bar{B} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \end{bmatrix}$$

$$\bar{G} = [\bar{B} \quad \bar{A}\bar{B}] = \begin{bmatrix} 1 & \lambda \\ 1 & 0 \end{bmatrix}$$

$$\det \bar{G} = \begin{vmatrix} 1 & \lambda \\ 1 & 0 \end{vmatrix} \neq 0$$

The rank of matrix $\bar{G}$ is

$$h = 2$$

because the minors of matrix $\bar{G}$

$$M_{\bar{G}_1} = |\lambda| \neq 0 \qquad \text{and} \qquad M_{\bar{G}_2} = \begin{vmatrix} 1 & \lambda \\ 1 & 0 \end{vmatrix} \neq 0$$

The system (2.5) is controllable.

2.1.3 Derivation of the time optimal control law

For time optimal control the Hamiltonian function H_f is defined, /5/:

$$H_f[\bar{x}, \bar{\Psi}, \bar{u}] = 1 + \langle \bar{\Psi}, \bar{f} \rangle = 1 + \langle \bar{A}\bar{x}, \bar{\Psi} \rangle + \langle \bar{B}\bar{u}, \bar{\Psi} \rangle \quad (2.9)$$

where $\bar{\Psi}$ is the vector of conjugate system to (2.1) and expressions in < > brackets are scalar products. Suppo-sing that vector $\bar{u}$ exists, the necessary conditions for extrem H_f (for optimal control $\bar{u}^*$) are

$$H_f[\bar{x}^*, \bar{\Psi}^*, \bar{u}^*] = \min_{u} H_f[\bar{x}, \bar{\Psi}, \bar{u}] \quad (2.10)$$

$$\dot{\bar{x}}^* = \frac{\partial H_f}{\partial \bar{\Psi}^*} = \bar{A}\bar{x}^* + \bar{B}\bar{u}^* \quad (2.11)$$

$$\dot{\bar{\Psi}}^* = -\frac{\partial H_f}{\partial \bar{x}^*} = -\bar{A}^*\bar{\Psi}^* \quad (2.12)$$

where the eqs. (2.11) and (2.12) are Hamiltonian canonical equations and the variables with asterisk are optimal variables. $\bar{\psi}$ is a vector function and solution of the conjugate system:

$$\frac{d\psi_i}{dt} = \sum_{\alpha=0}^{n} \frac{\partial f_\alpha}{\partial x_i} \psi_\alpha \qquad (2.13)$$

to the system

$$\frac{dx_i}{dt} = f_i(x, u) \qquad i = 1, 2, \ldots, n \qquad (2.14)$$

Assuming that performance index I is the coordinate n + 1 of the system whose order is n and denoting it as X_0, then:

$$x_0 = I = \int_{t_0}^{t_1} 1 . dt \qquad (2.15)$$

The first derivative of (2.15) is:

$$\frac{dx_0}{dt} = f_0(x, u) = 1 \qquad (2.16)$$

The set of differential equations describing the reactor model A-1 and performance index has the following form:

$$\frac{dx_0}{dt} = 1 = f_0 \qquad (2.17)$$

$$\frac{dx_1}{dt} = \frac{dN}{dt} = \lambda \rho + u = f_1 \qquad (2.18)$$

$$\frac{dx_2}{dt} = \frac{d\rho}{dt} = u = f_2 \qquad (2.19)$$

Then Hamiltonian according to (2,9):

$$H_f = \sum_{\alpha=0}^{n} \psi_\alpha \cdot f_\alpha(x, u) = \psi_0 \cdot 1 + \psi_1(\lambda\rho + u) + \psi_2 u =$$
$$= \psi_0 \cdot 1 + \psi\lambda\rho + (\psi_1 + \psi_2) \cdot u \qquad (2.20)$$

According to (2.10) and (2.9)

$$\frac{\partial H_f}{\partial u} = u^* = -\mathrm{sgn}\left[\bar{B}^T \bar{\Psi}^*\right] \tag{2.21}$$

Combining eq. (2.21) with eq. (2.20) yields

$$\frac{\partial H_f}{\partial u} = \Psi_1 + \Psi_2 \tag{2.22}$$

Then

$$u^* = -\mathrm{sgn}\left(\Psi_1 + \Psi_2\right) \tag{2.23}$$

It is known that the actuating unit is supplied by the voltage u_{max}. Then

$$u = -\mathrm{sgn}\left(\Psi_1 + \Psi_2\right) u_{max} \tag{2.24}$$

The variables Ψ_1, Ψ_2 are determined from (2.12) and (2.20):

$$\begin{aligned} \dot{\Psi}_1 &= -\frac{\partial H_f}{\partial x_1} = -\frac{\partial H_f}{\partial N} = -\frac{\partial}{\partial N}\left[\Psi_0 + \Psi_1 \lambda \varrho + (\Psi_1 + \Psi_2) u\right] = 0 \\ \Psi_1 &= k_1 \end{aligned} \tag{2.25}$$

where k_1 = const.

Analogically:

$$\begin{aligned} \dot{\Psi}_2 &= -\frac{\partial H_f}{\partial x_2} = -\frac{\partial H_f}{\partial \varrho} = -\frac{\partial}{\partial \varrho}\left[\Psi_0 + \Psi_1 \lambda \varrho + (\Psi_1 + \Psi_2) u\right] = -\Psi_1 \lambda \\ \Psi_2 &= -\int \Psi_1 \lambda \, dt = -\int k_1 \lambda \, dt = -k_1 \lambda t + k_2 \\ \Psi_2 &= k_2 + k_3 t \end{aligned} \tag{2.26}$$

where k_2 = const, $k_3 = -k_1\lambda$ = const.

Substituting the eqs. (2.25) and (2.26) into (2.24), the optimal control variable is obtained:

$$\boxed{u^* = \mathrm{sgn}\left(k_1 + k_2 + k_3 t\right) u_{max}} \tag{2.27}$$

From eq. (2.27) is seen that the optimal control variable u^* of reactor MODEL 1 can take three possible values: +max, -max, and 0.

In phase plane representation:

The optimal control variable u^* controls the reactor model from the initial state x_0 to the terminal state x_1 in such a way that only one sign switching exists (in the eq. (2.27) the variable t is of the 1st order). The switching is realized on the switching curve.

2.1.4 <u>Determination of the switching curve</u>

The 2nd eq. (2.5) is integrated for u =const and for the initial state of reactor $\rho(0) = \rho_0$:

$$\rho = \int u\,dt = ut + \rho_0 \qquad (2.28)$$

After substitution of (2.28) into the 1st eq. (2.5) and after integration for u = const and N (0) = n_0:

$$N = \int [\lambda(ut + \rho_0) + w]\,dt = \frac{1}{2}\lambda u t^2 + \lambda\rho_0 t + ut + N_0 \qquad (2.29)$$

By elimination of parameter t from eqs. (2.28) and (2.29) the equation describing the set of phase trajectories in phase plane for u = const and for N_0, ρ_0 is obtained:

$$N = \frac{\lambda}{2u}\rho^2 - \frac{\lambda}{2u}\rho_0^2 + \rho - \rho_0 + N_0 \qquad (2.30)$$

Only one trajectory in phase plane is going through the origin. This trajectory is described by eq. (2.30) for $N_0 = \rho_0 = 0$ as an initial state and thus for the negative time t:

$$\operatorname{sgn} u = -\operatorname{sgn} \rho \qquad (2.31)$$

The equation describing the switching curve (the parabola):

$$N = -\frac{\lambda}{2|u_{max}|}|\varrho|\varrho + \varrho \qquad (2.32)$$

The auxiliary switching function is defined as:

$$\boxed{\mu(N,\varrho) = -N - \frac{\lambda}{2|u_{max}|}|\varrho|\varrho + \varrho} \qquad (2.33)$$

Switching curve with the explanation of switching process is in fig. 1.

2.1.5. Simulation on the analogue computer

The block schema of time optimal control of reactor is in the fig. 2.

Two examplex of time optimal control of reactor are in fig. 3 and 4.

2.2. THE OPTIMAL CONTROL OF THE REACTOR MODEL (MODEL 2) WITH QUADRATIC PERFORMANCE INDEX

The quadratic performance index is defined in the following way:

$$I = \int_{t_0}^{t_1}\left(\frac{1}{2}\bar{x}^T\bar{Q}\bar{x} + \frac{1}{2}\bar{u}^T\bar{R}\bar{u}\right)dt + \bar{x}^T(t_1)\bar{F}_x(t_1) \qquad (2.34)$$

where Q and R are symmetrical, positive, semidefinite weighting matrices of the order nxn and mxm, respectively and $\bar{F}_x(t_1)$ is the matrix of terminal state $\bar{x}_1$.

As a model of controlled process the MODEL 2 is prepared. With respect to (1.30), (1.31), (2.1) and (2.2) the controlled process is described in this way:

$$pN_1 = -\frac{1}{T_3}N_1 - \frac{T_1 + T_3}{T_3}(u + w_0) + w \qquad (2.35)$$

$$pN = \frac{1}{T_2T_3}N_1 - \frac{T_1}{T_2T_3}(u + u_0) + w \qquad (2.36)$$

$$N_e = N + v \qquad (2.37)$$

In matrix form:

$$p\begin{bmatrix} N_1 \\ N \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_3} & 0 \\ \frac{1}{T_2T_3} & 0 \end{bmatrix} \cdot \begin{bmatrix} N_1 \\ N \end{bmatrix} + \begin{bmatrix} -\frac{T_1 - T_3}{T_3} \\ \frac{T_1}{T_2T_3} \end{bmatrix} (u + u_0) + \bar{\phi}\bar{w} \tag{2.38}$$

$$N_e = [0 \quad 1] \cdot \begin{bmatrix} N_1 \\ N \end{bmatrix} + \bar{v} \tag{2.39}$$

where $\bar{w}$ is an s-dimensional random vector driving the process and $\bar{v}$ is a q-dimensional random vector of the additive noise contaminating the measurements, $\bar{\phi}$ is time invariant matrix.

From eqs. (2.38) and (2.39) the following matrices and vectors are evident:

$$\bar{A} = \begin{bmatrix} -\frac{1}{T_3} & 0 \\ \frac{1}{T_2T_3} & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} -\frac{T_1 - T_3}{T_3} \\ \frac{T_1}{T_2T_3} \end{bmatrix} \qquad \bar{C} = [0 \quad 1] \qquad \bar{D} = 0$$

$$\bar{x} = \begin{bmatrix} N_1 \\ N \end{bmatrix} \qquad \bar{u} = u + u_0$$

and because the stochastic parts (noise) of variables are neglected, then:

$$\bar{w} = 0 \qquad \bar{v} = 0$$

2.2.1. <u>The test of observability</u>

$$\bar{A}^T\bar{C}^T = \begin{bmatrix} -\frac{1}{T_3} & \frac{1}{T_2T_3} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{T_2T_3} \\ 0 \end{bmatrix}$$

$$\bar{H} = [\bar{C}^T \quad \bar{A}^T\bar{C}^T] = \begin{bmatrix} 0 & \frac{1}{T_2T_3} \\ 1 & 0 \end{bmatrix}$$

$$\det \bar{H} = \begin{vmatrix} 0 & \frac{1}{T_2T_3} \\ 1 & 0 \end{vmatrix} = -\frac{1}{T_2T_3} = -\frac{1}{\frac{\lambda l + \beta}{\lambda} \cdot \frac{l}{\lambda l + \beta}} = -\frac{\lambda}{l} \neq 0$$

The rank of matrix $\bar{H}$ is:

$$h = 2$$

because the minors of matrix $\bar{H}$

$$M_{\bar{H}_1} = \left| \frac{1}{T_2 T_3} \right| \neq 0 \quad \text{and} \quad M_{\bar{H}_2} = \begin{vmatrix} 0 & \frac{1}{T_2 T_3} \\ 1 & 0 \end{vmatrix} \neq 0$$

The system (2.38) and (2.39) is observable.

2.2.2. <u>The test of controllability</u>

$$\bar{A}\bar{B} = \begin{bmatrix} -\frac{1}{T_3} & \frac{1}{T_2 T_3} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} -\frac{T_1 - T_3}{T_3} \\ \frac{T_1}{T_2 T_3} \end{bmatrix} = \begin{bmatrix} \frac{T_1 - T_3}{T_3^2} + \frac{T_1}{(T_2 T_3)^2} \\ 0 \end{bmatrix}$$

$$\bar{G} = [\bar{B} \quad \bar{A}\bar{B}] = \begin{bmatrix} -\frac{T_1 - T_3}{T_3} & \frac{T_1 - T_3}{T_3^2} + \frac{T_1}{(T_2 T_3)^2} \\ \frac{T_1}{T_2 T_3} & 0 \end{bmatrix}$$

$$\det \bar{G} = \begin{vmatrix} -\frac{T_1 - T_3}{T_3} & \frac{T_1 - T_3}{T_3^2} + \frac{T_1}{(T_2 T_3)^2} \\ \frac{T_1}{T_2 T_3} & 0 \end{vmatrix} = -\frac{T_1}{T_2 T_3} \left(\frac{T_1 - T_3}{T_3^2} + \frac{T_1}{(T_2 T_3)^2} \right) \overset{?}{\neq} 0$$

After substituting (1.21) to (1.23) into the previous relation:

$$\frac{-\frac{1}{\lambda}\left(\frac{1}{\lambda} - \frac{\ell}{\lambda\ell + \beta}\right)}{\frac{\lambda\ell+\beta}{\lambda} \cdot \left(\frac{\ell}{\lambda\ell+\beta}\right)^3} - \frac{\left(\frac{1}{\lambda}\right)^2}{\left(\frac{\lambda\ell+\beta}{\lambda} \cdot \frac{\ell}{\lambda\ell+\beta}\right)^3} \overset{?}{\neq} 0$$

$$\frac{\lambda(\beta\ell + \lambda)}{\beta^2} \overset{?}{\neq} 1$$

$$\frac{7.727 \times 10^{-1}\left(6.274 \times 10^{-3} \times 10^{-3} + 7.727 \times 10^{-1}\right)}{\left(6.274 \times 10^{-3}\right)^2} \neq 1$$

The rank of matrix $\bar{G}$ is h = 2 because the minors of matrix $\bar{G}$:

$$M_{\bar{G}_1} = \left| -\frac{T_1 - T_3}{T_3} \right| = \left| -\frac{\frac{1}{\lambda} - \frac{\ell}{\lambda\ell+\beta}}{\frac{\ell}{\lambda\ell+\beta}} \right| \neq 0$$

$$M_{\bar{G}_2} = \det \bar{G} \neq 0$$

The system (2.38) and (2.39) is controllable.

2.2.3. Derivation of the optimal control

The calculus of variations, which is concerned with the finding of trajectories that maximize or minimize a given functional, is used for the derivation of optimal control law. The eq. (2.1) is often written for time-variant systems, /6/:

$$\dot{\bar{x}} = f(\bar{x}, \bar{u}, t) \quad , \quad \bar{x}(t_0) = \bar{x}_0 \tag{2.40}$$

A control variable $\bar{u}$ is looked for, such, that the performance index

$$I = \int_{t_0}^{t_1} f_0(\bar{x}, \bar{u}, t)\,dt + f_k[\bar{x}(t_1)] \tag{2.41}$$

is equal to extremal (minimal) value. The expression (2.41) is the criterion of optimality. From the mathematical point of view the determination of min I represents the determination of extreme with adjacent condition (2.40). The solution of this problem is performed by Euler differential equation with adjacent condition (Lagrangian coefficients λ_k^T are applied). Then :

$$I = \int_{t_0}^{t_1} \left[f_0(\bar{x}, \bar{u}, t) + \bar{\lambda}_k^T(t) \cdot \left(\dot{\bar{x}} - f(\bar{x}, \bar{u}, t)\right) + \frac{\partial f_k}{\partial \bar{x}} \cdot \dot{\bar{x}} \right] dt \tag{2.42}$$

The variations of functional I equal to 0 is the necessary condition for the extreme determination:

$$\delta I = 0 \tag{2.43}$$

The variation is given as

$$\delta I = \frac{\partial I}{\partial \bar{\alpha}} \delta \bar{\alpha} \qquad \text{where } \bar{\alpha} = \bar{x}, \dot{\bar{x}}, \bar{u}, \bar{\lambda}_k^T \tag{2.44}$$

Then:

$$\delta I = \int_{t_0}^{t_1} \Big[\left(\frac{\partial f_0}{\partial \bar{x}} - \bar{\lambda}_k^T \frac{\partial f}{\partial \bar{x}} \right) \delta \bar{x} + \left(\bar{\lambda}_k^T + \frac{\partial f_k^T}{\partial \bar{x}} \right) \delta \dot{\bar{x}} + \left(\frac{\partial f_0}{\partial \bar{u}} - \bar{\lambda}_k^T \frac{\partial f}{\partial \bar{u}} \right) \delta \bar{u} + \delta \bar{\lambda}_k^T \left(\dot{\bar{x}} - f(\bar{x}, \bar{u}, t) \right) \Big] dt = 0 \tag{2.45}$$

After integration the 2nd term by method "per partes" and after substitution back into (2.45) it is obtained:

$$\delta I = \int_{t_0}^{t_1} \Big[\left(\frac{\partial f_0}{\partial \bar{x}} - \bar{\lambda}_k^T \frac{\partial f}{\partial \bar{x}} - \dot{\bar{\lambda}}_k^T \right) \delta \bar{x} + \left(\frac{\partial f_0}{\partial u} - \bar{\lambda}_k^T \frac{\partial f}{\partial \bar{u}} \right) \delta \bar{u} + \delta \bar{\lambda}_k^T \left(\dot{\bar{x}} - f(\bar{x}, \bar{u}, t) \right) \Big] dt + \left(\bar{\lambda}_k^T + \frac{\partial f_k^T}{\partial \bar{x}} \right) \delta \bar{x} \Big|_{t_0}^{t_1} = 0 \tag{2.46}$$

The eq. (2.46) must be satisfied for arbitraty variations $\delta \bar{x}$, $\delta \bar{u}$, $\delta \bar{\lambda}_k^T$ and thus the expressions in parentheses must be equal to 0.

$$\begin{aligned} \dot{\bar{x}} - f(\bar{x}, \bar{u}, t) &= 0 \\ - \dot{\bar{\lambda}}_k^T - \bar{\lambda}_k^T \frac{\partial f}{\partial \bar{x}} + \frac{\partial f_0}{\partial \bar{x}} &= 0 \\ \left(\bar{\lambda}_k^T + \frac{\partial f_k^T}{\partial \bar{x}} \right) \delta \bar{x} \Big|_{t_0}^{t_1} &= 0 \\ \frac{\partial f_0}{\partial \bar{u}} - \bar{\lambda}_k^T \frac{\partial f}{\partial \bar{u}} &= 0 \end{aligned} \tag{2.47}$$

Since

$$\left(\bar{\lambda}_k^T + \frac{\partial f_k^T}{\partial \bar{x}} \right) \delta \bar{x} \Big|_{t = t_0} = \left(\bar{\lambda}_k^T + \frac{\partial f_k^T}{\partial \bar{x}} \right) \delta \bar{x} \Big|_{t = t_1} = 0$$

then for all variations $\delta \bar{x}$ must be

$$\bar{\lambda}_k^T + \frac{\partial f_k^T}{\partial \bar{x}} = 0 \tag{2.48}$$

Applying the condition (2.48) into (2.47) and after defining

$$H_f = f_0 - \bar{\lambda}_k^T f(\bar{x}, \bar{u}, t) \qquad (2.49)$$

it is obtained:

$$\begin{aligned} \dot{\bar{x}} &= -\mathrm{grad}_{\bar{\lambda}_k^T} H_f \\ \dot{\bar{\lambda}}_k^T &= \mathrm{grad}_{\bar{x}} H_f \\ \mathrm{grad}_{\bar{u}} H_f &= 0 \\ \bar{\lambda}_k^T(t_1) &= -\frac{\partial f_k^T}{\partial \bar{x}} \end{aligned} \qquad (2.50)$$

where H_f is scalar Hamiltonian function and the first two equations are Hamiltonian canonical equations which are similar to (2.11) and (2.12). The vectors $\bar{\psi}$ and $\bar{\lambda}_k^T$ in equations (2.11), (2.12) and (2.50) have the same significance.

It is possible to re-write the equations (2.40) and (2.41) for controlled linear, time-invariant system and for quadratic performance index into the forms of the eqs. (2.1) and (2.34).

Then the Hamiltonian according to (2.49):

$$H_f = \frac{1}{2}\bar{x}^T\bar{Q}\bar{x} + \frac{1}{2}\bar{u}^T\bar{R}\bar{u} - \bar{\lambda}_k^T\left(\bar{A}\bar{x} + \bar{B}\bar{u}\right) \qquad (2.51)$$

The minimum H_f is determined applying the third eq. of (2.50):

$$\mathrm{grad}_{\bar{u}} H_f = \frac{\partial H_f}{\partial \bar{u}} = \frac{\partial}{\partial \bar{u}}\left[\frac{1}{2}\bar{x}^T\bar{Q}\bar{x} + \frac{1}{2}\bar{u}^T\bar{R}\bar{u} - \bar{\lambda}_k^T(\bar{A}\bar{x} + \bar{B}\bar{u})\right] =$$

$$= \frac{\partial}{\partial \bar{u}}\left(\frac{1}{2}\bar{u}^T\bar{R}\bar{u}\right) - \frac{\partial}{\partial \bar{u}}\left(\bar{\lambda}_k^T\bar{B}\bar{u}\right) = 0 \qquad (2.52)$$

The first term of (2.52) must be differentiated according to components $u_1, u_2 \ldots\ldots u_n$ of vector $\bar{u}$. It is possible to obtain with respect to the relation $\bar{R} = \bar{R}^T$ for symmetrical matrixes $\bar{R}$ and $\bar{R}^T$:

$$\mathrm{grad}_{\bar{u}} H_f = \frac{1}{2}\bar{R}\bar{u} + \frac{1}{2}\bar{R}^T\bar{u} - \bar{\lambda}_k^T\bar{B} = \bar{R}\bar{u} - \bar{\lambda}_k^T\bar{B} = 0 \quad (2.53)$$

Then the optimal control u:

$$\bar{u} = \bar{R}^{-1}\left(\bar{\lambda}_k^T\bar{B}\right) = \bar{R}^{-1}\bar{B}^T\bar{\lambda}_k \quad (2.54)$$

Then the set of eqs. (2.47):

$$\begin{aligned} &\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{R}^{-1}\bar{B}^T\bar{\lambda}_k \\ &\dot{\bar{\lambda}}_k^T = -\bar{\lambda}_k^T\bar{A} + (\bar{Q}\bar{x})^T \Rightarrow \dot{\bar{\lambda}}_k = -\bar{A}^T\bar{\lambda}_k + \bar{Q}\bar{x} \\ &\lambda_k(t_1) = -\bar{F}_x(t_1) \\ &x(t_0) = x_0 \end{aligned} \quad (2.55)$$

The solution of the set of eqs. (2.55) is performed with the help of the following substitutions:

$$\bar{\lambda}_k(t) = -\bar{K}_c(t)\cdot\bar{x}(t) \quad (2.56)$$

$$\dot{\bar{\lambda}}_k = -\dot{\bar{K}}_c\bar{x} - \bar{K}_c\dot{\bar{x}} \quad (2.57)$$

On applying the eqs. (2.56) and (2.57) into (2.55) the Riccati differential equation is obtained:

$$\dot{\bar{K}}_c = -\bar{K}_c\bar{A} - \bar{A}^T\bar{K}_c + \bar{K}_c\bar{B}\bar{R}^{-1}\bar{B}^T\bar{K}_c - \bar{Q} \quad (2.56)$$

The solution of this eq. is for the condition

$$\lim_{t\to\infty} \bar{K}_c(t) = \bar{K}_c = \mathrm{const.} \quad (2.57)$$

simplyfied into the solution of algebraic equation

$$\bar{K}_c\bar{A} + \bar{A}^T\bar{K}_c - \bar{K}_c\bar{B}\bar{R}^{-1}\bar{B}^T\bar{K}_c + \bar{Q} = 0 \quad (2.58)$$

The matrix $\bar{K}_c$ is a function only of the matrices $\bar{A}$ and $\bar{B}$.

The values of the matrix $\bar{K}_c$ as well as the values of the expression

$$-\bar{R}^{-1}\bar{B}^{T}\bar{K}_{c} = K_{R}\,, \qquad (2.59)$$

where $\bar{K}_R$ is the matrix of coefficients of optimal control law, are determined off-line with the help of program "ACRY" written in Fortran IV - ICT 1905 (inputs: matrices $\bar{A}$, $\bar{B}$, $\bar{Q}$, $\bar{R}$. Outputs: $\bar{K}_C$, $\bar{K}_R$, $\bar{K}_{R1}$, $\bar{K}_{R2}$), /7/. The results of solution for chosen combinations of weighting matrices $\bar{Q}$ and $\bar{R}$ of the quadratic performance index I are in TABLE 1.

TABLE 1

The matrix $\bar{K}_R$ of coefficients of feedback optimal control law

R \ Q	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 10\end{bmatrix}$	$\begin{bmatrix}10 & 0\\0 & 1\end{bmatrix}$	K_{Ri}
1	0.136	0.925	0.077	0.101	0.659	K_{R1}
	0.998	0	0.999	3.161	0.985	K_{R2}
10^{-2}	0.877	9.923	0.072	0.169	7.851	K_{R1}
	9.968	0	9.999	11.470	9.693	K_{R2}
10^{-3}	1.037	31.542	0.073	00.896	9.218	K_{R1}
	11.973	0	11.365	100.232	11.403	K_{R2}
10^{-5}	25.944	283.587	0.073	8.291	251.287	K_{R1}
	315.168	0	316.229	999.663	306.087	K_{R2}

According to eq. (2,56) and the first eq. of (2.55) the optimalized system is described by the equation.

$$\dot{\bar{x}} = \bar{A}\bar{x} - \bar{B}\bar{R}^{-1}\bar{B}^{T}\bar{K}_{c}\bar{x} = \bar{A}\bar{x} + \bar{B}\bar{K}_{R}\bar{x} \qquad (2.60)$$

The optimal control variable

$$\boxed{\bar{u}^* = -\bar{R}^{-1}\bar{B}^T\bar{K}_c\,\bar{x} = -\bar{K}_R\,\bar{x}} \qquad (2\ 61)$$

Since the real controlled process is driving by the noise (random vector $\bar{w}$) and measurements are contaminated by the additive noise (random vector $\bar{v}$) and since not all state variables of the controlled process are measurable (N_1 in MODEL 2), it is necessary to place the estimator (Kalman's filter) before the regulator. This estimator determines the nonmeasurable variables and makes filtering. The description (structure) of the estimator was derived by Kalman, /6/ and has the following form:

$$\dot{\hat{\bar{x}}} = \bar{A}\dot{\bar{x}} + \bar{K}_f\bar{C}^T\bar{\xi}^{-1}\left(\bar{y} - \bar{C}\bar{x}\right) \qquad (2.62)$$

$$\dot{\bar{K}}_f = \bar{A}\bar{K}_f + \bar{K}_f\bar{A}^T - \bar{K}_f\bar{C}^T\bar{\xi}^{-1}\bar{C}\bar{K}_f + \bar{G}\bar{\eta}\bar{G}^T \qquad (2.63)$$

$$\bar{A}\bar{K}_f + \bar{K}_f\bar{A}^T - \bar{K}_f\bar{C}^T\bar{\xi}^{-1}\bar{C}\bar{K}_f + \bar{G}\bar{\eta}\bar{G}^T = 0 \qquad (2.64)$$

These equations have the analogical form to equations (2.56), (2.58) and (2.59) and therefore the values of matrices $\bar{K}_f$ and $\bar{K}_e$ defined as

$$\bar{K}_f\bar{C}^T\bar{\xi}^{-1} = \bar{K}_e \qquad (2.65)$$

can be determined in the same way as in the case of control law derivation - by the programm "ACRY" (inputs: $\bar{A}$, $\bar{C}$, $\bar{\xi}$, $\bar{\eta}$; outputs: $\bar{K}_f$, $\bar{K}_e$, $\bar{K}_{e1}$, $\bar{K}_{e2}$). It is evident that the matrix $\bar{K}_f$ depends only on the matrix $\bar{A}$ and $\bar{C}$. The symbol $\wedge$ means the estimated value of a state variable.

The matrixes

$$\bar{\xi} = \mathrm{cov}\ \bar{v} \qquad (2.66)$$

$$\bar{\eta} = \mathrm{cov}\ \bar{w} \qquad (2.67)$$

ought to have been known from real measurement on the controlled process (reactor). Since the values of $\bar{\xi}$ and $\bar{\eta}$ had not been measured in the time of the simulation, the values of matrix $\bar{K}_r$ and $\bar{K}_e$ were computed for several chosen combinations of $\bar{\xi}$ and $\bar{\eta}$ (see TABLE 2).

TABLE 2

The matrix $\bar{K}_e$ of coefficients of the estimator

$\bar{\xi} = 1$

$\bar{K}_e$ / $\bar{\eta}$	K_{e1}	K_{e2}
10^3	26.356	63.897
10^2	7.231	33.490
10	1.794	16.643
1	0.372	7.611
10^{-1}	0.067	2.623
10^{-2}	0.00852	0.387
10^{-3}	0.000881	0.0406

For the system described by equations (2.35) and (2.36) whose order is 2, the matrix $\bar{K}_R$ ($\bar{K}_e$) must have two components K_{R1} and K_{R2} (K_{e1} and K_{e2}).

The block schema of the feedback optimal control of reactor model A-1 is introduced in Fig. 5.

2.2.4. The simulation on the analogue computer

The following set of equations is simulated on the analogue computer Tesla AP3-M:

$$\left.\begin{aligned} &pN - \frac{\delta k - \beta}{l} N - \sum_{i=1}^{6} \lambda_i c_i - \frac{\delta k_0 N_0}{l} = 0 \qquad (2.68) \\ &pc_i - \frac{\beta_i}{l} N + \lambda_i c_i = 0 \qquad (2.69) \end{aligned}\right\} \text{real controlled process}$$

$$\left.\begin{aligned} &p\hat{N}_1 + 6.372\,\hat{N}_1 + 82.454\,(u + w_0) - K_{e_1}(\hat{N} - N) = 0 \qquad (2.70) \\ &p\hat{N} - 77.519\,\hat{N}_1 + 1003.101\,(u + w_0) - K_{e_2}(\hat{N} - N) = 0 \qquad (2.71) \end{aligned}\right\} \textit{estimator}$$

$$K_{R_2}(\hat{N} - N_z) + K_{R_1}\hat{N}_1 + (u + w_0) = 0 \qquad \text{feedback optimal control law} \qquad (2.72)$$

$$\delta k_m - \frac{u + w_0}{\hat{N}} = 0 \qquad \text{division element} \qquad (2.73)$$

$$\hat{N} - N = 0 \qquad \text{deviation element} \qquad (2.74)$$

The classical regulator of reactor A-1 (SUZ) is simulated in this way:

$$N_z - N = \Delta N \qquad \text{deviation element} \qquad (2.75)$$

$$\left.\begin{aligned} &E = \Delta N \quad \text{for } |\Delta N| < 0.0207 \\ &E = +0.0207 \quad \text{for } \Delta N \geq 0.0207 \\ &E = -0.0207 \quad \text{for } \Delta N \leq -0.0207 \end{aligned}\right. \qquad \text{electronic amplifier} \qquad (2.76)$$

$$0.1\,p f_g + f_g = 483\,E \qquad \text{low frequency generator} \qquad (2.77)$$

$$\frac{1}{1.5}\,ps = f_g \qquad \text{synchronous motor} \qquad (2.78)$$

$$\delta k = 1.5 \times 10^{-5}\,s \qquad \text{regulating rod} \qquad (2.79)$$

$$(\delta k = 0.01\,s) \qquad \text{two reg. rods of 1 regulator} \qquad (2.80)$$

The comparison of optimal and conventional (classical) control of neutron density (N^{OC} and N^{SUZ}) in dependence on the set-point neutron density N_S is evident from fig. 6. The referenced reactivity functions (δk^{OC} and δk^{SUZ}) are drawn in the upper part of the same figure. The quality of neutron density control by means of optimal regulator is higher (e.g. quicker and with neglectable overshoot) than with the classical regulator (SUZ). When comparing, however, it is necessary to realize that the optimal control analysis has been solved provided that the random vectors driving process ($\bar{w}$) and the additive noise ($\bar{v}$) contaminating the measurements are neglected.

3. CONCLUSIONS

The first stage of optimal control studies is described in this report. The optimal control as a part of Modern Control Theory is applied on nuclear reactor. Neutron density time-optimal control and optimal control with quadratic performance index are determined on the analogue models.

The continuation of studies (in digital form) is assumed to be performed on the reactor models and in practical applications with respect to state and control variables constraints and the influence of stochastic character of signals.

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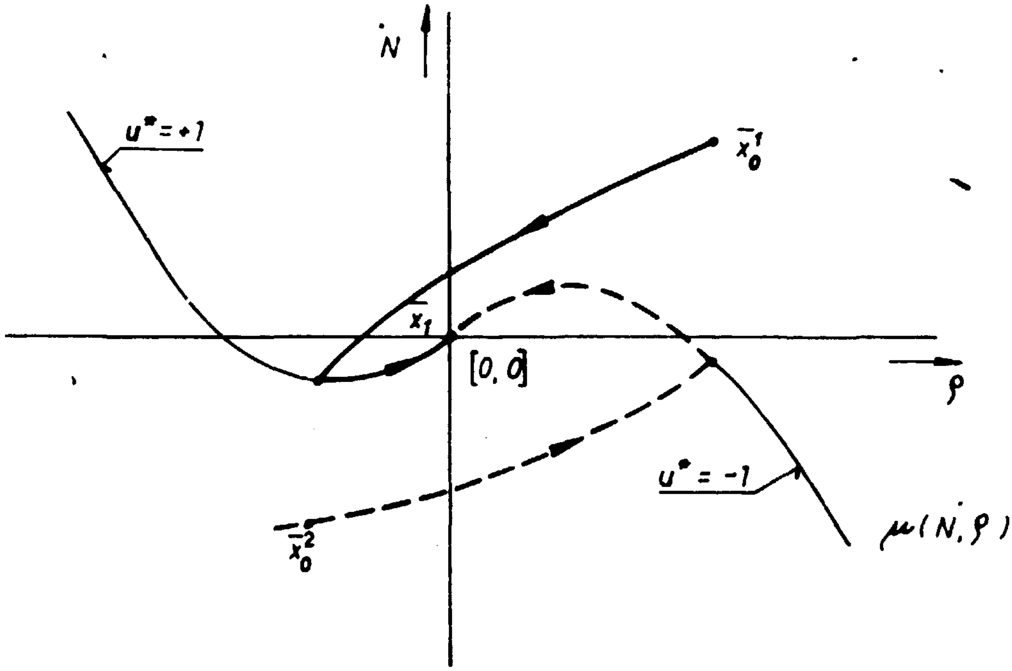

Fig. 1: Switching curve $\mu(\dot{N}, \rho)$

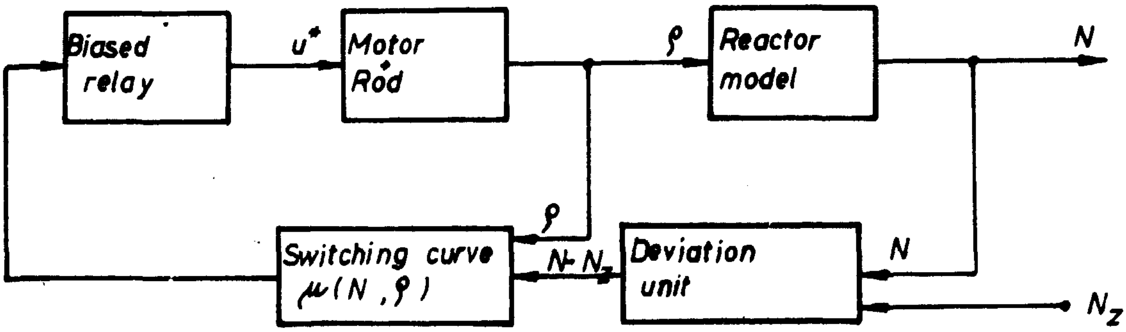

Fig. 2: Block schema of time optimal control of reactor model A1 (N = neutron density deviation).

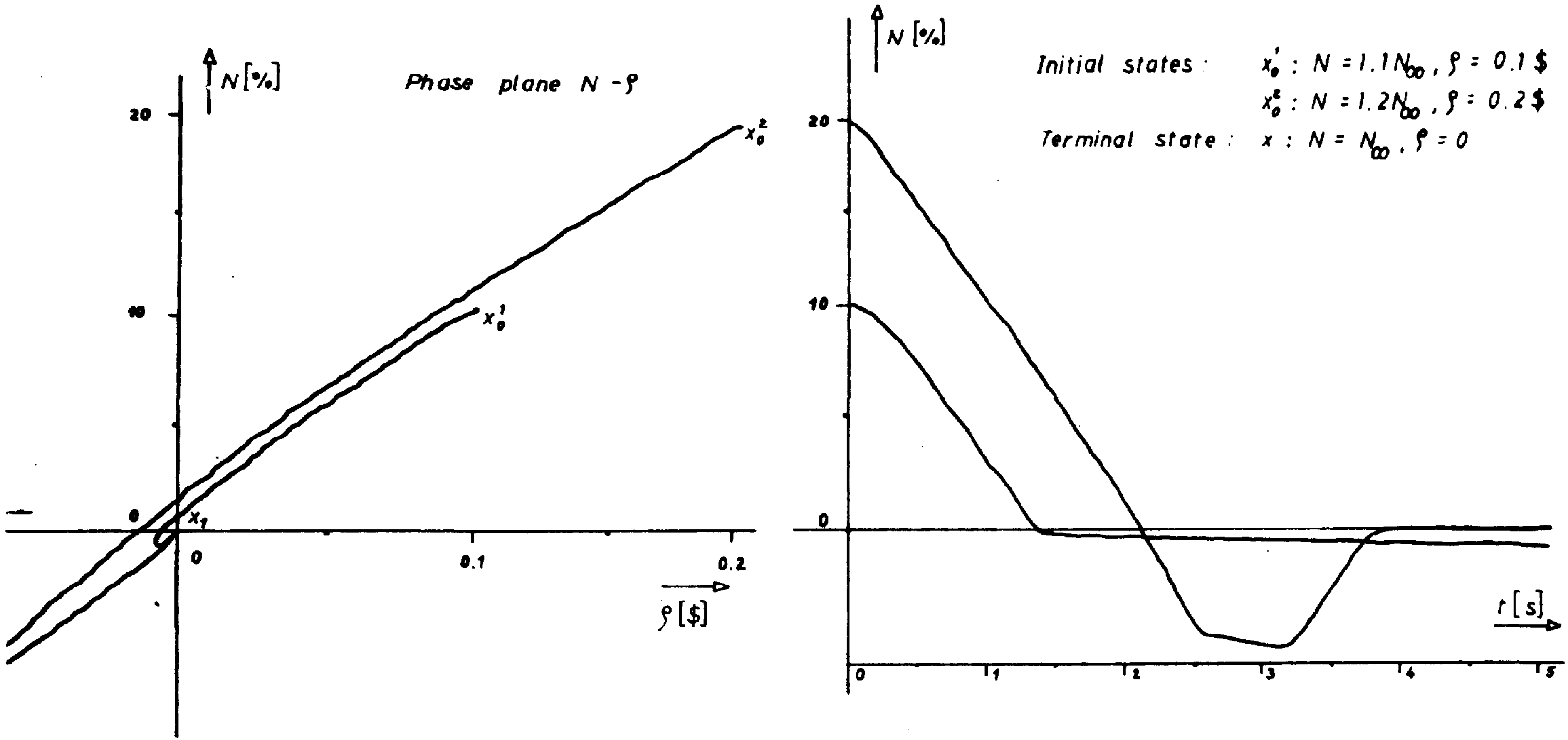

Fig. 3 : Time optimal control of neutron density reactor model A 1
(Disturbance : prompt jump of neutron density to the initial states).

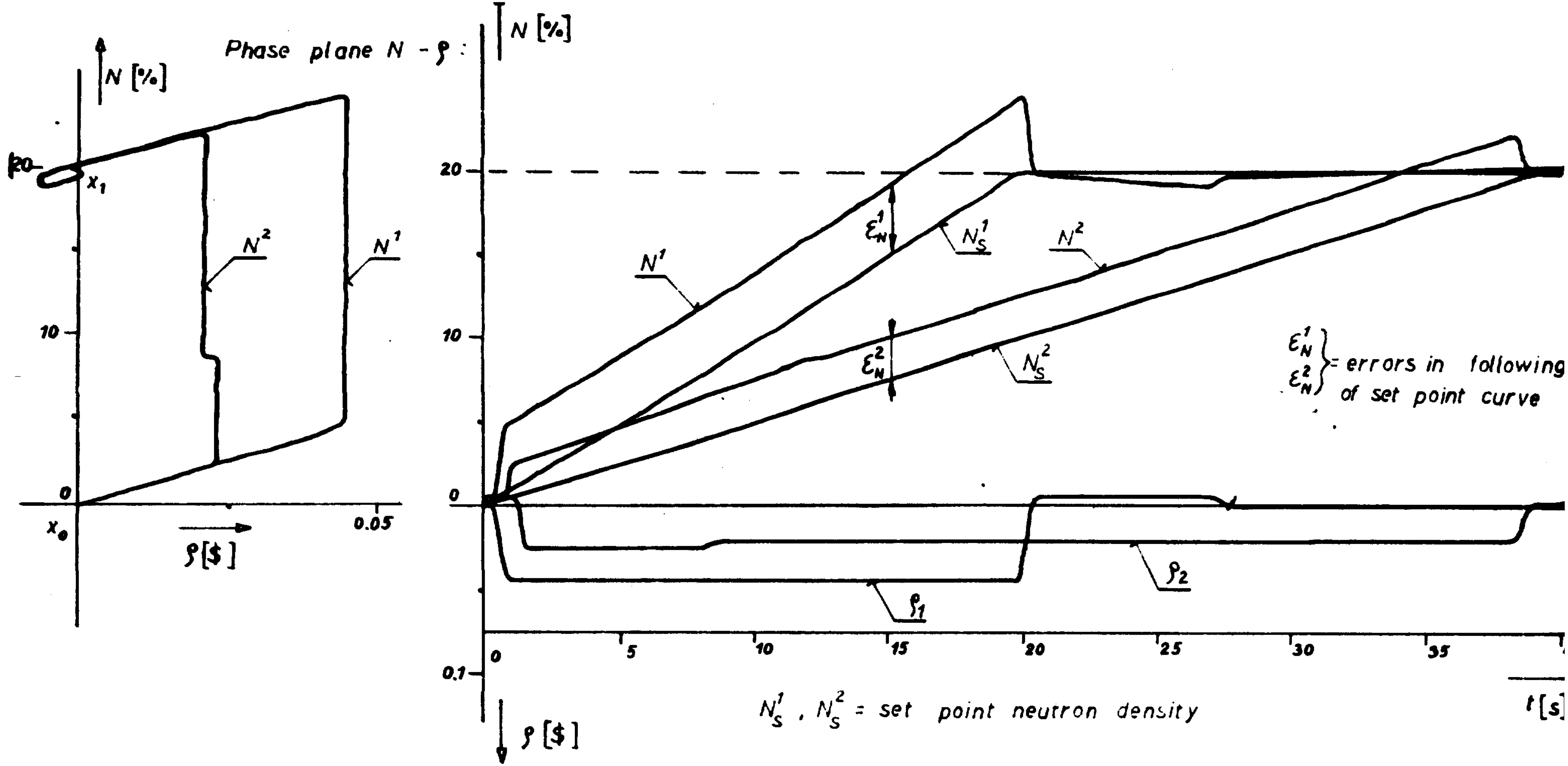

Fig. 4. Time optimal control of neutron density reactor A 1
(Disturbance : linear growth of neutron density).

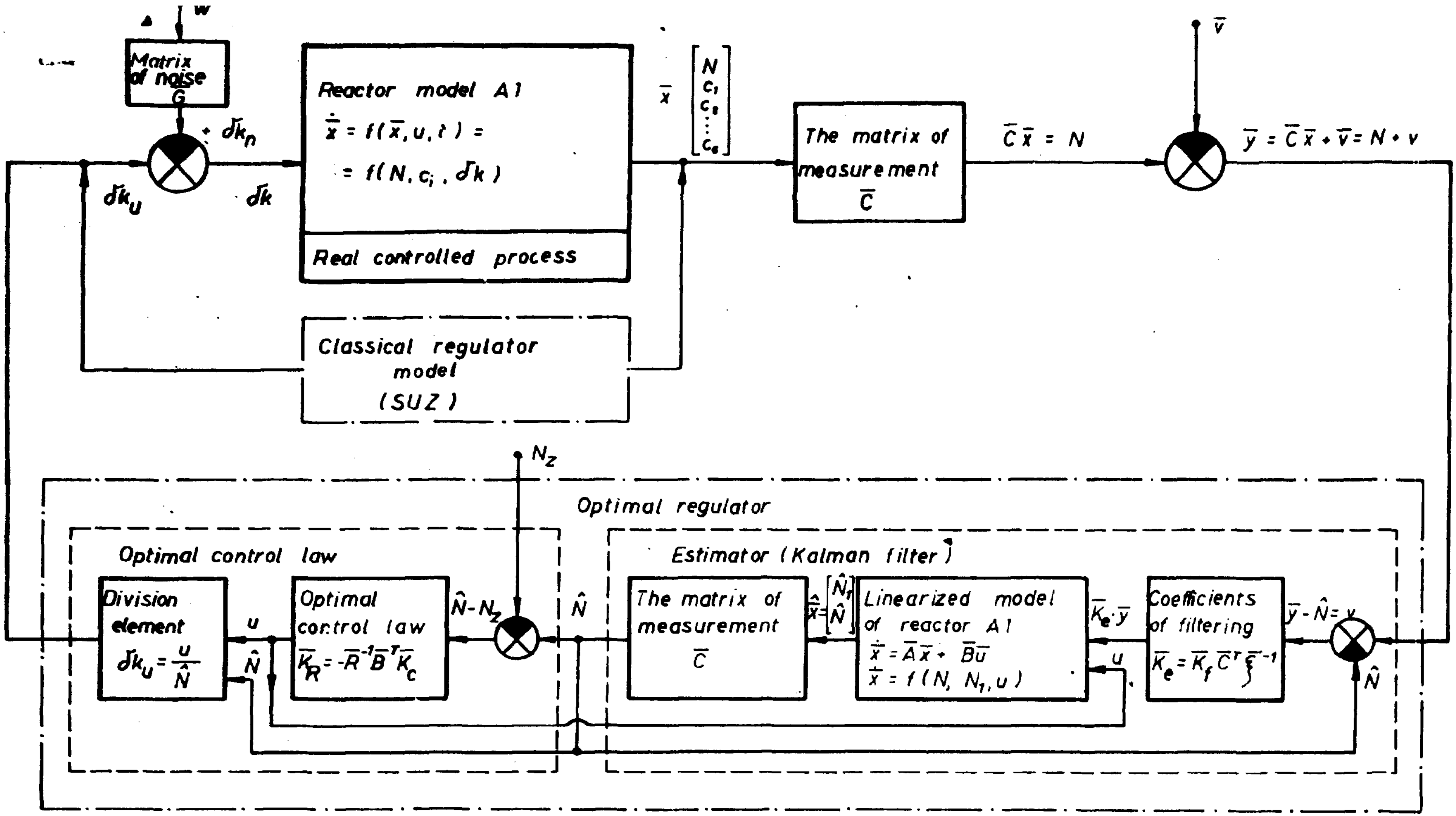

Fig. 5: **Block - schema of the feedback optimal control of reactor model A1 with quadratic performance index (N = neutron density deviation).**

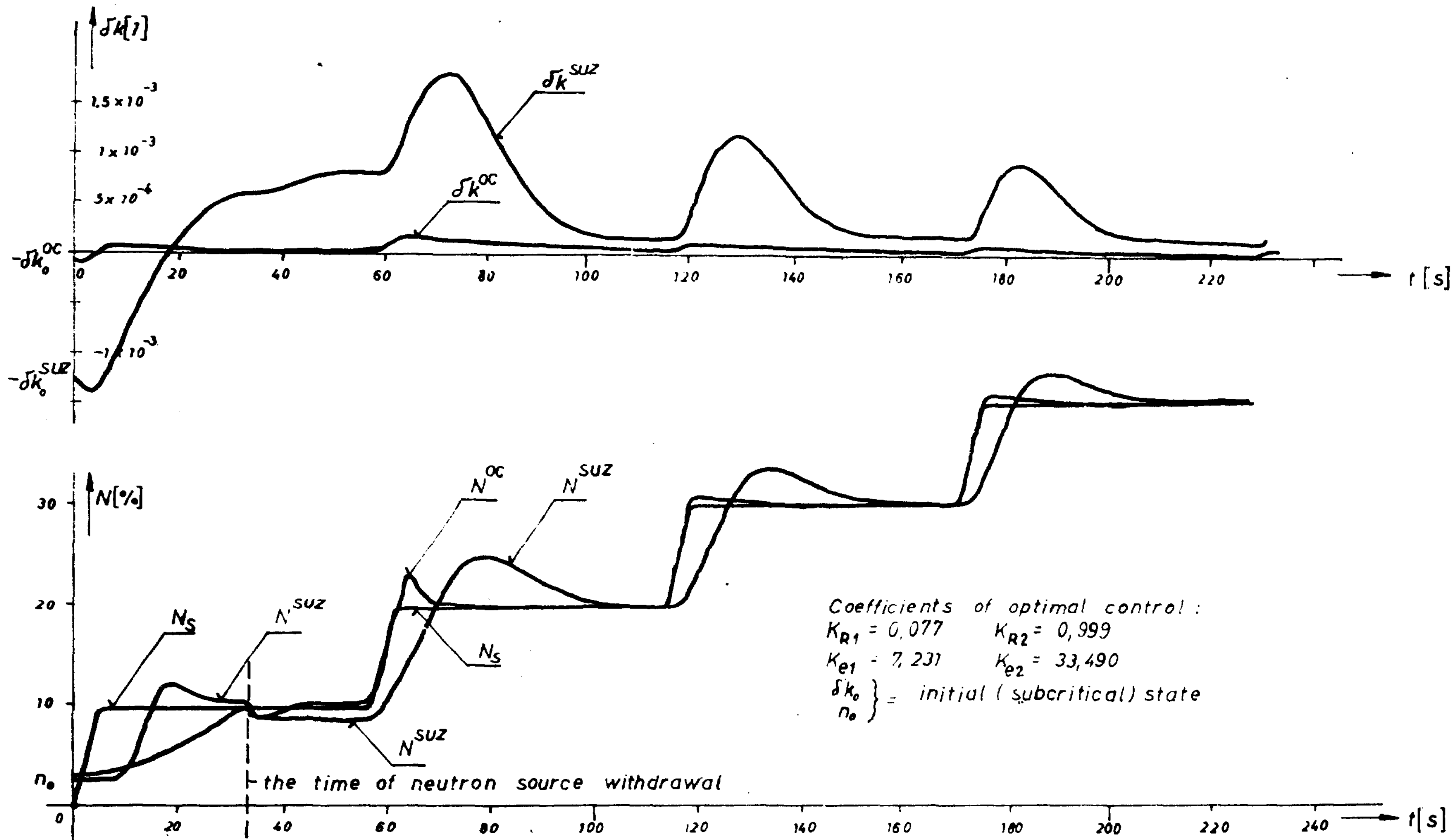

Fig. 6 : Neutron density deviation N and reactivity δk of reactor model A 1 controlled by optimal regulator and classical regulator SUZ.