CNRS - CPT - 75 - P. 743
FR7600894

2

RELATIVISTIC BOUND STATES : A MASS FORMULA FOR VECTOR MESONS

J.L. RICHARD*   P. SORBA *

ABSTRACT : In the framework of a relativistic description of two particle bound states, a mass formula for vector mesons considered as quark-antiquark systems bound by harmonic oscillator like forces is proposed. Results in good agreement with experimental values are obtained.

JULY 1975
75/P.743

* Centre de Physique Théorique - CNRS Marseille

POSTAL ADDRESS : Centre de Physique Théorique - C.N.R.S.
31, chemin Joseph Aiguier
13274 MARSEILLE CEDEX 2 (France)

1115

The problem of interaction in relativistic quantum mechanics remains certainly one of the most attractive in particle physics. Interesting approaches have been proposed by Dirac[1], Bakamjian and Thomas[2], Foldy[3]. More recently some hope had been born with the development of theories at infinite momentum[4], using the non relativistic structure of the Poincaré group ; but, if the success of such an approach is evident in the current algebra scheme, its use for describing interacting relativistic systems in the general case seems still uncertain.

The success of the theory of quarks in the hadrons physics is one of the motivations for the interest of a relativistic description of bound states. The concept of quarks prisoners of the hadrons they build up has led physicists to introduce appealing notions such as the colour, the gluon theory and more recently the bag theories. To these attempts one must add the symmetric harmonic oscillator quark model[5] for baryons in which the quarks constituting baryons are assumed linked by harmonic oscillator forces*: first presented in the non relativistic scheme[7], its generalization to the relativistic case[8] does not appear so easy. Note at this point a recent paper[9] in which a model for relativistic bound state perturbation theory is presented.

However, it appears very important to have a relativistic treatment of quarks. Let us recall the transformation from current to constituent quarks obtained by Melosh[10] in a relativistic free quark model : this transformation can be simply interpreted as a Wigner's rotation[11], and so could not appear in a non relativistic treatment.

The aim of this note is both. First, using the approach of Ref.[2], itself reconsidered by Coester[12], we try to obtain a relativistic description of two particle bound states. Then, in this framework, a mass operator for the composite particle appears naturally. From this mass operator we propose a mass formula for the vector mesons composed

---

* Harmonic oscillator interactions for mesons have also been used : see for example Ref.[6].

with a quark and an antiquark bound by harmonic oscillator like forces ; the so obtained mass spectrum seems in good agreement with the mass table of vector mesons.

Now, we would like to exhibit a very nice parallelism one can establish between the non relativistic description of interacting particles and the relativistic approach of Bakamjian and Thomas. This parallelism appears very clear when one is dealing with wave functions defined in momentum space. As we all know, a relativistic description of interacting particles in terms of coordinates in space-time has been unsuccessfull [13] when the good number of degrees of freedom are used in contradistinction with Feynman's approach [7] in which some internal time parameter occurs.

Let us briefly examine the non relativistic case for two particles. Let $\vec{p}_1$ and $\vec{p}_2$ be the momentum of particle (1) and (2) respectively with masses m_1 and m_2 . New variables are then defined which isolate the momentum $\vec{p}$ of the whole system. They read

$$\begin{aligned} \vec{p} &= \vec{p}_1 + \vec{p}_2 \\ \vec{k} &= \frac{m_2 \vec{p}_1 - m_1 \vec{p}_2}{m_1 + m_2} \end{aligned} \tag{1}$$

For non interacting particles, we could as well write $\vec{k}$ as

$$\vec{k} = \frac{1}{2} L_G \left(\frac{\vec{P}}{m_1 + m_2} \right)^{-1} \left(\vec{p}_1 - \vec{p}_2 \right) \tag{2}$$

where the Galilean boost L_G is defined according to

$$L_G(\vec{v})\, \vec{p}_i = \vec{p}_i + m_i \vec{v} \qquad i = 1,2 \tag{3}$$

as usual.
Then, the momentum variable $\vec{k}$ is seen to be the momentum of the particle (1) in the center of mass frame. Using these new variables, the generators of the Galilean group read

a) Translations :

$$\vec{P} = \vec{p} \qquad H_0 = \frac{\vec{p}^2}{2M} + \frac{\vec{k}^2}{2\mu} \tag{4a}$$

with $M = m_1 + m_2$ and $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

b) Rotations

$$\vec{J} = \vec{x} \times \vec{p} + \vec{y} \times \vec{k} \tag{4b}$$

with $\vec{x} = i\frac{\partial}{\partial \vec{p}}$ and $\vec{y} = i\frac{\partial}{\partial \vec{k}}$

c) Boosts :

$$\vec{K} = M\vec{x} \tag{4c}$$

If now we add to the free hamiltonian H_0 an operator V depending only on $\vec{y}$ and $\vec{k}$ in a scalar way, the commutation relations of the generators are still the same. We then introduce an interaction between the particles compatible with the Galilean symmetry group.

In the relativistic framework, we shall follow exactly the same steps. Consider two non interacting particles of mass m_1 and m_2 and spin s_1 and s_2. Define the variables

$$p = p_1 + p_2 \qquad \text{with} \qquad p_1^2 = m_1^2 \;, \; p_2^2 = m_2^2 \tag{5}$$

$$k = \frac{1}{2} L(p)^{-1}(p_1 - p_2)$$

as in (1) except that now p and k are four vectors and $L(p)$ is the relativistic boost mapping $\mathring{p} = (\sqrt{p^2}, \vec{0})$ into p, leaving invariant the vectors orthogonal to $\mathring{p}$ and p. As one can easily see the spatial components of the momentum variable k correspond to the momentum of particle (1) in the rest frame of the system as in the Galilean case. By means of the variables $\vec{p}$ and $\vec{k}$ so defined, one rewrites the generators of the Poincaré group as

a) Translations : $\vec{P} = \vec{p}$, $H_0 = \sqrt{\vec{p}^2 + M_0^2}$ (6a)

with $M_0 = \sqrt{\vec{k}^2 + m_1^2} + \sqrt{\vec{k}^2 + m_2^2}$

b) Rotations

$$\vec{J} = \vec{x} \times \vec{p} + \vec{y} \times \vec{k} + \vec{S} \tag{6b}$$

with $\vec{x} = i \frac{\partial}{\partial \vec{p}}$, $\vec{y} = i \frac{\partial}{\partial \vec{k}}$ and $\vec{S} = \vec{S}_1 + \vec{S}_2$
where $\vec{S}_1$ and $\vec{S}_2$ are the spin representations for the particles (1) and (2) respectively.

c) Boosts

$$\vec{K} = \frac{1}{2}\left[\vec{x} H_0 + H_0 \vec{x}\right] - \left(\vec{y} \times \vec{k} + \vec{S}\right) \times \vec{p} \left[M_0 + H_0\right]^{-1} \tag{6c}$$

As one can see, if in (6 a,b,c) we replace the mass M_0 by a "mass" M depending only on scalars made with $\vec{y}$, $\vec{k}$ and the spin variables, the commutation relations of the new generators are still the same. In doing so, we introduce some kind of an interaction between the particles compatible with Poincaré symmetries.

As we said, we intend to introduce in this framework an interaction which would exhibit an harmonic oscillator like spectrum in order to describe mesons as quark-antiquark bound systems. The "mass" M that we can introduce in (6) in place of M_0 is quite arbitrary and our only guide will be simplicity. We only require that M becomes M_0 when the coupling constant of the "interaction" is put to be zero. We shall then define the mass operator M in the very naïve form

$$M = \left(\vec{k}^2 + \omega^2 \vec{y}^2 - E_0 + m_1^2\right)^{\frac{1}{2}} + \left(\vec{k}^2 + \omega^2 \vec{y}^2 - E_0 + m_2^2\right)^{\frac{1}{2}} \tag{7}$$

in order to get a harmonic oscillator like spectrum. As we see ω plays the role of a coupling constant while m_1 and m_2 are to be connected with the quark masses. The constant E_0 is set to be the energy of the ground state for the harmonic oscillator. Its spectrum is well-known[14] and so is the spectrum of M . The eigenvalues of M are then

$$M_n = \left(2n\omega + m_1^2\right)^{\frac{1}{2}} + \left(2n\omega + m_2^2\right)^{\frac{1}{2}} \quad n = 0, 1, 2\ldots \qquad (8)$$

As we see, we did not introduce terms connected with spin-spin and spin-orbit couplings as one could do ; our goal is just to show that one can formulate a mass formula in such a framework.

In order to illustrate our purpose, we tried to classify the vector mesons according to the mass spectrum given in (8). We shall assume that a quark and the corresponding antiquark have the same mass. We denote by m_n, m_p, m_λ and m_c the masses of the neutron, proton, strange and charmed quarks respectively. Further, we suppose that $m_n = m_p$ as usual.

For isospin one mesons, this leads to a linear law in the $(\text{mass})^2$, which has been frequently observed[15]. More precisely, we get :

$$M^2_{n, I=1} = 4\left(2n\omega + m_p^2\right) \qquad (9)$$

From the mass values[16] of the $J^{PC} = 1^{--}$ $\rho(750)$ and $\rho(1600)$ mesons, corresponding to $n = 0$ and $n = 2$ respectively, we derive the values of m_p and ω :

$$m_p = 375 \text{ Mev.} \qquad \omega = \frac{1}{8} \times 10^6 \text{ Mev}^2$$

So we find for the $J^{PC} = 1^{+-}$, $n = 1$, B (1220) meson the mass of 1262 Mev.

We use the mass value of the $I = 0$, $J^{PC} = 1^{--}$ $\phi(1020)$ meson considered as a bound state $\lambda\bar{\lambda}$, $n = 0$, to deduce :

$$m_\lambda = 510 \text{ Mev.}$$

Then we obtain for the $I = \frac{1}{2}$, $J^{PC} = 1^{--}$, $n = 0$, K(892) the mass of 885 Mev, and for the $I = \frac{1}{2}$, $J^{PC} = 1^{+-}$, $n = 1$, K(1320) the mass of 1339 Mev.

Thus we can see that the masses of the excited states are found to be in very good agreement with the experimental values.

In order to have also the masses of the ψ particles as bound states of charmed quarks $(c\bar{c})$, one can see that if the ψ (3100) gives the mass of the charmed quark :

$$m_c = 1550 \text{ Mev.}$$

then the mass of the excited state ψ' (3700) is found to be $M_{\psi'}$ = 3400 Mev by taking n = 2 ; we should obtain the value of 3700 Mev by taking n = 4.

We predict in this way the masses of the charmed vector mesons[17] $D^*(c\bar{p})$ and $F^*(c\bar{\lambda})$:

$$m_{D^*(n=0)} \simeq 1925 \text{ Mev.} \qquad m_{D^*(n=1)} \simeq 2230 \text{ Mev.}$$

$$m_{F^*(n=0)} \simeq 2060 \text{ Mev.} \qquad m_{F^*(n=1)} \simeq 2345 \text{ Mev.}$$

Thus this simple mass formula for vector mesons predicts masses in a fairly good agreement with the experimental values. Let us emphasize on the elementary expression we have chosen for the mass operator. Indeed the mass formula does not involve quantum numbers such as the angular momentum ℓ and the "total quark spin" s , which leads to a degeneracy. This degeneracy could have been removed by introducing in the mass operator some kind of spin-spin and spin-orbit couplings. In this case one might think that pseudoscalar mesons could have been also treated. However, a more elaborate mass operator needs certainly some new insights in the physics of quarks.

We can remark we find again in Eq.(8) the advantages of the Gell-Mann-Okubo formula[18] and its generalization to SU(4)[19]: SU(4) and SU(3) breakings appear here by the quark mass differences : $m_p = m_n < m_\lambda \ll m_c$.

The generalization of such a mass formula for three bound particles system (baryons) does not seem to present too much difficulty. In the case of results in accordance with experimental values, an interesting discussion with the help of Refs. [7] - [8] on one hand and of Ref. [20] on the other hand could be made.

ACKNOWLEDGEMENTS

We wish to thank A. Sciarrino for discussions.

- REFERENCES -

[1] P.A.M. DIRAC
Rev. of Mod. Phys. 21, 392 (1949)

[2] B. BAKAMJIAN & L.H. THOMAS
Phys. Rev. 92, 1300 (1953).

[3] L.L. FOLDY
Phys. Rev. 122 , 275 (1961)

[4] K. BARDAKCI & M.B. HALPERN
Phys. Rev. 176, 1686 (1968)
and references therein.

[5] M. GELL-MANN
Phys. Letters 8, 214 (1964).

G. ZWEIG
Cern Reports n° TH 401 and TH 412 (1964).

R.H. DALITZ
in "Proceedings of the Berkeley Conference, 1966"
(University of California Press, Berkeley, Cal. 1967).

[6] F. BUCCELLA, C.A. SAVOY & E. CELEGHINI
Nuovo Cim. 7A, 281 (1972).

[7] D. FAIMAN & A.W. HENDRY
Phys. Rev. 173, 1720 (1968).

[8] R. FEYNMAN, M. KISLINGER & F. RAVNDAL
Phys. Rev. D.3, 2706 (1971).

[9] Y.S. KIM
Technical Report n° 75-081, University of Maryland.

[10] H.J. MELOSH
Phys. Rev. D.9, 1095 (1974).

[11] F. BUCCELLA, C.A. SAVOY & P. SORBA
Lettere al Nuovo Cim. 10, 455 (1974).

[12] F. COESTER
Helv. Phys. Acta, 38, 7 (1965).

[13] D.G. CURRIE, T.F. JORDAN & E.C.G. SUDARSHAN
Rev. Mod. Phys. 35, 350 (1963).

[14] M.E. ROSE
in "Spectroscopic and Group Theoretical Methods in Physics"
North Holland Publ. Co., Amsterdam 1968, p. 137.

[15] M.N. FOCACCI, W. KIENZLE, B. LEVRAT, B.C. MAGLIC & M. MARTIN
Phys. Rev. Letters 17, 890 (1966).

[16] N.P. SAMIOS, M. GOLDBERG & B.T. MEADOWS
Rev. of Mod. Phys. 46, 49 (1974).

[17] M.K. GAILLARD, B.W. LEE & J.L. ROSNER
Rev. of Mod. Phys. 47, 277 (1975).

[18] M. GELL-MANN & Y. NE'EMAN
"The Eightfold Way"
Benjamin (1964).

S. OKUBO
Prog. Theoret. Phys. 27, 949 (1962).

[19] S. OKUBO
Preprint Rochester U.R.-524 (1975),
and references therein.

[20] F. BUCCELLA, A. SCIARRINO & P. SORBA
Preprint Marseille 75/P.729.