ANALYSIS OF THE ANGULAR DISTRIBUTIONS OF ELASTICALLY SCATTERED NEUTRONS FOR ^{235}U

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ABSTRACT

Experimental data on the angular distributions of 0.5-15 MeV neutrons elastically scattered by ^{235}U nuclei are analysed on the basis of Bessel functions and Legendre polynomial expansions. The advantages of the method are that there are no negative cross-sections and relatively few expansion coefficients and that experimental data on scattering at 0^{o} and 180^{o} are not needed.

A knowledge of the angular distributions of elastically scattered neutrons is required for accurate prediction of the behaviour of neutrons passing through matter and for obtaining more precise parameters for the optical model of the nucleus. The standard method for analysing the angular distributions of elastically scattered neutrons is Legendre polynomial expansion. The degree of expansion of the scattering amplitude is equal to the highest momentum of the neutron undergoing scattering, i.e. approximately 30 expansion terms are necessary for an incident neutron energy of the order of 14 MeV. Measurements are usually made at 15-20 different angles, i.e. the set of Legendre polynomial expansion coefficients carries more information than is contained in the experimental data.

The physical inaccuracy of such a description is well known. At a sufficiently high degree of Legendre polynomial expansion the curve describes the experimental points, but in the spaces between them it can behave quite unphysically, giving negative cross-section values. In principle, moreover, experiments on scattering do not permit measurement of the differential cross-sections at extremely small and large angles, and for this reason the fitting procedure must permit extrapolation to these angles. Precisely because of the orthogonality of the Legendre polynomials this is impossible, although scattering at small angles constitutes a large portion of the total cross-section at energies above 8 MeV.

The Bessel functions have certain advantages for describing angular distributions. Such a description requires a smaller number of expansion terms, and use of the Bessel functions enables one to trace the dependence of the angular distributions on neutron energy, nucleus dimensions, scattering angles, and also to obtain the values of the scattering cross-sections at angles of 0^o and 180^o.

It is well known that in the approximation of small angles and for an absolutely black scatterer the series of the scattering theory is summed accurately (see Ref. [1]), and for the differential scattering cross-section we have

$$\frac{d\sigma}{d\Omega} = (KR^2)^2 \left[\frac{J_1(x)}{x}\right]^2 , \qquad (1)$$

where $J_1(x)$ is the Bessel functions, $K = 2\pi/\lambda$, $x = 2KR\sin\theta/2$, R is the radius of the nucleus and θ is the scattering angle. Even in such simple form the formula (1) correctly describes the value of the leading cross-section peak and the location of the second.

The experimental data on elastic scattering always contain a component from the inelastic scattering at low levels. Besides, taking into account the diffuseness of the nucleus boundaries leads to a certain complication of formula (1) [2]. A simple theory [3] gives the following expressions for the differential inelastic scattering cross-sections associated with quadrupole and octupole oscillations of the nucleus:

$$\left(\frac{d\sigma}{d\Omega}\right)_{квадр.} = (KR^2)^2 \frac{5}{8\pi} \frac{E_2}{C_2} \left[\frac{1}{4} J_0^2(x) + \frac{3}{4} J_2^2(x)\right] \qquad (2)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{октуп.} = (KR^2)^2 \frac{7}{8\pi} \frac{E_3}{C_3} \left[\frac{3}{8} J_1^2(x) + \frac{5}{8} J_3^2(x)\right], \qquad (3)$$

where E_i is the excitation energy and C_i is the surface tension energy. We shall therefore describe the angular distributions of the elastically scattered neutrons by the following formula:

$$\frac{d\sigma}{d\Omega} = (KR^2)^2 \left\{ D\left[\frac{J_1(x)}{x}\right]^2 + \sum_{i=0}^{M-1} A_i J_i^2(x) \right\}, \qquad (4)$$

where M is the number of Bessel functions and D and A_i are the fitting parameters. Integrating (4) we obtain the integral scattering cross-section

$$\sigma_s = \pi R^2 \Big\{ D\left[1 - J_0^2(2KR) - J_1^2(2KR)\right] + $$
$$+ (2KR)^2 \sum_{m=0}^{M-1} A_m \left[J_m^2(2KR) + J_{m-1}^2(2KR) - \frac{2m}{2KR} J_m(2KR) J_{m-1}(2KR)\right]\Big\} \qquad (5)$$

We have written a program enabling us to perform fitting to the experimental data in formula (4). It was found that in expansion of the angular distributions by Bessel functions a significantly smaller number of terms from series (4) is necessary than if Legendre polynomial expansion is used. Moreover, no previous knowledge of the differential scattering cross-sections at the angles 0° and 180° is needed. The scattering cross-section values at these angles obtained by Bessel function fitting were applied by us to obtain the Legendre polynomial expansion. The Bessel function expansion has yet another important advantage. It explicitly embodies energy dependence, thus permitting interpolation into energy regions for which there is no experimental information.

The approach outlined above was used to analyse the experimental data on the angular distributions of elastically scattered neutrons for ^{235}U. The following six sets of experimental data in this region are available: Allen et al. [4], Batchelor and Wild [5], Knitter et al. [6], Cranberg [7], Smith and Guenther [8] and Kammerdiener and Luther [9]. Smooth curves were drawn through the experimental points on the basis of Bessel function expansion. The integral scattering cross-sections obtained by integrating these smooth curves were regarded as experimental values and used to evaluate the integral cross-section of elastic neutron scattering. The standard practice is to represent differential elastic scattering cross-sections as Legendre polynomial expansions. To obtain such an expansion we used, instead of the experimental values of the differential cross-sections, smoothly interpolated cross-sections obtained from Bessel function expansion, considering them "true" and, in keeping with this, assigning the same relative weight to them. In each case 101 points were used, uniformly distributed along cos θ over the range -1 to 1. Figure 1 shows the characteristic situation that arises when the experimental data are described by differential cross-sections using either Bessel functions or Legendre polynomials.

The elastic neutron scattering distributions obtained are sufficiently reliable in the energy region up to 6 MeV, but in the 6-14 MeV energy range, where experimental data are entirely lacking, the distributions were obtained by using the energy dependence of the Bessel function expansion and interpolating between the points 5.5 MeV and 14 MeV.

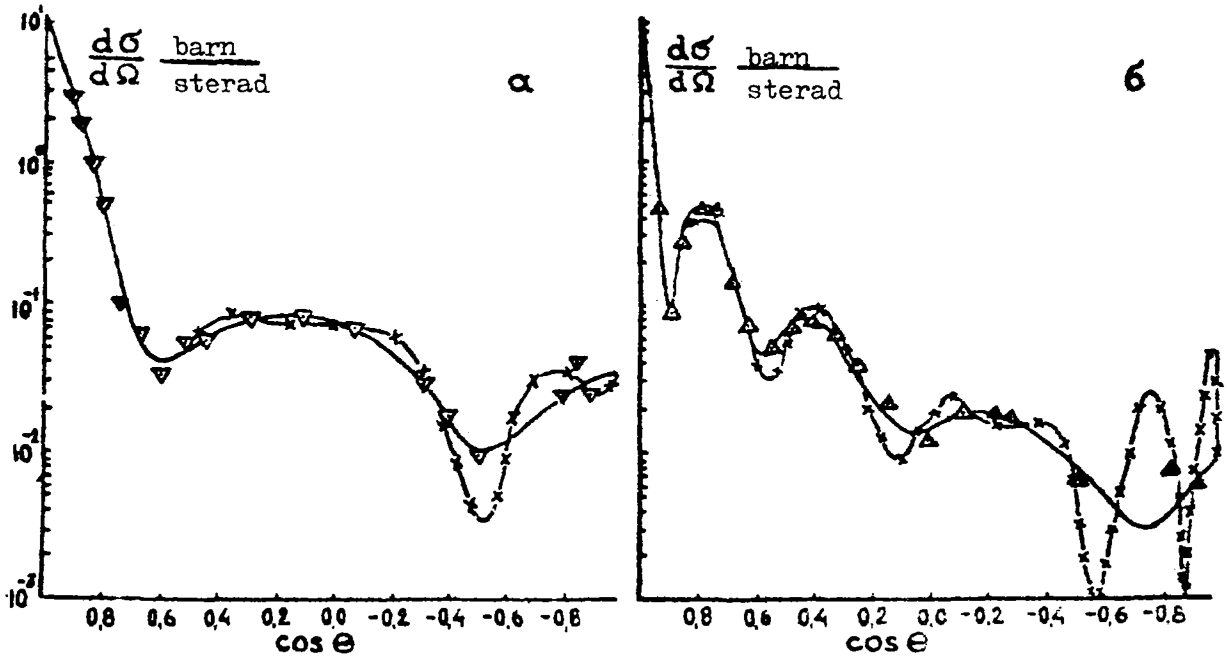

Fig. 1 Angular distributions of elastically scattered neutrons

a) Measurements by Knitter [6] at an energy of 4.5 MeV;

b) Measurements by Kammerdiener [9] at an energy of 14 MeV.

——x——x—— Legendre polynomial expansion not using additional information;

———————— Bessel function expansion.

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