BARYONIC SPECTROSCOPY AND ITS IMMEDIATE FUTURE

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ABSTRACT

We review briefly the quark model for baryons and the various versions of SU(6) symmetry which have been proposed and used in connection with baryon spectroscopy. We draw attention to a series of basic questions which experimental work in this field should aim to settle, as a minimal program. We also herald the beginning of a new baryon spectroscopy associated with ψ physics.

1. DO WE HAVE A THEORY OF HADRONIC RESONANCE STATES?

The hypothesis that hadronic interactions obey an SU(6) symmetry in the product space of unitary symmetry SU(3) and Pauli spin $SU(2)_\sigma$, first proposed by Gursey and Radicati in 1964, has been very fruitful for the analysis and classification of baryonic properties, especially when combined with the three-quark model for baryons. This model gives a simple description of the lowest octet and the lowest decuplet of baryons, the spins of the three quarks summing up to give $J = 1/2$ and $J = 3/2$, and the unitary spins of the three quarks combining to give octet and decuplet for the total unitary spin, respectively. Together, this $J = 1/2+$ octet and $J = 3/2+$ decuplet constitute a 56-dimensional representation of SU(6) symmetry, a representation which has even permutation symmetry with respect to the labels 1, 2 and 3 attached to the three quarks. The requirement that the proton should obey Fermi statistics implies that the overall three-quark wave function for a baryon must have odd permutation symmetry. Since it would be very exceptional for a ground-state space wave function to have odd permutation symmetry (since this would imply the presence of nodes in the space wave function), it is natural to conclude that the quark must have some further degree of freedom, associated with some further quantum number, and that this additional wave function factor for the three-quark system has odd permutation symmetry. This additional quantum number is now known as <u>colour</u>, and it can provide the antisymmetrical factor required for the three-quark wave function to describe a baryon if it operates in a three-dimensional space. As Lipkin[1] has pointed out, the qualitative systematics of the multiquark systems which describe hadronic states can be understood in terms of the colour variable if it has an SU(3)' symmetry and if the forces between quarks q (and their antiparticles $\bar{q}$) are mediated by a colour-octet of vector mesons. With these assumptions, the strongly-bound multiquark states are limited to colour-singlet states with the structures $\bar{q}q$ and qqq, which correspond with the mesons and baryons observed empirically.

With this qqq model for baryons, it is natural to expect that there should be excited baryonic states corresponding to excitation of the internal motions of the three quarks. The coordinate vectors which describe these motions may be chosen as

$$\underline{\rho} = (\underline{r}_1 - \underline{r}_2)/\sqrt{2}, \qquad \lambda = (2\underline{r}_3 - \underline{r}_1 - \underline{r}_2)/\sqrt{6}, \qquad (1.1)$$

and there will be corresponding internal angular momenta ℓ_{12} and ℓ_3, which sum to give the total orbital angular momentum L and the parity P, as

$$\underline{L} = \underline{\ell}_{12} + \underline{\ell}_3, \qquad P = (-1)^{\ell_{12} + \ell_3} \qquad (1.2)$$

It is most convenient to consider base wave functions which correspond to a harmonic form for the qq interaction. In this case, the total wave function will be a sum of terms of the type $\varphi(n_{12}\ell_{12}, \underline{\rho})\,\varphi(n_3\ell_3, \underline{\lambda})$ where $\varphi(n\ell, \underline{r})$ denotes a harmonic oscillator wave function and the parameters n_{12} and n_3 are such that $(2n_{12} + \ell_{12} - 2)$ and $(2n_3 + \ell_3 - 2)$ specify the number of quanta of excitation for the variables $\underline{\rho}$ and $\underline{\lambda}$, the total number of quanta of excitation being given by

$$n = 2n_{12} + \ell_{12} + 2n_3 + \ell_3 - 4 \quad . \qquad (1.3)$$

Finally, then, the total spin and parity of the qqq configuration considered are given by

$$\underline{J} = \underline{L} + \underline{S}, \qquad P = (-1)^n \quad , \qquad (1.4)$$

where S denotes the spin associated with the substate $^{2S+1}(\alpha)$ of the SU(6) multiplet considered and α characterizes its unitary spin. Thus, this model leads naturally to supermultiplets of baryonic resonances, classified according to an SU(6) x 0(3) symmetry. The notation we shall use for these supermultiplets is $(\underline{N}, LP)_n^{**\cdots}$, where N gives the dimensionality of the SU(6) representation, L and P characterize the internal orbital motion, n gives the number of quanta of excitation energy, and the number of stars specifies the degree of radial excitation involved for the sumpermultiplet considered, according to an appropriately chosen convention.

All of the supermultiplets $(\underline{N}, LP)_n$ possible for excitations up to N = 4 quanta are given in Table I. With this model, the observed resonant states correspond to substates of these supermultiplets, after taking into account the effects of the SU(6)- and SU(3)-breaking interactions, which separate and mix the various states which have the same spin J, parity P, hypercharge Y, and isospin I within these supermultiplets. These interactions can also mix states with the same (J, P, Y, I) but different n, but such interconfigurational mixing has not been included in the theoretical work to date. Despite all these distorting effects, the basic patterns of these supermultiplets still survive. The supermultiplets $(\underline{56}, 0+)_0$ and $(\underline{70}, 1-)_1$ are now established and well-known.

These remarks have all been made in a nonrelativistic language. This is natural, because Pauli spin is a nonrelativistic notion. It is well-known that, for rather technical reasons,[2] there exists no four-dimensional version of SU(6) symmetry, which could hold for the complete relativistic Lagrangian appropriate to hadronic interactions. It follows that SU(6) symmetry cannot be an exact symmetry for the hadrons, and can hold only in some approximate sense, and perhaps only for the low-lying hadronic states.

The applications of SU(6) symmetry in baryonic spectroscopy come under two headings, (a) the baryonic mass spectrum, and (b) the transitions between baryonic states, involving the emission of a pion or other meson, or of a (real or virtual) photon, or involving an interaction with a (lepton, neutrino) pair through the weak interactions. Let us consider these in turn:

(a) <u>Baryonic mass spectrum:</u> This work has been carried within the nonrelativistic framework for the qqq states. It represents essentially an extension of the Gell-Mann-Okubo mass formulae to include qqq states with $n \neq 0$, within the assumption of harmonic shell-model wave functions for these states. Of course, it is not to be expected that all of these baryonic mass values are to be given by a neat and simple formula; rather, all that we can aim for is to lay down a calculational procedure involving a series of matrix diagonalisations and depending on a limited number of parameters, through which a baryon mass spectrum can be computed, for a definite set of values for these parameters. In this procedure, the parameters which occur are mostly expectation values of certain qq potential terms, and a significant constraint arises from the fact that a given potential expectation value occurring for a supermultiplet with n_0 quanta of excitation will generally occur also in the discussion for the supermultiplets with $n > n_0$.

The first SU(6)-breaking qq interactions which arise are spin-spin and unitary spin-unitary spin interactions, of central form and SU(3)-invariant. Next come the SU(3)-invariant spin-orbit qq interactions, and finally the SU(3)-breaking qq interactions, both central and spin-orbit in form. Calculations of this kind have been carried out by Greenberg and co-workers[3], by Horgan and Dalitz[4], by Chen-Tsai et al.[5] and by others, and now cover all the n = 0, 1 and 2 supermultiplet states. Less complete calculations have also been carried through for some of the n = 3 and n = 4 supermultiplets.[6] Following the lead by Greenberg and Resnikoff[3], these calculations have mostly proceeded by expressing the qq interactions considered as a sum of SU(6) tensors ${}^{m}V_{D}^{\mu,\sigma}$, where D is the dimensionality of the tensor, (μ, σ) denote the SU(3) x $SU(2)_\sigma$ representation for the component of this SU(6) tensor being considered, and the superfix m distinguishes between even (m = 21) and odd (m = 15) parity for the relative motion between the interacting qq pair. In the published work, SU(3) breaking has been limited to the choice $\mu = 8$, the case of octet dominance proposed originally by Gell-Mann, and the noncentral interactions were limited to $\sigma = 3$, corresponding to spin-orbit couplings.[7] This procedure has the advantage of economy in that extensive use can then be made of the Wigner-Eckart theorem in the construction of the many matrix-elements required for the calculation of the energy matrices for all possible interactions for all possible qqq states. However, with the present asymmetric situation regarding the baryonic data available (extensive data for $Y = 1$ states, considerable data for $Y = 0$ states, very little data for $Y = -1$ states, and only one state known for $Y = -2$), Lee and Chen-Tsai[8] have argued the case for an alternative representation for the qq potentials, where it is much more apparent which parameters are well determined by the present data and which are poorly determined.

The most complete fit to the experimental data at present is that reported by Horgan.[4] Beyond the well-known states in the $(\underline{56}, 0+)_0$ supermultiplet and the negative parity states belonging to the $(\underline{70}, 1-)_1$ supermultiplet, he found it necessary to invoke the $(\underline{70}, 2+)_2$ and $(\underline{70}, 0+)_2$ supermultiplets, as well as the $(\underline{56}, 2+)_2$ and $(\underline{56}, 0+)_2^*$ supermultiplets, four of the five supermultiplets expected for $n = 2$ with the quark shell-model, in order to accommodate quantitatively all of the known baryonic resonance states in this mass region. Particular features of this fit are that:

(i) the SU(3)-breaking terms and the SU(6)-breaking terms are of the same order of magnitude, i.e., SU(3) symmetry does *not hold significantly better* than SU(6) symmetry,

(ii) the spin-orbit interactions are complicated. The smallness of spin-orbit effects for N states and their largeness (~ 115 MeV) for the ΛD03(1520)-ΛS01(1405) pair require that the SU(3)-breaking spin-orbit interactions are very strong, and

(iii) mixing is strong for the Λ, Σ and Ξ states, whenever allowed, but very weak for N, Δ and Ω states. *In particular, strong mixing is predicted for the ΛD03(1520) state, as is known empirically to be the case.*

No simple picture emerges for the qq interactions, from this analysis. In particular, such complicated spin-orbit forces would be difficult to account for, on any fundamental basis. However, this fit to the data then leads to many predictions which can be tested by experiment in due course.

No relativistic version of these calculations has yet been worked out. As mentioned earlier, there are some fundamental difficulties about this, and probably the best that we could hope for is that the SU(6) patterns might emerge as the result of an approximate symmetry, based on the use of *some relativistic model*. The above calculations have assumed nonrelativistic quarks, which means "heavy quarks." At present, there is much interest in "bag models" for qqq systems, and these assume "light quarks", in fact essentially zero mass quarks.[9] For the $L = 0+$ states, the discussion of the mass formulae is quite similar to that of Greenberg, Horgan and others, but for the $L = 1-$ states, the discussion will be quite different, even with the shell-model approach, since the $p_{1/2}$ and $p_{3/2}$ quark orbitals are quite different.[10] It is far from obvious at present whether or not the bag model with relativistic quarks will lead to patterns of mass spectra similar to those appropriate to SU(6) x O(3) for $n = 1$.

(b) Baryonic transitions. The pionic transitions $B^* \to \pi B$ and $B^* \to \pi D$, where B and D denote the octet and decuplet states of $(\underline{56}, 0+)_0$, are those best known. A simple model for the discussion of these transitions in the present SU(6) framework is by introducing an interaction term

$$(f_q/m_\pi)\,\bar{q}\,(\underline{\sigma}\cdot\underline{k})\,\underline{\tau}\cdot\underline{\phi}_\pi\,q \qquad (1.5)$$

describing the process $q \to q + \pi$ and by calculating its matrix-element between the initial and final qqq states, the procedure followed in the earliest calculations, by Mitra and Ross.[11] It has been found that this model gives quite good agreement with the data for those SU(3) multiplets with $L = n$ whose pionic decay is possible only when the omitted pion has angular momentum, $\ell_\pi = (L + 1)$, i.e., the maximum ℓ_π possible for the transition $(\underline{N}, LP)_n \to \pi B$, where B denotes the states $({}^2 8; \underline{56}, 0+)_0$. The most extensive comparison with the data on the basis of this model is that given by Faiman and Plane[12], for the case $n = 0$, 1 and 2.

We may note here that, for the transitions $({}^4(a); \underline{N}, LP) \to \pi D$, where D denotes the states $({}^4 10; \underline{56}, 0+)_0$, the requirements of angular momentum and parity conservation alone are not sufficient to exclude pionic decay with $\ell_\pi = (L + 3)$. It is a specific property of the SU(6) x O(3) model and the *interaction (1.5) that ℓ_π is limited to the values $(L \pm 1)$*, and the experimental data does not require a non-zero value *for* the transition amplitudes

($^4 8$; $\underline{70}$, $1-)_1 \to \pi D$ with $\ell_\pi = 4$ (possible for the states with $J = 5/2$, such as $\Sigma D15$ (1770)) and ($^4 10$; $\underline{56}$, $2+)_2 \to \pi D$ with $\ell_\pi = 5$ (possible for the states with $J = 7/2$, such as $\Delta F37$ (1925)). Of course, the negligible values found for these transitions with higher ℓ_π might well follow from the greater centrifugal barrier then appropriate, in the case of some other model for these baryonic resonance states, so that these results are not necessarily a significant success for the SU(6) x 0(3) three-quark model.

The kinetic energy released in these transitions,

$$\text{(a) } B^* \to \pi B, \qquad \text{(b) } B^* \to \pi D \quad , \tag{1.6}$$

is generally quite large and the recoil baryon then has relativistic velocity, so that we are again faced with the necessity for a relativistic treatment. Lipkin and Meshkov[13] proposed that the $SU(6)_W$ group should be used for the discussion of these decay processes. This is the "collinear SU(6) group", defined with respect to a definite axis in space. The spin states are then specified in terms of helicity states with respect to this axis, and this description is unaffected by any Lorentz transformation along this chosen axis. But which axis to choose? For two-body decay transitions, it is natural to consider the decay in the rest frame of the parent particle and to choose the reference axis to be parallel to the final pion momentum $\underline{k}$ in this frame. Since the interaction (1.5) is simply related with the quark helicities when the reference axis is $\underline{k}$, it is easy to see, as was first pointed out by Lipkin[14], that the predictions given by $SU(6)_W$ are identical with those given by the Mitra-Ross model with the interaction (1.5).

At this point, we must point out that the $SU(6)_W$ symmetry group violates relativistic invariance. If we consider a particular decay event, and apply an arbitrary Lorentz transformation (even a simple rotation), then the reference axis has a different description in the new coordinate space. But the general laws of physical processes cannot depend on a reference axis whose description varies from one Lorentz frame to another; they must have the same form with respect to all Lorentz frames. Hence $SU(6)_W$ symmetry cannot be derived from any Lorentz-invariant Lagrangian for the hadronic interactions. It cannot hold as a fundamental symmetry.

As remarked already, the Mitra-Ross model, and therefore $SU(6)_W$, agrees well with the data for decay processes with $\ell_\pi = (L + 1)$, for the supermultiplet $(\underline{70}, 1-)_1$. However, this is not the case for the transition for which $\ell_\pi = (L - 1)$. First of all, the momentum dependence of the matrix-element is predicted to be k^{L+1}, in both cases, whereas we know that the correct dependence (for sufficiently small k) is k^{ℓ_π}; this could be corrected by an ad hoc adjustment of the k-dependence. What is worse is that $SU(6)_W$ gives definite relationships between the matrix-elements for the processes $B^*(\underline{N}, LJP)_n \to \pi B$ which involve different ℓ_π (similarly, also for the processes $B^*(\underline{N}, LJP)_n \to \pi D$) and these relationships do not agree with the data. However, Faiman and Plane found that the application of $SU(6)_W$ to the decay processes with $\ell_\pi = (L - 1)$ gave quite good agreement, when the k-dependence k^{ℓ_π} was adopted and the parameters were not constrained to fit the decay processes $\ell_\pi = (L + 1)$ at the same time. Thus we have "ℓ-broken $SU(6)_W$", which simply means "for the decays (1.6) for a supermultiplet $(\underline{N}, LP)$, apply $SU(6)_W$ separately for each possible value ℓ_π, and forget the relationships which the three-quark model (or $SU(6)_W$) would imply between the different ℓ_π transitions."

Next, we recall that, for the decays $B^* \to \pi D$, both $\ell_\pi = (L + 1)$ and $(L - 1)$ can often occur, for example in the decays $ND13 \to \pi\Delta$, and these two transition amplitudes will generally interfere in the decay angular distributions observed, so that their relative sign comes into question. This relative sign is specified by $SU(6)_W$. Empirically it is found that some classes of transition do have the $SU(6)_W$ sign, whereas other classes of transition systematically require the opposite sign, for the relationship between the $\ell_\pi = (L - 1)$ and the $\ell_\pi = (L + 1)$ amplitudes. These two patterns are referred to as "$SU(6)_W$-like" and "anti-$SU(6)_W$", respectively.

Some theoretical basis for these prescriptions has been given by the work of Melosh[15] who distinguishes between "current quarks" and "constituent quarks." The qqq model discussed above is concerned with the quarks as constituents of hadrons, of course, whereas the algebra appropriate to hadronic interactions is concerned with quark currents, and there are a number of arguments which indicate that the currents associated with the constituent quarks are not those involved in the $SU(6)_W$ current algebra.[16]

What is needed is the transformation V connecting the current quark state vector with the constituent quark state vector, and this is the transformation which Melosh has obtained for the case of the free quark model and has applied to physical problems.

First, the transition amplitude for $B' \to B\pi$ is related with a matrix-element of the axial vector current (charge) operators by appeal to PCAC, thus

$$\langle B'|B\pi\rangle \sim (M'^2 - M^2)\,\langle B'|Q^5_\pi|B\rangle \quad . \tag{1.7}$$

Here the states B and B' are expressed in terms of constituent quarks, so the Melosh transformation is used to relate this matrix-element to one expressed in terms of current quarks.

$$\langle B'_{const.}|Q^5_\pi|B_{const.}\rangle = \langle B'_{curr.}|\tilde{Q}^5_\pi|B_{curr.}\rangle \tag{1.8}$$

where $\tilde{Q}^5_\pi = (V^{-1}Q^5_\pi V)$. The central question at this point is the form of $\tilde{Q}^5_\pi$ in terms of the $SU(6)_W$ currents; when this is settled, all that remains is pure algebra. The relationship between $\tilde{Q}^5_\pi$ and Q^5_π is neither simple nor explicit. According to the discussion by Gilman et al.[17,18], $\tilde{Q}^5_\pi$ transforms as the (8, 3) component of an operator transforming according to the 35-representation of $SU(6)_W$, as does Q^5_π. However, whereas Q^5_π allows only $\Delta L_z = 0$, $\tilde{Q}^5_\pi$ has both $\Delta L_z = 0$ and $\Delta L_z = \pm 1$ pieces, where $\hat{z}$ here denotes the reference axis for the $SU(6)_W$ group. For a given transition $(\underline{N}', L'P') \to \pi + (\underline{N}, LP)$, the calculation of the decay amplitudes then reduces to the evaluation of SU(6) x 0(3) Clebsch-Gordan coefficients, apart from the two parameters which measure the strengths of the $\Delta L_z = 0$ terms and of the $\Delta L_z = \pm 1$ terms. In phenomenological analyses of the data, these two parameters are regarded as freely disposable, varying in an unknown way from one supermultiplet to another, since they cannot be calculated at present for a known configuration of constituent quarks. The presence of those two disposable parameters for each transition $(\underline{N}'L'P') \to \pi + (\underline{N}, LP)$ then gives just the same degree of freedom as that which had been adopted phenomenologically in the $SU(6)_W$ analysis of baryonic decay processes. It is in this sense only that the introduction of the Melosh transformation has represented a justification for the procedures which have been adopted in the phenomenological analyses.

We may now answer the question entitling this Section. There exists no fundamental dynamical theory which could lead to SU(6) being an exact symmetry for the hadrons and there are deep reasons for this. The analysis of baryonic decay processes involves a series of prescriptions based on the $SU(6)_W$ group algebra, which are derived from our experience with the data, not from any fundamental standpoint. We know that $SU(6)_W$ cannot be a fundamental hadronic symmetry, since it is not even compatible with Lorentz invariance, and the best we can say is that the prescriptions adopted do not go beyond the freedom allowed by the Melosh transformation and by our inability to calculate any of the coefficients which multiply the $SU(6)_W$ Clebsch-Gordan coefficients in the expressions for the baryonic decay amplitudes. Thus, no predictions are possible, except at this group-theoretical level. For example, the theoretical situation, as we have just discussed it here, does not enable us to calculate the elastic width Γ_{el} for the decays $\Delta(\underline{56}, LJ+)_L \to \pi N$ for the states on the leading Δ trajectory (i.e., for L = even and J = L + 3/2) as function of their position L along this trajectory, a quantity which is relatively well-known empirically and which shows an interesting regularity, as we shall discuss later.

The most extensive analyses of the empirical decay data on baryonic resonance states are those given by Hey et al.[19,20] They find an excellent fit to the $(\underline{70}, 1-)_1$ supermultiplet for the negative parity states below mass 1.9 GeV, for Y = +1 and 0; all of the N^* and Δ^* are now identified, and all of the Λ^* and Σ^* states except for one ΛD03, one ΛS01 and two ΣS11 states. They also find an excellent fit to the $(\underline{56}, 2+)_2$ supermultiplet for the positive parity states below mass 2 GeV whose spin-parity allows their assignment to this supermultiplet; all of the N^* and Δ^* states are identified, and all of the Λ^* and Σ^* states except for one ΣF15, one ΣP11, and two ΣP13 states.

For the $(\underline{70}, 1-)_1$ states, both the mass spectrum analysis and the decay analyses indicate strong mixing, in general. This has been particularly well-known for the ΛD03(1520) for a long time; although this state is clearly a singlet state, isolated in mass, its decay processes show clearly that it has a substantial octet admixture.[21] The mixing matrices obtained by Faiman and Plane[12], Hey et al.[13], and Horgan[4] are compared for the ΛD03 states in Table II, and they show quite good qualitative agreement. However, this is not the case for the three ΛS01 mixing matrices they have determined, also shown there. Hey et al. give quite a different structure for Λ(1670), finding it to be dominantly $^4 8$, from those found by Faiman and Plane, who find strong contributions from $^4 8$, $^2 8$, and $^2 1$ states, and by Horgan, who finds mostly $^2 8$ and $^2 1$. This last situation would be much clarified by the identification of the missing ΛS01 state and of its properties. These examples serve to illustrate well that strong mixing between states with the same (J, P, Y, I) in the same supermultiplet is the rule and not the exception, as some authors have assumed.[22]

2. THE QUARK SHELL MODEL AND OTHER PATTERNS

As indicated in Table I, the number of baryon resonance states per unit mass interval increases very rapidly with increasing mass value, according to the quark shell model. The patterns provided by these predicted states may be shown on a Regge-plot. However, since each $(\underline{N}, LP)_n$ supermultiplet gives rise to such a large number of SU(3) multiplets with different J values, it is more economical to plot L versus M^2, rather than J. This is done on Fig. 1(a), corresponding to the approximation in which all SU(6)- and SU(3)-breaking interactions are neglected, and the qq interactions are of harmonic Wigner type, without any space-exchange component, so that all SU(6)-L supermultiplets with the same n are still degenerate. The ordinate plotted is n, since M^2 is expected to be a linear function of n, given by $M_0^2 + nC$ for some constant C, in this case. This shows well the rapid increase in the density of resonance states with increasing n.

We note some regularities apparent from Fig. 1(a):

(i) Regge recurrences (= rotational excitations). For each supermultiplet $(\underline{N})$ shown at (n, L), there is at least one corresponding supermultiplet at (n + 2, L + 2). These sequences are expected to occur, from general considerations, as Regge recurrences of less massive states, the sequence to infinity being known as a Regge trajectory. For example, the lowest supermultiplet $(\underline{56}, 0+)_0$ begins a Regge trajectory of $(\underline{56}, 2+)_2$, $(\underline{56}, 4+)_4$, $(\underline{56}, 6+)_6$, etc., and the next supermultiplet $(\underline{70}, 1-)_1$, begins a Regge trajectory consisting of $(\underline{70}, 3-)_3$, $(\underline{70}, 5-)_5$, etc. These two sequences form the "leading Regge trajectories."

We note that even the leading trajectories become multiple as the order of the recurrence increases. Thus, for n = 6, we find two states $(\underline{56}, 6+)_6$; one of these is the Regge recurrence from the ground supermultiplet, the other is the beginning of a second leading 56-trajectory. At n = 12, there begins a third leading 56-trajectory, and so on. The same holds for the leading 70-trajectory, and for all other trajectories, with varying degrees of complication. The mathematical reason why these trajectories become multiple at steps $\Delta n = 6$, after some point, is discussed for the harmonic quark shell-model by Karl and Obryk.[24]

As implied by these remarks, the Regge trajectories for this model do not necessarily begin at the lowest possible value for L. For example, there is a supermultiplet trajectory starting with $(\underline{56}, 3-)_3$; there is no corresponding state $(\underline{56}, 1-)_1$, since LP = (1-) with symmetry S is inconsistent with the Pauli principle. However, it is well known in Regge theory that any finite number of states can be absent from any particular Regge trajectory.

(ii) Radial excitations. For each supermultiplet $(\underline{N})$ shown at (n, L) there is at least one corresponding supermultiplet at (n + 2, L). These sequences are described as radial excitations of the lower supermultiplets. However, for given Δn, these excitations may be multiple. For example, Fig. 1(a) gives one state $(\underline{56}, 0+)_2^*$ corresponding to radial excitation of $(\underline{56}, 0+)_0$, but two doubly excited states $(\underline{56}, 0+)_4^{**}$. Thus again, a sequence of radially excited states does not begin with the lowest conceivable value for n. For example, the state $(\underline{56}, 1-)_3$ does not have a corresponding lower state $(\underline{56}, 1-)_1$, for the reasons mentioned above; similarly, the state $(\underline{20}, 1+)_3$ has no corresponding state $(\underline{20}, 1+)_1$. Again, there are three states $(\underline{70}, 2+)_4$, whereas there is only one state $(\underline{70}, 2+)_2$ and one state $(\underline{70}, 0+)_2$.

In terms of Regge recurrences and radial excitations, the pattern of supermultiplets shown on Fig. 1(a) is very complicated. Although this description is quite convenient for consideration of the supermultiplets with low values for n and L, it is not obvious that it has any real value when both n and L are large, especially as the radial and rotational excitation energies are of the same order of magnitude, which implies that these two sequences lie one over the other and that there is therefore likely to be much interference between them.[25]

Hey *et al.*[19] have raised the question whether it is really the case that all of the very many supermultiplets predicted with the quark shell-model should necessarily exist. Perhaps baryonic states do occur in SU(6) x 0(3) supermultiplets, but for reasons other than the qqq shell-model or with qq interactions quite different from those tacitly assumed in present models. At present, the $(\underline{56}, 0+)_0$ and $(\underline{70}, 1-)_1$ supermultiplets appear well established. The evidence is also very strong for both the $(\underline{56}, 2+)_2$ and $(\underline{56}, 0+)^*_2$, which represent a rotational excitation and a radial excitation of the $(\underline{56}, 0+)_0$. Starting with the established $(\underline{56}, 0+)_0$ and $(\underline{70}, 1-)_1$, and adding the rotational and radial excitation sequences required from rather general considerations, leads to the pattern of supermultiplets shown on Fig. 1(b). This may be regarded as the minimal pattern, no matter what dynamics give rise to these baryonic supermultiplets (provided that they lead to Regge trajectories which are linear) and Hey *et al*[19] have asked whether there is really any evidence requiring the existence of any supermultiplet beyond those given by the minimal pattern of Fig. 1(b).

Some theoretical schemes have been put forward which do appear to lead to this minimal pattern, even within the qqq model. This pattern results for the low-lying supermultiplets from a quark-diquark model discussed by Lichtenberg[26] and by Ono.[27] Capps[28] has discussed a particular colour structure for qq forces which can lead to such a quark-diquark picture, and to this minimal pattern, and Mitra[29] has discussed a model in which the qq forces are strongly attractive in states with even orbital angular momentum ℓ_{12} and negligible in states with odd ℓ_{12} with the same conclusion. All of these models involve effectively the assumption of qq forces with special features designed to push the supermultiplets not included in the minimal pattern of Fig. 1(b) to high mass values, although it is not always clear just how high their mass values will finally be for each particular model.

3. QUESTIONS FOR THE IMMEDIATE FUTURE

Experimental effort towards the study of baryonic spectroscopy may be expected to dwindle in the near future, for a number of reasons. In any case, the number of supermultiplets expected on the basis of the quark shell model rises so rapidly with increasing mass as to preclude any serious attempt to establish the existence of all of them, even up to the value n = 4 as depicted on Table 1. The question then arises, as to what limited objectives experiment should aim to reach in this next phase of research concerning baryon spectroscopy. Thus, we shall discuss here a series of topics where more detailed knowledge would do much to clarify the status of our understanding of baryonic spectra and which could mostly be settled using established experimental techniques with accelerators at present in existence.

(i) The $(\underline{70}, 1-)_1$ supermultiplet. At present, it appears that all baryonic states with negative parity and mass below about 1.9 GeV are consistent with this assignment. However, there are clearly some large gaps and uncertainties in our knowledge of this supermultiplet. The patterns followed by its states are expected to be quite different from those for the well-established $(\underline{56}, 0+)_0$, and it is highly desirable that these should be established just as firmly as those for the $(\underline{56}, 0+)_0$.

(a) Ξ^* and Ω^* states. Seven Ξ^* states and two Ω^* states are predicted, and some substantial knowledge of them is really necessary before a satisfactory and stable analysis of the $(\underline{70}, 1-)_1$ spectrum can be achieved. Despite enormous effort with bubble chamber work using a K^- beam at 4.2 GeV/c, only the one state $\Xi^*(1823)$ with width $\Gamma = 19 \pm 4$ MeV has become established.[30] Its main decay modes are known and it has spin $J \geq 3/2$. As for Ω^* states, this beam energy allows production only for Ω^* with masses

$\leq$ 2.0 GeV, which lies below the mass value predicted by Horgan.[4] Further experimentation is really necessary, and at significantly higher K^- momentum, but it must involve selection of the few relevant events by counter techniques, perhaps used with a rapid cycling bubble chamber or other track chambers of the Omega type.

(b) Mixing of states. The situation is unclear in many respects. Horgan's analysis[4] allows very little mixing for N^* states, whereas Hey et al.[19] point out that the observed decay NS11 (1665) $\to K\Lambda$ requires strong admixture of $^2 8$ (amplitude ~ 0.5) with the $^4 8$ state and that the data on the ND13 states requires mixing with amplitude about 0.2. Detection and decay measurements for the missing ΛS01 and ΛD03 states will be necessary to establish firmly their mixing patterns. This may need work on production reactions rather than on formation reactions, however. Even more remains to be done for the Σ states ΣS11 and ΣD13. There have been persistent reports over the years, now very clearly confirmed by the work of the ACNO group reported at this meeting,[31] that there appear to be two independent $\Sigma^*(1660)$ states, both with $J = 3/2$ and with closely comparable widths, since the production characteristics observed for $\Sigma^* \to \pi\Sigma$ are quite different from those observed for $\Sigma^* \to \pi\Lambda(1405)$. The state Σ(1580) has a convenient interpretation as ΣD13, according to Hey et al. However, the existence and properties of this state have not yet been confirmed, and Hey et al. assign it to the $^2 10$ configuration, which is not at all acceptable in terms of the systematics of the mass spectrum for $(70, 1-)_1$. It appears very difficult indeed to accommodate the second $\Sigma^*(1660)$ in this supermultiplet, and its proper interpretation is not known. It is also not clear that ΣD13(1940) necessarily belongs to $(70, 1-)_1$. Even less is known about ΣS11 states. There is much work still to be done here and it is important work, necessary to establish definitely whether or not these negative parity states do fit accurately the $(70, 1-)_1$ supermultiplet pattern.

(c) Selection rules. There is no intraband mixing for the $J = 5/2$ states of this supermultiplet. The selection rule $\Lambda D05 \not\to N\bar{K}$ is definitely violated since ΛD05(1830) is best known from the analysis of data on the reaction $K^-p \to \pi\Sigma$. The Moorhouse selection rule $\gamma p \not\to$ pD15(1670) also appears to be violated, according to the data analyzed by Walker and Metcalf.[32] With the SU(6) × O(3) scheme, these violations are only possible as a result of admixtures from higher configurations, i.e., interband mixing. This means that much more detailed study of the $(5/2^-)$ states could be extremely informative. Hey et al. already note that the fit of these states to the supermultiplet (70, 1-) is outstandingly poor.

(d) Λ(1405). The relationship between the properties of this state and the K^-p scattering and reaction processes at low energies is still not settled. The latter data (from 1967) fits well a simple potential model consistent with an unstable bound state at mass 1405 MeV. Improved scattering and reaction data is desirable.

(ii) Other n = 2 supermultiplets or not?, and higher supermultiplets. After making assignments of the excited positive parity states to $(56, 2+)_2$ and $(56, 0+)^*_2$, there are a number of states left over. Horgan assigned all of these states to the other n = 2 supermultiplets, NF17(2000), ΛF07(2020), ΛF05(2110), ΛP03(1890), and ΣF17(2022) to $(70, 2+)_2$, and NP11(1750) and ΛP01(1770) to $(70, 0+)_2$. If these $(7/2^+)$ octet states are n = 2 states, then they are clear indicators for a $(70, 2+)_2$ supermultiplet.

Cashmore et al.[20] contest these assignments. There are a number of well established states, NH19(2250), ΔH311(2400) and ΛH09(2350) which fit naturally into the exptected n = 4 supermultiplet $(56, 4+)_4$, and they propose that the states NF17(2000) and ΛF07(2020) should also be assigned here. Their elastic widths are consistent with these assignments, and the coupling $\Lambda F07(2020) \to \bar{K}N$ is predicted non-zero (as observed) whereas this coupling is forbidden for the $(70, 2+)_2$ assignment. They reject the other states as not well established.

Cashmore et al.[20] also argue that the photoproduction data indicates that NP11(1750) belongs to some (56, 0+) supermultiplet and they propose its interpretation as $(56, 0+)^{**}_4$. However, the photoproduction parameter involved is (A^p_{1-}/A^n_{1-}) which would have the value +3 for the (70, 0+) assignment and +3/2 for the (56, 0+) assignment. The empirical value[32] is $+1.43 \pm 1.3$, which really allows either possibility. Cashmore et al. reject the state ΛP01(1770) as not well established.

It appears that the minimal scheme of supermultiplets proposed by Cashmore et al. cannot really be ruled out by the data at present available. The crucial question is whether the properties of the $(7/2^+)$ states, which are unmixed within the n = 2 states, indicate assignment to the ($\underline{70}$, 2+) or to the $(\underline{56}, 4+)_4^{aa}$. There is also the qualitative question, whether or not the data gives any indication for the large number of positive parity resonances the $(\underline{70}, 2+)_2$ assignment requires for N, Δ, Λ and Σ states in the mass range about 1850-2000 MeV.

One further characteristic of the quark shell model is its prediction of supermultiplets belonging to the 20-dimensional representation of SU(6). The lightest of these is the $(\underline{20}, 1+)_2$ supermultiplet, in the n = 2 band. The shell model structure of this state is $(1p)_\rho(1p)_\lambda$, in which only the p-wave qq interactions are effective. Horgan's analysis indicates that the $(\underline{20}, 1+)_2$ supermultiplet should be higher in mass than the other n = 2 supermultiplets, with mean mass about 2050 MeV. No states belonging to this supermultiplet are known but there are good reasons for this. First of all, its formation from πB interactions is clearly forbidden on the basis of the Mitra-Ross decay model, since the transition $(1p)_\rho(1p)_\lambda \to (1s)_\rho(1s)_\lambda$ involves a two-quark jump. In fact, this forbiddenness holds more generally than this, since the reduction of the product of SU(6) representations $\underline{35} \times \underline{56}$ does not include a $\underline{20}$ representation. Both production $\pi B \to B^*$ and decay $B^* \to \pi B$ or πD are SU(6)-forbidden. However, the product $\underline{35} \times \underline{70}$ does include a $\underline{20}$ representation in its reduction, so that the transitions

$$(\underline{70}, LP) \to \pi + (\underline{20}, 1+), \qquad (\underline{20}, 1+) \to \pi + (\underline{70}, 1-) \quad , \qquad (3.1)$$

can both occur, as pointed out by Faiman[33], who suggested that ($\underline{20}$, 1+) states might be sought in πp collisions which excite some ($\underline{70}$, LP) resonance which can then decay by the successive pion emissions (4.1) passing through a ($\underline{20}$, 1+) state whose presence can be deduced from the kinematics of the (3 mesons + baryon) final state. Unfortunately, we cannot give a definite prescription for experiments at present since the most massive $\underline{70}$ state known is NG17(2190) and this mass is rather close to the threshold expected for the first step of (4.1). Although not yet proven, it is generally expected that N(2650) will turn out to belong to $(\underline{70}, 5-)_5$, as a Regge recurrence from the ND13(1520) of $(\underline{70}, 1-)_1$; if this proves to be correct, then $\pi N \to N(2650)$ would be the most natural excitation in which to seek evidence for the $(\underline{20}, 1+)_2$ supermultiplet which is characteristic of the quark shell model for baryons. The theories of the quark-diquark type do not include ($\underline{20}$, LP) representations, because they all assume (sometimes only tacitly) that the p-wave qq interactions is negligible relative to the s-wave qq interaction.

(iii) States on the leading Regge trajectories. Although it is quite unlikely that the supermultiplets $(\underline{N}, LP)_n$ for n ≥ 5 will ever be analyzed in detail, these are nevertheless a few states in those high l ing supermultiplets which have an appreciable degree of simplicity. These are the states which have the greatest spin value for each value of n, since they undergo either no mixing or only a minimal mixing within their band n. As n varies, these particular states form a number of "leading Regge trajectories." Two of these trajectories are believed to be well-known, the Δ-trajectory consisting of the Δ states with spin-parity (2r + 3/2, +) from the sequence of supermultiplets $(\underline{56}, 2r+)_{2r}$ where L = 2r and r is integral, and the N-trajectory consisting of N states with spin-parity (2r + 3/2, -) from the sequence of supermultiplets $(\underline{70}, (2r+1)-)_n$ with n = (2r + 1). The latter case is less clear, because the leading N trajectory should really consist of the N states for spin-parity (2r + 5/2, -). Here we confine our remarks to the leading Δ-trajectory, characteristic of the $(\underline{56}, 2r+)_{2r}$ sequence, although the leading N-trajectory, characteristic of the $(\underline{70}, (2r+1)-)_{(2r+1)}$ sequence, holds equal interest.

The states on the leading Δ-trajectory are plotted on Fig. 2(a) showing $(\text{mass})^2$ versus J. The spin-parity $(11/2^+)$ for the second Regge recurrence has been established only recently.[34] The third and fourth recurrences are deduced only from the π^+p total cross sections measured by Citron et al.[35] in 1966; their spin values have not been determined. These "leading trajectory" states stand out (relatively) strongly in $\sigma(\pi^+p)$ data because they have a (relatively) high elasticity, which is a consequence of their high spin. For example, for mass 3.23 GeV, the value of pR is approximately 7.5, where p denotes the c.m. πN momentum and we have taken $R \approx 1$ fm., whereas the spin expected for $\Delta(3230)$ is 19/2. The determination of the elasticity from $\sigma(\pi^+ p)$ is also rather uncertain, both because of uncertainties in the subtraction of background beneath a small and broad resonance bump and because our supermultiplet picture leads us to expect three other Δ resonances to occur at approximately the same mass value, with spin values $L + 1/2$, $L - 1/2$ and $L - 3/2$. Although a complete $\pi^+ p$ phase-shift analysis cannot be expected for such high energies, it does seem quite possible that the spin-parity and elasticity could be determined for the high-spin Δ states of interest here, from accurate data on $\pi^+ p$ scattering and polarization, as has proved possible for the state $\Delta(2420)$. It is important that the spin-parity values should be checked for these "leading trajectory" Δ and N states, in order to confirm the general belief that these trajectories are indeed straight lines on the (M^2, J) plot.

The magnitude of the resonance bump in total cross section gives the quantity $(J + 1/2)x_J$, where x_J denotes the elasticity Γ_{el}/Γ_{tot}, and the width of the resonance gives directly Γ_{tot}. It is noteworthy that Γ_{tot} does not increase rapidly with J along the Δ-trajectory; it rises from about 110 MeV for the $J = 3/2$ $\Delta(1232)$ to about 450 MeV for the $J = 19/2$? $\Delta(3230)$. For assumed J, these two parameters x_J and Γ_{tot} lead us directly to an empirical estimate for Γ_{el}. The values obtained for the Δ states on the leading trajectory are plotted versus J on Fig. 2(b). They range from about 110 MeV for $\Delta(1232)$ to about 2 MeV for $\Delta(3230)$. The experimental uncertainties in Γ_{el} for the cases $J \geq 11/2$ are not well known. The value used for $\Delta(2420)$ is probably correct to ±20% (which means ±0.1 for the data point on Fig. 2(b)). For $\Delta(2850)$, there is much uncertainty in the background subtraction, which leaves the value for x_J rather uncertain. The $\Delta(3230)$ appears as a bump only after subtraction of the tails from the lower resonances.

Abdullah[36] and Shapiro[37] have made calculations of Γ_{el} for the Δ- and N-trajectories on the basis of the nonrelativistic quark model. For the Δ state with $J = L + 3/2$, from the supermultiplet $(\underline{56}, L+)_L$, where $L = 2r$ and r is integral, Abdullah obtained the following expression:

$$\Gamma_{el}(\Delta, J = 2r + 3/2) = 32 \frac{f_q^2 k^3}{4\pi m_\pi^2} \left(\frac{E}{M_\Delta}\right) \frac{((2r+1)!)^2}{(4r+3)!(r!)^2} \left(\frac{k^2}{3a^2}\right)^{2r} \mathrm{Exp}\left(-\frac{k^2}{3a^2}\right) \quad (3.2)$$

where E and k denote the total energy and the momentum of the recoiling nucleon, f_q is the coupling constant in expression (1.5)[38], and a is given by the form of the Gaussian factor $(\mathrm{Exp}(-\frac{1}{2}a^2(\rho^2+\lambda^2))$ common to the wave functions for all baryon states in the harmonic quark shell model. For assigned value of a, the appropriate value for $f_q^2/4\pi$ may be deduced by fitting expression (3.2) to the elastic width Γ_{el}^* known empirically for the Δ resonance with $J^* = L^* + 3/2$, for some arbitrarily chosen value for L^*. We may note here that expression (3.2) for a given J greater (less) than J^* then has a maximum (minimum) with respect to a at a_{min}, where

$$a_{min}^2 = (k^2 - k^{*2})/3(L - L^*) . \quad (3.3)$$

For large L, the pion c.m. momentum k is approximately $M_\Delta/2$ and M_Δ^2 is a linear function of L, as shown in Fig. 2(a), with slope β about 1.1 GeV^2. In this approximation a_{min} takes the value $\sqrt{(\beta/12)} \approx 0.3$ GeV. It happens that, with this value for a, the expression (3.2) gives correctly the general trend observed for $\Gamma_{el}(\Delta, J)$ along the Δ trajectory, as shown in Fig. 2(b), where the value of the coupling constant $f_q^2/4\pi = 0.055$ has been chosen to fit the $\Delta(1232)$ width. It gives an even better fit to these data if $f_q^2/4\pi$ is increased a little[38] to fit the value $\Gamma_{el} = 92$ MeV for $\Delta F37(1950)$, to the value $f_q^2/4\pi = 0.06$. The maximum (minimum) property mentioned just above has the consequence that this excellent fit to the data is not sensitive to the precise value of a; a good fit is obtained with any value of a in the range 0.3 ± 0.05 GeV. It also has the consequence that for a outside this range the elastic widths predicted for the states $J > J^*$ (or $J < J^*$) by expression (3.2) are systematically and significantly smaller (or larger) than those observed empirically; i.e. we cannot hope to find better over-all agreement by varying a further.

It would be of much interest now to have more reliable determinations of the elastic widths for the states Δ(2850) and Δ(3230) from the analysis of $\pi^+ p$ scattering and polarization data, as well as for the higher states along the Δ trajectory. The values given for $(J+1/2)x_J$ with the use of expression (3.2) are as follows (the numbers enclosed by brackets are estimates based on extrapolation from the data):

J =	15/2	19/2	23/2	27/2	31/2	
M^* =	2.85	3.23	(3.57)	(3.87)	(4.15)	GeV
Γ^*(total) =	400	440	(480)	(520)	(550)	MeV
$(J+1/2)x_J$ =	0.10	0.043	0.012	0.0031	0.0009	

The value calculated for Δ(2850) is significantly smaller than the empirical value of 0.25 but the latter appears surprisingly high, being even larger than the value 0.22 reported for the lower state ΔH311(2420), contrary to the general trend. The calculated value falls rapidly with increasing J, and it seems unlikely that empirical values will ever become available, even through amplitude and partial wave analysis, for spin values $J \geq 27/2$.

We must emphasize here that the prediction (3.2) for the elastic widths along the Δ trajectory does depend on the adoption of particular shell model wavefunctions, and not just on the SU(6) algebraic structure. It also depends on the definite form (1.5) adopted for the calculation of the $\Delta^* \to \pi N$ amplitude, independent of which supermultiplet is considered. In contrast, the Melosh viewpoint is that the coefficients of the $\Delta L_z = 0$ and $\Delta L_z = \pm 1$ parts of any decay amplitude must be regarded as quantities which may vary from one supermultiplet transition to another, in an unspecified way; consequently, this point of view gives no means for relating the properties of the various baryonic states lying on a leading Regge trajectory. The degree of agreement found to date between the harmonic shell-model calculations and this decay data does give some support for the notion that these transitions are induced by a one-quark operator[39] whose intrinsic strength remains essentially constant for the states along the Δ trajectory, from J = 3/2 to J = 19/2.

A similar discussion can be given for all the N, Λ, Σ, Ξ and Ω trajectories but there is much less data available concerning them. For example, the N^* states ND13(1520), NG17(2190), N(2650) and N(3030), are often considered to form a negative-parity trajectory, with L = 1, 3, 5 and 7 for these states in turn, since their mass values are consistent with a form $M^2 = M_1^2 + \beta L$, with $\beta = 1.15\ (\mathrm{GeV})^2$. However, one must make several remarks:

(i) the spin-parity values are not yet determined for the states N(2650) and N(3030), and

(ii) these states do not form a leading trajectory. For the <u>70</u> supermultiplets, the leading trajectory consists of ND15(1670) and its rotational excitations, but no further states on this trajectory have yet been identified. The states which form the Regge trajectory based on ND13(1520) are non-unique, for each value n = 1, 3, 5, . . for the quanta of excitation energy. Even for n = L = 1, there are two ND13 states which can mix together (although Horgan's analysis of the mass spectrum would imply rather small mixing). For n = 3, there are four NG17 states which can mix, whereas the (unidentified) NG19 state is unique, and the mixing possibilities become increasingly complicated as n increases.

If mixing is neglected and we identify ND13(1520) with the $^2 8$ component of $(\underline{70}, 1-)_1$, then Abdullah's width calculation gives the following result for this trajectory,

$$\Gamma_{el}\,(N(^2 8),\ J = 2r + 3/2) = \frac{32}{3}\,\frac{f_q^2 k^3}{4\pi m_\pi^2}\,\left(\frac{E}{M^*}\right)\frac{((2r+1)!)^2}{(4r+3)!\,(r+1)!\,r!}$$

$$\times \left(\frac{k^2}{3\alpha^2}\right)^{2r+1} \mathrm{Exp}\left(-\frac{k^2}{3\alpha^2}\right) \qquad (3.4)$$

where L = (2r + 1). Using the value $f_q^2/4\pi = 0.55$ deduced from the Δ(1232) width, the predicted elastic widths for this trajectory are:

J =	3/2	7/2	11/2	15/2
M^*, Γ(total) =	(1.52, 0.125)	(2.19, 0.25)	(2.65, 0.36)	(3.03, 0.4) GeV
$(J + 1/2)x_J$ (data) =	1.1	1.0	0.45	0.05
$(J + 1/2)x_J$ (calc.) =	2.0	1.4	0.34	0.045

The agreement is qualitatively quite good, with no free parameters. Other trajectories can be discussed on the same basis, but at most three states are known for them, and the identification of the trajectories is subject to the same uncertainties as have been mentioned above, for the ND13(1520) trajectory. Further data of this kind, for the strange particle trajectories as well as for the N and Δ trajectories, would be of much interest in that the comparison of states along these trajectories provides a different kind of test for the harmonic quark shell model for baryonic states. Of course, information of this kind, for high spin baryonic states, would also have much intrinsic interest, even apart from this particular model, for example to provide tests of the SU(6) and SU(3) aspects of the SU(6) × 0(3) scheme for the baryonic supermultiplets.

To conclude this section, we must emphasize that the two models considered are by no means the only possibilities, with or without the shell model. Our central aim should be to determine what is the pattern followed by the supermultiplets and what are the characteristic patterns of their substates. These two models have been regarded as extreme cases (although much more complicated possibilities also exist, giving an exponential rise in the number of supermultiplets with increasing n, as has appeared clearly in the work published on dual models) which serve to illustrate some of the simplest questions which should be settled before experimental work in this field falls to a significantly lower level of activity.

4. THE NEW SPECTROSCOPY AHEAD

Owing to the fruitful possibilities for the study of lepton and hadronic phenomena at higher energies and/or larger momentum transfer, the research effort devoted to hadron spectroscopy has been falling away over the past few years, to such an extent that the 1974 Review of Particle Properties[40] will not be revised by the Particle Data Group (PDG) until 1976. However, in November, 1974, there emerged dramatic evidence for a new degree of freedom in hadronic physics, with the discovery of the remarkable ψ/J vector particle of mass 3.1 GeV, which was quickly followed by the discovery[41] of further vector particles, ψ(3.7) and ψ(4.1), which stimulated PDG to issue a Supplement to their 1974 Review. It was quickly realized that this sequence of ψ particles must represent part of a new hadronic spectroscopy, involving at least one new quantum number. For historical reasons,[42] which may yet have little to do with these particles, we denote the new quantum number generically by C, where C might well be multi-dimensional. Even the existence of a new quantum number was not sufficient to explain the most striking property of ψ/J (3.1) and ψ(3.7), namely their lifetimes of order 10^{-20} sec, remarkably long when measured on the time-scale (unit $\sim 10^{-23}$ sec) appropriate to hadronic phenomena; to account for this, it has been necessary to appeal to the "Zweig rule", whose operation is best illustrated by the relatively long lifetime ($\sim 10^{-22}$ sec) known for the φ(1020) meson. To emphasize the bizarre nature of the present situation, we may add here that, since these ψ particles are produced by the reaction

$$e^+ + e^- \to \text{"}\gamma\text{"} \to \psi \qquad (4.1)$$

they all necessarily have quantum numbers compatible with those of the photon; in particular, they have the value C = 0 for this new quantum number. In other words, no hadronic state which has a non-zero value for this quantum number is yet know.

The precise nature of the ψ particles is not yet known. The simplest possibility not yet excluded is to suppose that there is a fourth quark, denoted by c, in addition to the quarks (u, d, s) with which we have been interpreting the mesonic and baryonic spectroscopy admirably summarized in the 1974 Review of Particle Properties.[40] Of course, more complicated possibilities also exist, but it will be sufficient for our purpose here to confine attention to the simple case of one additional quark. This quark c has value +1 for the quantum number C. Its introduction closely parallels the introduction of the strange quark, with value -1 for the strangeness quantum number s. Thus, for any hadronic system, the value of C gives (number of c quarks-number of

$\bar{c}$ quarks) just as the value of s gives (number of s quarks-number of $\bar{s}$ quarks). In this respect, we can expect ψ physics to parallel strange-particle physics, apart from some important practical differences. For example, the ψ particles themselves are interpreted as $^3S_1$ states of the $\bar{c}$-c system, with radial excitations, just as the φ(1020) meson is interpreted as the lowest $^3S_1$ state of the $\bar{s}$-s system and ω(785) as the I = 0 component of the lowest $^3S_1$ states for the $\bar{u}$-u and $\bar{d}$-d systems. When there is only one quark c, it is necessarily a unitary singlet, since SU(3) symmetry is concerned only with unitary transformations in the space of (u, d, s) quarks. The charge of the c quark can differ from +2/3 by any integer, in principle; but we shall follow the usual assumption,[43] that $Q_c = +2/3$. In this case, the charge of any state is given by

$$Q = I_3 + (B + s + C)/2 \quad . \qquad (4.2)$$

We recall that B = 1/3 for the quark c, as for the quarks (u, d, s).

Mesonic states with C≠0 will result from the combination of quark c with the ($\bar{u}$, $\bar{d}$, $\bar{s}$) antiquarks, forming an isospin doublet D and a strange isospin singlet S, given by

$$(C, s) = (+1, 0) \; : \qquad D = (D^+, D^0) = (c\bar{d}, c\bar{u}), \qquad (4.3a)$$

$$(C, s) = (+1, +1) \; : \qquad S = (c\bar{s}). \qquad (4.3b)$$

These names have been introduced by Gaillard et al.[44] The antiparticle states are denoted by $\bar{D} = (\bar{D}^0, \bar{D}^-)$ and $\bar{S}^-$. Their mass relationship with the normal states $\omega = (\bar{u}u + \bar{d}d)/\sqrt{2}$, $\varphi = (\bar{s}s)$, $K^* = (\bar{s}u, \bar{s}d)$, and $\rho = (\rho^+, \rho^0, \rho^-)$ with $\rho^+ = (u\bar{d})$, etc., depends first of all on the mass relationship between the c quark and the (u, d, s) quarks, and on the relationship of the $\bar{c}$-c and $\bar{c}$-(u, d, s) forces with those of ($\bar{u}$, $\bar{d}$, $\bar{s}$) with (u, d, s). In the literature,[44] much emphasis has been placed on the possibility that these forces obey an SU(4) symmetry, i.e. invariance with respect to all unitary transfomations in the space of all four quarks Q = (u, d, s, c). In this case, the mass separations between the states ω, ρ, K*, φ, D, S, and ψ are due primarily to "mass differences" between the four quarks, although SU(4)-breaking Q-Q forces could also contribute. The simple model, in which each quark contributes a "one-body operator" or "mass" to the net $(\text{mass})^2$ for the state, works extremely well for the subset (ρ, ω, K*, φ) of vector mesons. If we follow the same prescription for the remaining vector mesons, then we have

$$2m(D)^2 = m(\omega)^2 + m(\psi)^2 \qquad m(D) = 2.26 \text{ GeV} \quad , \qquad (4.4a)$$

$$2m(S)^2 = m(\varphi)^2 + m(\psi)^2 \qquad m(S) = 2.30 \text{ GeV} \quad . \qquad (4.4b)$$

The use of a linear mass formula would give slightly lower values, m(D) = 1.94 GeV and m(S) = 2.06 GeV. Since the ψ mass is so disparate with the ω and φ masses, it is far from obvious that this simple calculation should be correct, even if its physical assumptions were correct; the observation of C≠0 mesons will give us illuminating guidance in this respect. In the interim, the estimates (4.4) give a useful orientation, and it is generally assumed that the masses of the D and S mesons are in the vicinity of 2 GeV and that the rise of the ratio $R = \sigma(e^+e^- \to \text{had.})/\sigma(e^+e^- \to \mu^+\mu^-)$ between about 3.5 and 5.0 GeV may be connected with the onset of $e^+e^- \to D\bar{D}$ pair creation.

We now discuss the baryonic states in terms of the quark shell model, as above. As there, we assume that the c quark is endowed with colour, transforming according to the SU(3)' group in this colour space, and that all low-lying three-quark states are colour singlets. Further, we suppose that the orbitals for the c quark are the same as those for the q = (u, d, s) quarks. Then, the three-quark states which belong to the ($\underline{56}$, 0+)$_0$ supermultiplet, and which have C=0 and represent the known octet and decuplet baryons, are supplemented by the following states:

(i) the C=+1 states cqq. Their SU(3) character is given by 3 x 3 = 6 + $\bar{3}$. The 6 states have even permutation symmetry for the SU(3) variables, and this requires parallel spin, $S_{qq} = 1$, since we are concerned with the ground configuration $(\ell_c, \ell_{qq}) = (0, 0)$. The $\bar{3}$ states have odd permutation symmetry for the SU(3) variables, which requires $S_{qq} = 0$. The 6 states thus have J = 3/2 or J = 1/2, whereas the $\bar{3}$ states necessarily have J = 1/2. The J = 3/2 states correspond to the C=0 baryon decuplet, whereas the J = 1/2 states correspond to the C=0 baryon octet.

(ii) the C=2 states ccq. These form a $\bar{3}$ representation. The cc symmetry requires $S_{cc}=1$, which allows both $J = 3/2$ and $J = 1/2$. Again the $J = 3/2$ states correspond to the decuplet, while the $J = 1/2$ states correspond to the octet.

(iii) the C=3 states ccc. The cc symmetries require all three spins to be parallel, giving $J = 3/2$, corresponding to the decuplet.

We see two distinct patterns of states, those with $J = 3/2$ and corresponding to the known decuplet, and those with $J = 1/2$ and corresponding to the known octet.

If we go to the limit of SU(4) symmetry, these $J = 3/2$ states are the substates of a 20-dimensional representation with even permutation symmetry, which we denote by $\underset{\sim}{20}_S$, whereas these $J = 1/2$ states are the substates of a 20-dimensional representation with mixed permutation symmetry, which we denote by $\underset{\sim}{20}_M$. Thus,

$$\underset{\sim}{20}_S = (10)_0 + (6)_1 + (3)_2 + (1)_3, \quad \underset{\sim}{20}_M = (8)_0 + (6)_1 + (\bar{3})_1 + (3)_2 \qquad (4.5)$$

where the notation $\underset{\sim}{N}$ denotes an SU(4) representation, its suffix giving the permutation symmetry for a three-quark state, and the $(a)_C$ denote (SU(3), C) representations. There is one further SU(4) representation appropriate to the qqq system, namely $\bar{4}_A = (1)_0 + (\bar{3})_1$.

To a first approximation, the mass spectrum for baryonic states results from the "quark mass differences" (= the one-body operators), and is thus given roughly by

$$M(s, C) = M_0 - s\delta + C\Delta \qquad (4.6)$$

where $\delta \approx 0.18$ GeV and $\Delta \approx 1.2$ GeV, this estimate for Δ coming from a comparison between the ψ and φ masses, giving $(\Delta - \delta) \approx (m(\psi) - m(\varphi))/2$. These estimates would suggest a mass of about 2.2 GeV for the lightest baryon with $C \neq 0$. A somewhat higher estimate would be obtained if (4.6) were replaced by a quadratic mass relation, and the corresponding $(\Delta - \delta)$ were estimated from $(m(\psi)^2 - m(\varphi)^2)/2$; the lightest $C \neq 0$ baryon would then have mass about 3.0 GeV. More detailed structure in the mass spectrum will arise from the properties of the c-(u, d, s) interactions, their strength and their spin-dependence, their SU(3)-breaking character and their spin-orbit character, as well as on the corresponding properties for the (u, d, s)-(u, d, s) interactions which are already effective in the C=0 baryonic states. Expressions for these effects on the mass spectrum for the lowest supermultiplet states, with configuration $(1s)_\lambda(1s)_\rho$, have already been provided by Hendry and Lichtenberg.[45]

In these remarks, we have tacitly assumed that the c quark moves in the same orbitals as do the (u, d, s) quarks. This would be the case if SU(4) symmetry held for the four quarks Q and for the Q-Q forces, but even if SU(4) symmetry held for the Q-Q forces, the large mass difference between the c quark and the (u, d, s) quarks might well cause their orbitals to differ significantly.[46] However, it has been found in physical problems that there is a general tendency for the wave functions of the low-lying states of few-particle systems to have a greater degree of permutation symmetry than holds for the binding forces themselves. Hence, it would appear a reasonable first approximation to calculate the baryon mass matrix taking the low-lying orbitals $(n_{12}\ell_{12})_\rho(n_3\ell_3)_\lambda$ to be the same for all four quarks, i.e. to use base wave-functions which are the same as SU(4) symmetry would require. Further, if the spin-dependence of the c-(u, d, s) central interaction is not too strong, the base supermultiplets may be taken to be those for an SU(8) symmetry,[45] acting in the space of SU(4) x $SU(2)_\sigma$. The SU(8) representations appropriate to the QQQ system are 120-, 168-, and 56-dimensional; these correspond to permutation symmetry S, M and A, respectively, and their SU(4) x $SU(2)_\sigma$ content is as follows:[47]

$$\underline{\underline{120}} = {}^4\underset{\sim}{20}_S + {}^2\underset{\sim}{20}_M, \qquad \underline{\underline{56}} = {}^4\bar{\underset{\sim}{4}}_A + {}^2\underset{\sim}{20}_M,$$
$$\underline{\underline{168}} = {}^4\underset{\sim}{20}_M + {}^2\underset{\sim}{20}_S + {}^2\underset{\sim}{20}_M + {}^2\bar{\underset{\sim}{4}}_A. \qquad (4.7)$$

From expressions (4.5), we see that the $\underline{\underline{120}}$ representation has the states $^4(10)$ and $^2(8)$ for C=0, while the $\underline{\underline{168}}$ representation has the states $^4(8)$, $^2(10)$, $^2(8)$ and $^2(1)$ for C = 0. These states are the $^{2S+1}(a)$ content of the $\underline{56}$ and $\underline{70}$ representations of SU(6), respectively; the $\underline{\underline{120}}$ and $\underline{\underline{168}}$ representations are simply the direct generalization of the $\underline{56}$ and $\underline{70}$ representations required to include the c quark. In the same way, the 56 representation generalizes the 20 representation, for which no baryon is yet established.

These QQQ configurations with $C \neq 0$ can also undergo internal excitations. Again, with the harmonic shell model, the first excited configuration will have LP=1-, which requires M symmetry. This must be taken together with the SU(8) representation which has M symmetry, namely the 168 representation, and so we have $(\underline{168}, 1-)_1$ for the first excited supermultiplet, the ground supermultiplet being denoted by $(\underline{120}, 0+)_0$. Then there are at least the Regge recurrences and radial excitations of these configurations, and then the other types of excitation discussed already for the C=0 baryonic states. In fact, in the limit of SU(8) symmetry with an harmonic quark shell model, the pattern of supermultiplets expected is exactly that given on Fig. 1(a) where the term SU(6) is replaced everywhere on the figure by SU(8) and the entries dot, square and cross there are now to be identified with the SU(8) representations 120, 168 and 56, respectively. When the quark mass differences are turned on, this pattern of supermultiplets will become greatly distorted on a mass plot, because the coefficient Δ in Eq. (4.6) is so much larger than the coefficient δ. However, since the additive quantum numbers s and C are both conserved by the strong interactions, the patterns of SU(3) multiplets for definite values (C, s) will still remain recognizable despite the fact that each supermultiplet has now become stretched out over a wide range of masses by this simple SU(4)-breaking mechanism. Other SU(4)-breaking mechanisms (e.g. through the Q-Q potentials) can cause mixing between the SU(3) multiplets for the same (J, P, s, C) and between different SU(8) x 0(3) supermultiplets (e.g. between the $(\underline{120}, 2+)_2$ and $(\underline{168}, 2+)_2$ supermultiplets), in complete analogy with the situation discussed for SU(6) x 0(3) supermultiplets before. The main difference is that the mass differences breaking SU(8) symmetry are an order of magnitude greater than those breaking SU(6) symmetry. However, it is still possible that the major SU(8)-breaking interactions are these mass differences (the "one-body operators") and that the Q-Q forces at short distances are still well-approximated by SU(8) symmetry, in which case the SU(8)-mixing effects would be very much less than the comparison of Δ and δ would suggest.

Consider first the lowest baryon states with C=+1. Whether the $^4(6)_1$ states, the $^2(6)_1$ states or the $^2(\bar{3})_1$ states of the $(\underline{120}, 0+)_0$ supermultiplet lies lowest will depend on the details of the SU(8)-breaking, SU(6)-breaking, SU(4)-breaking and SU(3)-breaking character of the q-q interactions. However, the limit of stability for baryons $B_1(s)$ will be given by the mass of $(B_0(s) + D)$, where $B_C(s)$ denotes the lightest baryon for given C and s, and D denotes the lightest C=+1, s=0 meson. With the mass estimates given above, the baryon $B_1(s{=}0)$ will be stable with respect to $(B_0(0) + D_V)$, whether the linear or quadratic mass formula is used, where D_V denotes the vector D particle. We have no estimate for the mass of D_P, the pseudoscalar D-particle with structure $^1S_0$; if it were more than 0.2 GeV lighter than D_V, and the quadratic mass formula were used for the baryons in place of (4.6), then $B_1(0)$, the lightest baryon, would be unstable with respect to strong interactions. With the linear mass formula (4.6), on the other hand, the baryons $B_1^*(s)$ would be stable with respect to $(B_0(s) + D)$ up to quite high excitation energies above $B_1(s)$. However, $B_1^*(s)$ will be unstable with respect to $(B_1(s) + \pi)$ for excitation energy above 0.14 GeV. Thus, in general, only the lowest baryon $B_1(s)$ for each value of s would be expected to be semistable, decaying through weak interaction processes.

Much discussion is being given in the literature at present, concerning the decay processes expected for the D and S mesons, both vector and pseudoscalar, and for the C=+1 baryons. The decay processes may be leptonic, leading to $(\ell + \nu_\ell + \text{hadrons})$, or non-leptonic, leading to hadrons alone. For the D and S mesons, the general opinion appears to be that the non-leptonic decay processes will dominate, giving lifetimes of order 10^{-13} sec. However, after examination of many possible "all-charged" final states for D meson decay, no evidence has been found for the production and decay of D mesons in e^+e^- collisions at energy 4.8 GeV,[48] which is above the $D_V\bar{D}_V$ threshold for the D_V mass estimates given above. One difficulty for experiment is that there is a large number of possible nonleptonic decay modes, and further, that only those decay modes where all the final particles are charged (or otherwise visible, such as K_S^0 mesons undergoing the charged decay mode $\pi^+\pi^-$) can contribute to the search for these particles. For leptonic decay modes, the situation is even more difficult since the dominant modes are those which arise through the charged lepton current and which therefore include a neutrino. For the semistable C=+1 baryons $B_1(s)$, again the general expectation is that nonleptonic decay modes will dominate. These may involve the following transitions:

(i) $\Delta C=-1$, $\Delta s=0$, e.g. as follows,

$$B_1(0) \to \pi N, \text{ or } \pi\Delta, \text{ or } \pi\pi N, \text{ etc.} \quad (4.8a)$$

$$B_1(-1) \to \bar{K}N, \text{ or } \pi\Lambda, \text{ or } \pi\Lambda(1520), \text{ or } \bar{K}\pi N, \text{ etc.} \quad (4.8b)$$

(ii) $\Delta C=-1$, $\Delta s=+1$, e.g. as follows,

$$B_1(0) \to KN, \text{ or } K\Delta, \text{ etc.} \quad (4.9a)$$

$$B_1(-1) \to \pi N, \text{ or } \pi\pi N, \text{ or } K\Lambda, \text{ etc.} \quad (4.9b)$$

(iii) $\Delta C=-1$, $\Delta s=-1$, e.g. as follows,

$$B_1(0) \to \bar{K}N, \text{ or } \pi\Lambda, \text{ or } \pi\Sigma(1385), \text{ etc.} \quad (4.10a)$$

$$B_1(-1) \to \pi\Xi, \text{ or } \bar{K}\Lambda, \text{ or } \bar{K}\bar{K}N, \text{ etc.} \quad (4.10b)$$

(iv) $\Delta C=0$, $\Delta s=+1$, which we know already from the decay processes of the C=0 baryons and which will give rise to corresponding processes for the C=+1 baryons, such as

$$B_1(-1) \to \pi B_1(0) \quad . \quad (4.11)$$

The relative rates for all these decays will naturally depend on the details of the weak interactions, concerning which present views are in a rather fluid state.

The lightest $B_1(s)$ baryons will therefore be seen as narrow resonances, with decay widths of order 10^{-2} eV corresponding to typical lifetimes of order 10^{-13} sec, in many final hadronic states. None have been established to date and they pose a problem of a different order of magnitude of difficulty in comparison with current investigations of the ψ particles. From the above remarks, we see that they can, in principle, be formed directly in meson-baryon collisions. For example, from Eqs. (4.8-11), the baryons $B_1(0)$ can be formed in πN, KN and $\bar{K}N$ collisions, and $B_1(-1)$ can be formed with πN and $\bar{K}N$ collisions. However, the integrated cross sections for these formation processes are negligibly small relative to the cross section for the non-resonant scattering integrated over the energy resolution ΔE; for $J=1/2$, the former is $4\pi\bar{\lambda}^2\Gamma_{\pi N}$, to be compared with $4\pi\bar{\lambda}^2 x(\Delta E)$ where $x \leq 1$ measures the strength of the non-resonant scattering relative to the geometric limit and is typically of order 10^{-2}, while ΔE is of order 1 MeV compared with $\Gamma_{\pi N} \approx 10^{-7}$ MeV. The situation would be rather comparable with attempting to establish the existence of the ψ particle by studying s-wave πN scattering in the neighbourhood of incident pion laboratory momentum 119.3 MeV/c, appropriate to the process $\pi N \to \psi$. The $e^+e^- \to \psi$ process has the great advantage that the background reactions are only electromagnetic in origin and with cross-section of order $(1/137)^2 4\pi\bar{\lambda}^2$, while the convenient entrance channel e^+e^- has a substantial branching ratio, $\Gamma(\psi\to e^+e^-)/\Gamma(\psi\to \text{all}) \approx 0.07$, with $\Gamma(\psi\to\text{all}) \approx 80$ keV being about one-tenth of the energy resolution for the electron and positron beams circulating in the storage ring. In this case, we have to compare $\Gamma(\psi\to e^+e^-) \approx 5$ keV with $(1/137)^2\,\Delta E \sim 5\times 10^{-2}$ keV, a very favourable ratio.

Comparison with the case of strangeness is instructive. Here, the most productive studies have been for formation experiments $\bar{K}N \to \Lambda^*$ and Σ^*, where the resonance state of interest is formed with quite a low velocity in the laboratory frame, as has also been the case for $\pi N \to N^*$ and Δ^* resonance formation. For the quantum number C, we lack any $C\neq 0$ meson with lifetime sufficiently long to permit the construction of a $C\neq 0$ meson beam for cross section measurements and resonance formation. The lifetime $\tau \sim 10^{-13}$ sec allows a mean path length of only $30(E/m)$ microns and the probability for a nuclear collision in this distance through normal matter is rather small. In assessing the situation, it is a chastening thought to compare our state of knowledge concerning N^*, Δ^*, Λ^* and Σ^* resonances, where formation experiments are possible, with our meagre knowledge of Ξ^* and Ω^* resonances, where our information can come only from production processes.

Nevertheless, we turn now to consider production processes and mention briefly three classes of experiment.

(i) <u>Associated Production.</u> The direct analogues of the $\pi^- p \to \Lambda K^0$ reaction through which we gained a large part of our early knowledge about strange particles, are the processes

$$\pi^- + p \to \overline{D}^0 + B_1^0(0) \quad , \tag{4.12a}$$

$$\to \overline{D}^- + B_1^+(0) \quad , \tag{4.12b}$$

$$\pi^+ + p \to \overline{D}^0 + B_1^{++}(0) \quad , \tag{4.12c}$$

for $\overline{D}$ and $B_1(0)$ production, where we note that there are three states $B_1^+(0)$ belonging to the ground supermultiplet $(\underline{120}, 0+)_0$, two with J=1/2 belonging to the $^2(6)_1$ and $^2(\bar{3})_1$ unitary multiplets and one with J=3/2 belonging to the $^4(6)_1$ unitary multiplet. The analogues to $\overline{K}N \to \pi\Lambda$ and $\pi\Sigma$ are the processes

$$K^- + p \to \overline{D}^0 + B_1^0(-1) \quad , \tag{4.13}$$

for $B_1(-1)$ production, where we note that there are three states $B_1^0(-1)$ belonging to $(\underline{120}, 0+)_0$, two with J=1/2 belonging to the $^2(6)_1$ and $^2(\bar{3})_1$ unitary multiplets and one with J=3/2 belonging to the $^4(6)_1$ unitary multiplet. Of course, these $B_1(s)$ production reactions (4.12) and (4.13) may proceed with associated production of vector or pseudoscalar $\overline{D}$ mesons, or of $\overline{D}$ mesons with higher spin values. Their thresholds will lie high, at laboratory π or $\overline{K}$ momentum of 10 GeV/c or more. A reasonable expectation would be that the cross section rises rapidly near threshold to some fraction (perhaps as high as 1% - probably much less since there are very many channels open at this energy) of $\pi\lambda^2 \approx 0.3$ mb, the geometric limit on the s-wave reaction cross section at this energy, and then falls away gradually as the energy rises further - thus giving a maximum cross section of no more than about 1 μb, as a reasonably optimistic estimate.

(ii) Pair Production. The most favourable reaction for experimentation is with an electron-positron storage ring, namely:

$$e^+e^- \to D^+ + \overline{D}^- \quad , \tag{4.14a}$$

$$\to S^+ + \overline{S}^- \quad . \tag{4.14b}$$

The energies at present available are more than adequate for these processes. Although the cross sections are small, being electromagnetic, it is reasonable to expect these cross sections to be a non-negligible fraction of the total cross section, not far above threshold. The major uncertainty is whether the $\gamma D\overline{D}$ form factor may reduce the cross section far below expectation. However, [illegible] and [illegible] pair production (with [illegible]) has already been detected at SPEAR with a branching ratio of order 1%. It is interesting to reflect that $\Omega\overline{\Omega}$ production may similarly be expected to have a comparable branching ratio, perhaps of order 0.1%. In this case the SPEAR storage ring will ultimately provide Ω and $\overline{\Omega}$ particles for experiment [illegible] favourable situation, namely (i) [illegible] ratio, when this is compared with Ω production in high energy K^-p interactions, and (ii) producing them with laboratory momentum which is not highly relativistic. It seems quite a hopeful possibility that one may be able conveniently to carry out work in C=1 baryon spectroscopy, as well as baryon spectroscopy for C=0 (and perhaps for C=2, and higher), with electron-positron storage rings.

Pair production may also be studied in other interactions, such as

$$p + p \to p + p + B_1(s) + \overline{B}_1(-s) \quad . \tag{4.15}$$

With proton beam on stationary target, the threshold is high, at incident momentum ≈20 GeV/c, and the C=1 baryon and C=-1 antibaryon are produced with highly relativistic momentum, and with a low cross section, as we know already for $p\overline{p}$ production. Of course, with a 20 GeV proton storage ring, the proton-proton collisions have far more than enough energy for $B_1\overline{B}_1$ pair production and the difficulty is rather that the relevant events are swamped by the high yield of pions and other light particles in these pp collisions.

(iii) Neutrino-Induced Reactions. Since the weak interaction violates almost all of the nongeometric selection rules we know in hadronic physics (leaving only baryon number and charge as rigorously conserved quantum numbers), neutrino reactions are especially favourable for the production of particles with new quantum numbers. It will be sufficient to give one illustration, as follows. The reaction

$$\nu + p \to \mu^- + B_1^{++}(0) \quad , \tag{4.16}$$

is already energetically possible for rather low neutrino energy, but a much higher neutrino energy is needed before the cross section becomes reasonably large for experiment. The $B_1^{++}(0)$ produced may have either J=3/2, as a member of the $^4(\bar{6})_1$ unitary multiplet, or J=1/2, as a member of the $^2(\bar{6})_1$ unitary multiplet, of the $(\underline{\underline{120}}, 0+)_0$ supermultiplet.

It is remarkable that one neutrino-induced event has recently been reported[49] which could be an example of this production reaction (4.16). The event is fitted as

$$\nu + P \to \mu^- + \Lambda + \pi^+ + \pi^+ + \pi^+ + \pi^- \quad , \qquad (4.17)$$

with ($\Lambda\pi\pi\pi$) mass 2426 MeV. The suggested interpretation is that $B_1^{++}(\bar{6})$ is formed and decays by strong interactions to ($\pi^+ B_1^+(\bar{6}, ^2(\bar{3}), I=1/2)$), the latter baryon is semi-stable, decaying through the $\Delta C=-1$, $\Delta S=-1$ weak interaction to ($\Lambda\pi^+\pi^+\pi^-$). However, although plausible, this interpretation cannot be demonstrated, on the basis of one event. Nevertheless, this event does illustrate well the kind of event which would result from neutrino excitation of a C=+1 baryonic state, followed by its decay through strong, electromagnetic or weak decay processes. In the course of time, we may expect a gradual collecting of statistics for the very many reaction processes which can result from the excitation of C=+1 baryons by neutrinos and their subsequent decay and which may be used to demonstrate their existence, their mass values and their unitary multiplet patterns, but the event rate for specific configurations will be exceedingly low and it is likely to be a long time before any spin-parity values for C=+1 baryons will be determined from such data.

5. CONCLUSION

We see that there are a number of questions central to our understanding of baryon spectroscopy, which could be settled by experiment using accelerators currently operating and techniques which are well understood. It is desirable that those experiments should be carried out in the near future, in order to decide whether or not our present notions about this field of hadronic physics are well conceived. At the same time, we see that a new field of baryon spectroscopy is emerging, as experimental work in ν physics develops and suggests the existence of some deeper structure for hadrons, corresponding to an energy scale of typically ~1 GeV, about an order of magnitude greater than that we know for strange particle physics. Work in this new area of baryonic spectroscopy, associated with a new additive quantum number (or, perhaps, several new quantum numbers) appears to be arduous and to involve a very complicated phenomenology (as we are learning at present from the study of ψ mesons and their decay products), but it will be essential for our deeper understanding of this new field of physics.

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Addendum I. Jones et al.[50] have recently published an up-dated analysis of the mass spectrum for the states of the $(\underline{56}, 0+)_0$ and $(\underline{70}, 1-)_1$ supermultiplets, using the expressions given by Greenberg et al.[3] This analysis calculates χ^2, taking into account their assessment of the experimental error to be assigned to each input datum. They attempted to fit the decay data at the same time as the mass spectrum but no satisfactory solution could be found. In fitting the mass spectrum, they found two distinct local minima, lying rather close together. The minima appear to be distinct from the minimum found by Horgan,[4] who included also $(\underline{56}, 2+)_2$ and $(\underline{70}, 2+)_2$ states in his fit. For comparison with Table II, the ΛS01 and ΛD03 mixing matrices found by Jones et al. are given for their solution 1. The individual elements of their mixing matrices for solution 2 all agree with those for solution 1 within ±0.05.

ΛS01			ΛD03			
1.0	0.1	0.0	-1.0	0.1	0.0	$\Lambda(^4 8)$
-0.1	0.9	-0.4	0.1	0.9	0.4	$\Lambda(^2 8)$
0.0	0.4	0.9	0.1	0.4	-0.9	$\Lambda(^2 1)$

The AD03 matrix agrees rather closely with that given by Cashmore et al.[19] It agrees qualitatively with those of Faiman and Plane[12] and of Horgan,[4] the main difference being in the magnitude of the $^4 8 - ^2 8$ mixing. The AS01 matrix disagrees completely with that of Cashmore et al. It has a rough qualitative agreement with those of Faiman and Plane and of Horgan, but in detail the agreement is poor, especially concerning the $^4 8 - ^2 8$ mixing.

Addendun II. An important paper including discussion of the $C \neq 0$ baryons in considerable detail, for the case of broken SU(4) symmetry, has just been published by De Rujula et al.[51] This discussion is given in the context of an interpretation of all hadrons and hadronic properties in terms of the simplest gauge model possible. The quantum number C is there referred to as "charm", the name given to it by Bjorken and Glashow[42] in their discussion of SU(4) symmetry in 1964.

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38. In principle f_q is directly related with the pion-nucleon coupling constant, as follows: $f_\pi = f_q(5/3)$, this factor 5/3 being well known as the SU(6) value for the ratio G_A/G_V between the axial vector and vector beta-decay coupling constants. However, $f_\pi^2/4\pi = 0.082$ is a well established parameter and the corresponding value $f_q^2/4\pi = 0.082(3/5)^2 = 0.03$ is too small to fit the $\Delta^* \to \pi N$ partial widths observed. It is of interest to remark that if the physical ratio $G_A/G_V = 1.23$ were used for the ratio f_π/f_q in place of the SU(6) value of (5/3), the predicted value for f_q would be $f_q^2/4\pi = 0.052$, in good agreement with the value required here.

39. If the decays $B^* \to \pi B$, $B^* \to \pi D$, $B^* \to \gamma B$, and $M^* \to \pi M$ (where M denotes meson) are mediated dominantly by a one-quark operator, then the conclusion from the Melosh transformation that the processes take place through operators which are part of a 35 $SU(6)_W$ representation, becomes a trivial consequence of this fact, since the transition $q \to q$ can only change the $SU(6)_W$ character of the state by those $SU(6)_W$ representations which are contained in $q\bar{q}$, namely $6 \times \bar{6} = 1 + 35$.

40. Particle Data Group, Phys. Letters 50B (1974) 1.

41. Particle Data Group, Revs. Modern Phys. 47 (1975) 535.

42. J. D. Bjorken and S. L. Glashow, Phys. Letters 11 (1964) 255; S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.

43. The charge assignment which is physically correct for the quark c will become settled directly when the charge states are determined for the hadrons having C≠0.

44. M. Gaillard, B. W. Lee and J. L. Rosner, Revs. Modern Phys. 47 (1975) 277.

45. A. W. Hendry and D. B. Lichtenberg, "Hadron Mass Relations in a Charmed Quark Model with Broken SU(8) Symmetry", Indiana Univ. preprint COO-2009-89 (February, 1975).

46. This remark assumes "light quarks", as is commonly accepted at present. If the truth corresponds to "heavy quarks", these remarks no longer follow, since these mass differences would then be small relative to the mean quark mass.

47. J. W. Moffat, Phys. Rev. 140B (1965) 1681; S. Iwao, Ann. Phys. (N. Y.) 35 (1965) 1.

48. A. Boyarski et al., Phys. Rev. Letters 35 (1975) 195.

49. E. Cazzoli, A. Cnops, P. Conolly, R. Loutit, M. Murtagh, R. Palmer, N. Samios, T. Tso and H. Williams, Phys. Rev. Letters 34 (1975) 1125.

50. M. Jones, R. Levi Setti and T. Lasinski, Nuovo Cimento 19A (1974) 365.

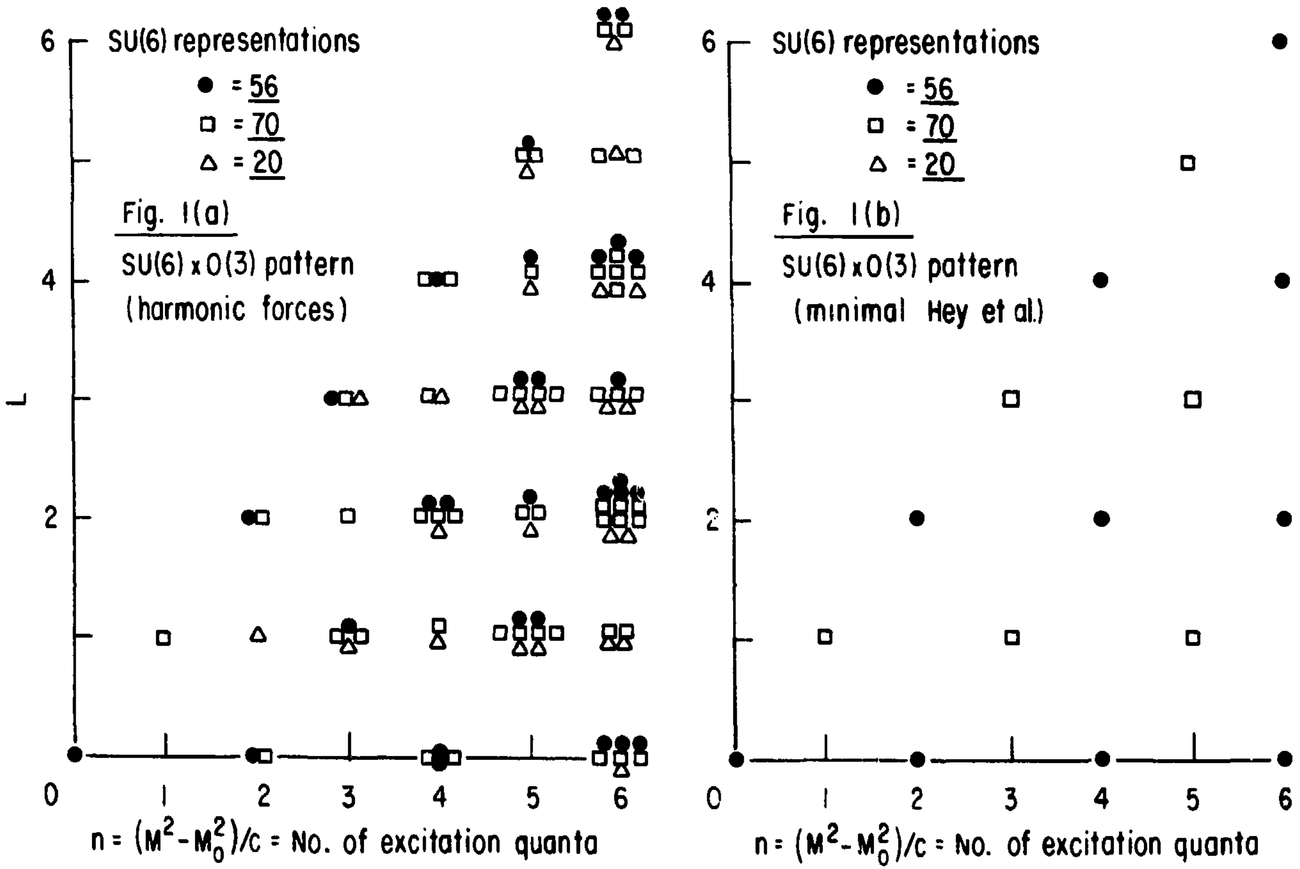
51. A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D12 (1975) 147.

[illegible]	n [illegible]	S, [illegible]	M, [illegible]	A, [illegible]	[illegible] multiplets
[illegible]	0	S	-	-	2
[illegible]	1	-	P	-	4
[illegible]	2	S	S	-	
		-	-	P	
		D	D	-	17
[illegible]	3	P	P + P	P	
		F	F	F	56
[illegible]	4	S + S	S + S	-	
		-	P	P	
		D + D	D + D + D	D	
		-	F	F	
		G	G + G		116

Table I. The states of internal motion for a three-quark harmonic oscillator system with [illegible] quanta of excitation [illegible] are tabulated according to their permutation symmetry, S [illegible], M [illegible] and A (implies [illegible]), and their internal orbital angular momentum [illegible] with the usual notation S, P, D, F, G, ... for L = 0, 1, 2, 3, 4, ... The SU(3) x [illegible] content for each SU(6) representation is given at the head of the appropriate column, with the notation [illegible], and the total spin J is given by the vector sum J = L + S, as usual. The total number of independent [illegible] multiplets characterized by J and [illegible] is given in the right-hand column, for values of N up to N = 4.

	Faiman & Plane [42] (Decay Analysis)			Cashmore et al. [49] (Decay Analysis)			Horgan [*] (Fit to Mass Spectrum)			Base States
D03 states	-0.9	0.4	[illegible]	-1.0	[illegible]	0.04	[illegible]	0.5	[illegible]	[illegible]
	0.4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.8	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	0.04	[illegible]	-0.9	[illegible]	[illegible]	[illegible]	[illegible]
S01 states	0.8	[illegible]	[illegible]	-0.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	0.6	-0.6	-0.45	0.95	0.05	[illegible]	[illegible]	0.65	[illegible]	[illegible]
	0.0	[illegible]	0.8	-0.25	[illegible]	0.85	[illegible]	[illegible]	[illegible]	[illegible]

Table II. The mixing matrices obtained by Faiman and Plane and by Cashmore et al. from their SU(6)$_W$ analyses for the decay transitions for the D03 and S01 states of the (70, 1$^-$)$_1$ supermultiplet are compared with the mixing matrix obtained by Horgan from his fit to the mass spectrum. The D03 states used were [illegible], [illegible](1690) and [illegible](1520), and the S01 states used were [illegible], [illegible](1670) and [illegible](1405). In both cases, the uppermost state is not known and the entries given are determined from the requirements of orthogonality.



SU(6) representations
● = 56
□ = 70
△ = 20
Fig. 1(a)
SU(6) x O(3) pattern
(harmonic forces)
L
0
1
2
3
4
5
6
n = $(M^2-M_0^2)/c$ = No. of excitation quanta
SU(6) representations
● = 56
□ = 70
△ = 20
Fig. 1(b)
SU(6) x O(3) pattern
(minimal Hey et al.)
n = $(M^2-M_0^2)/c$ = No. of excitation quanta

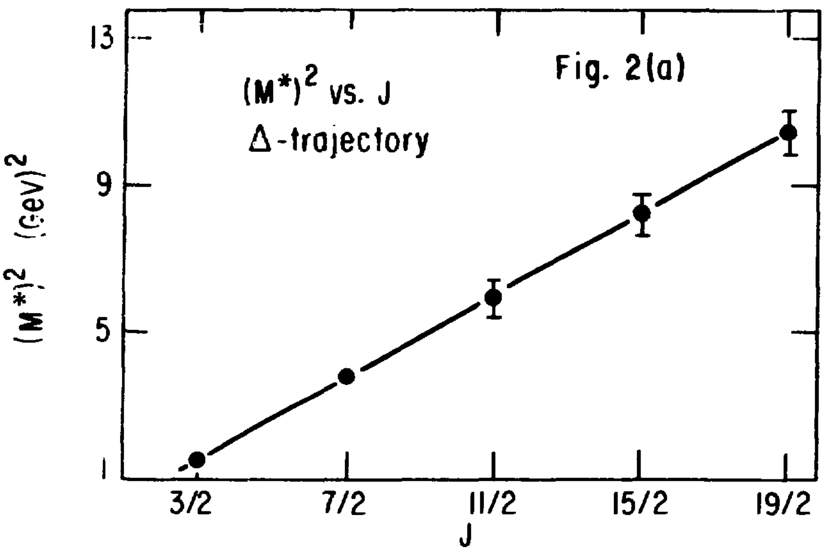
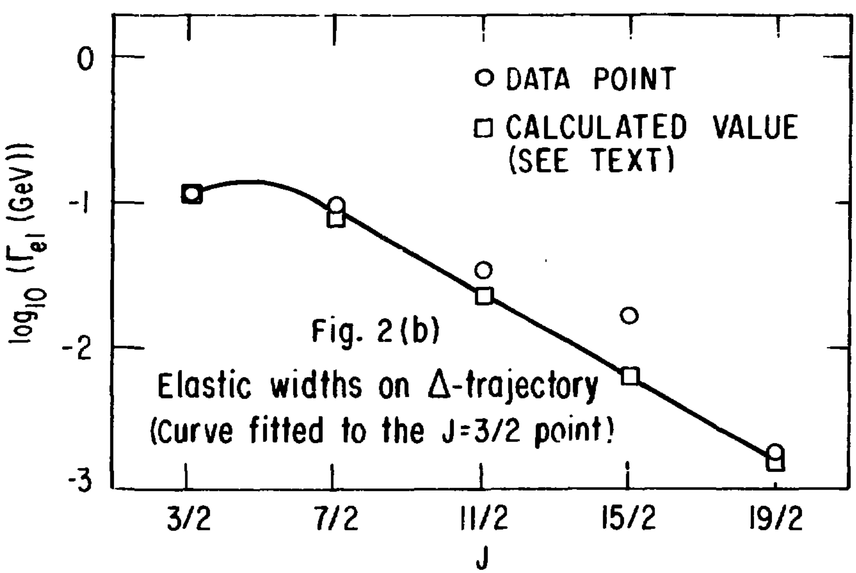


Fig. 2(a)
(M*)^2 vs. J
Δ-trajectory
(M*)^2 (GeV)^2
13
9
5
1
3/2
7/2
11/2
15/2
19/2
J



DATA POINT
CALCULATED VALUE (SEE TEXT)
Fig. 2(b)
Elastic widths on Δ-trajectory
(Curve fitted to the J=3/2 point)
log10 (Γel (GeV))
0
-1
-2
-3
3/2
7/2
11/2
15/2
19/2
J

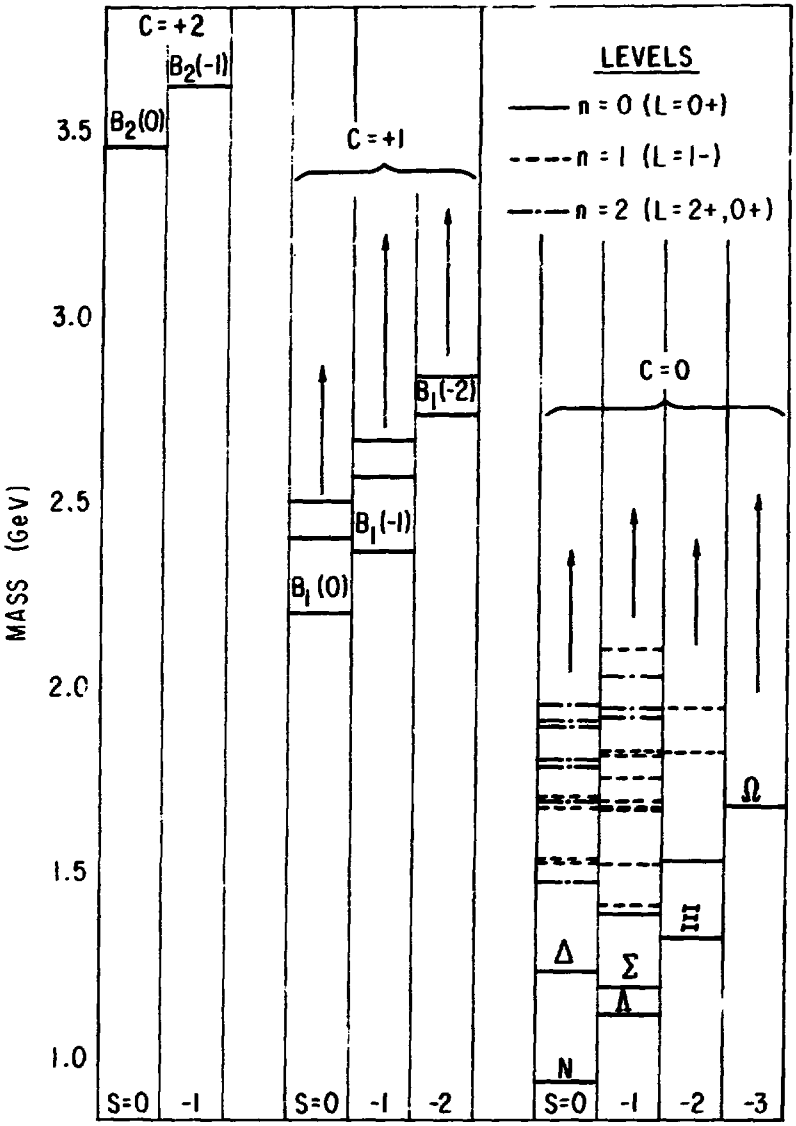


Fig. 3. Schematic representation of the mass distribution of the baryonic unitary multiplets as function of C and s. For C=0, the established states listed in the 1974 PDG Review are shown. For C=+1 and +2, the masses shown are taken from Eq. (4.15) with some arbitrariness; the C=+3 masses are off the mass scale.