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GOVERNMENT OF INDIA
ATOMIC ENERGY COMMISSION

A RANDOM SAMPLING PROCEDURE FOR ANISOTROPIC DISTRIBUTIONS

by

P. S. Nagarajan, P. Sethulakshmi, C. P. Raghavendran
and D. P. Bhatia
Division of Radiological Protection

BHABHA ATOMIC RESEARCH CENTRE
BOMBAY, INDIA
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OXYGEN

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IRON

LEAD

ABSTRACT

A procedure is described for sampling the scattering angle of neutrons as per specified angular distribution data. The cosine of the scattering angle is written as a double legendre expansion in the incident neutron energy and a random number. The coefficients of the expansion are given for C, N, O, Si, Ca, Fe and Pb and these elements are of interest in dosimetry and shielding.

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In neutron dosimetric and shielding calculations employing Monte Carlo methods, random sampling of the neutron direction after scattering events presents some problems because the angular distributions of scattered neutrons are generally anisotropic in the centre of mass system. This paper presents a sampling scheme developed by us for this purpose and which has been in use for a few years now.

A number of schemes can be developed including the one due to Coveyou[1] if there were just one particualr angular distribution to sample from. But the nature of angular distributions of scattered neutrons changes considerably with incident neutron energy. Thus any scheme should take into account this energy dependence. The only other technique for this purpose is due to Textor and Verbinsky[2].

The differential scattering cross section data is readily available in compilations[3, 4] wherein $(d\sigma / d\Omega)$ is given as a function of μ, the cosine of the scattering angle in c. m. system, for various neutron energies and target nuclides.

$f(\mu, E)$, the probability density in μ at incident neutron energy E is given by

$$f(\mu, E) = 2\pi \cdot \left(\frac{d\sigma\ (\mu, E)}{d\Omega}\right) \Big/ \sigma(E)$$

where σ, the scattering cross section, is given by

$$\sigma(E) = 2\pi \int_{-1}^{+1} \frac{d\sigma(\mu, E)}{d\Omega} \cdot d\mu$$

The straightforward sampling of any value μ_1 requires solution of the equation

$$F(\mu_1, E) = \int_{-1}^{\mu_1} f(\mu, E) d\mu = R_1$$

for μ_1 where F is the cumulative distribution function of μ and R_1 is a random number. Since F is generally a polynomial of degree six to ten or higher this approach is not convenient. Therefore an inverse approach was followed.

There are two options which are

(i) the normal polynomial expansion given by,

$$\mu(R, E) = \sum_{0}^{\infty} C_m(E) R^m$$

(ii) the Legendre polynomial expansion, given by,

$$\mu(R, E) = \sum_{0}^{\infty} a_m(E) P_m(R)$$

where R is a random number and E is incident energy. Both these were considered truncating the series at about ten terms, in addition to a third alternative. The behaviour of $C_i(E)$, and $a_i(E)$ obtained by a least square method, is shown in fig. 1a and 1b for nitrogen for illustration. This shows that C_i's do not lend themselves convenient for a second expansion in E, over a finite number of terms. Similarly cumulative $\phi_i(E)$ of Coveyou technique[1] are presented in fig. 2, for carbon as an illustration. This figure is taken from ref (5). Incidentally the sampling of μ, for any energy is done in the Coveyou technique in the following manner.

$$\mu = \bar{\mu}_i$$

where $\bar{\mu}_i$ may be the mid-points of the n intervals given by the (n+1) roots of the Legendre polynomial equation in the interval (-1, +1)

$P_{n+1}(\mu) = 0$, and where the index 'i' is decided by a random number R, and is given by a check

$$\phi_{i-1} \leq R < \phi_i$$

The cummulative ϕ_i's and the cummulative distribution function $F(\mu, E)$ at E = 5.0 MeV for carbon are given in fig. 3 to illustrate the method. From the above it is clear that neither the normal polynomials nor the Coveyou coefficients lend themselves convenient for function fitting in the energy variable. Therefore a double legendre expansion was considered since a_i's were somewhat smooth in E.

Data from ref (3) was graphically interpolated in the energy range 0.5 to 15 MeV so as to obtain the $(d\sigma/d\Omega)$ data at uniform intervals in energy. A double least square fit was made to obtain the coefficients a_{ij}'s of the expansion

$$\mu = \sum_j \sum_i a_{ij} P_i(E') P_j(R)$$

where R is a random number in the interval (-1, +1) and $E' = (E-7.5)/7.5$ and P's are the Legendre polynomials. The coefficients a_{ij}'s were obtained for seven elements, C, N, O, Si, Ca, Fe and Pb. In the case of lead, data was taken from ref (6) in addition to that from ref. (2) since no other recent data was available in the energy range, 6 MeV to 12 MeV. To increase the speed of sampling, coefficients b_{ij}'s are obtained from a_{ij}'s thus enabling sampling of μ, given by

$$\mu = \sum_j \sum_i b_{ij} E^i P_j(R)$$

Tables 1 through 14 give coefficients a_{ij} 's and b_{ij} 's for the seven elements of interest in dosimetry and shielding for elastic scattering for use in the energy range, 0.5 to 15 MeV. Usually inelastic scattering is taken to be isotropic for these elements over the range of energies considered. Further, the effect of inelastic events themselves, in this energy range, is usually negligible in dosimetry and shielding. However, similar coefficients could be obtained for these events too.

Figures 4 through 19 show the data used (broken lines) and the sampled distribution (histograms). Generally, the results show good agreement with the data source, except a few cases, notably in the case of lead (14 and 15 MeV). It should, of course, be noted that because of

(i) the abrupt changes in the nature of distributions with energy, and (ii) the smoothing that is inherent in the method; deviations from the data used are anticipated. However, in spite of the above, the results show that the method yields accurate enough samples and can be used in Monte Carlo calculations.

ACKNOWLEDGEMENTS

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Tables 1-7 :'Double-Legendre' Coefficients a_{ij}'s

Table 1 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

CARBON

j \ i	0	1	2	3
0	3.70814977505-001	3.93627512117-001	-1.31748598168-001	-2.96914090816-002
1	-9.59590721549-001	1.85409072958-001	8.28481169324-002	-9.73960422259-002
2	-3.74435248255-001	-4.06943234557-001	1.71776919032-001	4.14919665083-002
3	-2.82056556531-002	-2.40898032195-001	-1.06467492197-001	1.01975555229-001
4	5.09166666528-002	1.34793570239-002	-1.25554202026-001	-5.44094835687-002
5	3.52483041695-002	1.00349350437-001	-3.86313836963-003	-9.01751774107-002
6	-1.04820648907-002	1.44455890969-002	4.24454983254-002	-1.51756230451-002
7	-4.61365734378-002	-5.03570926702-002	6.84124173620-002	9.34276155825-002
8	-2.51327618834-002	-6.07222424366-002	-1.81450237625-002	6.17754669220-002

j \ i	4	5	6	7
0	3.11688606668-002	-3.17666488362-003	-6.49489047716-003	-1.22931599888-003
1	1.80862185152-002	2.22071269224-003	-1.82607907095-002	5.02870145440-004
2	-5.90338137327-002	8.73909162823-003	3.16323116346-003	4.29487002850-003
3	9.25211989298-003	-1.69177673219-002	2.22018564784-003	4.39038669504-003
4	6.70791328209-002	1.55444314537-002	-1.26682302915-002	-1.86296444081-003
5	-5.69298162009-003	6.37426520430-002	-9.29995663976-003	-2.82144608942-002
6	-3.15199755278-002	5.91883233888-002	8.99796372489-002	1.99059941425-002
7	-3.38921060722-002	-7.33970827097-002	3.29058767803-002	2.49898239016-002
8	6.18075302534-002	-5.42422700324-002	-3.95575558324-002	3.30481426389-002

j \ i	8
0	3.44313716656-004
1	4.70575284259-003
2	2.52975745918-003
3	-1.08841474724-004
4	-6.22916325217-003
5	2.76744422369-003
6	-2.72758765524-002
7	-2.40460198827-002
8	1.60926998127-002

Table 2 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

NITROGEN

j \ i	0	1	2	3
0	3.47658584703-001	3.14304245793-001	9.58289786801-002	-9.53225190658-002
1	-9.83715069504-001	2.35574334347-001	1.60762130804-001	-2.89455102227-002
2	-3.18847709545-001	-2.66217307257-001	-2.85500458832-002	9.86385863856-002
3	5.40477064135-002	-1.98592304805-001	-2.07321489049-001	2.82274769805-002
4	2.77930008905-002	-7.45350749115-003	-6.18432314205-002	-4.19477027794-002
5	-6.86525846284-002	1.39785551737-003	9.69311130163-002	5.06151809730-003
6	-6.23807243397-002	-5.34372747631-002	3.88503928343-002	3.57328885596-002
7	-7.24756645365-003	-4.42673752934-002	-6.24537492823-002	1.02050390598-002

j \ i	4	5	6	7
0	1.45925697329-001	-9.75457552960-002	4.27701441001-003	6.34470737423-002
1	2.45148689323-002	4.40093401750-002	-9.68136579706-002	5.81065105472-002
2	-1.39847545285-001	1.48753728220-001	-1.77618438579-002	-1.03438709493-001
3	-1.24476362148-002	-2.39903140994-002	9.65149013000-002	-8.01383670699-002
4	-2.54635995813-003	-9.28756359871-002	5.95747339621-002	4.39226896397-002
5	-3.87734026770-002	-6.63863663503-002	-9.97405376891-003	5.90447262127-002
6	7.99778773123-003	1.04145444816-002	-2.58126644377-002	-2.51422752062-002
7	5.38866821700-002	5.34606766026-002	-4.48160989175-004	-3.18120311195-002

j \ i	8	9	10
0	-4.19540642282-002	0.00000000000+000	0.00000000000+000
1	1.98614322854-002	-5.04372846009-002	3.40835951292-002
2	9.88084867573-002	-1.07009005436-002	-6.02110560029-002
3	0.00000000000+000	0.00000000000+000	0.00000000000+000
4	-6.23235821840-002	1.62016986887-002	0.00000000000+000
5	-2.40757966496-002	0.00000000000+000	0.00000000000+000
6	2.09030981577-002	3.69800767652-003	0.00000000000+000
7	8.71130327927-003	9.79982149671-003	-1.04656328404-002

Table 3 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

OXYGEN

j \ i	0	1	2	3
0	3.30999741016-001	2.51903997641-001	-1.59364401334-002	2.73214644491-002
1	-1.02211990242-002	1.20647500182-001	9.73636889225-002	-2.31043120072-003
2	-3.48331424856-001	-2.82684672985-001	8.34901693463-002	1.05372169157-002
3	5.16174161481-002	-1.43984520796-001	-1.05339694762-001	5.48777769448-002
4	3.97826707305-002	3.75843842386-002	-1.06988275768-001	2.22750760114-003
5	-4.07333905366-002	2.26534271863-002	1.03596513913-002	-8.15978100803-002
6	-4.04496165051-002	-1.39383916988-002	3.18690680858-002	-5.26318917249-002
7	-4.76342041628-003	-1.95177876175-002	-4.29501006333-003	2.32083416468-002

j \ i	4	5	6
0	-7.43451518216-003	-1.37867811776-002	2.59052968747-003
1	1.59043438866-002	-1.39747815087-002	-7.57142258564-003
2	1.06104044619-002	1.00692053759-002	-1.51076089463-002
3	-1.70646486318-002	1.04239422589-002	-3.73345983174-003
4	1.72655229157-002	-9.79197025090-004	3.14721850312-003
5	3.30389431812-002	2.83938252559-002	1.97258958098-003
6	-3.96423666499-002	4.25685447478-002	7.00399087043-003
7	-5.52648435545-002	-1.19093476376-002	2.50889739245-002

Table 4 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

SILICON

j \ i	0	1	2	3
0	4.97249999968-001	6.45697915694-002	-8.40667535109-002	9.84391947370-002
1	-8.90283333394-001	-6.06156248297-002	4.08299827215-002	-3.36330176308-002
2	-5.46933333331-001	-1.06521249886-001	1.22864027828-001	-1.22924121539-001
3	-1.59566666660-001	3.07925000467-002	4.27036458242-002	-4.59396104678-002
4	1.71833333344-002	6.34252083325-002	-1.51786284940-002	5.05527838226-002
5	5.14166666637-002	-6.64145834337-003	-3.04299131938-002	2.47629087913-002
6	4.16999999993-002	-2.18431250099-002	-2.66003819340-002	1.59061723039-002

j \ i	4	5	6
0	-9.60210521286-002	8.51207476831-002	-6.13970299414-002
1	2.91609432496-002	-1.94021356702-002	5.39377482224-003
2	1.08824820485-001	-8.24841765338-002	4.14624296292-002
3	4.18022505520-002	-3.23199340136-002	2.34223557939-002
4	-2.13879262412-003	1.64786488874-003	-1.52310414275-004
5	-1.74802204370-002	8.10460160370-003	-6.06460765225-004
6	-1.47549693601-002	1.16812097703-002	-7.82057540596-003

Table 5 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

CALCIUM

j \ i	0	1	2	3
0	6.18145102216-001	2.32161594572-001	-2.23427597777-001	9.12029687967-003
1	-7.27195996465-001	2.34315057530-001	-2.53490889817-001	-5.49705878025-002
2	-5.57148169610-001	-1.15051849966-001	1.22147708668-001	-1.16238923132-001
3	-3.24850000005-001	-2.09712708258-001	2.41207864642-001	1.60768061361-003
4	-1.29816666673-001	-1.37206458309-001	2.24818802130-001	1.12036907612-001
5	3.93999999820-003	-6.06228333374-002	9.24856041931-002	1.02998862253-001
6	6.44666666701-002	-2.81125001155-003	-4.60579861095-002	3.12618156697-002
7	4.13666666660-002	3.56241666578-002	-1.08432881936-001	-4.81903508503-002
8	3.13833333348-002	4.31210416639-002	-8.40113368211-002	-6.60585790104-002

j \ i	4	5	6	7
0	4.73009023396-002	-2.61664150452-002	1.05580506442-003	3.74794718152-003
1	1.09890584016-001	-3.68185530277-002	-1.05110864041-002	1.14143675982-002
2	7.71993605071-002	-1.38772453720-002	-2.36987276992-002	2.71475507331-002
3	-1.99070173039-002	2.49759385287-002	-9.77790455078-003	-9.18052957160-003
4	-1.51865764157-001	7.63292892824-002	-1.51315315338-002	-2.16603128758-002
5	-1.64575221561-001	3.70914501848-002	2.79950009700-002	-3.45384012483-002
6	-6.51849615935-002	-1.70828128752-002	4.47082830942-002	-1.87146121897-003
7	5.16933022335-002	-4.28728100820-002	2.51905145147-002	1.76116015809-002
8	1.11824985076-001	-2.11654817773-002	-2.26560338534-003	2.13510867389-002

Table 6 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

IRON

j \ i	0	1	2	3
0	7.02015819168-001	3.25064854696-001	-2.75976923812-001	9.25714545208-002
1	-5.59211458336-001	3.87960210355-001	-2.79801788297-001	-4.01557088352-002
2	-4.45000409428-001	-2.46444689075-002	6.46621562087-002	-2.15517760225-001
3	-3.04638584825-001	-2.20882984495-001	1.61529754158-001	-7.64028554095-002
4	-2.03093730356-001	-2.33727568545-001	1.59793522325-001	7.02133142203-002
5	-1.20196077038-001	-1.91481489138-001	1.32418053236-001	1.10250142237-001
6	-7.69266687462-002	-7.49909321498-002	5.22542465862-002	1.04820835448-001
7	-2.94088677276-002	-1.38380216595-002	1.46906439334-002	4.62615889765-002

j \ i	4	5	6	7
0	1.66443143770-002	-3.37744363205-002	1.13063041220-002	4.36728977016-003
1	1.63559320164-001	-1.13399517586-001	2.94054434035-002	1.43278987898-002
2	1.78773960957-001	-5.69339223730-002	-1.88481794790-002	2.77547213587-002
3	-6.03526569984-002	1.41314643133-001	-1.03780467718-001	1.82772221434-002
4	-1.75290072965-001	1.63752621770-001	-4.45097904257-002	-2.30587656773-002
5	-1.50049927837-001	5.69611312973-002	4.05362214625-002	-4.83956099290-002
6	-3.50346306339-002	-7.63294559089-002	1.08754370559-001	-4.16855530394-002
7	8.72665643669-003	-6.60636383749-002	4.00298279594-002	1.51035503525-002

j \ i	8	9	10
0	-7.17324635049-003	5.31678442599-003	2.01288795442-003
1	-1.49559348120-002	9.03393440973-003	-7.85508970555-003
2	-6.92125043669-003	-3.78266326134-003	-2.45653085929-003
3	3.68761977216-002	-3.87188981712-002	1.36180515683-002
4	3.89146496635-002	-6.89276500617-003	-2.06707409432-002
5	5.84517334693-003	2.83815800486-002	-2.29145350336-002
6	-3.46073436295-002	4.62339692097-002	-5.71534525510-003
7	-3.69336392084-002	9.46390059497-003	1.90467253845-002

Table 7 DOUBLE LEGENDRE COEFFICIENTS a_{ij}

LEAD

j \ i	0	1	2	3
0	6.80737873178-001	2.96703434353-001	-3.85953527945-001	2.56930644478-002
1	-5.98703676148-001	3.22741994879-001	-4.26660152548-001	-1.53626520580-001
2	-4.65118877264-001	-9.85373659104-002	8.37311584083-002	-1.85544650379-001
3	-2.91841799801-001	-2.43294023926-001	2.81622447970-001	1.11733556044-002
4	-1.78101339744-001	-1.52124997377-001	2.13909061422-001	1.27409408908-001
5	-1.06055854014-001	-6.57241372543-002	1.22222555440-001	1.20695610303-001
6	-3.99398798228-002	-3.04439895175-002	4.54453769859-002	9.04235292971-002
7	-1.18581741658-002	-3.78759925324-002	3.00356938126-002	2.84487537302-002
8	-1.00000000000-002	0.00000000000+000	0.00000000000+000	0.00000000000+000
9	-4.72831987543-003	1.14639828412-002	-1.46951740331-002	1.82737137648-002

j \ i	4	5	6	7
0	7.47450018197-003	-1.48307764556-001	5.43347640114-002	2.32430872688-003
1	8.42679426866-002	-2.25336991769-001	3.36787905783-002	6.70605089085-002
2	3.32979458902-003	1.08536800960-001	-1.75982006185-001	1.26979926167-001
3	-1.01095298081-001	2.63262924505-001	1.19546195111-001	9.83649141737-003
4	-7.85899926396-002	7.98148656776-002	5.82236979296-002	-1.24989105607-001
5	1.04785104215-002	-3.48877614416-002	1.20704512417-001	-7.33299115766-002
6	-4.43412207940-002	-4.51371308591-002	4.36496278634-002	-1.73792859699-002
7	-2.80671873217-002	4.01457872178-002	1.41593963301-002	7.51264402492-003
8	0.00000000000+000	0.00000000000+000	0.00000000000+000	0.00000000000+000
9	-2.12437827449-002	1.54394138954-002	6.91830420727-004	-1.69097242169-002

j \ i	8	9	10
0	-3.82288960653-002	3.83166064793-002	0.00000000000+000
1	-4.76852554036-002	3.16742930864-002	-1.27914314999-002
2	-3.07733986754-003	-5.78798452439-002	5.79311019753-002
3	7.38945811626-002	-1.11170513221-001	1.00260382290-002
4	4.82441588223-002	0.00000000000+000	0.00000000000+000
5	-2.19434627582-002	9.44368829601-002	-3.96760636949-002
6	-1.58645618739-002	2.00142354466-002	0.00000000000+000
7	0.00000000000+000	0.00000000000+000	0.00000000000+000
8	0.00000000000+000	0.00000000000+000	0.00000000000+000
9	1.64068033083-002	-4.96767856683-003	0.00000000000+000

Tables 8-14 ; 'Normal-Legendre' Coefficients b_{ij}'s

Table 8 NORMAL LEGENDRE COEFFICIENTS b_{ij}

CARBON

j \ i	0	1	2	3
0	4.56501400894-001	4.34897507829-001	-3.60518181658-001	-7.06348637003-002
1	-9.87239221686-001	3.34566944151-001	-1.09709837609-001	-2.53021035634-001
2	-4.82758167105-001	-4.67190415704-001	4.74898584566-001	1.11818118210-001
3	2.77740660495-002	-4.35186149663-001	-1.78755311314-001	4.39405000197-001
4	1.41103977613-001	1.28214626067-001	-4.61694986644-001	-3.08714596571-001
5	3.87079647265-002	4.16848722339-001	-7.27190210947-002	-1.33865834776+000
6	-7.91016890115-002	1.04642761832-001	1.04085643497+000	-1.63937627586-001
7	-1.09910491952-001	-3.82783285910-001	6.82361848404-001	1.36778067023+000
8	2.38796576455-002	-3.27382511153-001	-6.77004748027-001	1.27969383929+000

j \ i	4	5	6	7
0	2.82873281502-001	2.82285132430-002	-1.26081671612-001	-3.29610352200-002
1	6.93408923090-001	-4.29245072303-003	-7.05245659191-001	1.34832057746-002
2	-1.83586398673-001	-1.17201211537-001	-1.91732776638-001	1.15156202639-001
3	-9.12462587760-003	-3.23386041389-001	4.22680224343-002	1.17717243261-001
4	2.05626198255-001	2.03102045040-001	4.01670464112-001	-4.90597340693-002
5	3.08017261681-001	1.72401222232+000	-3.93975467854-001	-7.56500232726-001
6	-3.38610700524+000	-3.96070324611-001	3.85875155299+000	5.33729467946-001
7	-2.09797895839+000	-1.66037377408+000	2.73164727438+000	5.70039653361-001
8	1.92046615110+000	-1.85855555455+000	-2.08131151038+000	9.86103324505-001

j \ i	8
0	1.73098341147-002
1	2.36574371422-001
2	1.27179603514-001
3	-5.47183507695-004
4	-3.13161449435-001
5	1.39128934214-001
6	-1.37125207512+000
7	-1.20887607770+000
8	8.09035338239-001

Table 9 NORMAL LEGENDRE COEFFICIENTS b_{ij}

NITROGEN

j \ i	0	1	2	3
0	3.41657850420-001	1.35599259400-001	3.75753298490-002	1.86433332547+000
1	-1.02760569029+000	1.10279115609-001	-2.20311381148-001	2.50699716537+000
2	-3.09629429720-001	6.46759831607-002	-1.42257096682+000	-2.70521262012+000
3	1.11629680701-001	-1.10612681247-001	4.81575442013-001	-1.29724016087+000
4	2.21010227498-002	-1.74883286518-001	9.21240956462-001	9.87740448172-001
5	-1.35124500984-001	-2.59829197126-001	4.62338324974-001	1.75597754812+000
6	-6.50246018187-002	-2.34100439197-002	-3.46876597603-001	-6.30258590513-001
7	4.92843881325-002	1.34369401035-001	-5.26102605959-001	-1.42227909244+000

j \ i	4	5	6	7
0	-1.71719805400+000	-3.51622420442+000	3.99887611047+000	1.70117466471+000
1	-9.09583675308-001	-9.27000057435+000	8.73287795350+000	1.17006035143+001
2	1.21504456271+001	4.14530576156+000	-3.07181128753+001	-6.21566179504-001
3	-2.08584552778+000	3.28206930018+000	1.39343388752+000	-2.14870096706+000
4	-4.55825559139+000	-3.53154886445-001	6.70878888715+000	-2.08038322972+000
5	-1.27674813112+000	-3.08016733910+000	2.11536264054+000	1.58313672158+000
6	1.67488395113+000	1.69154169450+000	-2.33429546056+000	-1.41777348517+000
7	1.94388059790+000	3.17833942398+000	-4.50697506090+000	-2.82364293850+000

j \ i	8	9	10
0	-2.10917502586+000	0.00000000000+000	0.00000000000+000
1	-1.35662431861+001	-4.78957183066+000	6.14955927899+000
2	3.06970847500+001	-1.01616754772+000	-1.08636268192+001
3	0.00000000000+000	0.00000000000+000	0.00000000000+000
4	-3.13322071371+000	1.53852849657+000	0.00000000000+000
5	-1.21037305813+000	0.00000000000+000	0.00000000000+000
6	1.05087059879+000	3.51166275844-001	0.00000000000+000
7	4.91016553820+000	9.30600236660-001	-1.88826998151+000

Table 10 NORMAL LEGENDRE COEFFICIENTS b_{ij}

OXYGEN

j \ i	0	1	2	3
0	3.35370477362-001	1.85071586259-001	2.09751228070-002	1.88937996427-001
1	-1.06247154837+000	9.79104316542-002	3.67167830906-002	1.16503260199-001
2	-3.81376480061-001	-2.79610738279-001	-1.36974464230-002	-6.17625047499-002
3	9.90547264895-002	-2.06756294478-001	-1.18517939920-001	4.59849475966-002
4	9.87178739255-002	3.24071284148-002	-2.04424503159-001	1.41367429724-002
5	-3.41400467835-002	1.98288564661-001	-9.54114407174-002	-4.52440496190-001
6	-7.34387851888-002	1.44825467291-001	2.42426167153-001	-5.04054495856-001
7	-3.14617860690-002	-7.66603269083-002	3.71353289613-001	1.62227645946-001

j \ i	4	5	6
0	-8.35270571440-002	-1.08570901774-001	3.74007723628-002
1	2.18643886658-001	-1.10051404381-001	-1.09312413580-001
2	3.43851570651-001	7.92949923351-002	-2.18116104162-001
3	-1.15534732674-003	8.20885452888-002	-5.39018263208-002
4	1.35757984759-002	-7.71117657259-003	4.54379671388-002
5	1.05710019042-001	2.23601373890-001	2.84792620754-002
6	-3.11326424355-001	3.35227289889-001	1.01120118192-001
7	-7.53441614690-001	-9.37861126462-002	3.7521581103 -001

Table II NORMAL LEGENDRE COEFFICIENTS b_{ij}

SILICON

j \ i	0	1	2	3
0	5.22462054030-001	7.65124013693-002	-1.68939193775-001	-4.98708555384-001
1	-9.01448525670-001	-4.65451027651-002	-1.27119158331-002	8.56861430372-002
2	-5.80513048824-001	-7.67928985787-002	4.83001593641-002	4.14426240823-001
3	-1.72562131801-001	3.91020394728-002	6.10062390637-002	1.67950396449-001
4	2.40181973518-002	5.89320374255-002	-1.57470074943-002	-1.78062182083-003
5	6.02660595857-002	-2.85896935235-002	1.59260580761-002	-9.00799205425-003
6	5.19110072705-002	-2.38001151465-002	-3.58919639023-002	-6.24451547305-002

j \ i	4	5	6
0	7.88661923908-001	6.70325888003-001	-8.86419619780-001
1	2.13981849040-002	-1.52791818403-001	7.78726239960-002
2	-3.40182993702-001	-6.49562890205-001	5.98613827770-001
3	-2.78242783528-001	-2.54519480357-001	3.38160261774-001
4	-6.35860644949-003	1.29769359988-002	-2.19898160610-003
5	-6.45362630966-002	6.38237376291-002	-8.75577729794-003
6	8.94145873542-002	9.19895269411-002	-1.12909557424-001

Table 12 NORMAL LEGENDRE COEFFICIENTS b_{ij}

CALCIUM

j \ i	0	1	2	3
0	7.47266800399-001	1.61220486583-001	-5.05591059705-001	3.25544583980-001
1	-5.55956868051-001	2.22767223185-001	-8.61305029313-001	4.09456231575-001
2	-5.81866411349-001	-2.60985675694-002	-2.61798939426-001	3.65295994232-001
3	-4.49863468643-001	-1.45211936000-001	3.72295613238-001	-3.95261936533-001
4	-2.94447125693-001	-1.14762467907-001	8.07424143091-001	-8.14226421935-001
5	-1.12766947987-001	-7.00219048902-002	9.39602681007-001	-7.47027808021-001
6	4.90799606603-002	-7.76404262408-002	4.68754734617-001	1.90784759083-001
7	1.07095060186-001	-1.10022044290-002	-1.91186454778-001	6.01389617215-001
8	1.16031372207-001	5.58181296054-002	-5.60228721484-001	4.40401038197-001

j \ i	4	5	6	7
0	1.86155285530-001	-3.68393480781-001	1.52431856175-002	1.00491833804-001
1	6.87708318650-001	-7.84330901691-001	-1.51753809959-001	3.06047731226-001
2	8.04315903796-001	-1.28511159843+000	-3.42150381158-001	7.27893704031-001
3	1.05409295139-001	5.94317202982-001	-1.41168496952-001	-2.46152949138-001
4	-3.66510691115-001	1.53925545453+000	-2.18461486519-001	-5.80767138983-001
5	-1.27116817593+000	1.78803967427+000	4.04177826504-001	-9.26060883471-001
6	-1.16537853039+000	-5.34694873458-002	6.45475837173-001	-5.01785539337-002
7	-2.69780057237-001	-1.10042587287+000	3.62688053305-001	4.72211057288-001
8	5.33838376356-001	-1.09144711337+000	-3.27096488759-002	5.72476013186-001

Table 13 NORMAL LEGENDRE COEFFICIENTS b_{ij}

IRON

j \ i	0	1	2	3
0	8.40255885231-001	1.26411432615-001	-3.04327547093-001	4.21033283580-001
1	-3.69321445133-001	2.26459347492-001	-7.99174293946-001	8.47868444110-001
2	-4.05689188612-001	1.21858716138-001	-6.62218767137-001	6.42328999038-001
3	-3.68872294131-001	2.34200425865-002	-3.91119839944-001	3.20832776875-001
4	-3.19087294703-001	1.46701197991-003	-5.79152336582-002	-1.44249586727+000
5	-2.48103982359-001	-7.43438899410-002	6.59643025377-001	-2.19997974862+000
6	-1.58233954105-001	-1.70373859276-001	1.18676973935+000	-1.55950682321+000
7	-6.07773368142-002	-2.16848675603-001	8.73372665333-001	6.49474293685-001

j \ i	4	5	6	7
0	-7.74259220164-001	2.93258558519-001	1.54476268036+000	-9.52074161203-001
1	2.48367543723-001	-2.41930893930-001	-9.36254432934-001	-1.43250096141+000
2	1.06665137304+000	-2.10294996569+000	-4.87092249911-001	1.50484340664+000
3	2.17817419252+000	-5.12906926980+000	-1.66547331886-001	6.27618644787+000
4	4.64103069095+000	1.31802031076+000	-1.15688311738+001	7.67828808236-001
5	1.54991841523+000	6.53985461804+000	-9.02723840291+000	-7.00496565413+000
6	-3.49760460400+000	7.71256462277+000	2.80651376136+000	-1.04150561367+001
7	-4.98378859545+000	1.5[illegible]68205341-001	1.07467323530+001	-1.49816731644+000

j \ i	8	9	10
0	-1.22077952463+000	5.04886833576-001	3.63176881744-001
1	2.60478381761+000	8.57870880861-001	-1.41726069692+000
2	7.01780079029-001	-3.59205249544-001	-4.43221499453-001
3	-3.96543002594+000	-3.67678286931+000	2.45704759331+000
4	1.07894814322+001	-6.54543426954-001	-3.72953458369+000
5	1.00857927382+001	2.69514144915+000	-4.12437288543+000
6	7.02475319022-001	4.39042106050+000	-1.03119563277+000
7	-9.99590804709+000	8.98700872905-001	3.43652030775+000

Table 14 NORMAL LEGENDRE COEFFICIENTS b_{ij}

LEAD

j \ i	0	1	2	3
0	8.49084747195-001	6.85971275560-002	1.25927916866-001	2.44954139293-002
1	-3.74188789102-001	6.19285085859-002	-4.38710241861-001	1.76464212957+000
2	-4.65839348813-001	-3.69211586883-002	-2.27372826397-001	3.17455932633+000
3	-4.15466878216-001	-6.15370937410-002	-5.74703725292-001	1.93059943593+000
4	-3.19530261120-001	7.98274309213-002	5.22765643786-001	-2.84057956905+000
5	-1.97440078505-001	1.10630342871-001	6.28682178094-001	-4.29525694014+000
6	-9.77691056459-002	-8.72715248963-002	6.78567305693-001	-7.98989690440-001
7	-3.29769049646-002	-2.20078058989-002	5.73859547587-002	-1.31757324590-001
8	-1.00000000000-002	0.00000000000+000	0.00000000000+000	0.00000000000+000
9	-1.07711311510-003	3.53759775615-002	-9.93429086855-002	-2.71595353170-001

j \ i	4	5	6	7
0	-3.10675105422+000	4.12506579131+000	4.37200112054+000	-7.64290955646+000
1	-1.37559359632+000	-2.20435439558-001	4.59716101424-001	-4.57144248024+000
2	-3.48298531943+000	-1.27925833352+001	1.81348183401+001	1.50439243999+001
3	4.73610592345+000	-1.40018342792+001	5.13289277708+000	2.26194363191+001
4	1.12185864044+000	6.04213270381+000	-3.69680814062+000	-3.35127039409+000
5	1.01832417194+000	1.61948275267+001	-9.80872246515+000	-2.09568251869+001
6	-1.91226878005+000	3.53451119455+000	2.11898631710+000	-4.49071976442+000
7	1.55969170716-001	-9.23544498937-003	-2.04426284516-001	2.01432767918-001
8	0.00000000000+000	0.00000000000+000	0.00000000000+000	0.00000000000+000
9	7.81712624445-001	1.08007733187-001	-1.52968764626+000	5.73690037485-001

j \ i	8	9	10
0	-1.92189801704+000	3.63858087309+000	0.00000000000+000
1	3.06878676518+000	3.00782056613+000	-2.30790402168+000
2	-2.49100713465+001	-5.49632436672+000	1.04522643326+001
3	-5.68579697268-001	-1.05568561578+001	1.80859492875+000
4	2.42539970329+000	0.00000000000+000	0.00000000000+000
5	1.54240453992+001	8.96781494046+000	-6.97815900783+000
6	-7.97566059833-001	1.90057056135+000	0.00000000000+000
7	0.00000000000+000	0.00000000000+000	0.00000000000+000
8	0.00000000000+000	0.00000000000+000	0.00000000000+000
9	8.24826400693-001	-4.71070687343-001	0.00000000000+000

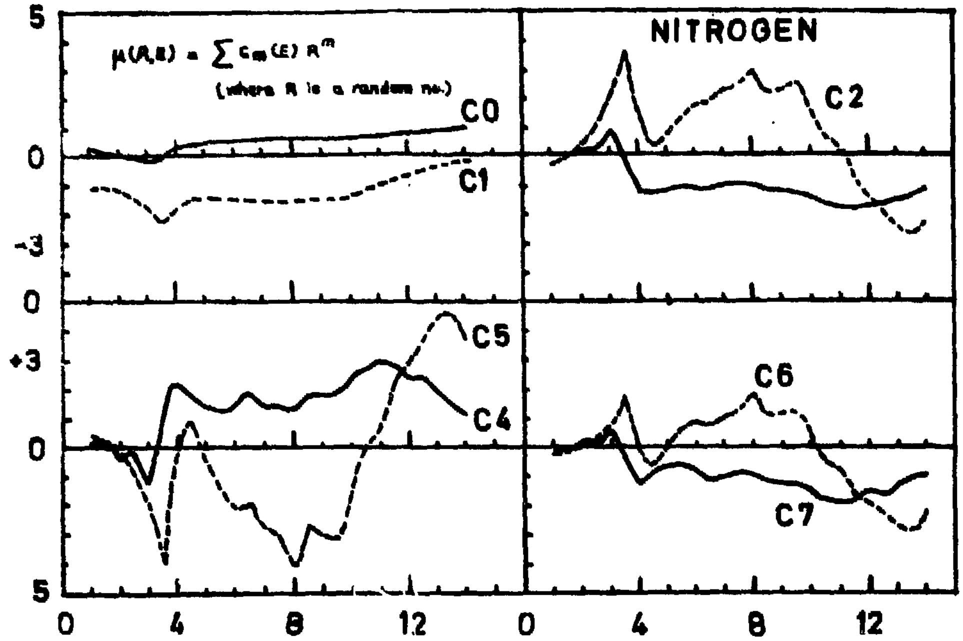


Fig.1a NEUTRON ENERGY (MeV)

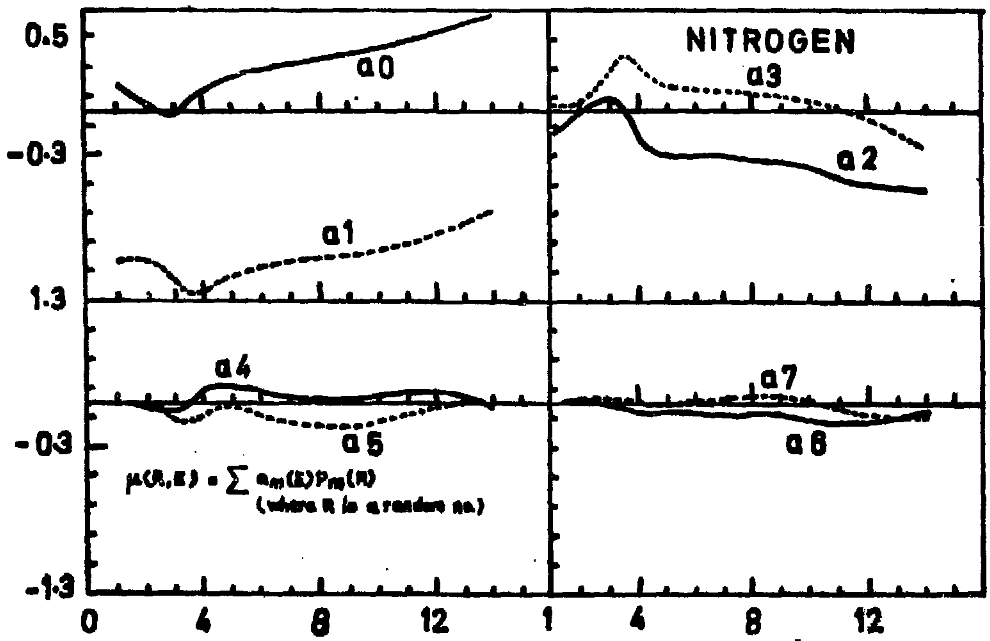


Fig.1b NEUTRON ENERGY (MeV)

Fig.1 Behaviour of the coefficients

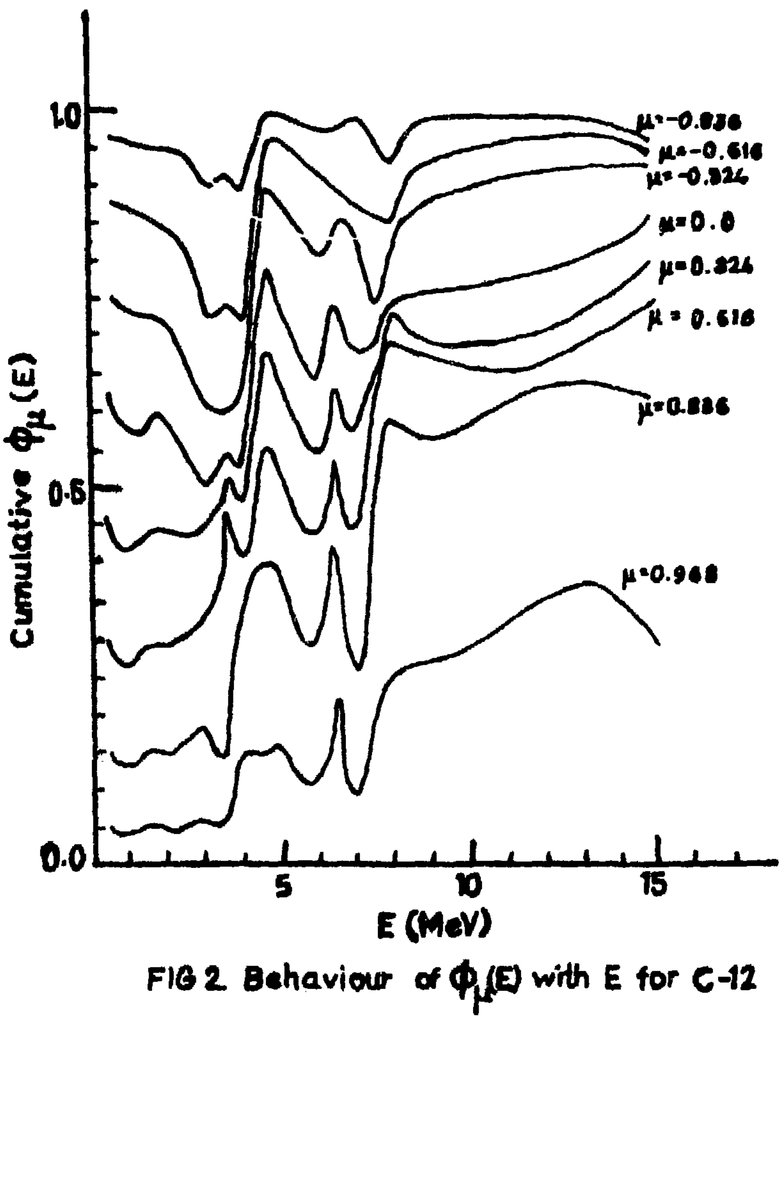


FIG 2. Behaviour of $\Phi_\mu(E)$ with E for C-12

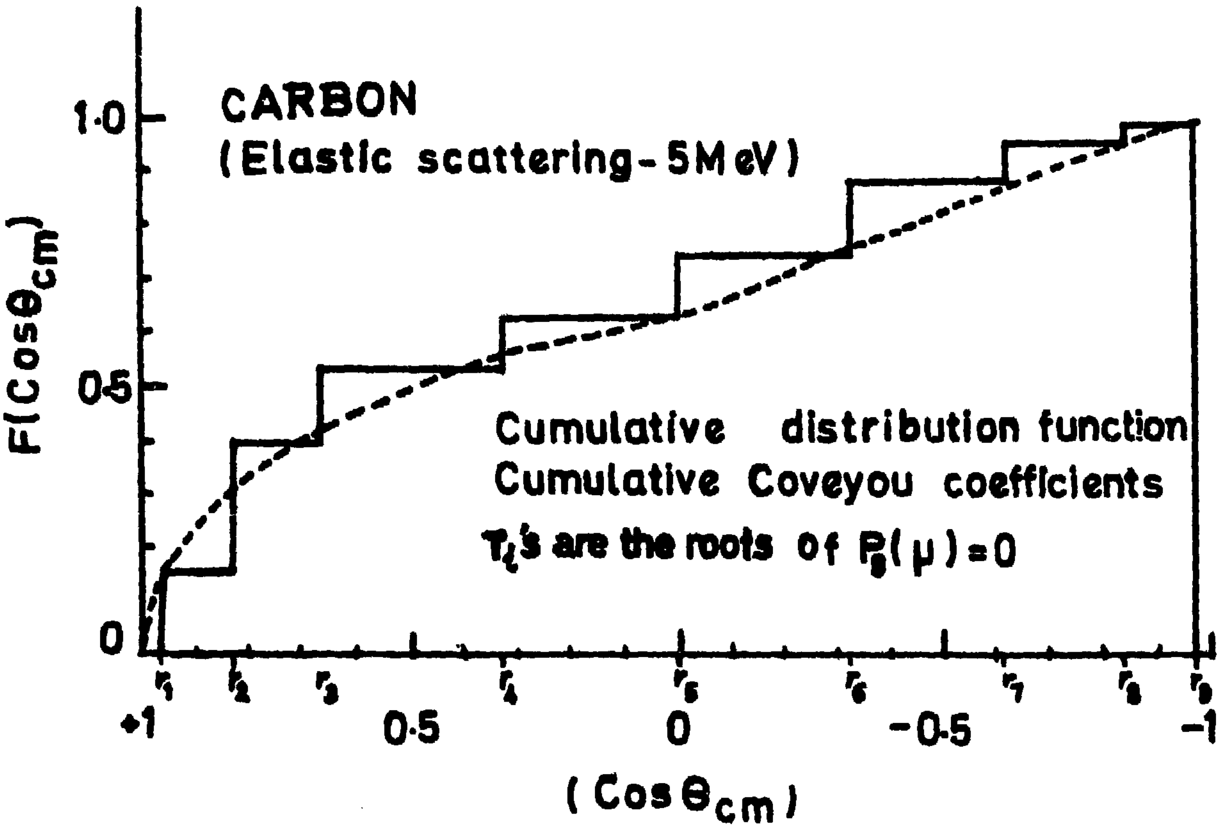


Fig. 3 Cumulative Coveyou coefficients and F(μ) for Carbon at incident neutron energy of 5 MeV

Figures 4-19 : Comparison of sampled data with the original data for C, N, O, Si, Ca, Fe and Pb

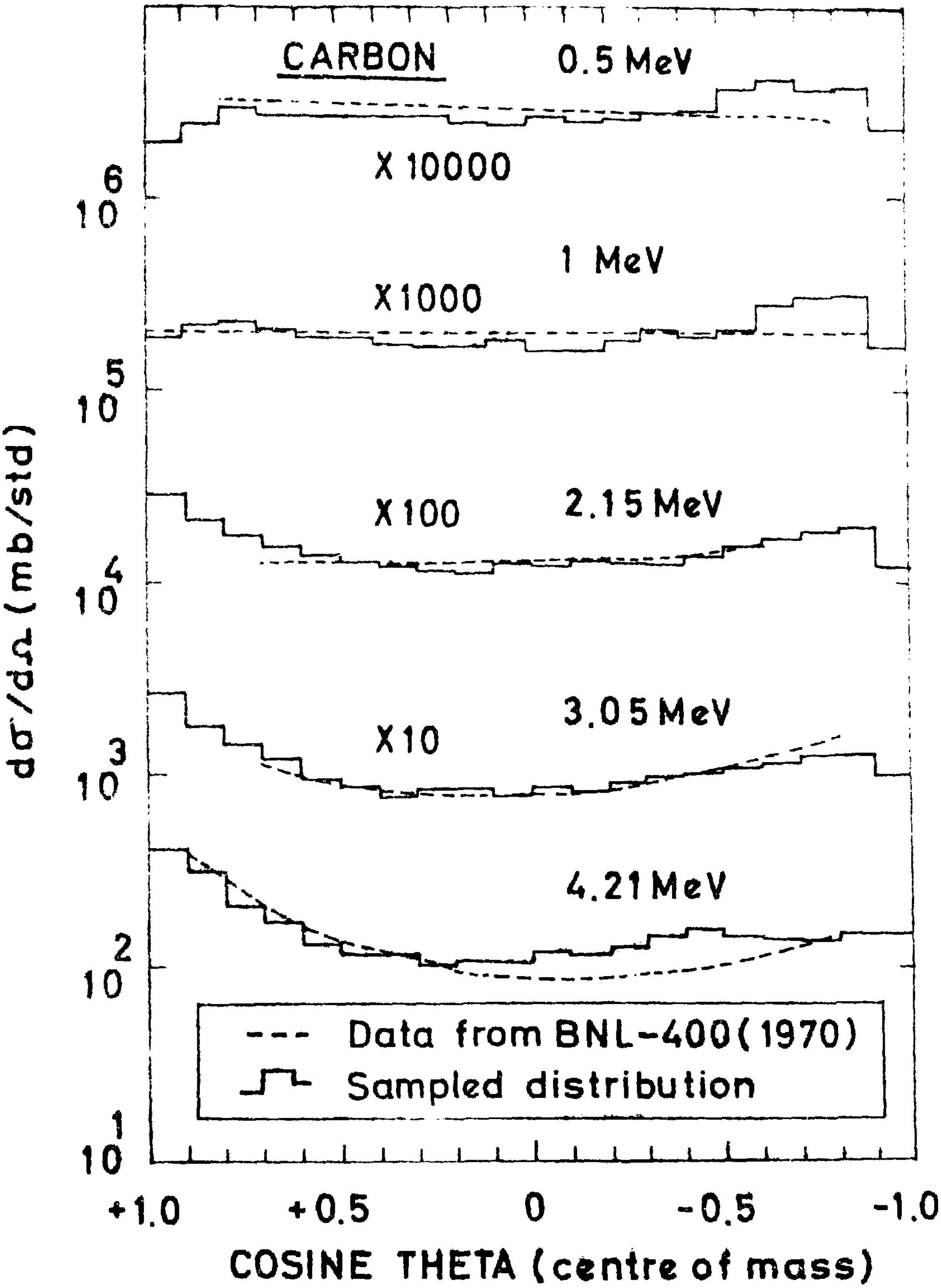

CARBON
0.5 MeV
X 10000
1 MeV
X1000
X100
2.15 MeV
X10
3.05 MeV
4.21 MeV
Data from BNL-400(1970)
Sampled distribution
dσ/dΩ (mb/std)
COSINE THETA (centre of mass)
+1.0
+0.5
0
-0.5
-1.0

Fig.4

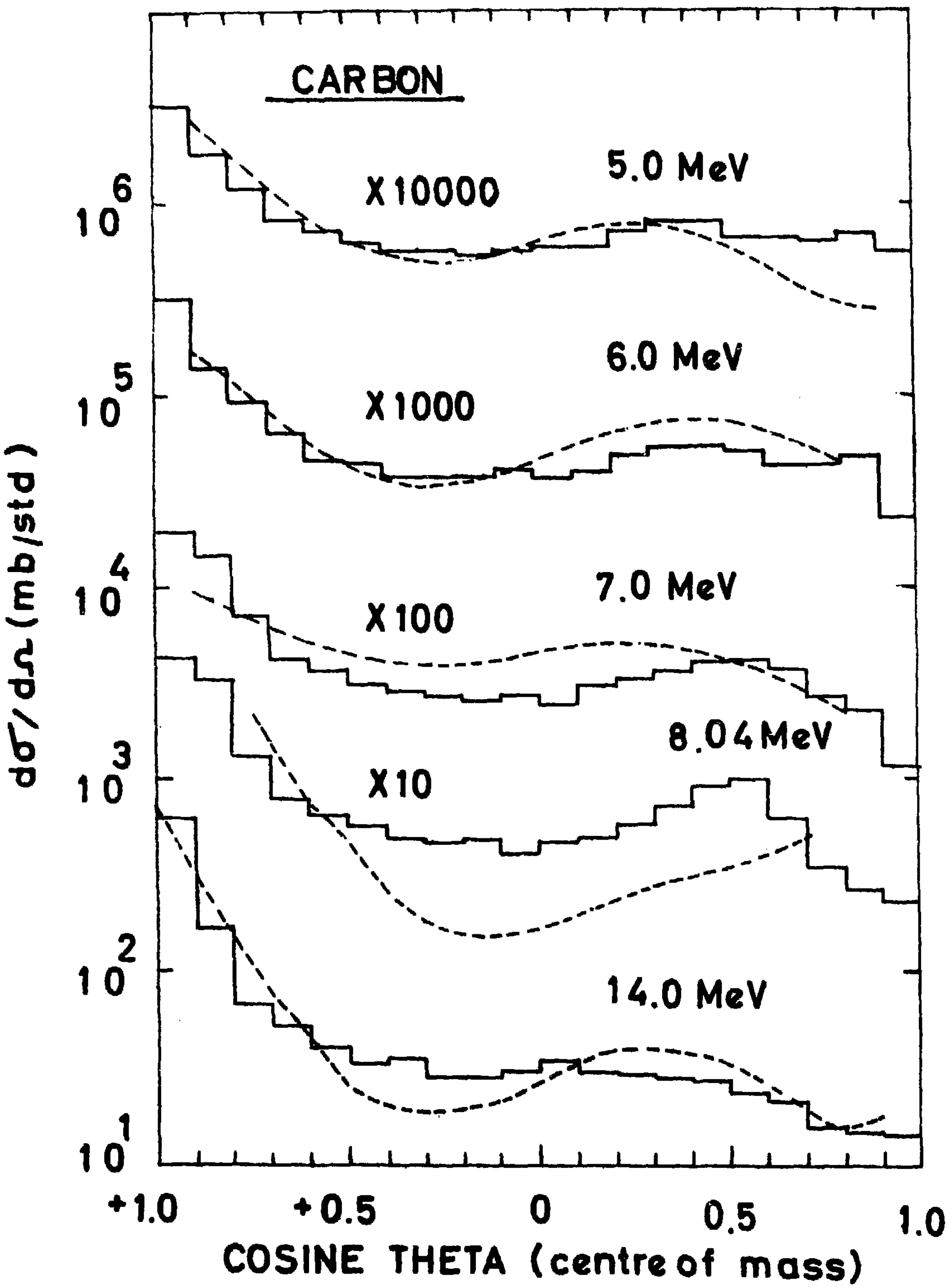

CARBON
5.0 MeV
X10000
6.0 MeV
X1000
7.0 MeV
X100
8.04 MeV
X10
14.0 MeV
10^6
10^5
10^4
10^3
10^2
10^1
dσ/dΩ (mb/std)
+1.0
+0.5
0
0.5
1.0
COSINE THETA (centre of mass)

Fig.5

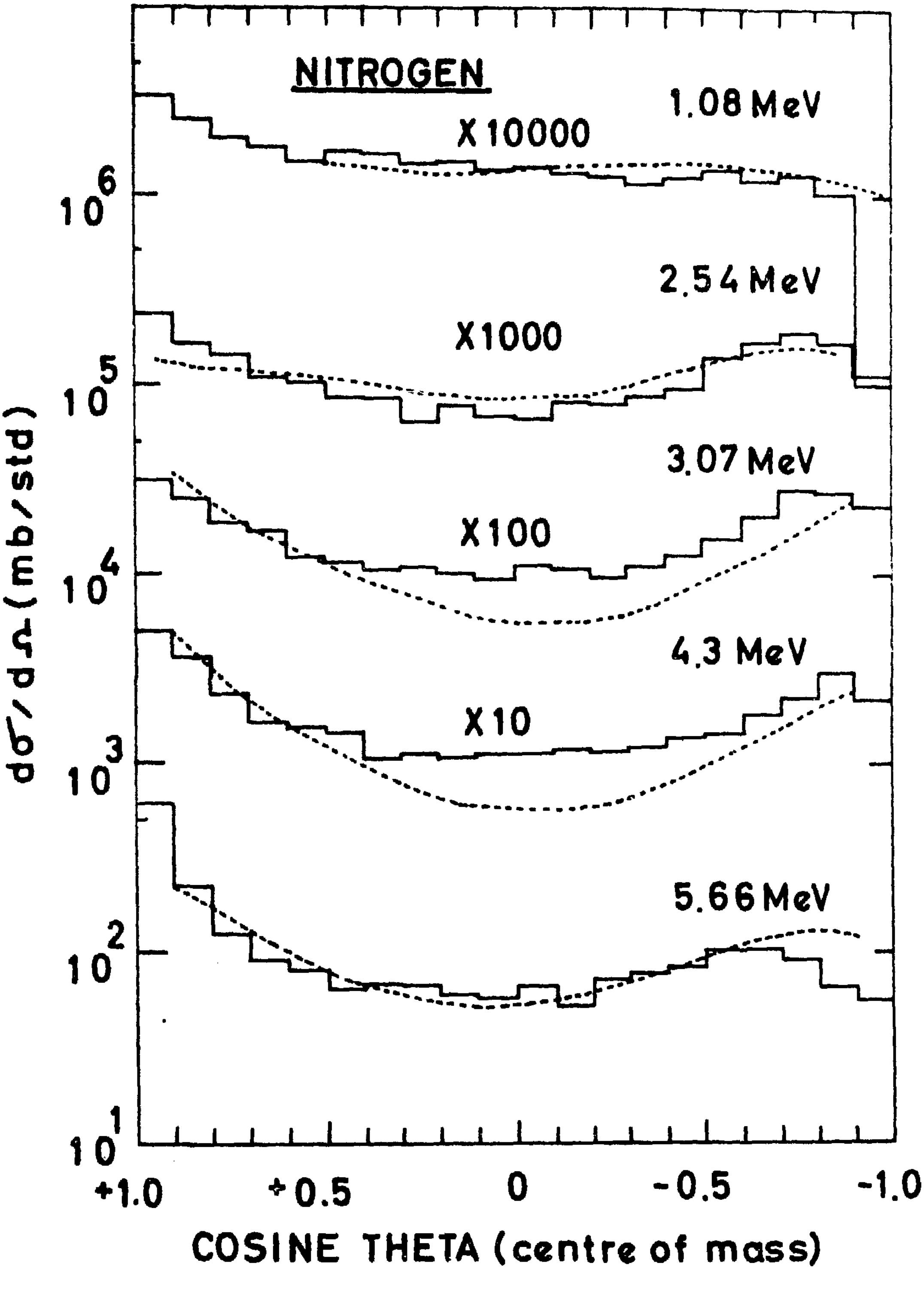

NITROGEN
1.08 MeV
X 10000
2.54 MeV
X1000
3.07 MeV
X100
4.3 MeV
X10
5.66 MeV
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 6

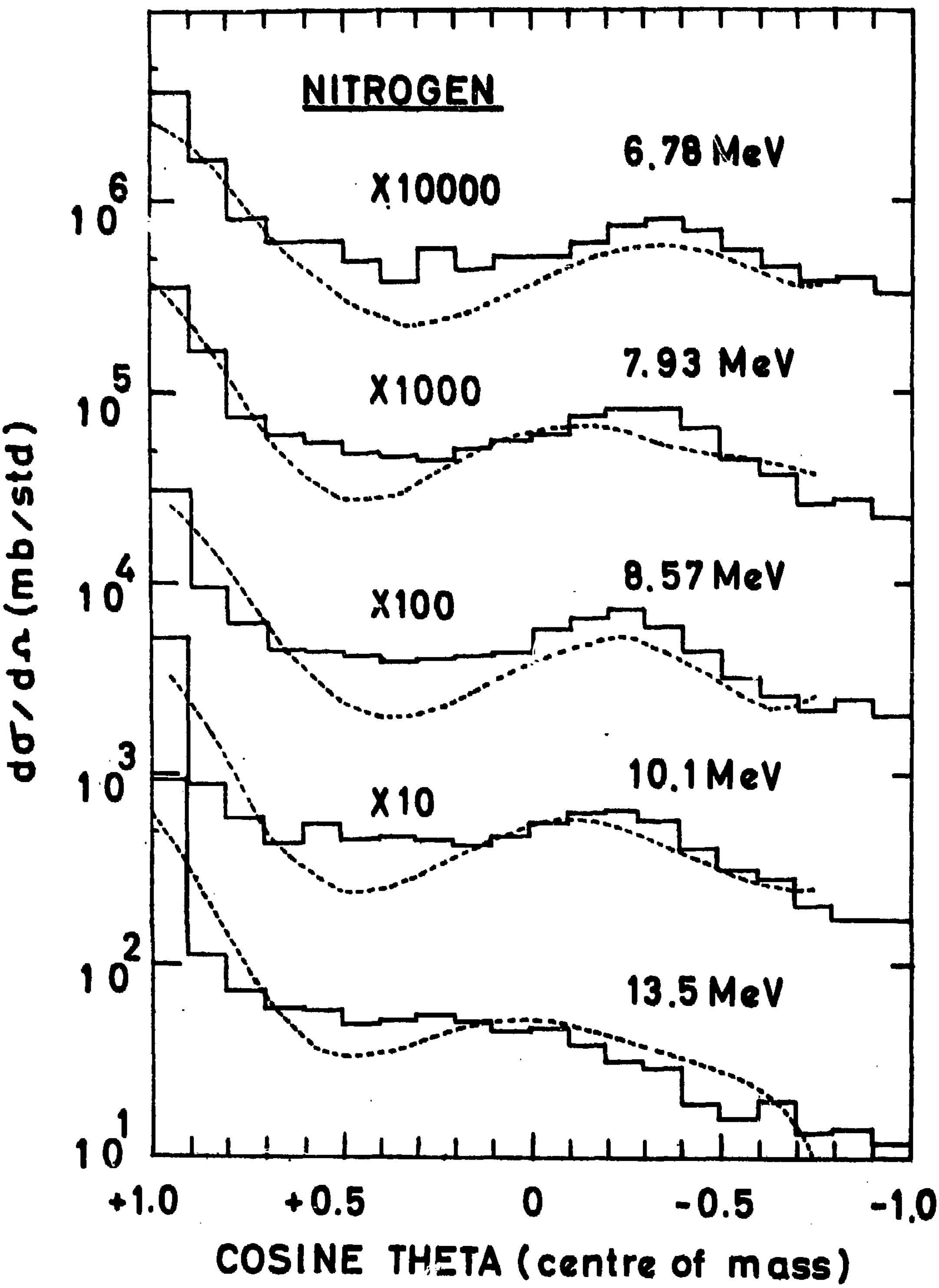

NITROGEN
6.78 MeV
X10000
7.93 MeV
X1000
8.57 MeV
X100
10.1 MeV
X10
13.5 MeV
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 7

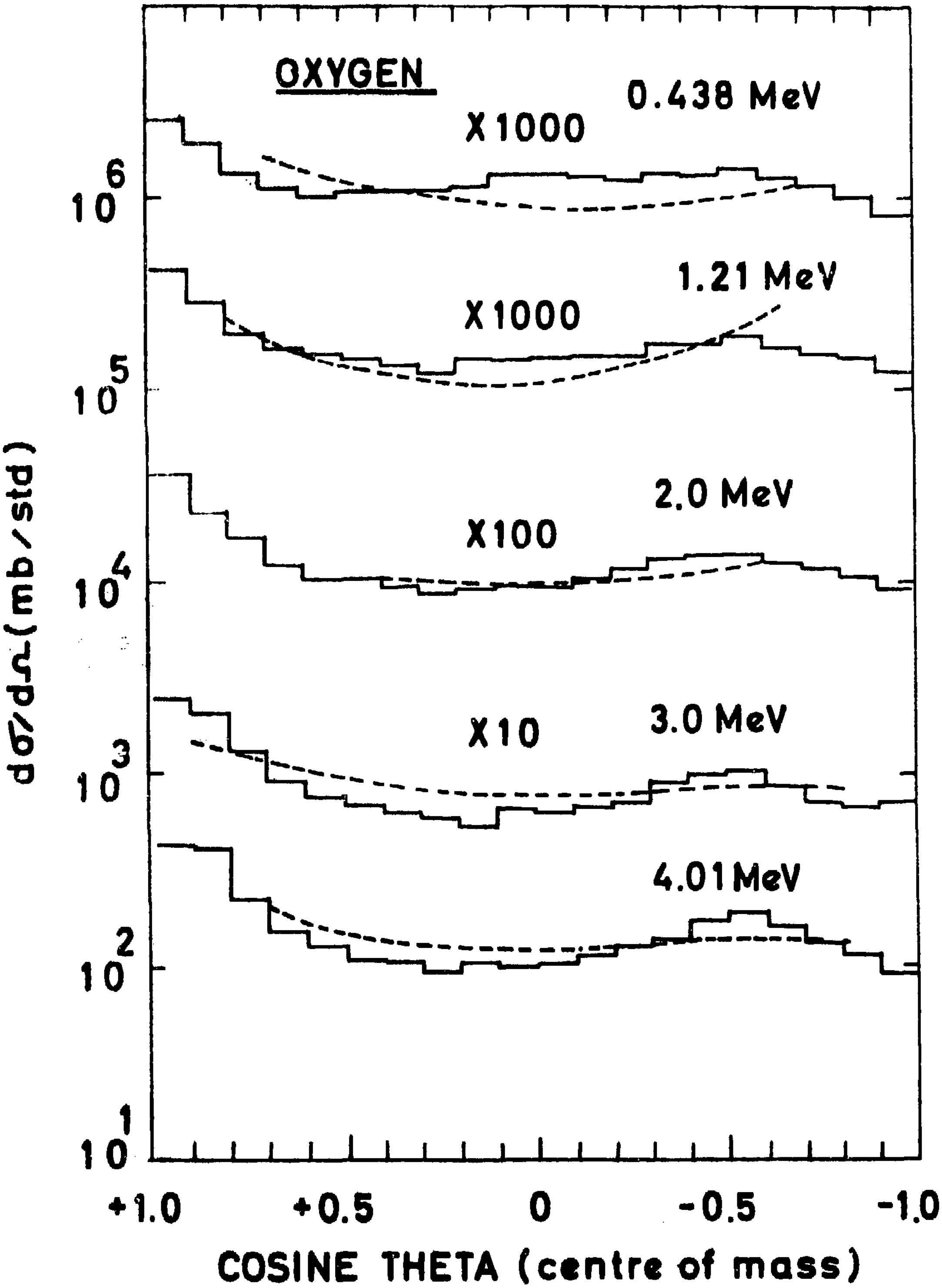

OXYGEN
0.438 MeV
X1000
1.21 MeV
X1000
2.0 MeV
X100
3.0 MeV
X10
4.01 MeV
10^6
10^5
10^4
10^3
10^2
10^1
$d\sigma/d\Omega$ (mb/std)
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 8

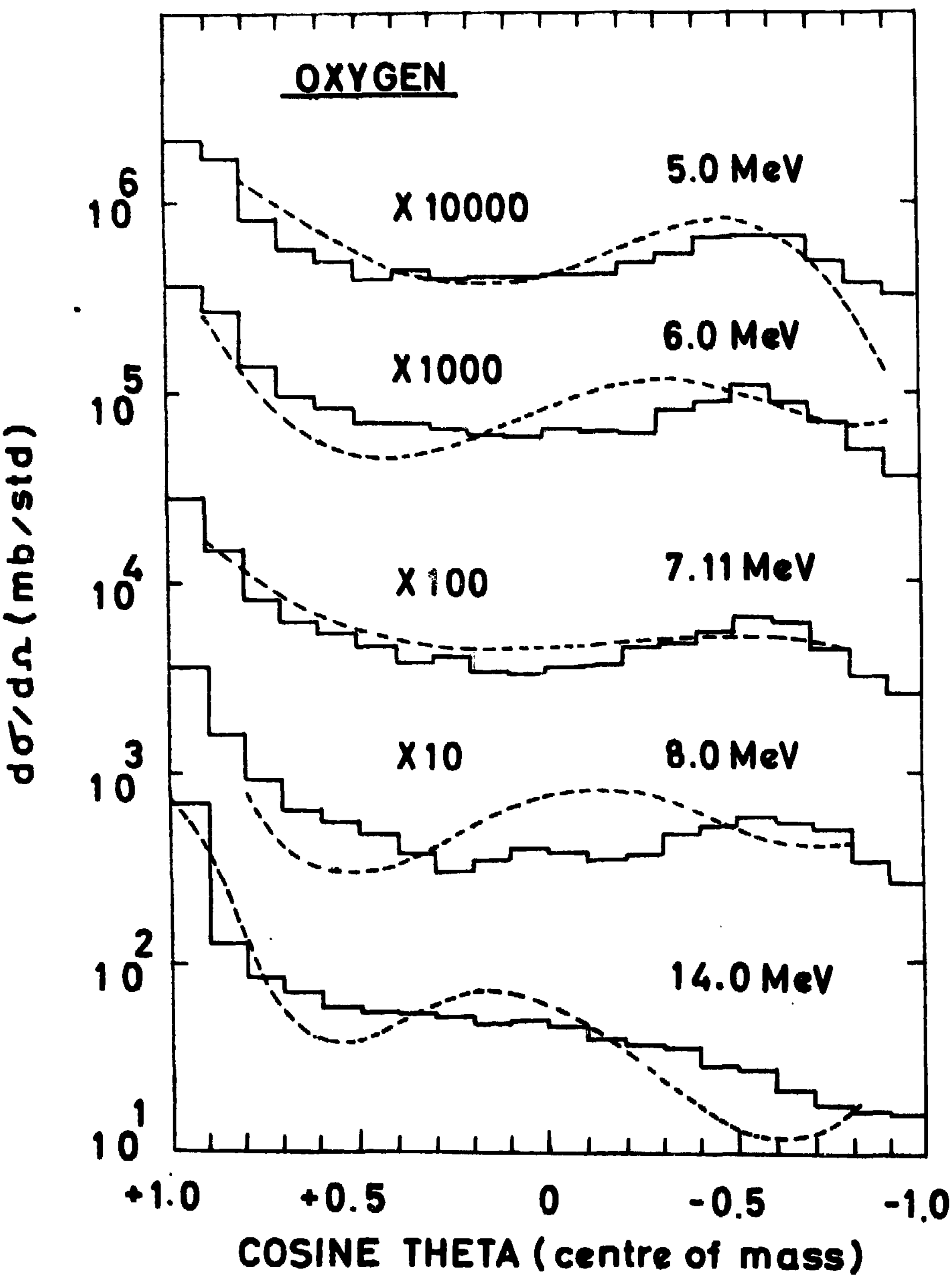

OXYGEN
5.0 MeV
X 10000
6.0 MeV
X 1000
7.11 MeV
X 100
8.0 MeV
X 10
14.0 MeV
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 9

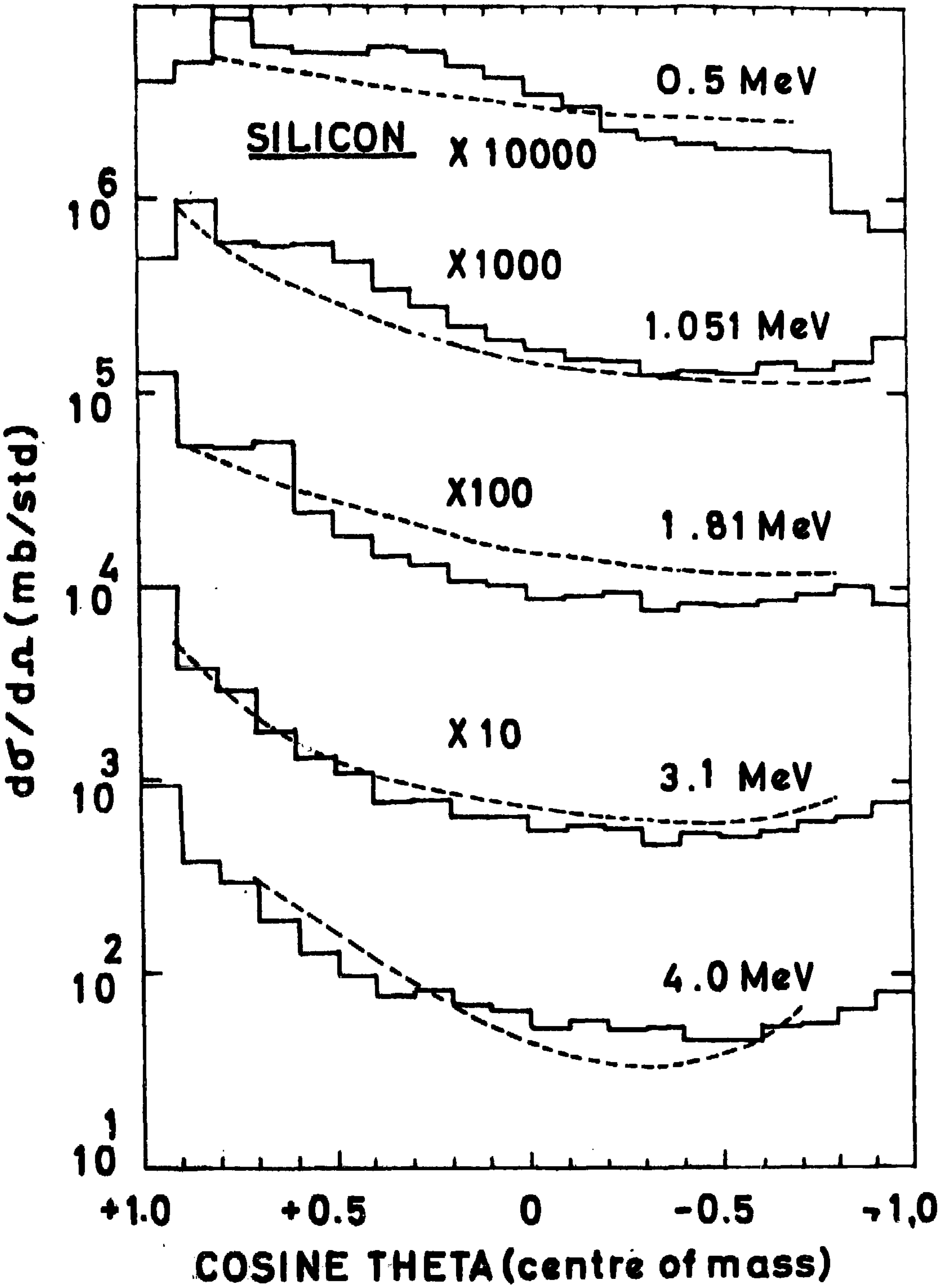

SILICON
0.5 MeV
X 10000
X1000
1.051 MeV
X100
1.81 MeV
X 10
3.1 MeV
4.0 MeV
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+1.0
+0.5
0
-0.5
-1,0
COSINE THETA (centre of mass)

Fig.10

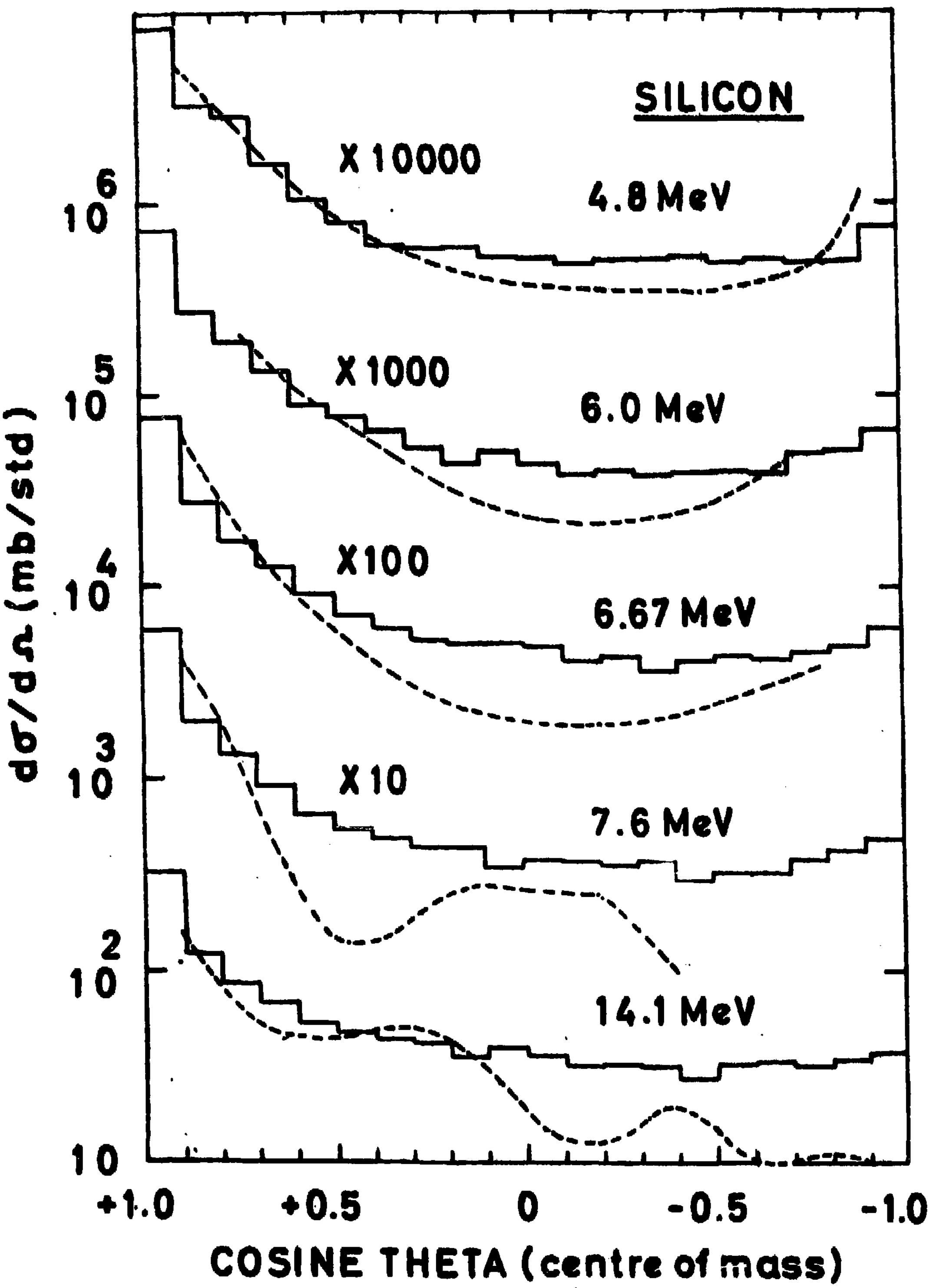
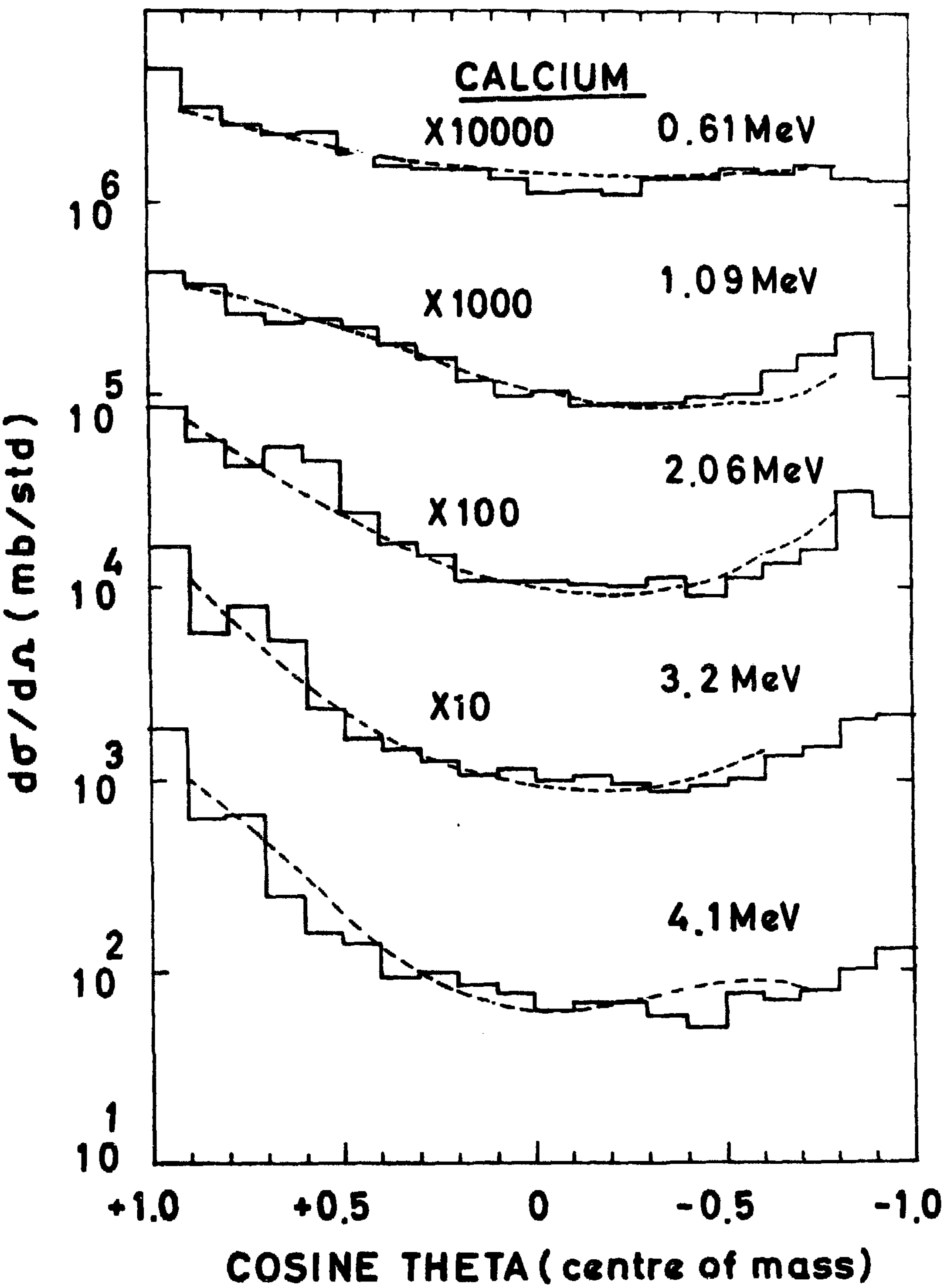

SILICON
X 10000
4.8 MeV
X 1000
6.0 MeV
X100
6.67 MeV
X 10
7.6 MeV
14.1 MeV
10^6
10^5
10^4
10^3
10^2
10
dσ/dΩ (mb/std)
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig.11



CALCIUM
X10000
0.61 MeV
1.09 MeV
X1000
2.06 MeV
X100
3.2 MeV
X10
4.1 MeV
10^6
10^5
10^4
10^3
10^2
10^1
dσ/dΩ (mb/std)
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig.12

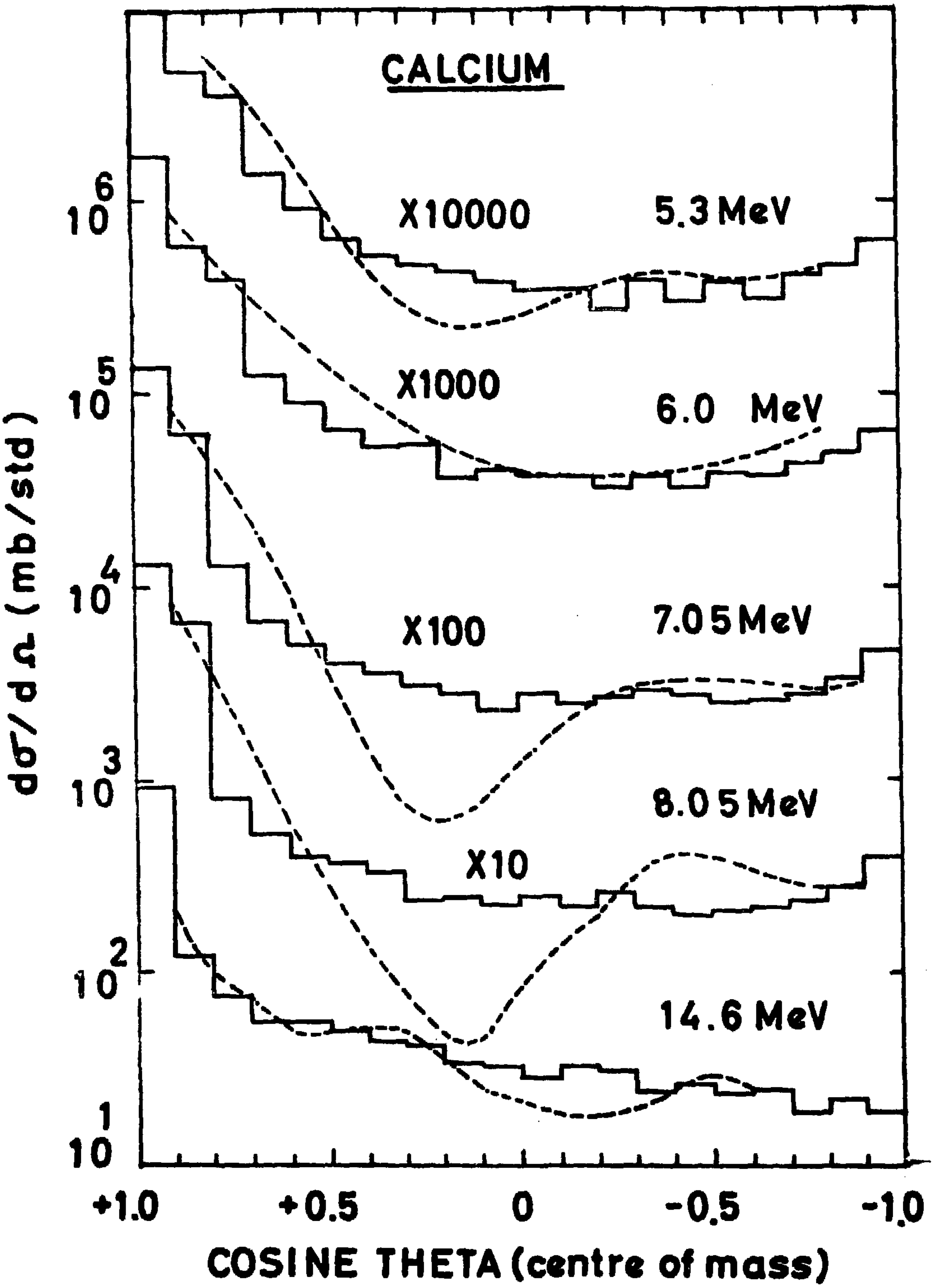
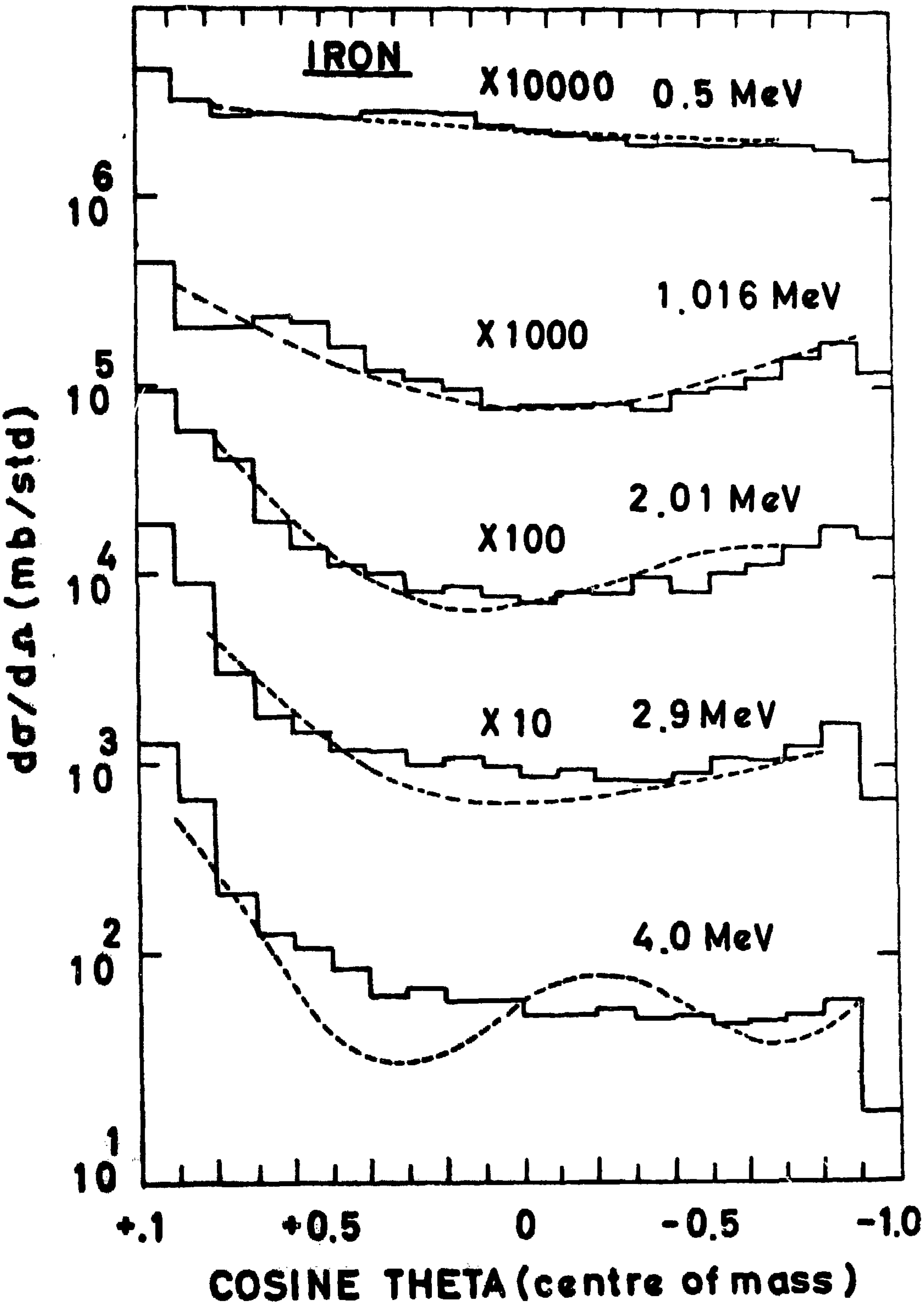
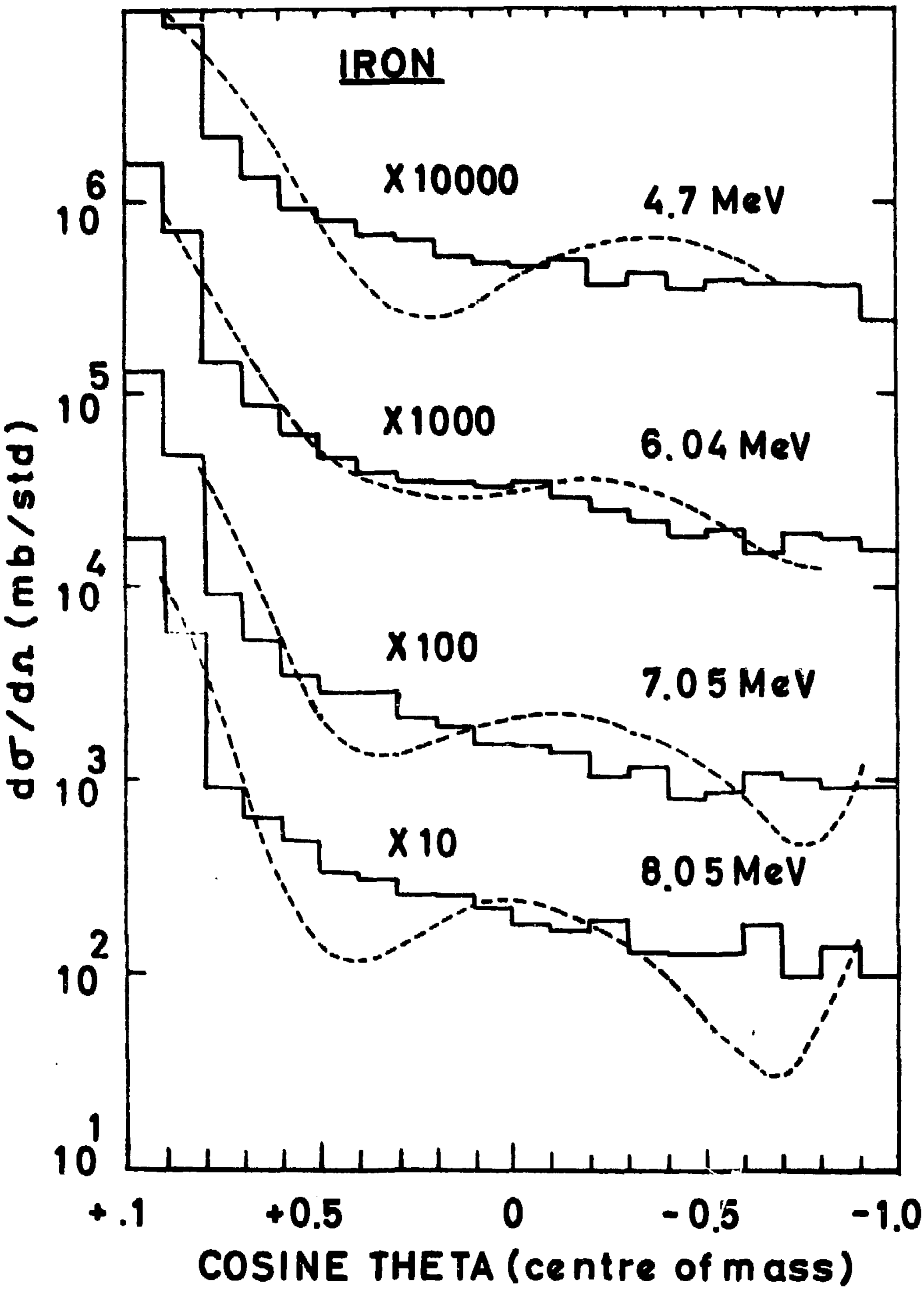

CALCIUM
X10000
5.3 MeV
X1000
6.0 MeV
X100
7.05 MeV
8.05 MeV
X10
14.6 MeV
10^6
10^5
10^4
10^3
10^2
10^1
10
dσ/dΩ (mb/std)
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 13



IRON
X10000
0.5 MeV
1.016 MeV
X1000
2.01 MeV
X100
X10
2.9 MeV
4.0 MeV
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+.1
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 14



IRON
X 10000
4.7 MeV
X 1000
6.04 MeV
X 100
7.05 MeV
X 10
8.05 MeV
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+.1
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 15

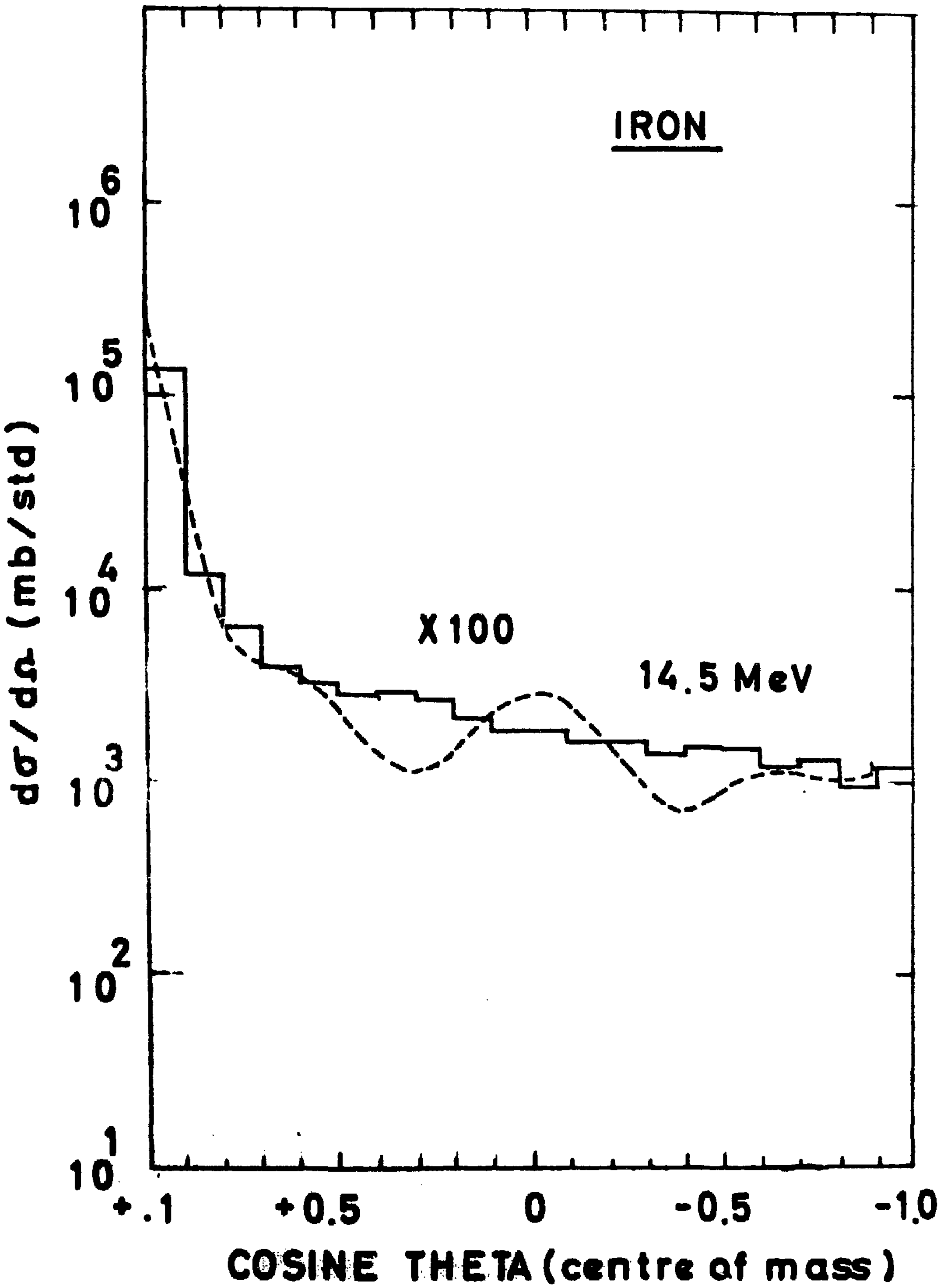

IRON
10^6
10^5
10^4
10^3
10^2
10^1
dσ/dΩ (mb/std)
X100
14.5 MeV
+.1
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 16

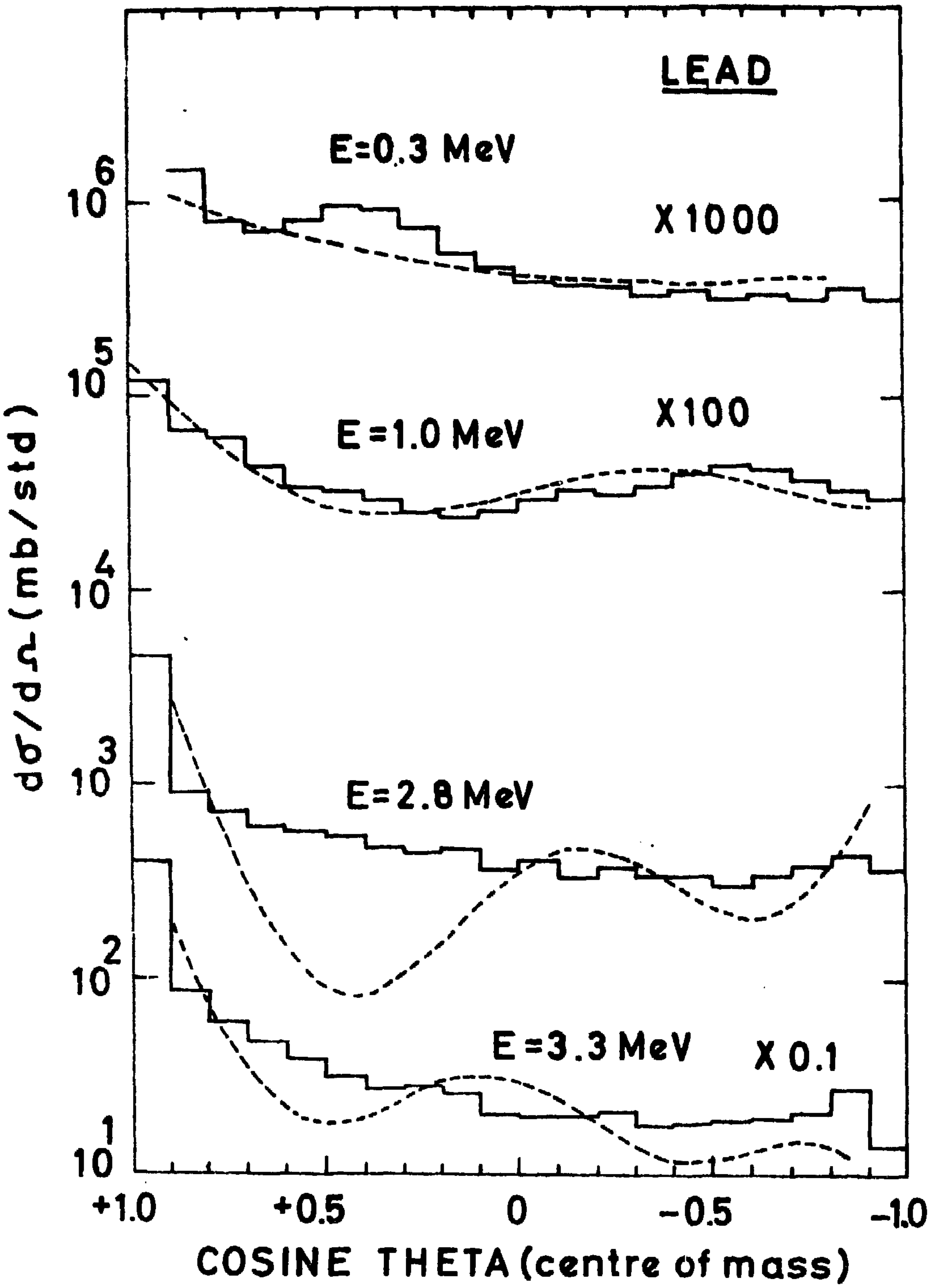
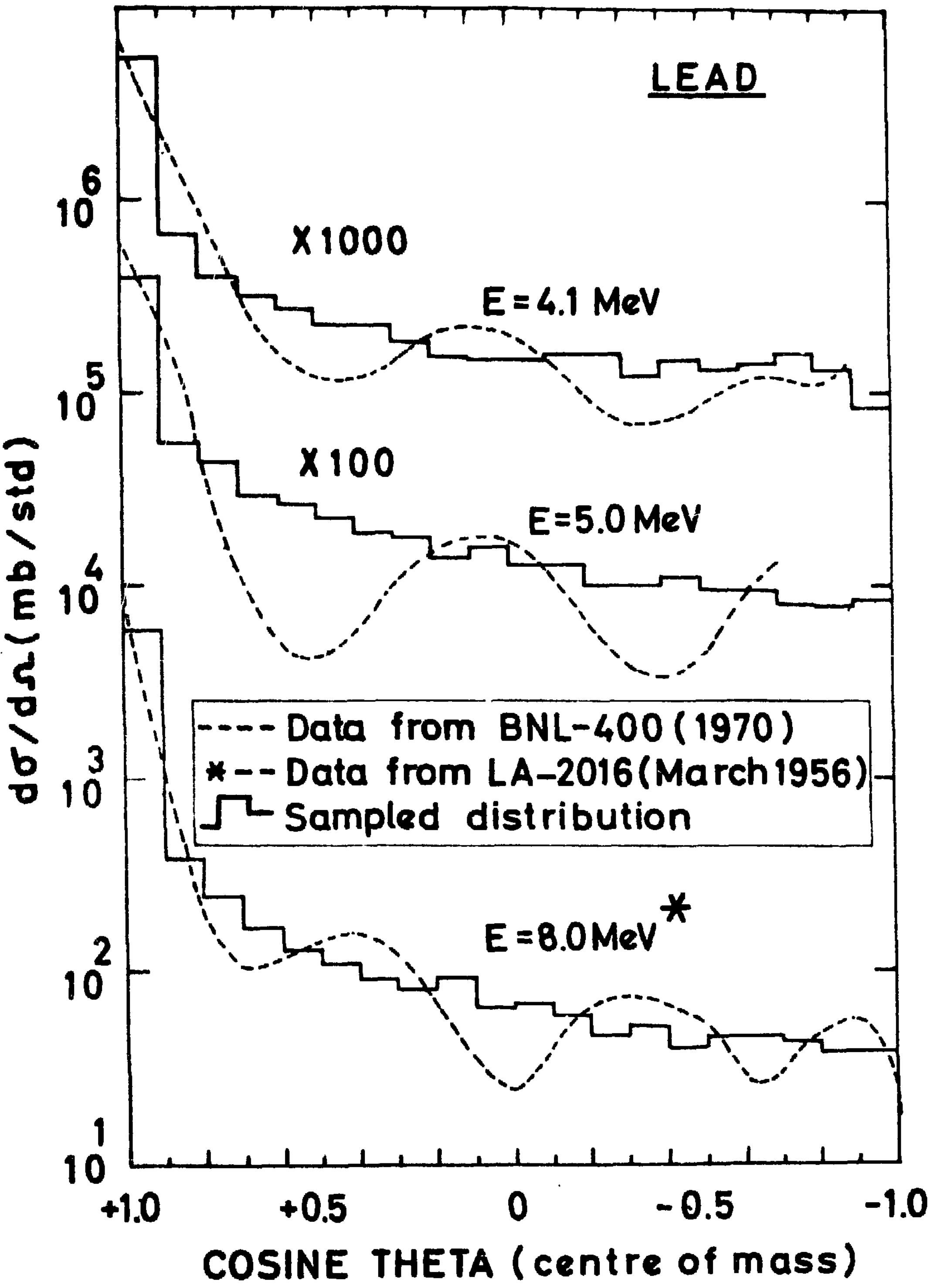

LEAD
E=0.3 MeV
X 1000
E=1.0 MeV
X100
E=2.8 MeV
E=3.3 MeV
X 0.1
10^6
10^5
10^4
10^3
10^2
10^1
dσ/dΩ (mb/std)
+1.0
+0.5
0
−0.5
−1.0
COSINE THETA (centre of mass)

Fig. 17



LEAD
X1000
E=4.1 MeV
X100
E=5.0 MeV
E=8.0 MeV*
----- Data from BNL-400 (1970)
*--- Data from LA-2016 (March 1956)
Sampled distribution
dσ/dΩ (mb/std)
10^6
10^5
10^4
10^3
10^2
10^1
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 18

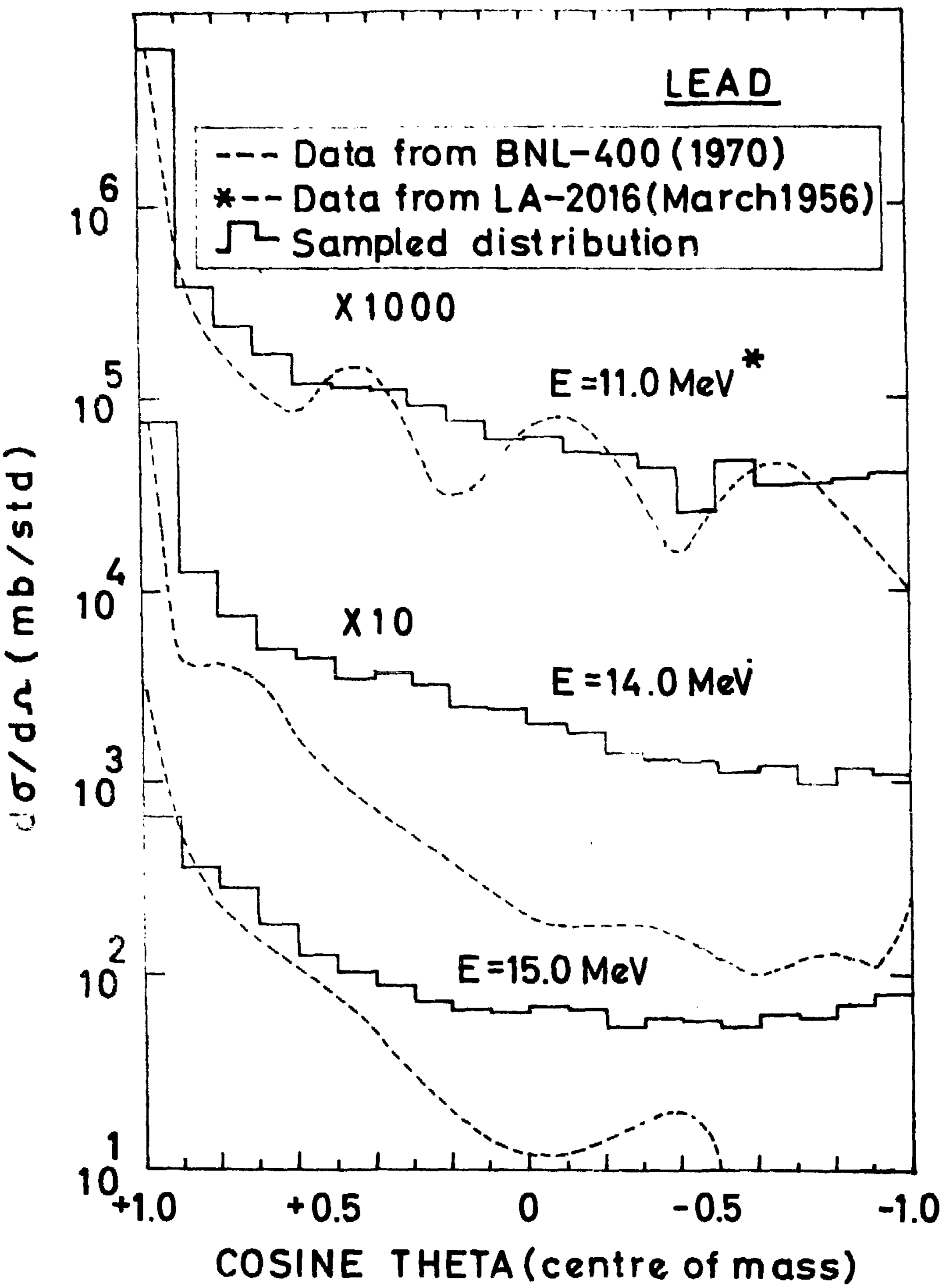

LEAD
--- Data from BNL-400 (1970)
*-- Data from LA-2016 (March 1956)
Sampled distribution
X 1000
E = 11.0 MeV*
X 10
E = 14.0 MeV
E = 15.0 MeV
10^6
10^5
10^4
10^3
10^2
10^1
dσ/dΩ (mb/std)
+1.0
+0.5
0
-0.5
-1.0
COSINE THETA (centre of mass)

Fig. 19