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THEORY OF AURORA FORMATION

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Abstract

A new theory of aurora formation is presented based on Alfvén wave-electron interation. The theory explains consistently

1) the electron acceleration process,

2) the formation of auroral layers and

3) the long wave formation in the longitudinal direction.

The aurora[1] is formed by precipitating electrons with energy of one to ten keV. It has a thin thickness (a few km) in the north south direction and stretched in the east west direction beyond a thousand kilometers. It appears often in many layers in the north south direction and propagates to the north. The each layer has a curtain shape with a wavelength of a few km as well as of a hundred km in the east-west direction.

There has been no theory that explains all of these features consistently. For example, anomalous resistivity[2] is assumed to explain the acceleration mechanism of the electrons. However it is unlikely that such a localized phenomena (existing at the top-side ionoshere) can explain the gross feature of the aurora. Nonlinear relaxation phenomena[3,4] at the ionosphere are assumed to explain the formation of the layers. Again, the approach seems too microscopic in dealing with the phenomena involving the entire magnetosphere.

The present theory bases entirely on the magnetospheric dynamics and explains all of the observed features of the aurora in a consistent way.

When the geomagnetic tail collapses as a substorm starts, the disturbance propagates towards the earth. Such a disturbance, when it hits the inner edge of the plasma sheet (Fig.1), will excite a strong oscillation in the earth magnetic field in this region. This oscillation may appear in a form of a surface wave propagating in the east-west direction along the surface of the inner edge of the plasma sheet. The surface wave accompanies the modified Alfvén wave[6] (by finite ion Larmor radius) which propagates toward the tail. This Alfvén wave has a parallel electric field and wavelength on the order of an ion Larmor radius. The present theory claims that the aurora forming electrons are accelerated by this wave.

Taking the finite ion Larmor radius and electron inertia into account, we obtain the wave equation in a non-uniform plasma. x,y,z coordinates represent earth-tail, west-east and geomagnetic field directions near the inner edge of the plasma sheet (Fig.2). Assuming the Fourier amplitude of the form $\phi(x)e^{i(k_y y+k_z z-\omega t)}$, and using Vlasov-Maxwell equations, the wave equation becomes[6], for $\omega \sim k_\parallel v_A$ (v_A: Alfvén speed) $\ll \omega_{ci}$,

$$\frac{\omega_{pi}^2}{\omega_{ci}^2}\left\{\frac{d}{dx}\left[1+\frac{3}{4}\rho_i^2\frac{d^2}{dx^2}\right]\left(g\frac{d\phi}{dx}\right)-k_y^2\phi\right\}-\frac{\omega_{pe}^2}{v_{Te}^2}\psi=0 \qquad (1)$$

(quasi-neutrality condition)

and

$$\left(\frac{d^2}{dx^2}-k_y^2\right)(\phi-\psi)=\frac{\omega^2}{c^2}\frac{\omega_{pe}^2}{k_z^2 v_{Te}^2}g\psi \qquad (2)$$

(Ampére's Law in the z direction).

In these expressions, ω_p is the plasma frequency corresponding to the maximum density n_{max}, ω_{ci}, ρ_i and v_{Te} are the ion cyclotron frequency, ion Larmor radius and the electron thermal speed. $g = n_0(x)/n_{max}$ is the local density normalized by the maximum density. The three components of the electric field are given by[7]

$$E_x=-\frac{\partial\phi}{\partial x},\quad E_y=-ik_y\phi,\quad E_z=-ik_z\psi, \qquad (3)$$

while the magnetic fields are given by

$$i\omega \underset{\sim}{B} = \underset{\sim}{\nabla} \times \underset{\sim}{E} \,. \tag{4}$$

In the limit of a sharp density gradient and the zero temperature for both electrons and ions, this wave equation gives that of the surface wave,

$$\left(\frac{d^2}{dx^2} - k_y{}^2\right)\phi = 0 \tag{5}$$

and the corresponding dispersion relation is given by[5]

$$\omega = k_z \left[\frac{B_I{}^2 + B_{II}{}^2}{\mu_0 m_i (n_I + n_{II})}\right]^{1/2}. \tag{6}$$

At the inner edge of the plasma sheet there is no jump in the magnetic field across the surface hence $B_I = B_{II}$, while there exists a density jump $n_I \sim 0$, $n_{II} = m_{max}$ (= plasma sheet density). This gives the eigen frequency of the surface mode given by

$$\omega_s = \sqrt{2}\, k_z \;\; v_A \tag{7}$$

where v_A is the Alfvén speed in the plasma sheet while the wave number k_z is given by π/L where L is the thickness of the plasma sheet (Fig.1) ω_s as calculated using (1) is $\sim 1\ \mathrm{sec}^{-1}$, for $L \sim 2R_E$, $B_0 \sim 100\,\gamma$ and $n_0 \sim 1\ \mathrm{cm}^{-3}$.

Now, when the finite temperature and a smooth density gradient are assumed, the wave equation becomes forth order and represents coupling of the surface wave to another wave whose dispersion relation (in a uniform plasma) is given by[8]

$$\omega^2 = k_z^2 v_A^2 [1 + (\frac{3}{4} + \frac{T_e}{T_i}) k_x^2 \ \rho_i^2] \quad . \qquad (8)$$

This wave is essentially the Alfvén wave but is modified by the finite ion Larmor radius and the finite electron enertia. When $k_x \sim \rho_i^{-1}$, electrons and ions can no longer move together and consequently small charge separation is set up. This small charge separation allows the wave to propagate across the magnetic field and produces a parallel electric field E_z. Because of the finite thickness of the plasma sheet, this parallel electric field is localized near the equatorial region and depending its phase, the electric field will accelerate (or decelerate) electrons along the magnetic field.

A synchronous acceleration occurs for those electrons whose bounce frequency ω_b in the magnetic mirror coresponds to the wave frequency given by (7). For an equatorial distance R_0 of the field line, the bounce frequency can be expressed using the perpendicular speed at the equator $v_\perp$ by

$$\omega_b = \frac{3v_\perp}{R_0} \qquad (9)$$

which is independent of $v_\parallel$.

If we calculate R_0 for the synchronous condition of $\omega_s = \omega_b$ for typical electron energy of the plasma sheet (~ 1 keV), R_0 becomes 7.5 R_E which just corresponds to the equatorial distance of the field line associated with the aurora formation.

To obtain the amount of energy gain by the electrons associated with the parallel field E_z, we express E_z in terms of the magnetic field perturbation B_y (east-west direction).

Using Eqs(1), (3) and (4)

$$E_z = (k_x\rho_i)^2 (\frac{T_e}{T_i})(\frac{k_z}{k_x}) E_x , \qquad (10)$$

and $E_x = v_A B_y$, hence for $k_x\rho_i \sim T_e/T_i \sim O(1)$, $k_z/k_x = \pi\rho_i/(2R_E) \sim 10^{-2}$, and $v_A \sim 10^{-2}c$

$$E_z \sim 3 \times 10^{-5} \quad B_y(\gamma) \qquad V/m \quad . \qquad (11)$$

While it can be shown that the amplitude of the magnetic field oscillation B_{sy} of the macroscopic surface wave is related to that of the modified Alfvén wave through[6] $B_{sy} = \sqrt{\kappa\rho_i} \; B_y$, ($\kappa$ is the measure of the density gradient of the plasma sheet) if we express E_z in terms of the surface wave amplitude, which is actually observable by the satellite,

$$E_z \sim 3 \times 10^{-5} \frac{B_{sy}(\gamma)}{\sqrt{\kappa\rho_i}} \sim 2 \times 10^{-4} \; B_{sy}(\gamma) \quad V/m \qquad (11')$$

for $\kappa \sim 0.3\ R_E$, $\rho_i \sim 45$ km(for T_i = 1keV and B_s = 100γ). For a conservative value of $B_{sy} \sim 5\ \gamma$ (=5×10^{-5} gauss), E_z becomes 10^{-3}V/m and the potential change over the distance of $L \sim 2R_E$ becomes 13 keV.

Acceleration mechanism is similar to that in a cyclotron accelerator in that the acceleration field is localized with respect to the distance over which the excusion of electrons takes place, (bounce distance >> L). If we simplify the localized field by $\bar{E}_z \; \delta(k_z z) \cos \omega_s t$, the equation of motion of electrons in the z direction is

$$\ddot{z} + \omega^2_b z = - \frac{e}{m_e} \bar{E}_z \, \delta(k_z z) \cos\omega_s t \quad . \tag{12}$$

Integrating this (noting that ω_b is independent of $\dot{z}$) over one excusion period, we have

$$\frac{\dot{z}^2}{2} + \omega_b{}^2 \frac{z^2}{2} = \frac{e}{m_e} V \cos[\omega_s t(z=0)] \tag{13}$$

where $V = -E_z/k_z$

For a synchronous electron the phase space trojectory becomes a circle with discontinuous jump at $z = 0$ (Fig.3).

Because the phase change of the wave in the x direction, the synchronous condition of acceleration is satisfied for each wavelength $\lambda = 2\pi/k_x$. For $k_x \sim \rho_i{}^{-1}$, the wavelength at the equater becomes $\sim$300 km for $T_i \sim 1$ keV. By taking into account of plane convergence of the field line $\sim(10^2\gamma/5\times10^4\gamma)^{1/2}$, this wavelength projected on the ground becomes $\sim$10 km in the north-south direction. This distance decides the spacing between two auroral sheets.

In the east-west direction, the auroral sheet has two or three basic patterns. The short wavelength pattern (~ a few km) may be explained by the Kelvin-Helmholtz instability[9]. However, the long wavelength pattern has not be explained yet.

If we take the present theory, the wavelength in the east-west direction should corresponds to that of the surface wave $2\pi/k_y$. The value of k_y is somewhat arbitrary. However, k_y that corresponds to the wave that gives the maximum convergion efficiency to the modified Alfvén wave can be shown[5] to be κ. Again taking $\kappa \sim 0.3\ R_E$, the wavelength λ_y on the ground becomes $\sim$500 km.

Because the present theory uses a wave whose frequency is $\omega_s/2\pi$ or the period of 6 sec. Hence it may give an impression that the aurora should pulsate at this period. However, because the wave propagates toward the tail at a speed of $v_{Ti}\omega/\omega_{ci}$ (Eq.(8)) ~ 50 km/s at equator (~ a few km/s on the ground), the whole pattern moves toward north at a speed of a few km/s. However, because this speed is rather small, the north-ward motion may also be governed by other factors such as the motion of the plasma sheet or that of precipitating electrons themselves.

It has been shown that electrons can be accelerated along the field line by the magnetic field perturbation alone. The acceleration takes place periodically in the north-south direction in the form of several layers with spacing of ~ 10 km on the ground. This process explains in a consistent manner all the essential features observed.

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7. B.B. Kadomtsev, Plasma Turbulence, Translated by L.C. Ronson, Accademic Press, New York, N.Y. (1965) p.82; this choice of ϕ and ψ decouples the magnetosonic wave by producing $B_z = 0$, which is convenient for a low β plasma with $1 >> \beta >> m_e/m_i$.

8. As shown in Ref.6, the coupling occurs at the position x for which $\omega(x)$ given by Eq.(8) corresponds to ω_s of Eq.(7).

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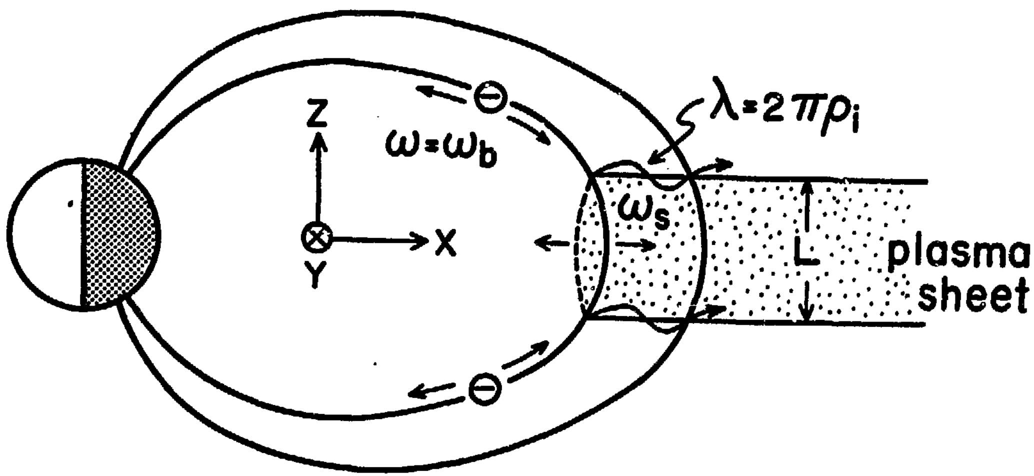
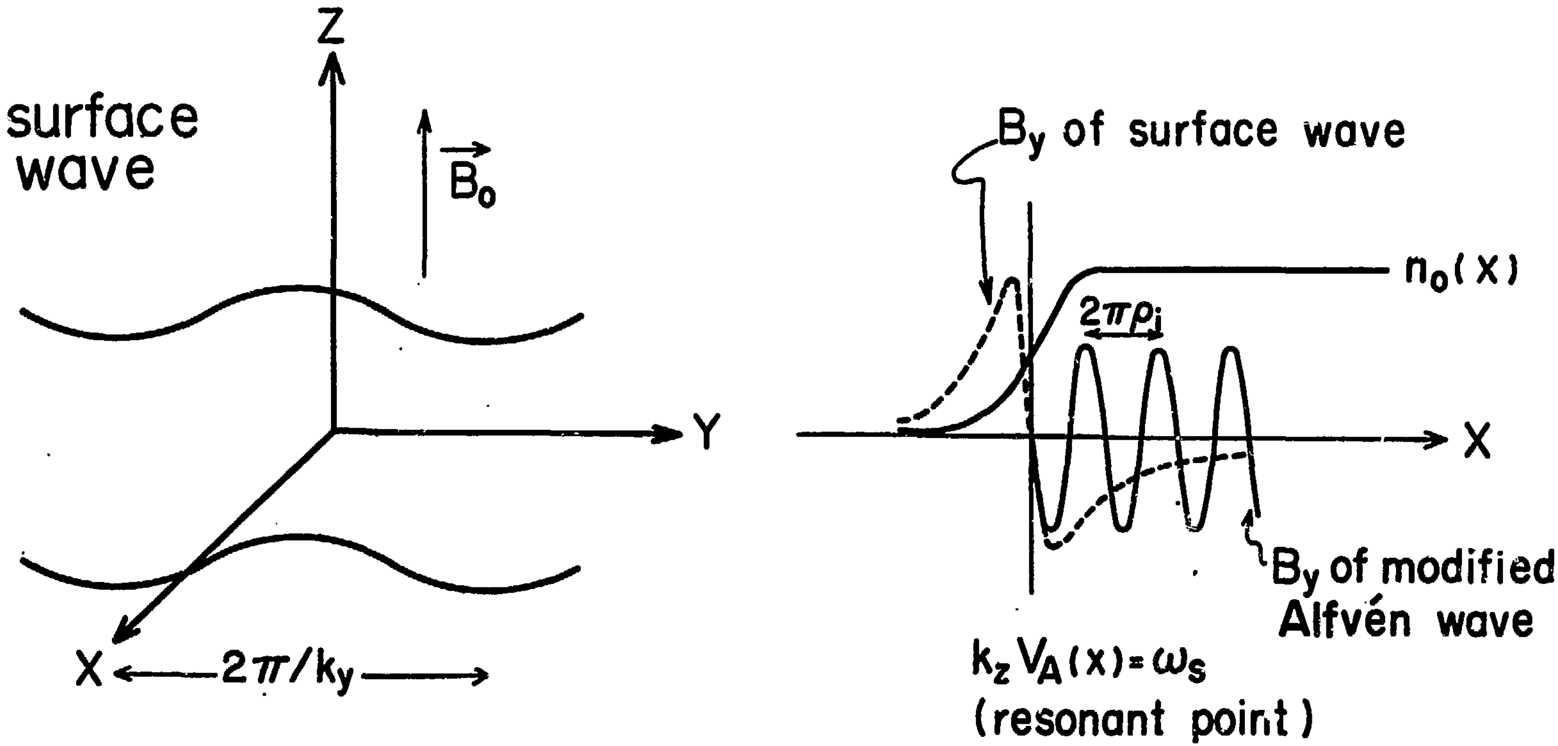
Figure Captions

Fig. 1 Resonant acceleration occurs when the bounce frequency ω_b coincides with the surface wave frequency ω_s.

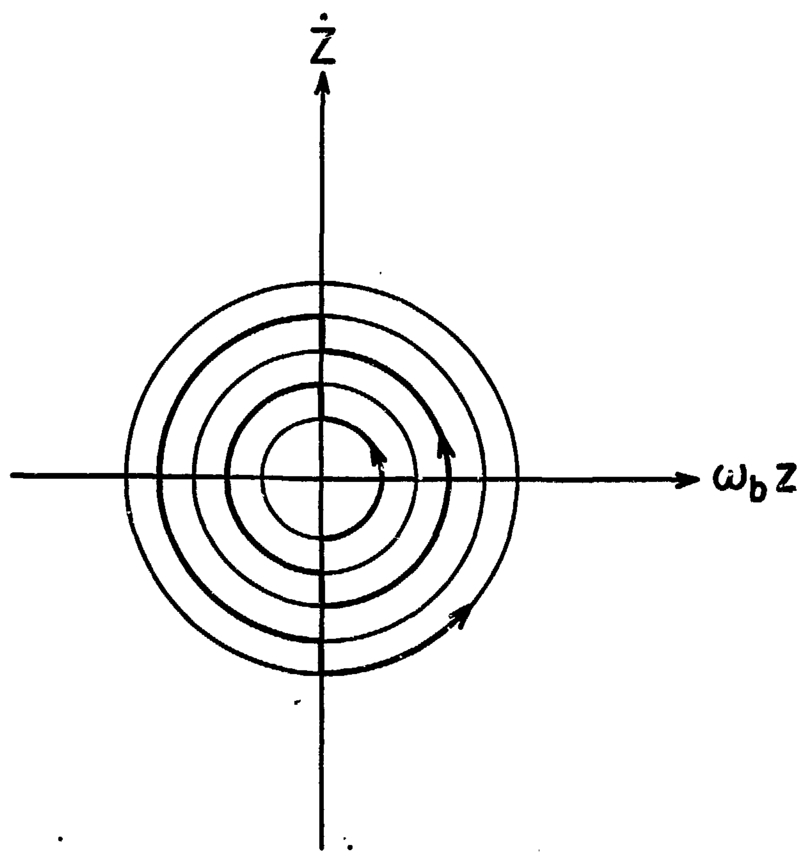
Fig. 2 Coordinate systems showing the surface wave, the density variation, and the modified Alfvén wave

Fig. 3 Phase diagram of electron trajectory along the magnetic field. Acceleration occurs locally at equator due to E_z associated with the modified Alfvén wave.


λ=2πρi
ω=ωb
ωs
Z
X
Y
L
plasma
sheet



surface wave
Z
$\vec{B}_0$
Y
X
$2\pi/k_y$
B_y of surface wave
$n_0(X)$
$2\pi\rho_i$
X
B_y of modified Alfvén wave
$k_z V_A(x) = \omega_s$
(resonant point)


$\omega_b Z$
$\dot{Z}$