CREEP ANALYSIS OF ORTHOTROPIC SHELLS

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Presented at the Symposium on Structural Mechanics, Bombay, 20-22 March 1975

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I INTRODUCTION

Creep deformation of shells made of isotropic materials has been found by large number of researchers(1,2,3,4). The assumption that a material starts off and remains in an isotropic state cannot be always justified. Zirconium alloys which are extensively used as structural materials in reactor core show an orthotropic behaviour. Creep behaviour of orthotropic shells has been investigated in the present paper. Numerical method has been evolved to find stress redistribution, strain rates, steady state stresses in a circular cylindrical shell subjected to uniform internal pressure.

Mackenzie(5) showed that the reference stress can be obtained by comparing the m-power solution with that for m=1. Results were obtained for beams, cylinders and circular plates. Sim(6,7) extended Mackenzie's approach to include the results of numerical solutions. The concept of reference stress has been extended to shells in the present paper.

II MATHEMATICAL FORMULATION

The formulation is valid for homogeneous, orthotropic material. The axes of orthotropy coincide with directions of principal stresses. Power creep law stated below is applicable to creep deformation in uniaxial stress field.

$$\frac{d\epsilon_c}{dt} = B(t)\,P(T)\,\sigma^{m} \qquad \ldots (1)$$

Von-Mises yield criterion and Prandtl-Reuss flow rate for orthotropic material as given by Bhatnagar and Gupta[5] are applicable. Stress-strain relations of the type given by equations [4] are used. These equations are based on the assumption that creep behaviours in tension and in compression are identical. Kirchoff-Love assumptions hold good. Deformations are small and structure is deformed elastically at time equal to zero.

Elastic deformation is obtained by solving the following equations (Ref. figure 1).

$$\frac{d^2 q}{d\eta^2} - \gamma = 0 \quad ; \quad \frac{d^2 \gamma}{d\eta^2} + q = 0 \qquad \ldots (2)$$

along with the boundary conditions stated below:

$$\lambda_\theta = \gamma = 0 \quad \text{at a fixed edge } (\eta = \text{constant}) \quad \ldots (3)$$

Relation between creep strain rates and stresses are given by the following equations :

$$\left.\begin{aligned} \sigma_e &= \left(A_{xx}\sigma_x^2 + A_{\theta\theta}\sigma_\theta^2 + 2A_{x\theta}\sigma_x\sigma_\theta\right)^{\frac{1}{2}} \\ \frac{d\epsilon_{xc}}{dt} &= \sigma_e^{m-1}\left(A_{xx}\sigma_x + A_{x\theta}\sigma_\theta\right) P(T)\, B(t) \\ \frac{d\epsilon_{\theta c}}{dt} &= \sigma_e^{m-1}\left(A_{\theta\theta}\sigma_\theta + A_{x\theta}\sigma_x\right) P(T)\, B(t) \end{aligned}\right\} \quad (4)$$

where $A_{xx}, A_{\theta\theta}, A_{x\theta}$ are anisotropy coefficients of the material. A_{xx} is taken equal to one for the sake of simplicity.

Creep solution is obtained by solving the rate equations :

$$\frac{d^2\dot{\varphi}}{d\eta^2} - \dot{\gamma} = \dot{F}_c \quad ; \quad \frac{d^2\dot{\gamma}}{d\eta^2} + \dot{\varphi} = \dot{G}_c \qquad \ldots (5)$$

along with the following boundary conditions .

$$\dot{\varphi} = \dot{\gamma} = 0 \qquad \text{at a fixed edge } (\eta = \text{constant}) \ldots (6)$$

Expressions for finding out the stress rates, strain rates etc. are given in Appendix A.

III METHOD OF SOLUTION

Equations (3) and (5) can be written in finite difference form. When combined with boundary conditions, the following equations are obtained.

$$[A]\{X\} = \{B\} \qquad \ldots (7)$$

$$[A]\{\dot{X}\} = \{C\} \qquad \ldots (8)$$

Elements of matrix $[A]$ and vectors $\{X\}, \{\dot{X}\}, \{B\}, \{C\}$ are given in Appendix B.

Solution of equations (7) and (8) yields final result. Steps involved in the solution are as mentioned below. (i) Equation (7) is solved. (ii) Stresses, strains, stress resultants are obtained by using $\{X\}$. (iii) Using these values $\dot{F}_c, \dot{G}_c$ and then vector $\{C\}$ are obtained. (iv) The rate problem defined by equation (8) is solved. (v) Stress rates, strain rates etc. are evaluated by using $\{\dot{X}\}$. (vi) New stresses and strains after an interval $\Delta\tau$ are determined by using expressions of the type:

$$\Sigma_x\Big|_{\tau+\Delta\tau} = \Sigma_x\Big|_{\tau} + \Delta\tau * \dot{\Sigma}_x\Big|_{\tau} \quad \ldots (9)$$

(vii) Steps (iii) to (vi) are repeated till stationary state is obtained.

Equations (7) and (8) have been successfully solved by Choleski's unsymmetrical method[3]. As suggested by Penny[4] a suitable value of $\Delta\tau$ is obtained by

$$\Delta\tau = \frac{1}{f}\left|\frac{\Sigma}{\dot{\Sigma}}\right| \quad ; \quad f \gg 1 \quad \ldots (10)$$

Stationary state may be defined when stress rates at points of interest become very low.

IV CALCULATION OF REFERENCE STRESS

It is possible to choose unit stress σ_0 such that the parameters defining the stationary state behaviour are approximately constant over a range of values of m . Such a value of stress is reference stress. Now,

$$\frac{d\lambda_c}{d\tau} = \frac{1}{\epsilon_0}\frac{d\epsilon_c}{dt}\frac{dt}{d\tau} = \frac{1}{A(T)\,B(t)\,\sigma_0^{\,m}}\frac{d\epsilon_c}{dt} \quad \ldots . (11)$$

If $\bar{\sigma}_0 = \bar{\alpha}\,\sigma_0$, dimensionless rate $\dot{\lambda}_c$ associated with unit stress is related to $\dot{\bar{\lambda}}_c$ associated with reference stress by the following equations.

$$\dot{\Delta}_c = (\bar{\alpha})^m \, \dot{\bar{\Delta}}_c \quad \ldots . (12)$$

Hence $\bar{\alpha}$ may be calculated by using equation (12).

V RESULTS AND DISCUSSION

Results were computed for clamped cylindrical shells for the following ranges of parameters.

$3 \leq m \leq 9$; $0.6 \leq A_{\theta\theta} \leq 1.2$; $-0.3 \geq A_{x\theta} \geq -0.6$

The following observations may be made from results plotted in figures 2 through 6.

(i) Axial stress at clamped edge diminishes from its initial value at $\tau = 0$ to a stationary value at $\tau = \tau_{ss}$ by the process of creep (Figure 2). The higher the value of m, the higher is the strain rate and, therefore, the quicker is the stress redistribution.

(ii) The distribution of stress across the shell wall becomes non-linear, unlike the linear distribution in the elastic case. Higher the value of stress exponent, greater is the deviation from elastic solution. Also, there exists a skeletal point at $\xi = -0.46$ (Figure 3).

(iii) $A_{\theta\theta}$ and $A_{x\theta}$ are two independent variables. As explained in reference 10, if $A_{\theta\theta} < A_{xx}$, $A_{x\theta} > -0.5$. Similarly, if $A_{\theta\theta} > A_{xx}$, $A_{x\theta} < -0.5$. Keeping this in view the following four cases were studied for m = 3.

(a) $A_{\theta\theta} = 0.6$ $A_{x\theta} = -0.3$

(b) $A_{\theta\theta} = 0.8$ $A_{x\theta} = -0.4$

(c) $A_{\theta\theta} = 1.0$ $A_{x\theta} = -0.5$ (isotropic material)

(d) $A_{\theta\theta} = 1.2$ $A_{x\theta} = -0.6$

Figure 4 shows that the strain rates at stationary state are minimum for case (a) and maximum for case (d). Referring to equations (4) it is seen that decrease in the values of $A_{\phi\phi}$ and $A_{x\phi}$ results in decrease of creep strain rate. Comparing any two examples under study, it will be seen that an increase in the value of $A_{x\phi}$ and decrease in the value of $A_{\phi\phi}$ reduces strain rate. Thus it may be concluded that the value of $A_{\phi\phi}$ is more important.

For higher creep rate, stress redistribution is quicker. Therefore, time to reach steady state is minimum for case (d) and maximum for case (a) (Figure 5).

(iv) Figure 6 shows the dependence of stationary state strain rate on stress exponent m. The form of numerical results depends on the magnitude of unit stress $\sigma_0 (= \alpha\, pa/2h)$. As may be seen, there are particular values of unit stress at which the strain rates are approximately constant. Such a value of unit stress is the reference stress for a particular parameter. It may be seen that the computed results behave almost according to equation (12) within ± 4 percent.

VI LIST OF SYMBOLS

a = radius of shell	$U_1' = U_1\,F(T)$
E = modulus of Elasticity	$U_2' = U_2\,F(T)$
h = thickness of shell	w = radial displacement
$K = \{3(1-\nu^2)\,a^2/h^2\}^{1/4}$	Z = distance across shell wall
m = creep law constant	α = numerical constant

M = bending moment/length

$m_{\dots} = M/\sigma_0 \ell^2$ (non-dimensional)

N = stress resultant/length

p = internal pressure

$n_{\dots} = N/\sigma_0 \ell$ (non-dimensional)

Q = lateral shear force/length

t = time

T = temperature

u = displacement parallel to axis of shell

$U_1 = \Sigma_\ell^{m-1}\left(\Sigma_x + \frac{A_{x\theta} + \nu A_{\theta\theta}}{A_{xx} + \nu A_{x\theta}} \Sigma_\theta\right)$

$U_2 = \Sigma_\ell^{m-1}\left(\Sigma_\theta + \frac{A_{x\theta} + \nu A_{xx}}{A_{\theta\theta} + \nu A_{x\theta}} \Sigma_x\right)$

$q = Q\kappa a/\sigma_0 \ell$

$\tau = -d\lambda_0/d\eta$

Δ = distance between adjacent nodes

$\eta = \sqrt{3}\,\kappa x/a$ (non-dimensional)

$\zeta = 2z/\ell$ (non-dimensional)

$\epsilon_0 = \sigma_0/E\,(1-\nu^2)$

ϵ = strain

$\lambda = \epsilon/\epsilon_0$

ϵ_e = effective strain

σ = principal stress

$\sigma_0 = \alpha\, pa/2h$, unit stress

σ_e = effective stress

$\Sigma = \sigma/\sigma_0$ (non-dimensional)

ν = poisson's ratio

<u>Subscripts</u>

c = creep component

el = elastic component

x, θ, z = axial, circumferential, radial components

SS = stationary state

<u>Superscripts</u>

$\dot{(\dots)}$: differentiation with respect to τ

$\bar{(\dots)}$: value associated with reference stress

VII **REFERENCES**

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APPENDIX-A

$$n_\theta = \nu/\alpha - \omega/a\epsilon_0$$

$$m_x = -\frac{1}{2}\frac{d\gamma}{d\eta}\left\{\frac{1}{3(1-\nu^2)}\right\}^{\frac{1}{2}}$$

$$m_\theta = \nu\, m_x$$

$$\lambda_x = \frac{1}{\alpha} - \nu n_\theta - \frac{d\gamma}{d\eta}\left\{3(1-\nu^2)\right\}^{\frac{1}{2}}$$

$$\lambda_\theta = n_\theta - \nu/\alpha$$

$$\dot{\lambda}_\theta = n_\theta + \frac{1}{2}\int_{-1}^{1} V_2\, d\xi$$

$$\dot{\lambda}_x = \frac{1}{2}\int_{-1}^{1} V_1\, d\xi - \nu\dot{n}_\theta - \xi\frac{d\dot{\gamma}}{d\eta}\left\{3(1-\nu^2)\right\}^{\frac{1}{2}}$$

$$\Sigma_e = (A_{xx}\Sigma_x^2 + A_{\theta\theta}\Sigma_\theta^2 + 2A_{x\theta}\Sigma_x\Sigma_\theta)^{1/2}$$

$$V_1 = \dot{\lambda}_{xc} = \Sigma_e^{m-1}(A_{xx}\Sigma_x + A_{x\theta}\Sigma_\theta)$$

$$V_2 = \dot{\lambda}_{\theta c} = \Sigma_e^{m-1}(A_{\theta\theta}\Sigma_\theta + A_{x\theta}\Sigma_x)$$

$$\dot{\epsilon}_x = \dot{\epsilon}_{xel} + \dot{\epsilon}_{xc}$$

$$\dot{\epsilon}_\theta = \dot{\epsilon}_{\theta el} + \dot{\epsilon}_{\theta c}$$

$$\dot{n}_\theta = -\frac{\omega}{a\epsilon_0} - \frac{1}{2}\int_{-1}^{1} V_2\, d\xi$$

$$\dot{\Sigma}_x = \frac{\dot{\lambda}_x + \nu\dot{\lambda}_\theta}{1-\nu^2} - \frac{A_{xx} + \nu A_{x\theta}}{1-\nu^2} U_1 \qquad \dot{m}_x = -\frac{1}{2}\frac{d\dot{r}}{d\eta}\left\{\frac{1}{3(1-\nu^2)}\right\}^{\frac{1}{2}} - \frac{1}{4}\frac{A_{xx} + \nu A_{x\theta}}{1-\nu^2} * \int_{-1}^{1} U_1 \xi \, d\xi$$

$$\dot{\Sigma}_\theta = \frac{\dot{\lambda}_\theta + \nu\dot{\lambda}_x}{1-\nu^2} - \frac{A_{\theta\theta} + \nu A_{x\theta}}{1-\nu^2} U_2 \qquad \dot{m}_\theta = -\frac{1}{2}\nu \frac{d\dot{r}}{d\eta}\left\{\frac{1}{3(1-\nu^2)}\right\}^{\frac{1}{2}} - \frac{1}{4}\frac{A_{\theta\theta} + \nu A_{x\theta}}{1-\nu^2} * \int_{-1}^{1} U_1 \xi \, d\xi$$

$$\dot{F}_c = \frac{1}{2}\frac{d}{d\eta}\int_{-1}^{1}\frac{V_2}{2}\, d\xi$$

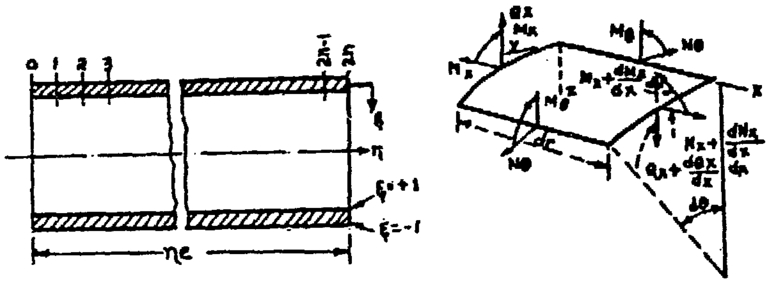
$$\dot{G}_c = -\frac{3}{2}\left(\frac{1-\nu^2}{2}\right)^{\frac{1}{2}} \frac{A_{xx} + \nu A_{x\theta}}{1-\nu^2} * \frac{d}{d\eta}\int_{-1}^{1} U_1 \xi \, d\xi$$

APPENDIX - B

$$\{X\} = \begin{Bmatrix} q_0 \\ r_{-1} \\ q_1 \\ r_1 \\ \vdots \\ q_{n-1} \\ r_{n-1} \\ q_{n+1} \\ r_{n+1} \end{Bmatrix} ; \quad \{B\} = \begin{Bmatrix} \frac{2\Delta}{\alpha}(2-\nu) \\ \vdots \end{Bmatrix} ; \quad \{C\} = \begin{Bmatrix} \Delta^2 \dot{F}_{c0} + 2\Delta I_0 \\ \Delta^2 \dot{G}_{c0} \\ \Delta^2 \dot{F}_{c1} \\ \Delta^2 \dot{G}_{c1} \\ \vdots \\ \Delta^2 \dot{F}_{cn} \\ \Delta^2 \dot{G}_{cn} \end{Bmatrix}$$

$$I_0 = \frac{1}{2}\int_{-1}^{1} \frac{V_2}{2}\, d\xi \quad \text{at } \eta = 0$$

$$[A] = \begin{bmatrix}
-2 & 0 & 2 & & & & & & & & & & \\
S & 1 & 0 & 1 & & & & & & & & & \\
1 & 0 & -2 & b & 1 & & & & & & & & \\
 & 0 & S & -2 & 0 & 1 & & & & & & & \\
 & & 1 & 0 & -2 & b & 1 & & & & & & \\
 & & & 1 & S & -2 & 0 & 1 & & & & & \\
 & & & & \ddots & & & & & & & & \\
 & & & & 1 & 0 & -2 & b & 1 & & & & \\
 & & & & & 1 & S & -2 & 0 & 1 & & & \\
 & & & & & & & \ddots & & & & & \\
 & & & & & & & & 1 & 0 & -2 & b & 0 \\
 & & & & & & & & & 1 & S & -2 & 0 & 0 \\
 & & & & & & & & & & 1 & 0 & 1 & 0 \\
 & & & & & & & & & & & 1 & 0 & 1
\end{bmatrix} \qquad S = \Delta^2, \quad b = -\Delta^2$$



…URE-1: CO-ORDINATE SYSTEM

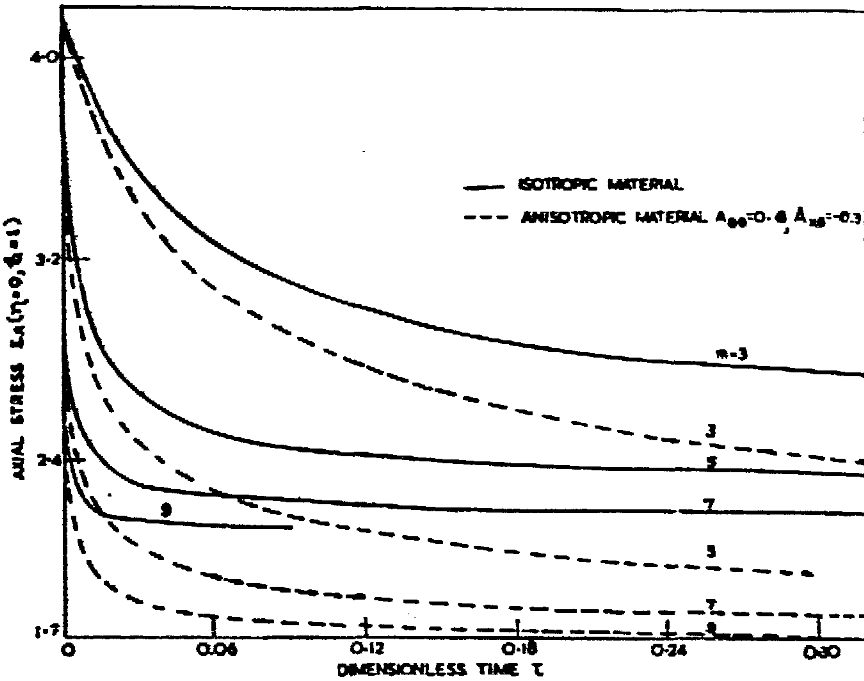


FIGURE-2 :- VARIATION OF MAXIMUM SURFACE STRESS IN CLAMPED CYLINDRICAL SHELL WITH TIME: INFLUENCE OF STRESS OF STRESS EXPONENT m.

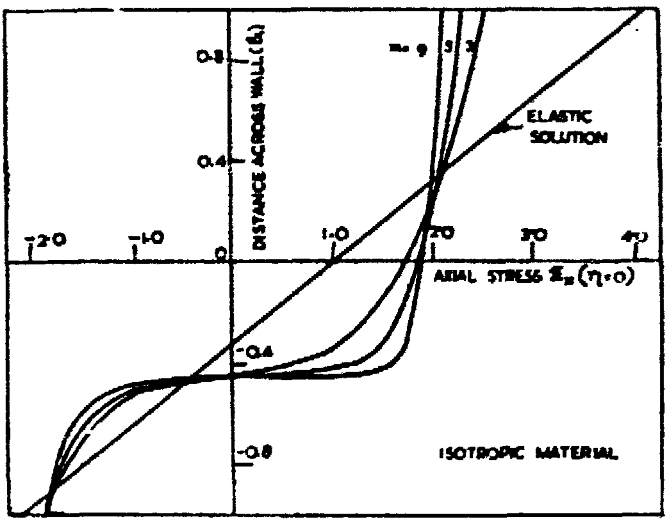
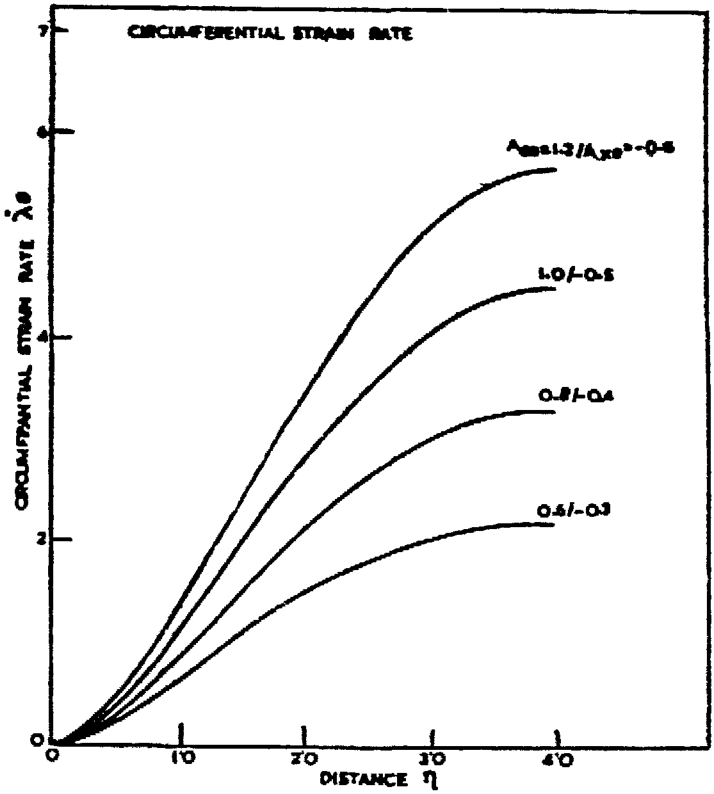


FIGURE-3 :- DISTRIBUTION OF AXIAL STRESS IN CLAMPED CYLINDRICAL SHELL AT ITS ROOT. VALUES AT STATIONARY STATE. INFLUENCE OF STRESS EXPONENT m



FIGURE·4·VARIATION OF STRAIN RATE IN A CLAMPED CYLINDRICAL SHELL AT STATIONARY STATE. INFLUENCE OF ANISOTROPIC COEFFICIENTS. STRESS EXPONENT m = 3

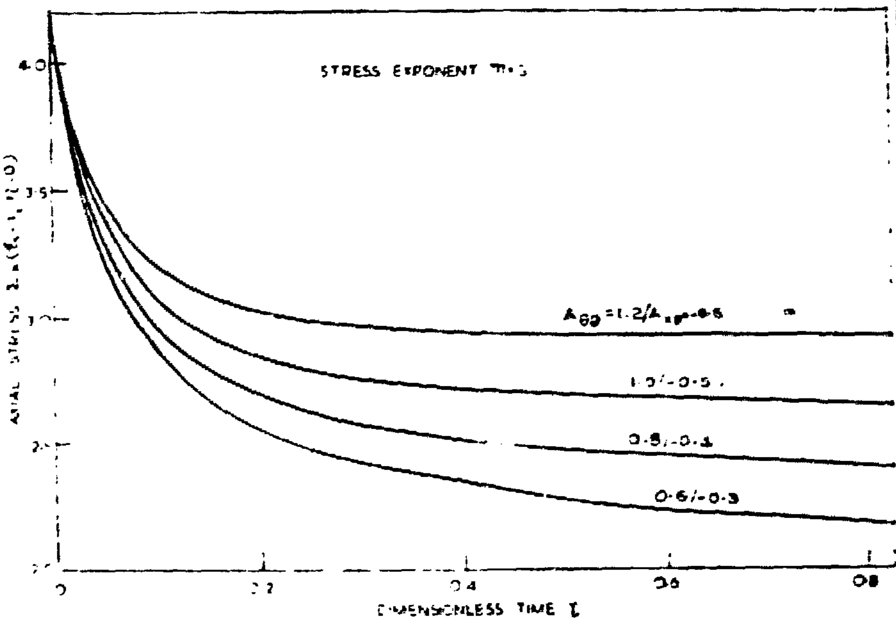


FIGURE-5. VERIATION OF AXIAL STRESS AT CLAMPED EDGE OF CYLINDRICAL SHELL WITH TIME: INFLUENCE OF ANISOTROPY.

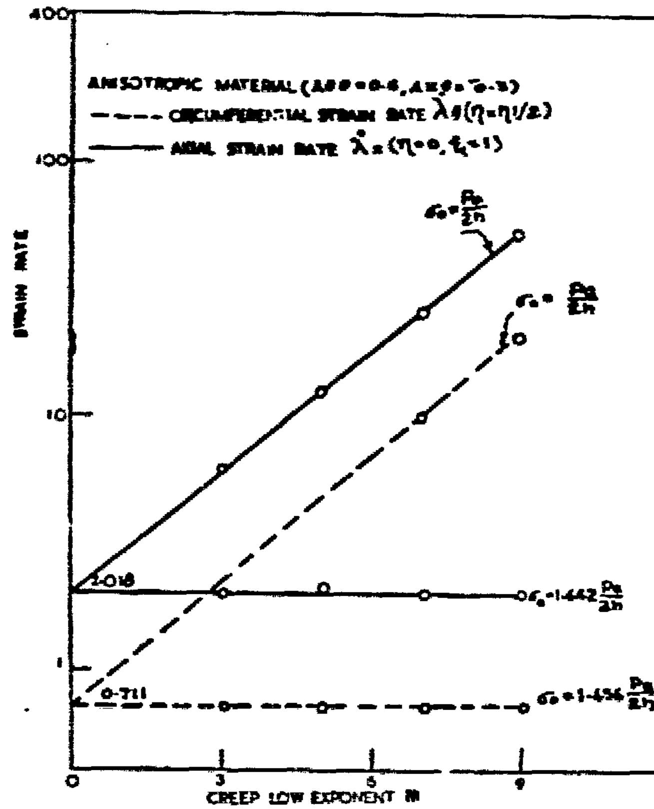


FIGURE-6-STRAIN RATE IN CLAMPED CYLINDRICAL SHELL AT STATIONARY STATE