ON THE RELIABILITY OF FINITE ELEMENT SOLUTIONS

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The finite element method has become the most widely applied tool for the analysis of Nuclear reactor structures. This is because of its unique potential to incorporate not only the physical features such as variations of geometry, material, loading pattern, nature of stress distributions etc., but also the accuracy requirements in the analysis procedure by way of judicious choice of wide range of elements with various geometric and functional variations, each with a typical mathematical model that defines the scope and extent of utility. These potentialities have been extensively exploited to hybridise the numerical and analytical techniques in rendering the hitherto intractable or cumbersome problems, solvable with ease and accuracy. The state of art for the finite element analysis of reactor vessels is thoroughly deliberated in SMIRT conferences(1). Certain salient features of the different versatile programmes such as 'ANSYS', 'NASTRAN', 'ASKA' and 'TITUS' are reviewed by Stoker(2). Hence without going into the details of reactor structural analysis proper, we shall restrict to discuss a single significant aspect of this analysis for the benefit of the vast majority of users who tend to look up on this as a brute force method of solving higher order magnitude problems.

Most of the advances in the field of finite element analysis[3] are by way of development of new elements more often labelled as 'improved', 'refined' or 'special', thereby indicating the possibilities of using these in preference to simple conventional displacement elements. The claims are often established by making comparisons in the case of simple problems on the plea that there exists an exact, if not atleast a reliable elastic solution for comparative purposes. Fortunately all FEM users, though uncomfortably are aware that the errors in stress estimations, generally based on displacement gradient fields generating considerable discontinuities at inter-element boundaries, are invariably minimised through an averaging process. The reliability of this minimising process in the case of sophisticated elements often leaves an element of suspicion on the results of each of the complex problems. This is precisely the reason why bulk of common users tend to play safe with conventional elements with known and proven convergence trends and be contented with obtaining increasingly accurate results by successively refining the mesh. Though this approach may be justified in a few exceptional cases where estimation of discretisation error[4] associated with the model is possible, in majority of the cases one has to examine, the basic issues involved in the mathematical as well as physical modelling of the problem before accepting the solution as a reliable one.

Firstly we shall discuss the conservative usage of established elements by brute force method to a problem involving dissimilar materials and large stress gradients arising out of cut outs and/or load discontinuities. Here the discretisation procedure adopted in the generation of the mesh and the relative sizes of the element play an important role on the efficiency of solution. Some of the note-worthy guide-lines are:

i) Nodes must be located on geometric, material and load discontinuities.

ii) Sharply curved boundaries should be either taken care of by use of isoparametric elements or by appropriately refining the mesh.

iii) Element size at the loading surface must have direct bearing on the load intensities. The higher the load intensities the finer the mesh should be.

iv) As far as possible the string of nodes should fall either on interface boundaries of dissimilar materials or along isostress lines. While it is difficult to know the latter apriori, a qualitative judgement is a must for FEM, lest the indiscreminate subdivision renders the procedure invariably inefficient.

v) In the known regions of stress concentration, use of large stack of finite elements of uniformly varying size is a must. However, the limit on the extravaganza of this stack of elements should really be based on the accuracy requirements.

vi. Wherever size of the matrices are exceeding the limits of handling, substructure analysis must be resorted to.

vii) Sometimes even refinement of mesh also may not lead to improved accuracy, unless the numerical errors resulting from the ill-conditioning arising out of the coexistance of small and very large elements within a single substructure are well within tollerable limits.

Even after observing such restrictions, one may finally come across formidable core memory requirements that limit the use of conventional elements necessitating the use of special elements for problems involving stress concentrations and singularities. Here is the case where variations of mathematical as well as physical models should be considered to check the reliability of the solution. While checking the use of unconventional special hybrid or refined elements one should use a parallel programme with conventional elements, the dispersion of which though limited would help to establish the credibility of the former. Having done this for a small part of a huge problem (where such a check is feasible), it is the user's ingenuity which finally guides his choice of special elements or otherwise, in which the variations of physical and mathematical models are used with care and caution to solve this difficult problem. Thus the guide-rules could be:

i) If a large number of stress raisers of similar nature are present in a problem it is worth-while to try appropriate special elements in one or two cases and test their reliability by isolating the parts and varying the models and then mechanically use these elements to save computer memories.

ii) If only a few stress raisers are present in the given problem and if it is possible to use established conventional elements it is preferable to use these, but one has to check the reliability by varying sizes and composition of the grid as well as type of element (with different displacement or stress fields).

It is worth-noting that for any procedure it is desirable to obtain bounds of error as suggested by G.V.Rao[5] by modifying a convergence curve by suitable introduction of a modification function. However this extrapolation technique is applicable to procedures permitting monotonic convergence. Some relevant remarks on error control are given by A.K.Rao et al[6].

Thus a finite element solution is reliable if

i) The bounds are established

ii) If the use of different elements with different physical models each of which saisfy the necessary reliability criterion of asymptotic accuracy lead at least to close solutions.

iii) In case the sophisticated elements used in the repeated analysis have proven convergence and with reliability checked at least in one case and further the repetitive analysis do not involve violations of basic assumptions of the hybrid formulations; the solutions thus obtained, though may not be very accurate, are certainly reliable. This is the justification for using special elements even if we have no means of obtaining the solution by refining the mesh by use of conventional FEM due to restrictions on memory and time of computers.

Finally the reliability of reactor vessel analysis have to be further checked through experimental analysis and testing procedures[2].

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