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WEAK CURRENTS

J. LEITE LOPES
Centre de Recherches Nucléaires
Université Louis Pasteur, Strasbourg

DIVISION DES HAUTES ENERGIES
CENTRE DE RECHERCHES NUCLAIRES
STRASBOURG

INSTITUT DE PHYSIQUE
UNIVERSITE DE BELGRADE
BELGRADE

[illegible]

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[illegible] field [illegible] strong interactions [illegible] gauge field allows [illegible] electromagnetic current of these particles, some examples [illegible] in Table I.

Hadrons exhibit strong interactions as well as electromagnetic and weak interactions. Because of strong interactions it is not possible at present to describe these particles by an elementary field (they would rather be [illegible] of elementary particles such as quarks).
If one calls :

$$h^{\lambda}_{(\gamma)}(x)$$

the hadron electromagnetic current, the electromagnetic coupling of hadrons with an electromagnetic field is of the form

$$e_0 h^{\lambda}_{(\gamma)}(x)\ A_{\lambda}(x)$$

where $A_{\lambda}(x)$ is the electromagnetic field.
The total charge of a hadronic system is :

$$Q^{(h)}_{(\gamma)} = \int h^{0}_{(\gamma)}(x)\ d^3x$$

and it is conserved :

$$\partial_{\gamma}\ h^{\lambda}_{(\gamma)}(x) = 0$$

TABLE 1

Field	Gauge transformation	Gauge-invariant Lagrangian $\mathcal{L}$	Electromagnetic current [illegible]
Scalar $\phi(x)$, $\phi^+(x)$	$\phi'(x)=\exp(ie\Lambda(x))\phi(x)$ $\phi^{+\prime}(x)=\exp(-ie\Lambda(x))\phi^+(x)$ $A^{\mu\prime}(x)=A^\mu-\partial^\mu\Lambda(x)$	$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+$ $+(D^\mu\phi)^+(D_\mu\phi)-m^2\phi^+\phi$; $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$; $D^\mu=\partial^\mu+ieA^\mu(x)$.	$ie(\phi^+\partial^\mu$ [illegible] $ie(\phi^+$ [illegible] $-2e^2A^\mu\phi^+\phi$
Proca Vector field $W^\mu(x)$, $W^{\mu+}(x)$	$W^{\mu\prime}(x)=\exp(ie\Lambda(x))W^\mu(x)$ $W^{\mu+\prime}(x)=\exp(-ie\Lambda(x))W^{\mu+}(x)$ $A^{\mu\prime}(x)=A^\mu-\partial^\mu\Lambda(x)$	$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-$ $-\frac{1}{2}G^{\mu\nu+}G_{\mu\nu}+$ $+m^2W^{\mu+}W_\mu$; $G^{\mu\nu}=D^\nu W^\mu-D^\mu W^\nu$	ie [illegible] $W^+_\nu\,G^{\nu}$ [illegible]
Lepton field : left-handed isospinor $L_1(x)=\frac{1}{2}(1-\gamma^5)\begin{pmatrix}\nu_1(x)\\ 1(x)\end{pmatrix}$ and right-handed isoscalar : $R_1(x)=\frac{1}{2}(1+\gamma^5)1(x)$ where : $1(x)=e(x),\mu(x)$	$L_1'(x)=\exp(ie\Lambda(x)\frac{1-\tau_3}{2})\cdot L_1(x)$ $R_1'(x)=\exp(ie\Lambda(x))R_e(x)$	$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+$ $+\frac{1}{2}\Sigma_1(\bar{L}_1 i\gamma^\alpha D_{\alpha 3}L_1+$ $+\bar{R}_1 i\gamma^\alpha D_\alpha R_e-$ $-m_1\bar{L}_1\Omega R_1)+h.c.$; $D_{\alpha 3}=\partial_\alpha-ieA_\alpha\frac{1-\tau_3}{2}$; $\Omega=\binom{0}{1}$	$e(\bar{L}_1$ [illegible] $\frac{1-\tau_3}{2}$ [illegible] $+$ $+\bar{R}_e\gamma^\mu R_e$ [illegible] $=e(\bar{e}(x)\gamma^\mu e(x)+$ [illegible]

The Gell-Mann-Nishijima relation between charge, hypercharge and isospin :

$$Q^{(h)} = I_3 + \frac{1}{2} Y$$

suggests that the current vector h^{λ} (x)(we omit the index ν)contains one part which transforms like a scalar under the group of isospin rotations and another part wich transforms like the third component of an isovector

$$h^{\lambda}_{(\gamma)} = h^{\lambda}_{(\gamma)} \ (I=1) + h^{\lambda}_{(\gamma)}(I=0)$$

where :

$$\frac{1}{2} Y = \int d^3x \quad h^{\lambda}_{(\gamma)} \ (I=0)$$

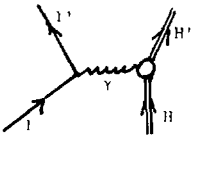
$$I_3 = \int d^3x \quad h^{\lambda}_{(\gamma)} \ (I=1)$$

Electromagnetic processes involving the emission of real photons by a hadron in a given state or the scattering of hadrons by leptons allow us to investigate the structure of the matrix elements of $h^{\lambda}_{(\gamma)}(x)$ between two states.

Thus consider the scattering of a charged lepton l by a hadron H. To lowest order in the electromagnetic interaction this process :

$$l + H \rightarrow l' + H'$$

has the graph



Now the S-Matrix in this lowest order is

$$S = -ie_0 \int d^4x \langle l'H'|\bar{l}_f(x)\gamma_\alpha A^\alpha(x) l_i(x)|lH\rangle$$

where $A^\alpha(x)$ is the field of the lepton created by the hadron :

$$\Box A^\alpha(x) = e_0 h^\alpha_{(\gamma)}(x)$$

$$A^\alpha(x) = e_0 \int D_F(x-y) h^\alpha_{(\gamma)}(y) d^4y$$

and $D_F(x-y)$ is Feynman's Green function, then :

$$S = -ie_0^2 \iint d^4x \langle l'H'|\bar{l}_f(x)\gamma_\alpha l_i(x) D_F(x-y) h^\alpha_{(\gamma)}(y)|lH\rangle d^4y$$

$$= -ie_0^2 \iint d^4x \langle l'|\bar{l}_f(x)\gamma_\alpha l_i(x)|l\rangle D_F(x-y)\langle H'|h^\alpha_{(\gamma)}(y)|H\rangle d^4y$$

Now :

$$\langle l'|\bar{l}_f(x)\gamma_\alpha l_i(x)|l\rangle = \frac{1}{(2\pi)^3}\, [\bar{u}_f(p'\sigma')\gamma_\alpha u_i(p\sigma)]\, e^{i(p'-p)x},$$

$$D_F(x-y) = -\int \frac{d^4K}{(2\pi)^4}\, \frac{e^{-ik(x-y)}}{K^2+i\varepsilon},$$

then :

$$S = (2\pi)^4 \delta^4\ (p'-p-K) M$$

where :

$$M = ie_0^2\ \frac{\bar{u}_f(p'\sigma')\gamma^\alpha u_i(p\ \sigma)}{(2\pi)^3}\ \frac{1}{K^2}\quad \langle H'|h^\alpha_{(\gamma)}(K)|H\rangle$$

and :

$$\langle H'|h^\alpha_{(\gamma)}(x)|H\rangle \;=\; \int d^4K \langle H'|h^\alpha_{(\gamma)}(K)|H\rangle e^{-iKx}$$

So the amplitude for the scattering process above is :

$$M = ie_0^2 \, [\bar{U}_f(p'\sigma')\gamma^\tau U_i(p\sigma)] \, \frac{1}{K^2} \langle H'|h^\tau_{(\gamma)}(K)|H\rangle$$

with $$U = \frac{u(p\sigma)}{(2\pi)^{3/2}} \, , \quad K^2 = (p'-p)^2$$

and the study of this process gives information on $\langle H'|h^\tau_{(\gamma)}(x)|H\rangle$.

<u>Electromagnetic form factors -</u>

If P is the momentum of a hadron in the incoming state $|H\rangle$ and P' the momentum of the out going hadron then we will have to express $\langle P'|h^\tau_{(\gamma)}(x)|P\rangle$ as a function of all vector quantities available and built with these states.

If the hadron is a <u>proton</u> one has five vectors namely :

1) $\bar{u}\,(P')\gamma^\mu u(P)$

2) $\bar{u}\,(P')\sigma^{\mu\nu}q_\nu u(P)$, $\quad q_\nu = P'_\nu - P_\nu$

3) $\bar{u}\,(P')\sigma^{\mu\nu}Q_\nu u(P)$, $\quad Q_\nu = P'_\nu + P_\nu$

4) $\bar{u}\,(P')u\,(P)\,q^\mu$

5) $\bar{u}\,(P')u\,(P)\,Q^\mu$

Out of these only three are independent and we leave it to the reader to show this by making use of Dirac's equation for the asymptotic protons

$$(\gamma^\mu P_\mu - M)\, u\,(P) = 0$$
$$(\gamma^\mu P'_\mu - M)\, u\,(P') = 0$$

and $$\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu}$$

The independent three quantities may be taken as :

I) $\bar{u}(P')\gamma^{\mu}u(P)$

II) $\bar{u}(P')i\sigma^{\mu\nu}q_{\nu}\,u(P)$

III) $\bar{u}(P')u(P)\,q^{\mu}$

If one writes :

$$\langle P'|h^{\alpha}_{(\gamma)}(x)|P\rangle = \langle P'|e^{iPx}h^{\alpha}_{(\gamma)}(o)e^{-iPx}|P\rangle$$

$$= e^{i(P'-P)x}\langle P'|h^{\alpha}_{(\gamma)}(o)|P\rangle$$

then one may express the matrix element of $h^{\alpha}_{(\gamma)}(o)$ for a proton :

$$e_o\langle P'|h^{\alpha}_{(\gamma)}(o)|P\rangle = \frac{e\bar{u}_p(P')}{(2\mu)^3}\left\{\gamma^{\alpha}F_1^{P}(q^2) + \right.$$

$$\left. + \frac{i\sigma^{\alpha\beta}}{2m_p}q_{\beta}F_2^{P}(q^2) + \frac{q^{\alpha}}{m_p}F_3^{P}(p^2)\right\}u_p(P)$$

as a combination of the three independent vectors above, the coefficients F_1, F_2, F_3, being invariant functions of the squared momentum-transfer q.

However, as $h^{\alpha}_{(\gamma)}(x)$ is a conserved current, F_3 has to be null. Indeed

$$\partial_{\alpha}h^{\alpha}_{(\gamma)}(x) = 0$$

implies :

$$\langle P'|\partial_{\alpha}h^{\alpha}_{(\gamma)}(x)|P\rangle = \langle P'|\partial_{\alpha}(e^{iPx}h^{\alpha}_{(\gamma)}(o)e^{-iPx})|P\rangle$$

$$= i(P'-P)_{\alpha}\langle P'|h^{\alpha}_{(\gamma)}(o)|P\rangle$$

Now

$$(P'-P)_\alpha < P'|h^\alpha_{(\gamma)}(o)|P> =$$

$$= \frac{\bar{u}_p(P')}{(2\pi)^3} \{ (P'-P)_\alpha \gamma^\alpha F_1(q^2) + i \frac{\sigma^{\alpha\beta}}{2m_p} (P'-P)_\alpha (P'-P)_\beta F_2(q^2) +$$

$$+ \frac{F_3^p(q^2)}{m_p} (P'-P)^2 \} u(P) \frac{e}{e_0}$$

The two first terms vanish because of Dirac's equation for the incoming and out going proton and symmetry of tensors and so the vanishing of the divergence of $h^\alpha_{(\gamma)}(x)$

$$q_\alpha < P' h^\alpha_{(\gamma)}(o)|P> =$$

$$= \frac{q^2}{m_P} F_3^p(q^2) \bar{u}_p(P') u_p(P) = 0$$

requires $F_3^p(q^2) = 0$

We have thus :

$$e_0 < P'|h^\alpha_{(\gamma)}(o)|P> = \frac{e\bar{u}(P')}{(2\pi)^3} \{ \gamma^\alpha F_1^p(q^2) + \frac{i\sigma^{\alpha\beta}}{2m_p} q_\beta F_2^p(q^2) \} u(P)$$

Now $h^\alpha(x)$ being hermitian one has :

$$<P'|h^\alpha_{(\gamma)}(x)|P>^* = <P|h^{\alpha\,+}_{(\gamma)}(x)|P'> = <P|h^\alpha_{(\gamma)}(x)|P'>$$

[illegible]

[illegible]

$$[illegible] + \frac{i\sigma^{\mu\nu}}{2m_p}(P-P')_\nu F_2^p(q^2)\} u(P')$$

[illegible] that :

$$[illegible] = \bar{u}(P)\gamma^\mu u(P')$$

$$(\bar{u}(P') i\sigma^{\mu\nu} q_\nu u(P))^* = \bar{u}(P) i\sigma^{\mu\nu}(P-P')_\nu u(P')$$

[illegible] that F_1 and F_2 are real functions.

$F_1(q^2)$ and $F_2(q^2)$ are the nucleon electro-magnetic form factors.

[illegible] the meaning of $F_1(0)$ we write :

$$e_0\langle P'|Q|P\rangle = e_0\langle P'|\int h^0_{(\gamma)}(x)\,d^3x|P\rangle =$$

$$= e_0\int\langle P'|h^0_{(\gamma)}(x)|P\rangle d^3x =$$

$$\frac{e}{(2\pi)^3}\{\bar{u}(p')\gamma^0 u(P)F_1^p(q^2) +$$

$$+ \bar{u}(P')\frac{i\sigma^{ok}}{2mP} q_k u(P)F_2^p(q^2)\}\int d^3x\, e^{i(P'-P)x}$$

$$= \frac{e}{(2\pi)^3}\{u^+(P')u(P)F_1^p(q^2) + \bar{u}(P')\frac{i\sigma^{ok}}{2m_p} q_k u(P)F_2^p(q^2)\}\,\cdot$$

$$\cdot\,(2\pi)^3\delta^3(P'-P)\,e^{i(P'_o - P_o)x_o}$$

Now P and P' are the energy-momenta of incoming and out going protons hence

$P_o = \sqrt{\vec{P}^2+m_p^2}$, $P_o' = \sqrt{\vec{P}'^2+m_p^2}$ therefore $\delta^3(P'-P)e^{i(P'_o-P_o)x_o} = \delta^3(P'-P)$ and :

$e_o\langle P'|Q|P\rangle = eu^+(P)u(P)F_1^P(o)\delta^3(P'-P)$

On the other hand if $|P\rangle$ is a proton state with charge e $e_o\langle P'|Q|P\rangle = e\langle P'|P\rangle =$

$= e\ 2P^o\delta^3(P'-P)$

where $\langle P'|P\rangle = 2P^o\delta^3(P'-P)$

so if $u^+(p)\ u\ (p) = 2P^o$

then $F_1^P(o) = 1$

<u>To see meaning of $F_2^P(o)$</u> we consider the interaction energy of a proton in the transition $|P\rangle \to |P'\rangle$ with a vector potential $\vec{A}_{ext}(x)$:

$$H_{int} = - e_o \int \langle\vec{P}'|\vec{h}(x)|P\rangle.\vec{A}_{ext}(x)d^3x$$

$$H_{int} = - \frac{e}{(2\pi)^3} \int \bar{u}(P')\{\vec{\gamma}\ F_1^P(q2) + \frac{i\vec{\sigma}x\vec{q}}{2m_p}\ F_2^P(q^2)\}\ u\ (P)\ \vec{A}_{ext}(x)e^{i(P'-P)x}$$

If one assumes that $P'_o \sim P_o$ and $|\vec{P}'|\sim|\vec{P}|$ then, as

$$\bar{u}(P')\vec{\gamma}u(P) = \bar{u}(P')u(P)\ \frac{\vec{P}+\vec{P}'}{2m_p} + \frac{i}{2m_p}\ \bar{u}(P')\vec{\sigma}x\vec{q}u(P)$$

we get considering that

$$\bar{u}\ (P')\ u\ (P) = 2m_p \text{ for } P'\sim P$$

$$H_{int} = -\frac{e}{(2\pi)^3}\int d^3x\, e^{-i(\vec{P}'-\vec{P}).\vec{x}} \Big\{ \frac{(\vec{P}+\vec{P}')}{2m_p}.\vec{A}_{ext}\, 2m_p +$$

$$+\ \bar{u}(P')\frac{i(\vec{\sigma}\times\vec{q})u}{2m_p}(P)\,(1+F_2(o)).\vec{A}_{ext}\Big\}$$

The first term is of form $e\langle\vec{V}\rangle.\vec{A}$; the 2nd is

$$-\frac{e}{2m_p}(1+F_2(o))\frac{1}{(2\pi)^3}\int d^3x\ e^{-i(\vec{P}'-\vec{P}).\vec{x}}\,\vec{\sigma}.\vec{\nabla}\,\vec{A}_{ext}$$

$$= -\frac{e}{2m_p}(1+F_2(o))\frac{1}{(2\pi)^3}\int d^3x\ e^{-i(\vec{P}'-\vec{P}).\vec{x}}\,\vec{\sigma}.\vec{B}_{ext}(x)$$

is the interaction of a magnetic dipole moment with an external magnetic field. The magnetic moment is :

$$\vec{M} = \mu\vec{\sigma}\ ;\ H_{int} = -\frac{1}{(2\pi)^3}\int \vec{M}.\vec{B}\ d^{i(\vec{P}'-\vec{P}).\vec{x}}\ d^3x$$

$$\mu = \frac{e}{2m_p}\ (1+F_2(o))$$

So if we call μ_p the anomalous proton magnetic moment we have :

$$\boxed{F_2(o) = \mu_P} \quad \simeq 1.79$$

For neutrons one finds : $F_1^{(n)}(o) = 0$, $F_2^{(n)}(o) = \mu_n \simeq -1.91$

So the <u>electro magnetic hadron current $h^{\alpha}_{(\gamma)}(x)$ is such that, for a nucleon</u> the matrix element is :

$$\langle P'|h^{\alpha}_{(\gamma)}(x)|P\rangle = e^{i(P'-P)x}\langle P'|h^{\alpha}_{(\gamma)}(o)|P\rangle$$

$$e_o\langle P'|h^{\alpha}_{(\gamma)}(o)|P\rangle = e\bar{U}(P')\Big\{\gamma^{\alpha}F_1^{N}(q^2) + \frac{i\sigma^{\alpha\beta}}{2m_N}q_{\beta}\,F_2^{N}(q^2)\Big\}U(P)$$

where : $$U(P) = \frac{1}{(2\pi)^{3/2}}\, u(p)$$

$F_1^N(o) = 1, \quad F_2^N(o) = \mu_p \sim 1.79$ for N = proton

$F_1^N(o) = 0, \quad F_2^N(o) = \mu_n \quad -1.91$ for N = neutron

or else :

$$e_o \langle P' | h^{\alpha}_{(\gamma)}(o)\, P\rangle = e\bar{U}(P')\Big\{\gamma^{\alpha}\, \frac{1}{2}\, (F_1^{s}(q^2) + \tau_3 F_1^{v}(q^2)) +$$

$$+ \frac{i\sigma^{\alpha\beta}}{2m_N}\; q_{\beta}\, \frac{1}{2}\, (F_2^{s}(q^2) + \tau_3 F_2^{v}(q^2))\Big\} U(P')$$

where :

$$F^{p}_{1,2}(q^2) = \frac{1}{2}\, (F^{s}_{1,2} + F^{v}_{1,2}) \qquad F_1^{s}(o) = F_2^{s}(o) = 1$$

$$F_2^{s}(o) = \mu_p + \mu_n$$

$$F^{n}_{1,2}(q^2) = \frac{1}{2}\, (F^{s}_{1,2} - F^{v}_{1,2}), \qquad F_2^{v}(o) = \mu_p - \mu_n$$

These form factors contain all information on the behaviour of the internal structure of nucleons under electromagnetic interaction. They are given by experiment. Thus if one sets :

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{2m_N}\, (F_2(q^2) | 2m_N)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

one gets from data the fit :

$$G_E^{p}(q^2) \simeq \frac{1}{(1 - \frac{q^2}{a^2})^2} \quad , \qquad a^2 \simeq o{,}71\ (\underline{gev})^2$$

$$G_M^{p}(q^2) \simeq \frac{1 + \mu_p}{(1 - \frac{q^2}{a^2})^2}$$

where : $U(P) = \frac{1}{(2\pi)^{3/2}} u(p)$

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$$+ \frac{i\sigma^{\alpha\beta}}{2m_N} q_\beta \frac{1}{2} (F_2^s(q^2) + \tau_3 F_2^v(q^2)) \} U(P')$$

where :

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$$F_2^s(o) = \mu_p + \mu_n$$

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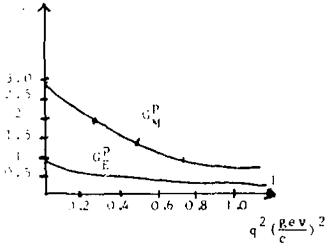
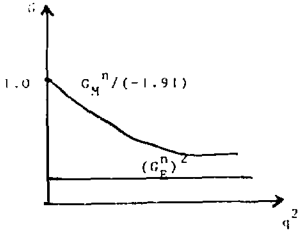
$$- [illegible] \; G_E^n(q^2) = [illegible] \frac{1}{\left(1 - \frac{q^2}{a^2}\right)^2}$$

$$G_M^n(q^2) = \frac{\mu_n}{\left(1 - \frac{q^2}{a^2}\right)^2}$$





It is known that for scattering the momentum transfer is a space-like vector where as for annihilation it is time-like.

Indeed let the reaction :

$$e + H \rightarrow e' + H'$$

where :

$$q = p_H' - p_H = p_e - p_e'$$

In the barycentric system :

$$\vec{p}_e + \vec{p}_H = \vec{p}_e' + \vec{p}_H' = 0$$

so :

$$\sqrt{\vec{p}^2+m^2} + \sqrt{\vec{p}^2+M^2} = \sqrt{\vec{p}'^2+m^2} + \sqrt{\vec{p}'^2+M^2}$$

where :

$$\vec{p} = \vec{p}_H = -\vec{p}_e, \quad \vec{p}' = \vec{p}'_H = -\vec{p}'_e$$

hence :

$$|\vec{p}| = |\vec{p}'|$$

$$E_H = E'_H, \quad E_e = E'_e$$

Now :

$$q^2 = (p'_H - p_H)^2 = p'^2_H + p_H^2 - 2(p'_H . p_H) =$$

$$= E'^2_H - \vec{p}'^2 + E_H^2 - \vec{p}^2 - 2(E'_H E_H - \vec{p}'_H . \vec{p}_H) =$$

$$= 2 \{ M^2 - E_H^2 + |\vec{p}|^2 \cos\theta \} =$$

$$= 2 \{ -\vec{p}^2 + \vec{p}^2 \cos\theta \} = -2\vec{p}^2 (1-\cos\theta)$$

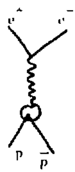
$$= -2\vec{p}^2 \sin^2\frac{\theta}{2}$$

So for elastic scattering one has :

$$\boxed{q^2 \leq 0}$$

For hadron-antihadron annihilation :

$$p + \bar{p} \rightarrow e^+ + e^-$$



we have in the barycentric system :

$$\vec{p} + \vec{\bar{p}} = 0$$

Then

$$q = (2E, 0) \rightarrow q^2 = 4E^2 \geq 4M^2$$

$\boxed{q^2 > 0}$ for annihilation.

II

WEAK CURRENTS AND WEAK FORM FACTORS OF HADRONS

[illegible] processes which involve hadrons and leptons are called [illegible] processes. Examples are the decays

$$n \to p + e + \bar{\nu}_e$$

$$\Lambda \to p + e + \bar{\nu}_e \qquad \text{(II, 1)}$$

$$K^+ \to \mu^+ + \nu_\mu$$

$$\pi^+ \to \pi^0 + e^+ + \nu_e$$

etc.

The purely leptonic weak processes involve only leptons, such as :

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e \; ; \; \mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e \; ; \; \nu_e + e \to \nu_e + e \qquad \text{(II, 2)}$$

In this case experiment allows one to describe the leptonic weak reactions by means of a phenomenological lagrangean of the form :

$$L = \frac{\mathcal{G}_F}{\sqrt{2}} \; l^{\lambda +}(x) \; l_\lambda(x) \qquad \text{(II, 3)}$$

where :

$$l^\lambda(x) = (\bar{\nu}_e \, \gamma^\lambda (1-\gamma^5) e) + (\bar{\nu}_\mu \gamma^\lambda (1-\gamma^5) \mu) \qquad \text{(II, 4)}$$

is the weak leptonic current :

$$l^{\lambda +}(x) = (\bar{e} \, \gamma^\lambda (1-\gamma^5) \nu_e) + (\bar{\mu} \gamma^\lambda (1-\gamma^5) \nu_\mu)$$

is its adjoint.

Thus the lagrangean for the decay of the muon is :

$$L = \frac{\mathcal{G}_F}{\sqrt{2}} \; (\bar{\nu}_\mu \gamma^\lambda (1-\gamma^5)\mu)(\bar{e}\gamma_\lambda \; (1-\gamma^5)\nu_e). \qquad (II, 5)$$

I shall remind you that the fact that the neutrino occurs in the interaction with the operator $(1-\gamma^5)\nu$, and the fact that both neutrinos are assumed to have zero mass, imply that <u>the neutrino is a polarised particle with negative helicity whereas the antineutrino has positive helicity</u>. And this holds for both neutrinos, ν_e and ν_μ. Why two neutrinos ? As the β-radioactivity of nuclei involves a neutrino or antineutrino which we call ν_e :

$$n \rightarrow p + e + \bar{\nu}_e \qquad (II, 6)$$

$$\nu_e + N_Z \rightarrow N'_{Z+1} + e^- \qquad (II, 7)$$

then if the pion-decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad (II, 8)$$

involved a neutrino ν_μ which would be identical to ν_e it would be possible to have the reaction (II, 7) by using as neutrino incident beam the one coming from the decay (II, 8). However, experiment shows that the reaction (II, 7) does not occur but only :

$$\nu_\mu + N_Z \rightarrow N'_{Z+1} + \mu^- \qquad (II, 9)$$

Therefore:

$$\nu_\mu \neq \nu_e \qquad (II, 10)$$

The experimental study of the allowed transitions in the nuclei β-radioactivity shows that the amplitude for the β-decay of the neutron is of the form :

$$M_{(\beta)} = \frac{1}{\sqrt{2}} \; (\bar{u}_p(p)\gamma^\lambda \; (G_V - G_A \; \gamma^5)u_n(p))(\bar{u}(p_e)\gamma_\lambda(1-\gamma^5)v(p_\nu)) \qquad (II, 11)$$

for vanishing momentum transfer. One has

$$G_V = \mathcal{G}_F \cos\theta \qquad \text{(II, 12)}$$

where

$$\mathcal{G}_F = 1.0[illegible] \times 10^{-5}\, m_p^{-2} \qquad \text{(II, 13)}$$

is the Fermi constant which occurs in the β-decay of the muon and

$$\frac{G_A}{G_V} \simeq 1.23 \;, \quad \theta = \text{Cabibbo's angle} \simeq 0.23 \qquad \text{(II, 14)}$$

As $\bar{u}_p \gamma_\lambda u_n$ is a polar vector and $\bar{u}_p \gamma_\lambda \gamma_5 u_n$ is an *axial vector, the form above* of the amplitude is said to result from a V-A interaction form.

In general for a semi-leptonic weak decay process one assumes a lagrangean of the form :

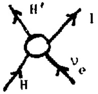
$$L = \frac{\mathcal{G}_F}{\sqrt{2}} \left\{ h^{\lambda +}(x)\; l_\lambda(x) + \text{h.c.} \right\} \qquad \text{(II, 15)}$$

where $h^\lambda(x)$ is the hadronic weak current and $l^\lambda(x)$ is the leptonic weak current:

$$l^\lambda(x) = \sum_{l=e,\mu} \left(\bar{\nu}_l \gamma^\lambda (1-\gamma^5) l \right) \qquad \text{(II, 16)}$$

If one considers a semi-leptonic process of the type

$$\nu_l + H \rightarrow l + H'$$



where H and H' are the hadronic initial and final systems then the lowest order amplitude for this transition is :

$$M = \frac{i\,\mathcal{G}_F}{\sqrt{2}} \langle H' | h^\lambda | H \rangle \left(\bar{u}_l \gamma_\lambda (1-\gamma^5) u_{\nu_l} \right) \qquad \text{(II, 17)}$$

One sees that the study of this reaction allows one to get information on the matrix elements $\langle H'|h^\lambda(x)|H\rangle$.

Experimentally it is known that :

1) there exist processes in which $\Delta S = 0$, there is no change in strangeness of the hadrons. Examples :

$$n \to p + e^- + \bar{\nu}_e$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

$$\pi^- \to e^- + \bar{\nu}_e$$

$$\pi^- \to \pi^0 + e + \bar{\nu}_e \quad \text{etc.}$$

2) there exist processes in which $\Delta S = 1$ such as :

$$\Lambda \to p + e^- + \bar{\nu}_e$$

$$K^0 \to \pi^- + e^+ + \nu_e \quad \text{etc.}$$

and for these $\Delta Q = \Delta S$ where ΔQ is the change in charge of the hadronic system. One has then to write :

$$h^\lambda(x) = h^\lambda_{\Delta S=0}(x) + h^\lambda_{\Delta S=1}(x)$$
$$\equiv h^\lambda_{(o)}(x) + h^\lambda_{(1)}(x). \qquad \text{(II, 18)}$$

<u>For the reactions with $\Delta S = 0$</u> it is known that :

a) there exist reactions where $h^\lambda_{(o)}(x)$ has to have a component which is a polar vector. Example :

$$\pi^- \to \pi^0 + e + \bar{\nu}_e$$

The matrix element of the hadronic current has the form

$$< \pi^{o} | h_{\lambda(o)}(o) | \pi^{-} >$$

and the amplitude for this reaction is :

$$M = \frac{\mathcal{G}_F}{\sqrt{2}} < \pi^{o} | h_{\lambda(o)}(o) | \pi^{-} > < \bar{u}(e) \gamma^{\lambda} (1-\gamma^5) v_{\nu_e} >$$

Now as the states π^{-} and π^{o} are spin zero states with parity - 1,

$$< \pi^{o} | h_{\lambda(o)}(o) | \pi^{-} >$$

may be either a polar vector if h_λ has a component V_λ which is such a vector ; or an axial vector if h_λ has a component A_λ which is an axial vector. Now with the initial and final pions the only vectors available are their momenta p and p' and these are polar vectors. Therefore h_λ has to have a component V_λ which is a polar vector so that, as we shall see later :

$$< \pi^{o} | h_{\lambda(o)}(o) | \pi^{-} > = < \pi^{o} | V_{\lambda(o)}(o) | \pi^{-} > =$$

$$= \sqrt{2}\, F_{\pi}(q^2)\ (p_\lambda + p'_\lambda)\ \frac{G_V}{\mathcal{G}_F} \qquad \text{(II, 19)}$$

b) There exist reactions for which $h_\lambda(o)$ has to have an axial vector component. Example :

$$\pi^{-} \rightarrow \mu^{-} + \bar{\nu}_{\mu}$$

The amplitude here is the following :

$$M = \frac{\mathcal{G}_F}{\sqrt{2}} < 0 | h^{\lambda}(o) | \pi^{-} > < \bar{u}(p_e) \gamma_\lambda (1-\gamma^5) v(p_{\nu e})$$

and since $|\pi^-\rangle$ is a spin zero state with parity-1, then $\langle 0|h^\lambda(o)|\pi^-\rangle$ is either an axial vector if $h^\lambda(o)$ consists of V^λ or a polar vector if $h^\lambda(o)$ consists of A^λ. Since there exists only a polar vector p^λ in a state with a pion of momentum p one has to have :

$$\langle 0|h_\lambda(o)|\pi^-\rangle = \langle 0|A_\lambda(o)|\pi^-\rangle = i f_\pi (p^2) p_\lambda \frac{G_V}{\mathcal{G}_F}$$

and so h_λ has to have an axial component A_λ.

The pion decay is a Gamow-Teller transition.

<u>In general therefore we shall write</u> for $\Delta S = 0$:

$$h^\lambda_{(o)}(x) = V^\lambda_{(o)}(x) - A^\lambda_{(o)}(x) \qquad (II, 20)$$

This structure has been extended to the current with $\Delta S = 1$ because of reactions such as $K^+ \to \pi^0 + e^+ + \nu_e$, $K^+ \to \mu^+ + \nu_\mu$:

$$h^\lambda_{(1)}(x) = V^\lambda_{(1)}(x) - A^\lambda_{(1)}(x)$$

so that one has for the weak hadronic current :

$$h^\lambda_{(x)} = (V^\lambda_{(o)}(x) - A^\lambda_{(o)}(x)) + (V^\lambda_{(1)}(x) - A^\lambda_{(1)}(x)). \qquad (II, 21)$$

To each semi-leptonic weak process there corresponds a hadronic current matrix element and to this one can associate weak form factors.

Examples are :

a) Neutron beta-decay :

$$n \to p + e^- + \bar{\nu}_e \qquad \Delta S = 0$$

then one defines :

$$\mathcal{G}_F \langle p|V^{\lambda}_{(o)}(o)|n\rangle = \frac{\bar{u}_p(P_f)}{(2\pi)^3} \left\{ g_V^{(1)}(q^2)\gamma^{\lambda} + g_V^{(2)}(q^2) i\sigma^{\lambda\eta} q_{\eta} + \right.$$

$$\left. + g_V^{(3)}(q^2) q^{\lambda} \right\} u_n(P_i), \qquad \text{(II, 22)}$$

$q = P_f - P_i$.

$$\mathcal{G}_F \langle p|A^{\lambda}_{(o)}(o)|n\rangle = \frac{\bar{u}_p(P_f)}{(2\pi)^3} \left\{ g_A^{(1)}(q^2)\gamma^{\lambda} + g_A^{(2)}(q^2) i\sigma^{\lambda\eta} q_{\eta} + \right.$$

$$\left. + g_A^{(3)}(q^2) q^{\lambda} \right\} \gamma^5 u_n(P_i) \qquad \text{(II, 23)}$$

since with one initial neutron and a final proton one can form three independent polar vectors and three independent axial vectors which are the ones written down ; the coefficients, functions of the invariant momentum transfer squared, are the nucleon weak form factors :

$$\mathcal{G}_F \langle p|h^{\lambda}_{(o)}(o)|n\rangle = \frac{\bar{u}_p(P_f)}{(2\pi)^3} \left\{ \left[g_V^{(1)}(q^2) - g_A^{(1)}(q^2)\gamma^5 \right] \gamma^{\lambda} + \right.$$

$$+ \left[g_V^{(2)}(q^2) - g_A^{(2)}(q^2)\gamma^5 \right] \sigma^{\lambda\eta} q_{\eta} + \qquad \text{(II, 24)}$$

$$\left. + \left[g_V^{(3)}(q^2) - g_A^{(3)}(q^2)\gamma^5 \right] g^{\lambda} \right\} u_n(P_i).$$

These form factors are real functions of q^2.

The constants G_V and G_A in 12) and 14) are defined by :

$$G_V = g_V^{(1)}(o)$$

$$\qquad \text{(II, 25)}$$

$$G_A = g_A^{(1)}(o)$$

so that in the limit of vanishing momentum transfer :

$$(2\pi)^3 \mathcal{G}_F \langle p|h^\lambda_{(o)}|p\rangle = (\bar{u}_p(p)\gamma^\lambda(G_V - G_A\gamma^5)u_n(p)) \qquad \text{(II, 26)}$$

as inserted in formula (II, 11).

b) Pion decays :

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$
$$\to e^- + \bar{\nu}_e$$
$$\to \pi^o + e^- + \bar{\nu}_e$$

Consider the first two reactions.
The amplitude for the reaction

$$\pi^- \to l^- + \bar{\nu}_l \qquad \text{(II, 27)}$$

is :

$$S_{\pi 2} = -(2\pi)^4 \, i\,\delta^4(p_\pi - p_l - p_{\nu l})\, M_{\pi 2}$$

where :

$$M_{\pi 2} = \frac{\mathcal{G}_F}{\sqrt{2}} \langle o|h^\lambda(o)|\pi^-\rangle\langle l\bar{\nu}_l|l_\lambda^{+}(o)|o\rangle =$$

$$= \frac{\mathcal{G}_F}{\sqrt{2}} \langle o|h^\lambda(o)|\pi\rangle(\bar{u}(p_l)\gamma_\lambda(1-\gamma^5)v(p_{\nu l})) \qquad \text{(II, 28)}$$

Now it is clear that as :

$$h^\lambda(o) = (V^\lambda_{(o)}(o) - A^\lambda_{(o)}(o))$$

then :

$$\langle o|V^\lambda_{(o)}(o)|\pi\rangle = o \qquad \text{(II, 29)}$$

$$\langle o|A^\lambda_{(o)}(o)|\pi\rangle = i\, f_\pi(p_\pi)p_\pi{}^\lambda \frac{G_V}{\mathcal{G}_F}$$

where

$$f_\pi(p_\pi) \quad f_\pi(p_\pi^2) \quad f_\pi(m_\pi^2) \quad f_\pi$$

So, if $\cos\theta = \dfrac{G_V}{\mathcal{G}_F}$ then :

$$M_{\pi 2} = - \frac{i\,\mathcal{G}_F\, f_\pi}{\sqrt{2}} \cos\theta\; p_\pi^\lambda \,(\bar{u}(p_l)\gamma_\lambda(1-\gamma^5)v(p_{\nu_l}))$$

From the momentum conservation :

$$p_\pi = p_l + p_{\bar{\nu}_l}$$

and from the equation for the final leptons :

$$\bar{u}(p_l)(\not{p}_l - m_l) = 0$$

$$\not{p}_{\nu_l}\, v(p_l) = 0$$

one gets :

$$M_{\pi 2} = - \frac{i\,\mathcal{G}_F\, f_\pi}{\sqrt{2}} \cos\theta \quad m_l(\bar{u}(p_l)(1-\gamma^5)v(p_{\nu_l}))$$

The transition probability is :

$$\Lambda_{\pi 2} \equiv \frac{1}{\tau_{\pi 2}} = \frac{\mathcal{G}_F^2\, |f_\pi|^2}{8\pi} \cos^2\theta\; m^3_\pi \left(\frac{m_l}{m_\pi}\right)^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)$$

From $\tau_{\pi 2} = (2.55 \pm 0.02).10^{-8}$ sec. and from $G_V = \mathcal{G}_F \cos\theta = (1.4173 \pm 0.0034)$ 10^{-49} erg cm^3 determined from nuclear physics one obtains

$$f_\pi = 0.97\; m_\pi \simeq m_\pi$$

The ratio :

$$\frac{\Lambda(\pi \to e + \bar{\nu}_e)}{\Lambda(\pi \to \mu + \bar{\nu}_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{1-\left(\frac{m_e}{m_\pi}\right)^2}{1-\left(\frac{m_\mu}{m_\pi}\right)^2}\right)^2 \sim 1.2\times10^{-4}$$

is understood on the basis of the polarization state of leptons. Both neutrinos, which are assumed to have an exactly vanishing mass, are left-handed polarised particles (helicity = - 1); anti-neutrinos are therefore right-handed particles. In the limit $m_e \to 0$ the electron must be left-handed. However in the pion-decay of a pion at rest one has that the electron and the anti-neutrino must have the same polarisation as seen in the graphs :

$$\overset{p_{\bar{\nu}}}{\longleftarrow} \;\bullet\; \longrightarrow p_e$$

for momentum and

$$\Longleftarrow \;\bullet\; \Longleftarrow\!\!\!\!\!\!\Longrightarrow$$

for spin.

As in the limit $m_e=0$ the electron must be left-handed and as in the pion decay the electron is righ-handed then

$$\lim_{m_l \to 0} \Lambda_{\pi 2} = 0$$

In fact $m_e \neq 0$ and the electron and muon have both states of polarization. As $m_e \ll m_\pi$ one understands the small value of the above ratio.

This is also a confirmation of the V-A coupling, since besides A, only a pseudoscalar coupling P could contribute to $\Lambda_{\pi 2}$ and for this coupling the above ratio would be ~ 5.

III

CONSERVATION OF THE WEAK VECTOR CURRENT (CVC)

Let us consider the decay of muon :

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$$

and the β-decay of the neutron :

$$n \to p + e^- + \bar{\nu}_e$$

The amplitudes for the first reaction is of the form :

$$\begin{aligned} M_\mu &= \frac{\mathcal{G}_F}{\sqrt{2}} \langle \nu_\mu | l^\lambda | \mu \rangle \langle e \bar{\nu}_e | l_\lambda^+ | 0 \rangle \\ &= \frac{\mathcal{G}_F}{\sqrt{2}} (\bar{u}_{\nu_\mu}(p) \gamma^\lambda (1-\gamma^5) u_\mu)(\bar{u}_e \gamma_\lambda (1-\gamma^5) v_{\nu e}) \end{aligned} \qquad \text{(III, 1)}$$

whereas for the second reaction one has

$$\begin{aligned} M_n &= \frac{\mathcal{G}_F}{\sqrt{2}} \langle p | h^\lambda | n \rangle \langle e \bar{\nu}_e | l_\lambda^+ | 0 \rangle = \\ &= \frac{\mathcal{G}_F}{\sqrt{2}} \frac{G_V}{\mathcal{G}_F} (\bar{u}_p \gamma_\lambda (1 - \frac{G_A}{G_V} \gamma^5) u_n)(\bar{u}_e \gamma_\lambda (1-\gamma^5) v_{\nu e}) \end{aligned} \qquad \text{(III, 2)}$$

Thus in both amplitudes the electron-neutrino current multiplies

$$\frac{\mathcal{G}_F}{\sqrt{2}} (\bar{u}_{\nu_\mu} \gamma^\lambda (1-\gamma^5) u_\mu) \qquad \text{(III, 3)}$$

in the muon case and

$$\frac{\mathcal{G}_F}{2}\frac{G_V}{\mathcal{G}_F}(\bar{u}_p\, \gamma^\lambda (1-\frac{G_A}{G_V}\,\gamma^5)u_n) \qquad \text{(III, 4)}$$

in the neutron case.

If we loke at the first term in both these expressions-the vector terms :

$$\mathcal{G}_F(\bar{u}_{\nu_\mu}\ \gamma^\lambda u_\mu)$$

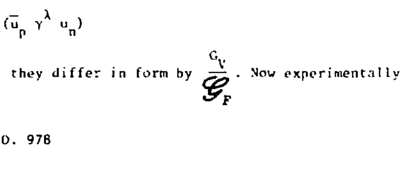
and $\qquad$ (III, 5)

$$\mathcal{G}_F\frac{G_V}{\mathcal{G}_F}\ (\bar{u}_p\ \gamma^\lambda\ u_n)$$

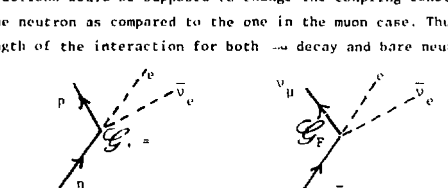
we see that they differ in form by $\frac{G_V}{\mathcal{G}_F}$. Now experimentally one has found

$$\frac{G_V}{\mathcal{G}_F} \simeq 0.\ 978 \qquad \text{(III, 6)}$$

The fact that G_V is almost equal to $\mathcal{G}_F$ is remarkable because the strong interactions would be supposed to change the coupling constant in the case of the neutron as compared to the one in the muon case. Thus if $\mathcal{G}_F$ is the strength of the interaction for both μ decay and bare neutron decay



then diagrams with exchange of pions in the neutron case



would give corrections which would change the value of the effective coupling constant.

[illegible] has led Feynman and Gell-man and Gershtein and Zeldovich to admit the hypothesis that <u>the vector-current $V^{\lambda}_{(o)}(x)$ for</u> [illegible] <u>is conserved</u> :

$$\partial_\lambda V^{\lambda}_{(o)}(x) = 0 \qquad \text{(III, 7)}$$

and thus that this current is not renormalized.

The conservation of $V^{\lambda}_{(o)}(x)$ garantees the cancellation of these graphs at zero lepton momentum transfers.

As $\frac{G_A}{G_V}$ [illegible], this means that the axial hadron current is, on the contrary, renormalized and $\partial_\lambda A^{\lambda}_{(o)}(x) \neq 0$.

The connection between the conservation of the vector-current and the non-renormalization of the corresponding coupling constant results from analogy with the fact that in electrodynamics the conservation of the electromagnetic current has the consequence that the strong interactions do not renormalize the charge of hadrons. First we remark that the time-derivative of the charge operator is :

$$\dot{Q} = i\left[H, Q\right]$$

the conservation of the electric current leads to

$$\dot{Q} = 0$$

and hence to :

$$\left[H, Q\right] = 0$$

There fore given two states $|p\rangle$ and $|p'\rangle$ one gets :

$$\langle p'|\left[H, Q\right]|p\rangle = (E'-E)\langle p'|Q|p\rangle = 0$$

where

$$H|p\rangle = E|p\rangle \text{ and } H|p'\rangle = E'|p'\rangle$$

Therefore Q can induce transitions only between states with the same energy- a consequence of charge conservation :

$$\langle p'|Q|p\rangle \neq \;\Rightarrow\; E' = E$$

Now :

$$\langle p'|Q|p\rangle = \int d^3 x \langle p'|j^o(x)|p\rangle = \int d^3 x \langle p'|j^o(\vec{x}, o)|p\rangle =$$

$$= \int d^3 x \langle p'|e^{i\vec{P}.\vec{x}} j^o(o) e^{-i\vec{P}.\vec{x}}|p\rangle =$$

$$= (2\pi)^3\delta^3(p' - p)\langle p'|j^o(o)|p\rangle =$$

$$= (2\pi)^3\delta^3(p' - p)\langle p|j^o(o)|p\rangle \quad \left(= \frac{e}{e_o} \delta^3(p' - p)u^+(p)u(p)F_1(o)\right)$$

So Q can also induce transitions only between states with the same momentum

$$\langle p'|Q|p\rangle = 0 \quad \text{if} \quad \vec{p}' \neq \vec{p}$$

We assume that the interaction lagrangean is $-e_o\, j^\mu(x)A_\mu(x)$ so that Q is the charge operator, $j^\mu(x)$ the current and e_o is the bare charge. Now in view of the vacuum polarization the observed charge in a small finite volume around and electron is not e_o but smaller, e, say.
If the states $|p\rangle$ are normalised in the following way

$$\langle p'|p\rangle = 2p^o\delta^3(p' - p)$$

then the <u>observed charge e</u> is by definition

$$\langle p'|e_o Q|p\rangle = 2p^o\delta^3(p' - p)e \qquad \text{(III, 7a)}$$

From the commutator between Q and a charged field $\psi(x)$

$$[Q, \psi(x)] = -\psi(x)$$

(which comes from $e^{iQ}\psi(x)e^{-iQ} = e^{-i}\psi(x)$) one obtains .

$$<0|[Q, \psi(x)]|p> = -<0|\psi(x)|p>$$

In[illegible] a complete set of states $|n>$:

$$\sum_n \{ <0|Q|n><n|\psi(x)|p> - <0|\psi(x)|n><n|Q|p> \} = - <0|\psi(x)|p>$$

[illegible] $Q|0> = 0$ then :

$$\sum_n <0|\psi(x)|n><n|Q|p> = <0|\psi(x)|p>$$

[illegible] in this [illegible] we have : a) the states which have the same three-momentum and same mass as the state $|p>$; b) degenerate states in mass with the same quantum numbers as $|p>$.

If we treat the electromagnetic interactions to lowest order we neglect the states b). This means that we want to see, for a hadron, the effect of the strong interactions on the charge. We shall have then :

$$\int \frac{d^3p_n}{2p_n^o} <0|\psi|p_n><p_n|Q|p> = <0|\psi|p>$$

Replacing $<n|Q|p>$ by $(2\pi)^3\delta^3(p - p_n)<p|j^o|p>$
we have :

$$\frac{(2\pi)^3}{2p^o} <0|\psi|p><p|j^o|p> = <0|\psi|p>$$

and so :

$$<0|j^o|p> = \frac{2p^o}{(2\pi)^3}$$

if $<0|\psi|p> \neq 0$.
Therefore :

$$<p'|Q|p> = (2\pi)^3\delta^3(p - p')<p|j^o|p> = \delta^3(p - p')2p^o$$

Comparing with

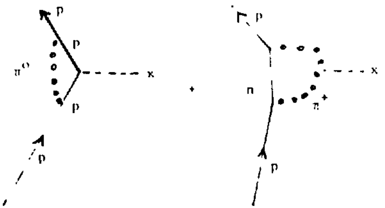
$$p' \, Q \, p = \delta^4(p - p')2p^0 \frac{e}{e_0}$$

we see that :

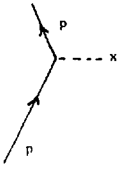
$$e = e_0$$

hence the strong interactions do not renormalize the charge. This is equivalent to Ward's identity which tells us that $Z_2 = Z_1$ and thus that when one does not renormalize the electromagnetic field one has $e = \frac{Z_2}{Z_1} e_0 = e_0$.

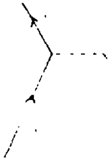
Current conservation makes the cancellation at zero momentum transfer of graphs such as :



so that there only remains the graph for bare charge



[illegible]



In these graphs we do not consider virtual photons.

If [illegible] the electromagnetic field the result is the renormalization of the charge of an electron :

$$e_{(electron)} = \sqrt{Z_3}\; e_o^{(electron)}$$

For any other particle such as a proton or a pion, as the *bare charge is not changed* by stong interactions, then if we assume that

$$e_o^{(proton)} = e_o^{(electron)}$$

then

$$e_{observed}^{(proton)} = \sqrt{Z_3}\; e_o^{(proton)} =$$

$$= \sqrt{Z_3}\; |e_o^{(electron)}| = |e_{observed}^{(electron)}|.$$

In other words if the parameter e_o is the same for all charged elementary particles interacting with the electromagnetic field, the fact that $\partial_\mu j^\mu_{electromgn} = 0$ insures that the observed charges of these particles are the same, up to the sign.

Thus in order to explain the near equality of $\mathcal{G}_F$ and G_V, Feynman and Gell-Mann [illegible] that $V_{(o)}^{\lambda}(x)$ [illegible] current. [illegible] vector [illegible] with some current which is conserved. [illegible] that conserved [illegible] according to Noether's theorem, from the invariance of [illegible] of a system of fields under certain transformation groups. As strong interactions are invariant under the group of isospin rotations SU(2), the isospin current

$$i_a^{\lambda}(x) \;, \quad a = 1,\, 2,\, 3$$

is conserved :

$$\partial_\lambda i_a^{\lambda}(x) = 0$$

and the generators of this group :

$$I_a = \int d^3x \; i_a^{o}(x) \;, \quad \left[I_a , I_b \right] = i\epsilon_{abc} I_c , \qquad (III,\ 8)$$

are the isospin operators.

One thus writes for $V_{(o)}^{\lambda}(x)$

$$V_{(o)}^{\lambda}(x) = \frac{G_V}{\mathcal{G}_F} \; i_+^{\lambda}(x) \qquad (III,\ 9)$$

where

$$i_+^{\lambda}(x) = i_1^{\lambda}(x) + i\, i_2^{\lambda}(x)$$

corresponds to a transition of hadrons with $\Delta Q = 1$

$$I_+ |n\rangle = |p\rangle$$

[illegible] relation :

$$[illegible]\ Y$$

[illegible] states with I_3 [illegible]

$$[illegible]$$

$$[illegible] = 0$$

[illegible]

$$\left[Q, j_+^{\lambda}(x) \right] = \left[I_3, j_+^{\lambda}(x) \right] = j_+^{\lambda}(x)$$

[illegible] and B are states with charges q_A and q_B :

$$Q|A> = q_A|A>, \quad Q|B> = q_B|B>$$

[illegible] :

$$<B| \left[Q, j_+^{\lambda}(x) \right] |A> = <B| (Qj_+^{\lambda} - j_+^{\lambda}Q)|A> =$$

$$= (q_B - q_A)<B|j_+^{\lambda}(x)|A> = <B|j_+^{\lambda}(x)|A>$$

hence :

$$q_B = q_A + 1 .$$

The identification :

$$V_{(o)}^{\lambda}(x) = \frac{G_V}{\mathcal{G}_F} \; j_+^{\lambda}(x) \qquad \text{(III, 10)}$$

is the conserved vector current hypothesis or simply CVC.

Feynmann and Gell-Mann supposed $G_V = \mathcal{G}_F$ and so $V^{\lambda}_{(o)}(x)$ would be $j^{\lambda}_{+}(x)$. Cabibbo has stated that $V^{\lambda}_{(o)}(x) \propto j^{\lambda}_{+}(x)$ and $\frac{G_V}{\mathcal{G}_F} = \cos\theta$ so that :

$$V^{\lambda}_{(o)}(x) = j^{\lambda}_{+}(x) \cos\theta$$

θ is called the Cabibbo angle. We shall come back to this later.

We shall see that the axial current $A^{\lambda}_{(o)}(x)$ is not conserved and thus the coupling constant is renormalised by the strong interactions.

II

CONSEQUENCES OF THE CVC HYPOTHESIS

Nuclear decays.

Consider the reaction :

$$O^{14} \to N^{14*} + e^+ + \nu_e$$

It is a pure Fermi transition the nuclei O^{14} and N^{14*} having both spin zero. The amplitude for this reaction will be :

$$\frac{\mathcal{G}_F}{\sqrt{2}} < N^{14*}(p')|V^{\lambda +}_{(o)}(0)|O^{14}(p) > \quad .$$

$$.(\bar{u}_{\nu e}(p_{\nu e})\ \gamma_\lambda (1 - \gamma^5)\ v(p_e))$$

Now according to the CVC hypothesis :

$$\mathcal{G}_F\ V^{\lambda +}_{(o)}(0) = G_V\, j^\lambda_{(-)}(0)$$

and the charge decreasing isospin operator is :

$$I_- = \int j^o_-\ (x)\ d^3x$$

Clearly we can express the matrix element of the hadronic current above in the following way :

$$< N'^{4*}(p')|j^{\lambda}_{(-)}(0)|O'^{4}(p)> = \frac{\sqrt{2}}{(2\pi)^3}\{f_+(q^2)\,P^{\lambda} +$$

$$+ f_-(q^2)\,Q^{\lambda}\}$$

where :

$$P^{\lambda} = p^{\lambda} + p'^{\lambda} \quad , \quad Q_{\lambda} = p^{\lambda} - p'^{\lambda}$$

If $j^{\lambda}_{(-)}(x)$ is conserved then :

$$f_-(q^2) = 0$$

and as q^2 is small :

$$f_+(q^2) \approx f_+(0)$$

So in a good approximation :

$$< N'^{4*}(p')|j^{\lambda}_{-}(0)|\,O'^{4}(p)> = \frac{\sqrt{2}}{(2\pi)^3}\,f_+(0)\,P^{\lambda}$$

To find $f_+(0)$ we compute the matrix element of I_- :

$$< N'^{4*}(p')|\int j^{o+}_{(+)}(x)\,d^3x|O'^{4}(p)> =$$

$$= < N'^{4*}(p')|\int j^{o}_{-}(x)\,d^3x|\,O'^{4}(p)> =$$

$$= < N'^{4*}(p')|I_-|O'^{4}(p)> = <p'|p><I,I_3-1|I_-|I\,I_3>$$

Now

$$ \langle N^{14}(p')|\int J^{o}(x)\,d^{3}x\,|O^{14}(p)\rangle = \frac{\sqrt{2}}{(2\pi)^{3}}\, f_{+}(0)\, P^{o}(2\pi)^{3}\delta^{3}(p - p') $$

Also

$$ 2\pi f_{+}(0)\, P^{o}\delta^{3}(p - p') = \langle p'|p\rangle \langle 1,I_{3}|I_{-}|1,I_{3}+1\rangle $$

$$ = (2p^{o})\,\delta^{3}(p'- p)\,\sqrt{2} $$

I ... N^{14} and O^{14} are I_{z} = { 0 for N^{14} ; 1 for O^{14} }

and :

$$ \langle 1\,I_{3}|I_{-}|1,I_{3}+1\rangle = \sqrt{2} $$

As :

$$ P^{o} = p^{o} + p'^{o} $$

then :

$$ P^{o}\delta^{3}(p - p') = 2p^{o}\delta^{3}(p'- p) $$

Therefore :

$$ f_{+}(0) = 1 $$

We thus see that

$$ \langle I\rangle \equiv \frac{1}{\langle N^{14*}|O^{14}\rangle}\langle N^{*14}(p')|I_{-}|O^{14}(p)\rangle = \sqrt{2}\; f_{+}(0) = \sqrt{2} $$

does not depend on the form factor, $f_{+}(0)$ being equal to 1, does not depend on the details of nuclear structure.

Now there exist nine pure Fermi transitions for systems with isospin $I =$ [illegible], from the equation for the mean life τ_F :

$$\frac{1}{\tau_F} = \frac{G_V^2 |\langle I_+ \rangle|^2 f}{2\pi^3}$$

one gets :

$$\frac{1}{f\tau_1} = \frac{G_V^2 |\langle I_+ \rangle|^2}{2\pi^3}$$

The product $f\tau_1$ does not depend therefore, [illegible] exact symmetry, on the reaction

Table

	$f\tau_1$ [illegible]
$C^{10} \to B^{10} + e^+ + \nu_e$	3035
$O^{14} \to N^{14*} + e^+ + \nu_e$	[illegible]
$Al^{26} \to Mg^{26} + e^+ + \nu_e$	[illegible]
$Cl^{34} \to S^{34} + e^+ + \nu_e$	[illegible]
$Sc^{42} \to Ca^{42} + e^+ + \nu_e$	3[illegible]22
$V^{40} \to Ti^{40} + e^+ + \nu_e$	3[illegible]32
$Mn^{50} \to Cr^{50} + e^+ + \nu_e$	[illegible]
$Co^{54} \to Fe^{54} + e^+ + \nu_e$	[illegible]

<u>Weak magnetism</u> :

The isospin operators :

$$I_a = \int j_a^o(x)\, d^3x$$

[illegible] the SU(2) commutation rules :

$$[I_a, I_b] = i\epsilon_{abc} I_c$$

[illegible]

$$[I_+, I_-] = 2I_3$$

$$[I_3, I_+] = + I_+$$

$$[I_3, I_-] = - I_-$$

[illegible]

$$[j_3(x), I_+] = j_+(x)$$

It follows from CVC for the neutron-proton transition :

$$\mathcal{G}_F \langle p', I_3 = \tfrac{1}{2} | V_{(\nu)}(0) | p, I_3 = -\tfrac{1}{2} \rangle = G_V \langle p', I_3 = \tfrac{1}{2} | j_+^\lambda(0) | p, I_3 = -\tfrac{1}{2} \rangle =$$

$$= G_V \langle p', I_3 = \tfrac{1}{2} | [j_3(0), I_+] | p, I_3 = -\tfrac{1}{2} \rangle$$

But as Y commutes with I_- we have :

$$\mathcal{G}_F \langle p(p') | V_{(\nu)}(0) | n(p) \rangle = G_V \langle p(p') | [j_3^\lambda(0) + \tfrac{1}{2} j_Y^\lambda(0), I_+] | n(p) \rangle$$

$$= G_V \langle p(p') | [j^\lambda_{(\gamma)}(0), I_+] | n(p) \rangle =$$

$$= G_V \{ \langle p(p') | j^\lambda_{(\gamma)}(0) | p(p) \rangle - \langle n(p') | j^\lambda_{(\gamma)}(0) | p(p) \rangle \}$$

since

$$I_+ | n(p) \rangle = | p(p) \rangle$$

$$\langle p(p') | I_+ = \langle n(p') |$$

We thus see that CVC implies a relationship between the vector weak form factors and the electromagnetic form factors. Indeed we know that :

$$\langle p(p')|j^{\lambda}_{(em)}(0)|p(p)\rangle = \frac{\bar{u}_p(p')}{(2\pi)^3}\left\{ F_1^p(q^2)\gamma^\lambda + \frac{i}{2m_p}\sigma^{\lambda\nu}q_\nu F_2^p(q^2)\right\} u_p(p)$$

$$\langle n(p')|j^{\lambda}_{(em)}(0)|n(p)\rangle = \frac{\bar{u}_n(p')}{(2\pi)^3}\left\{ F_1^n(q^2)\gamma^\lambda + \frac{i}{2m_n}\sigma^{\lambda\nu}q_\nu F_2^n(q^2)\right\} u_n(p)$$

and

$$\mathcal{G}_F \langle p(p')|V^{\lambda}_{(o)}(0)|n(p)\rangle = \frac{\bar{u}_p(p')}{(2\pi)^3}\left\{ g_V^{(1)}(q^2)\gamma^\lambda + g_V^{(2)}(q^2)\frac{i\sigma^{\lambda\nu}q_\nu}{2m_N} + \right.$$

$$\left. + g_V^{(3)}(q^2)q^\lambda \right\} u_n(p)$$

In the limit when $\partial_\lambda V^\lambda_{(o)}(x) = 0$ one has $g_V^{(3)}(q^2) = 0$ because in this limit $m_n = m_p$. So we can write, $u(p)$ designating the nucleon spinor

$$\mathcal{G}_F \langle p(p')V^{\lambda}_{(o)}(0)|n(p)\rangle = G_V \langle p(p')|j^{\lambda}_{+}(0)|n(p)\rangle =$$

$$= \frac{\bar{u}(p')}{(2\pi)^3}\left\{ g_V^{(1)}(q^2)\gamma^\lambda + g_V^{(2)}(q^2)\frac{i\sigma^{\lambda\nu}q_\nu}{2m_N}\right\}\tau_+ u(p)$$

and if we set :

$$F_i^p = \frac{1}{2}(F_i^S + F_i^V) \qquad i = 1,2$$

$$F_i^n = \frac{1}{2}(F_i^S - F_i^V) \qquad i = 1,2$$

then

$$\langle p'|j^{\lambda}_{(\gamma)}(0)|p\rangle = \frac{\bar{u}(p')}{(2\pi)^3}\Big\{\tfrac{1}{2}[F_1^S(q^2)+F_1^V(q^2)\tau_3]\frac{\gamma^{\lambda}}{2} +$$

$$+\tfrac{1}{2}[F_2^S(q^2)+F_2^V(q^2)\tau_3]\frac{i\sigma^{\lambda\eta}q_\eta}{2m_N}\Big\}u(p)$$

So we compare the weak vector matrix element :

$$\frac{\bar{u}(p')}{(2\pi)^3}\Big\{g_V^{(1)}(q^2)\gamma^{\lambda}\tau_+ + g_V^{(2)}(q^2)\frac{i\sigma^{\lambda\eta}q_\eta}{2m_N}\tau_+\Big\}u(p)$$

with the vector part of the electromagnetic current matrix element :

$$\frac{\bar{u}(p')}{(2\pi)^3}\Big\{F_1^V(q^2)\gamma^{\lambda}\frac{\tau_3}{2} + F_2^V(q^2)\frac{i\sigma^{\lambda\eta}q_\eta}{2m_N}\frac{\tau_3}{2}\Big\}u(p)$$

where $\tau_+ = \frac{1}{2}(\tau_1 + i\tau_2)$

The result is :

$$g_V^{(1)}(q^2) = G_V\,F_1^V(q^2)$$

$$g_V^{(2)}(q^2) = G_V\,F_2^V(q^2)$$

Therefore we may finally write as a consequence of CVC :

$$\langle p'|j^{\lambda}_{(\gamma)}(0)|p\rangle = \frac{\bar{u}(p')}{(2\pi)^3}\Big\{\tfrac{1}{2}[F_1^S+F_1^V\tau_3]\gamma^{\lambda}+\tfrac{1}{2}[F_2^S+F_2^V\tau_3]\frac{i\sigma^{\lambda\eta}q_\eta}{2m_N}\Big\}u(p)$$

$$\langle p'|j^{\lambda}_{+}(0)|p\rangle = \frac{\bar{u}(p')}{(2\pi)^3}\Big\{\tfrac{1}{2}F_1^V(\tau_1+i\tau_2)\gamma^{\lambda}+\tfrac{1}{2}F_2^V(\tau_1+i\tau_2)\frac{i\sigma^{\lambda\eta}q_\eta}{2m_N}\Big\}u(p)$$

So $j^V_{(e.m.)}(x)$, $j_+(x)$, $j_-(x)$ belong to the same triplet just as π^+, π^0, π^-.

From $g_V^{(2)}(q^2) = G_V F_2^V(q^2)$ it follows that

$$\frac{g_V^{(2)}(0)}{G_V} = \mu_p - \mu_n$$

$g_V^{(2)}$ is the weak magnetism term.

Pion beta-decay

The amplitude of the reaction

$$\pi^+ \to \pi^0 + e^+ + \nu_e$$

is :

$$M = \frac{\mathcal{G}_F}{\sqrt{2}} \langle \pi^0 | h^+_\lambda(0) | \pi^+ \rangle (\bar{\nu}_e \gamma^\lambda (1 - \gamma^5) e)$$

where :

$$\langle \pi^0 | h^+_\lambda(0) | \pi^+ \rangle = \langle \pi^0 | V^+_{(o)\lambda}(0) | \pi^+ \rangle =$$

$$= \frac{G_V}{\mathcal{G}_F} \sqrt{2}\, F_\pi(q^2)(p' + p)_\lambda$$

where $F_\pi(q^2)$ is the electromagnetic form factor of the pion.

We have indeed, by CVC :

$$\langle \pi^0|h^+(0)|\pi^+\rangle = \langle \pi^0|V^+_{(\mu)}(0)|\pi^+\rangle = \frac{G_V}{\mathcal{G}_F}\langle \pi^0|j^+(0)|\pi^+\rangle$$

$$= \frac{G_V}{\mathcal{G}_F}\langle \pi^0|[I_+, j(0)]|\pi^+\rangle =$$

$$= \frac{G_V}{\mathcal{G}_F}\langle \pi^0|[I_+, j_3(0) + \frac{1}{2} j_Y(0)]|\pi^+\rangle =$$

$$= \frac{G_V}{\mathcal{G}_F}\langle \pi^0|[I_+, j_{(\gamma)}(0)]|\pi^+\rangle$$

Remember

$$I_\pm|I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\;|I\, I_3 \pm 1\rangle$$

Now

$$I_+|\pi^0\rangle = \sqrt{2}|\pi^+\rangle \qquad \langle\pi^0|I_- = \langle\pi^+|\sqrt{2} \;;\; I_-|\pi^+\rangle = \sqrt{2}|\pi^0\rangle$$

and
$$\langle \pi^0|j_{(\gamma)}(0)|\pi^0\rangle = 0$$

So :

$$\langle \pi^0|h_\lambda^+(0)|\pi^+\rangle = \frac{G_V}{\mathcal{G}_F}\langle\pi^+|j^\lambda_{(\gamma)}(0)|\pi^+\rangle\sqrt{2}$$

Now $\langle\pi^+|j^\lambda_{(\gamma)}(0)|\pi^+\rangle = (p + p')^\lambda F_\pi(q^2)$

where $F_\pi(0) = 1$

As in this reaction q^2 is small one makes the approximation $F_\pi(q^2) \approx F_\pi(0) = 1$.

Experimentally one has

$$\mathscr{R} = \frac{\Lambda(\pi^+ \to \pi^0 + e^+ + \nu_e)}{\Lambda(\pi^+ \to \mu^+ + \nu_\mu)} \cong (1.03 \pm 0.07).10^{-8}$$

Theoretically by CVC above :

$$\mathscr{R} = 1.05.10^{-8}$$

<u>Sigma beta-decay</u>

In the reactions

$$\Sigma^\pm \to \Lambda + e^\pm + \begin{matrix} \nu_e \\ \bar{\nu}_e \end{matrix}$$

the allowed vector contribution vanishes according to the CVC hypothesis.

To see this we write :

$$\mathscr{G}_F < \Lambda(p')|V^\lambda_{(o)}(0)|\Sigma(p)> = G_V \frac{\bar{u}_\Lambda(p')}{(2\pi)^3} \{ F^\lambda_1(q^2)\gamma^\lambda +$$

$$+ \frac{i\sigma^{\lambda\eta} q_\eta}{2m_\Lambda} F^V_2(q^2) + g^{(3)}_V(q^2) q^\lambda \} u_\Sigma(p)$$

Now in the case of the reaction $n \to p + e + \bar{\nu}_e$, in the limit of exact SU_2 symmetry $m_n = m_p$ and this together with $\partial_\lambda V^\lambda(x) = 0$ gives $g^3_V(q^2) = 0$. Here however in the limit of exact SU_2 symmetry $m_\Lambda \neq m_\Sigma$ and thus we have in this limit :

$$\mathcal{G}_F \quad \langle p' | \partial_\lambda V^\lambda_{(o)}(0) | \Sigma(p) \rangle =$$

$$= G_V \bar{u}_\Lambda(p') \{ (m_\Lambda - m_\Sigma) F^\lambda_V(q^2) + g_V^{(3)}(q^2) \frac{q^2}{2m} \} u_\Sigma(p) = 0$$

If the form factor $g_V^{(3)}(q^2)$ had a pole at $q^2 = 0$ this would mean a scalar quantum of zero mass which would be coupled to the current vector in the transition $\Sigma \to \Lambda$.

Therefore if

$$\lim_{q^2 \to 0} q^2 g_V^{(3)}(q^2) = 0$$

one has that the CVC hypothesis implies

$$F^V_\Sigma (0) = 0.$$

As in the beta-decay of the pion we have here :

$$\langle \Lambda | V^\lambda_{(o)}(0) | \Sigma^- \rangle = \frac{G_V}{\mathcal{G}_F} \langle \Lambda | j^\lambda_+(0) | \Sigma^- \rangle =$$

$$= \frac{G_V}{\mathcal{G}_F} \langle \Lambda | [j^\lambda_3(0), I_+] | \Sigma^- \rangle = \frac{G_V}{\mathcal{G}_F} \langle \Lambda | [j^\lambda_3(0) + \frac{1}{2} j^\lambda_y(0), I_+] | \Sigma^- \rangle$$

$$= \frac{G_V}{\mathcal{G}_F} \langle \Lambda | [j^\lambda_{(\gamma)}(0), I_+] | \Sigma^- \rangle =$$

$$= \sqrt{2} \frac{G_V}{\mathcal{G}_F} \langle \Lambda | j^\lambda_{(\gamma)}(0) | \Sigma^0 \rangle \qquad \text{since} \qquad I_+ | \Sigma^- \rangle = \sqrt{2} | \Sigma^0 \rangle$$

$$I_+ | \Lambda \rangle = 0$$

Therefore :

$$< \Lambda | V^{\lambda}_{(x)}(0) | \Sigma^- > = \frac{\sqrt{2} G_V}{\mathcal{G}_F} \bar{u}_\Lambda \left\{ F_1^V(q^2) \gamma^\lambda + F_2^V(q^2) \frac{i\sigma^{\lambda\eta} q_\eta}{2m_\Sigma} \right\} u_{\Sigma}$$

As $$F_1^V(0) = 0$$

the first forbidden contribution will be, for small q :

$$\frac{\sqrt{2} G_V}{\mathcal{G}_F} F_2^V(0) \, \bar{u}_\Lambda \frac{i\sigma^{\lambda\eta} q_\eta}{2m_\Lambda} u_\Sigma$$

$F_2^V(0)$ is the transition dipole moment for the reaction $\Sigma^0 \to \Lambda^0 + \gamma$, $F_2^V(0) = \mu^{\Sigma\Lambda}$.

The term in $F_3^V(q^2) q^\lambda$ is omitted because it is small, of the order $\frac{m_e}{m_\Lambda}$, in the amplitude. For this term must be multiplied by the leptonic current $(\bar{u}_e \gamma_\lambda (1 - \gamma^5) v_\nu)$ and then

$$\frac{F_3^V}{m_\Lambda} q^\lambda (\bar{u}_e \gamma_\lambda (1 - \gamma^5) v) = F_3^V \frac{m_e}{m_\Lambda} (\bar{u}_e (1 - \gamma^5) v_e)$$

Thus

$$< \Lambda | V^{\lambda}_{(x)}(0) | \Sigma^- > = \frac{G_V}{\mathcal{G}_F} \frac{\sqrt{2}\, \mu^{\Sigma\Lambda}}{2m_\Lambda} (\bar{u}_\Lambda i\sigma^{\lambda\eta} q_\eta u_\Sigma)$$

and the dominant contribution comes from the axial current.

All experiments, at least those for small q, confirm the validity of the CVC hypothesis.

V

PCAC-PARTIAL CONSERVATION OF THE AXIAL CURRENT

The Goldberger Treiman relation

If we consider the amplitudes (III,1) and (III,2) for the muon decay and the neutron beta-decay respectively we see that the axial current terms which multiply the lepton current differ by the occurrence of the constant $\frac{G_A}{\mathcal{G}_F} \frac{G_A}{G_V}$ in the neutron case. We saw that G_V is nearly $\mathcal{G}_F$ and this was explained by the CVC assumption.

In the case of the axial current one has

$$\frac{G_A}{G_V} \cong 1.23$$

and this suggests that the coupling constant is renormalized in this case by strong interactions. The axial current must therefore not be conserved although the value of the ratio $\frac{G_A}{G_V}$ indicates that there is partial conservation, a notion which we shall make precise later.

Now, that the axial current is not conserved follows from the existence of the pion decay (II,27).

By analogy with the relation (III,9) we shall define an axial current $g_+^\lambda(x)$ such that

$$A^\lambda_{(o)}(x) = \frac{G_V}{\mathcal{G}_F} \; g_+^\lambda(x)$$

where $A^\lambda_{(o)}(x)$ is the hadron axial current with $\Delta S = 0$. In this notation the $\Delta S=0$ part of $h^\lambda(x)$, (II,20), has the form :

$$h^\lambda_{(o)}(x) = \frac{G_V}{\mathcal{G}_F}\left(j^\lambda_+(x) - g^\lambda_+(x)\right)$$

and the current $h^\lambda(x)$ is :

$$h^\lambda(x) = h^\lambda_{(o)}(x) + V^\lambda_{(1)}(x) - A^\lambda_{(1)}(x).$$

Now from (II,29) we see that for the pi-decay :

$$\langle 0|g^\lambda_+(x)|\pi^-\rangle = i f_\pi p^\lambda e^{-ipx} \qquad (V,1)$$

therefore, if p^λ is the pion-momentum :

$$< 0|\partial_\lambda g^\lambda_+(x)|\pi^- > = f_\pi p^2 e^{-ipx} = f_\pi m_\pi^2 \, e^{-ipx} \qquad (V,2)$$

since $p^2 = m_\pi^2$.

We see that this matrix elements is different from zero since f_π and m_π are non-vanishing ; it is impossible therefore to have $\partial_\lambda g^\lambda_+(x) = 0$. The conservation of g^λ_+ would be possible if the pion were massless. As the pion mass is small in the world of hadrons one may be led to imagine that there is a kind of " partial conservation " of the axial current (PCAC). This idea was formulated by Nambu who proposed to <u>assume that all matrix elements of $\partial_\lambda g^\lambda_+(x)$, and not only $\langle 0|\partial_\lambda g^\lambda_+(x)|\pi^-\rangle$, are proportional to some positive power of m_π</u> .

Consider the neutron-decay transition and its axial current matrix element :

$$G_V \langle P(p')|g^\lambda_+(0)|N(p)\rangle = \frac{1}{(2\pi)^3}\,\bar{u}_p(p')\,\Big\{ g_A^{(1)}(q^2)\gamma^\lambda + g_A^{(2)}(q^2)\,\frac{i\sigma^{\lambda\eta}q_\eta}{2M_N} +$$

$$+ g_A^{(3)}(q^2) q^\lambda \} \gamma^5 u_N(p)$$

Now :

$$\langle P(p')|\partial_\lambda g_\pi^\lambda(x)|N(p)\rangle = \langle P(p')|\partial_\lambda e^{iqx} g_\pi^\lambda(0)|N(p)\rangle =$$

$$= iq_\lambda \langle P(p')|g_\pi^\lambda(0)|N(p)\rangle$$

$q = p' - p$

Now :

$$\bar{u}_P(p')(p'_\lambda - p_\lambda)\gamma^\lambda \gamma^5 u_N(p) = (M_P + M_N)\bar{u}_P(p')\gamma^5 u_N(p) \cong$$

$$\cong 2M_N \bar{u}_P(p')\gamma^5 u_N(p)$$

so :

$$\langle P(p')|\partial_\lambda g_\pi^\lambda(0)|N(p)\rangle = \frac{i\bar{u}_P(p')}{(2\pi)^3} \{ 2M_N f_A^{(1)}(q^2) + q^2 f_A^{(3)}(q^2) \} \gamma^5 u_N(p)$$

where :

$$f_A^{(i)}(q^2) = \frac{g_A^{(i)}(q^2)}{C_V}$$

We see that :

$$\langle P(p')|\partial_\lambda g^\lambda(0)|N(p) = \frac{i\bar{u}(p')}{(2\pi)^3} \{ 2M_N f_A^{(1)}(q^2) + q^2 f_A^{(3)}(q^2) \} \gamma^5 \tau_+ u(p)$$

So for the component a, in isospin, of g^λ :

$$\langle P(p')|g_a^\lambda(0)|N(p)\rangle = \frac{i\bar{u}(p')}{(2\pi)^3}\left\{2M_N f_A^{(1)}(q^2) + q^2 f_A^{(3)}(q^2)\right\}\gamma_5 \frac{\tau_a}{2} u(p)$$

Call

$$D(q^2) = 2M_N f_A^{(1)}(q^2) + q^2 f_A^{(3)}(q^2)$$

so that :

$$\langle P(p')|\partial_\lambda g_a^\lambda(0)|N(p)\rangle = \frac{i\bar{u}(p')}{(2\pi)^3}\,\gamma_5\,\frac{\tau_a}{2}\,u(p)\,D(q^2) \qquad (4,28)$$

We see that the vanishing of the above matrix elements would require the vanishing of $D(q^2)$.

As $f_A^{(1)}(q^2) \neq 0$ since we know that

$$f_A^{(1)}(0) = \frac{G_A}{G_V} \simeq 1,23$$

the vanishing of $D(q^2)$ would require $M_N = 0$ and $f_A^{(3)}(q^2) = 0$ which is not realized in the physical world.

However it is possible that $D(q^2)$ vanish without $f_A^{(1)}(q^2)$ vanishing for $q^2 \to 0$. This is the case if $f_A^{(3)}(q^2)$ has a pole at $q^2 = 0$. We can then write :

$$2M_N f_A^{(1)}(q^2) + q^2\, f_A^{(3)}(q^2) = 0$$

and :

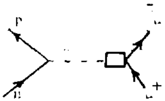
$$f_A^{(1)}(0) = -\frac{1}{2M_N}\lim_{q^2\to 0} q^2\, f_A^{(3)}(q^2) \neq 0$$

[illegible] $f_A^{(-)}(0) = \frac{g_A}{g_V}$

or if you wish :

$$f_A^{(3)}(q^2) = \frac{2M_N f_A^{(-)}(q^2)}{q^2}$$

which means that $f_A^{(3)}(q^2)$ has a pole at $q^2 = 0$ with residue $-2M_N f_A^{(-)}(0)$. This corresponds to an exchange of a zero mass pion between the nucleon and the axial current. Thus if in the muon-capture we consider the graph (we consider this just to take the charge increasing hadron current) :



then we see that the effective matrix-element $\langle p(p')|g_+^i|N(p)\rangle$ which multiplies the lepton current is

$$\langle P(p')|g_+^i(0)|N(p)\rangle = i(iq^i f_\pi(q^2)) \frac{i}{-q^2} \sqrt{2} g_{\pi N} \cdot$$

$$\frac{i}{(2\pi)^3} \cdot (\bar{u}(p') i\gamma^5 \tau_+ u(p))$$

The last term is the vertex with $g_{\pi N}$ the coupling constant nucleon-pion (pseudoscalar interaction), the second is the propagator for a zero mass pion and the first is the matrix element for absorption of this pion.

We see that this diagram gives a form factor

$$f_A^{(3)}(q^2) = \frac{\sqrt{2}\; g_{\pi N} f_\pi(q^2)}{-q^2}$$

If we assume that this graph dominates the form factor at $q^2 = 0$ we shall have :

$$\lim_{q^2 \to 0} \quad q^2 f_A^{(3)}(q^2) = -\sqrt{2}\; g_{\pi N} f_\pi(0)$$

and so as :

$$\lim_{q^2 \to 0} \quad q^2 f_A^{(3)}(q^2) = -2M_N\; f_A^{(\cdot)}(0)$$

we get the Goldberger-Treiman relation :

$$\boxed{g_{\pi N} f_\pi(0) = \sqrt{2} M_N \;\frac{G_A}{G_V}} \qquad \text{where } f_A^{(\cdot)}(0) \quad \frac{G_A}{G_V}$$

This relation would be exact if the pion mass were zero provided that the form factor $f_A^{(3)}$ is dominated by the pion pole. Note that the residue of this pole is $\sqrt{2}\; g_{\pi N} f_\pi(0)$. What is the value of $f_\pi(0)$?
What is experimentally known is $f_\pi(m_\pi^2)$ from the decay of a pion in its mass shell. In the diagram above the pion is virtual $q^2 \neq m_\pi^2$ and we do not know $f_\pi(q^2)$.

Now following Nambu, imagine that the pion mass is built up. Then $f_A^{(3)}(q^2)$ will acquire a pion pole at $q^2 = m_\pi^2$. The PCAC hypothesis consists in saying that in this process of building up the pion mass quantities such as $f_\pi(q^2)$ and $f_A^{(3)}(q^2)$ change slowly in the domain $0 \leq q^2 \lesssim m_\pi^2$.

For a massive pion the contribution of the virtual pion diagram to $f_A^{(3)}(q^2)$ is clearly :

$$f_A^{(2)}(q^2) = \frac{\sqrt{2}\; g_{\pi N} f_\pi(q^2)}{m_\pi^2 - q^2}$$

[illegible] the pole of $f_A^{(3)}$ [illegible] with residue $\sqrt{2}\, g_{\pi N} f_\pi(m^2)$. If we assume that [illegible] the value of the form factor down to $q^2 = 0$ this means [illegible] will be approximately equal to that at $q^2 = 0$ and so [illegible]

$$f(m^2) \; [illegible] \; f(0)$$

[illegible] this residue to that of $f_A^{(3)}(q^2)$ at $q^2 = 0$ one obtains the Goldberger-Treiman relation.

This is the essence of the PCAC assumption.

We [illegible] this in another way.

[illegible]

$$D(q^2) = 2M_N f_A^{(1)}(q^2) + q^2 f_A^{(3)}(q^2) .$$

[illegible] pion pole at $q^2 = m_\pi^2$ dominates $f_A^{(3)}(q^2)$ and hence $D(q^2)$ in the range [illegible] we can write a dispersion relation for $D(q^2)$ of the form :

$$D(q^2) = \frac{\sqrt{2}\, g_{\pi N} f_\pi m_\pi^2}{m_\pi^2 - q^2} + \frac{1}{\pi} \int_{(3m_\pi)^2}^{\infty} \frac{\sigma(t)dt}{t-q^2}$$

[illegible] unsubtracted dispersion relation one assumes $\lim_{q^2 \to \infty} D(q^2) = 0$ which is consistent with the notion that for high-momentum transfer one must neglect all [illegible] and thus the axial current is conserved.

Now the PCAC hypothesis can also be formulated by saying that the contribution of the integral above is negligible as compared to the pole term in the interval $0 \le q^2 \lesssim m_\pi^2$.

One will have then :

$$D(0) = \sqrt{2}\, g_{\pi N} f_\pi$$

and as

$$D(0) = 2m_N f_A^{(-)}(0)$$

one gets once again the Goldberger-Treiman relation. We also see that to assume with Nambu that all matrix elements of $\partial_\lambda A^\lambda(0)$ are proportional to m_π^2 means to state that the kernel $\sigma(t)$ of the integral is also proportional to m_π^2 . Therefore this integral will have the behaviour :

$$\lim_{m_\pi^2 \to 0} \int_{(3m)^2}^{\infty} \frac{\sigma(t)dt}{t} = 0$$

and so :

$$\lim_{m_\pi^2 \to 0} D(0) = \sqrt{2}\, g_{\pi N} f_\pi$$

Another derivation of the Goldberger-Treiman relation is obtained in the Gell Mann-Lévy model which assumes that

$$\partial_\lambda g_a^\lambda(x) = C \varphi_a(x)$$

where C is a proportionality constant and $\varphi_a(x)$ is the interpolating field for the pion π_a. Normalizing this field in such a way that :

$$\langle 0|\varphi_a(0)|\pi_b\rangle = \delta_{ab}$$

then from

$$g_+^\lambda = g_1^\lambda + i g_2^\lambda$$

and

$$\varphi(x) = \frac{1}{\sqrt{2}} (\varphi_1(x) + i\varphi_2(x))$$

one gets

$$\partial_\mu g^\mu_+(x) = \sqrt{2}\, C\, \varphi(x)$$

Let $j_a(x)$ be the source of the field φ_a :

$$(\Box + m^2)\,\varphi_a(x) = j^\pi_a(x)$$

or :

$$(\Box + m^2)\,\varphi(x) = j^\pi_+(x), \qquad j^\pi_+ = \frac{1}{\sqrt{2}}\,(j^\pi_1 + i j^\pi_2)$$

Then :

$$< P'|\partial_\mu g^\mu_+(0)|P > = C\sqrt{2} < P'|\varphi(0)|P > =$$

$$= C\sqrt{2}\,\frac{< P'|(m_\pi^2-q^2)\varphi(0)|P >}{m_\pi^2 - q^2} = C\sqrt{2}\,\frac{< P'|j^\pi_+(0)|P >}{m_\pi^2 - q^2}$$

Now by definition

$$< P'|j^\pi_a(0)|\,P > = \frac{g}{(2\pi)^3}\,\bar{u}(P')i\gamma^5\,\frac{\tau_a}{2}\,u(P) \text{ for } q^2 = m_\pi^2$$

One defines then $g(q^2)$ by :

$$< P'|j^\pi_a(0)|P >_{q^2 \neq m_\pi^2} = \frac{g(q^2)}{(2\pi)^3}\,\bar{u}(P')i\gamma^5\,\frac{\tau_a}{2}\,u(P), \qquad q^2 \neq m_\pi^2$$

Comparison with equation (V, 9) gives :

$$D(q^2) = \frac{C\sqrt{2}\, g(q^2)}{m_\pi^2 - q^2} = \frac{\sqrt{2} f_\pi m_\pi^2\, g(q^2)}{m_\pi^2 - q^2} \quad ; \quad C = f_\pi m_\pi^2$$

$g(q^2)$ is the coupling constant pion-nucleon outside of the mass shell $q^2 \neq m_\pi^2$.

The definition of $D(q^2)$ leads then to :

$$g(0)\, f_\pi = \sqrt{2}\, m_N \frac{G_A}{G_V}$$

which is the G-T relation if one assumes that $\langle P' | j_a^{\lambda}(0) | P \rangle$ varies slowly, as a function of q^2, in the interval $0 \le q^2 \lesssim m_\pi^2$ and thus replaces $g(0)$ by $g(m_\pi^2)$.

SU(2) ⊗ SU(2) symmetry

Another interpretation of the Goldberger-Treiman relation postulates that the strong interaction Lagrangian is composed of two terms

$$L_{strong} = L_o + \lambda\, L_i$$

The first term has a symmetry called chiral symmetry or SU(2) ⊗ SU(2) symmetry. The second term, for $\lambda \neq 0$, breaks this symmetry but has a simple SU(2) symmetry. What is the chiral symmetry ? This will be now defined. Physically we require that in the limit $\lambda \to 0$ both vector and axial vector currents be conserved. When L_i is included, only the vector current is conserved.

If only L_o remains, i.e., if $\lambda = 0$ then we have 6 generators of the symmetry group of L_o :

$$I_a = \int j_a^o(x)\, d^3x \; , \qquad a = 1,2,3$$

where we have set : $\quad V_a^{\lambda}(x) = \dfrac{G_V}{\mathcal{G}_F}\, j_a^{\lambda}(x)$;

and

$$I_a^5 = \int g_a^5(x)\, d^3x, \qquad a = 1,2,3,$$

where we have written

$$V_a^\lambda(x) = \frac{G_V}{\mathcal{G}_1} g_a^\lambda(x)$$

Now the I_a's define a subgroup of this group namely the SU(2) subgroup :

$$[I_a, I_b] = i\,\epsilon_{abc}\, I_c$$

As I_a^5 transform as a vector representation of SU(2) we have :

$$[I_a, I_b^5] = i\,\epsilon_{abc}\, I_c^5$$

To close the algebra of the 6 operators we need the commutation relations between I_a^5 and I_b^5. Based on the universality of weak interactions for the lepton world and for the hadron world Gell-Mann postulated that the I_a^5's obey the same commutation rules as the analogous operators for leptons, namely :

$$[I_a^5, I_b^5] = i\,\epsilon_{abc}\, I_c$$

Indeed in the lepton world the weak current is :

$$J_\mu(x) = \bar{\nu}_e \gamma_\mu (1-\gamma^5)e + \bar{\nu}_\mu \gamma_\mu (1-\gamma^5)\mu$$

and the weak charges are :

$$Q_W(t) = \int d^3x \{ \nu_e^+(1-\gamma^5)e + \nu_\mu^+(1-\gamma^5)\mu \}$$

$$Q_W^+(t) = \int d^3x \{ e^+(1-\gamma^5)\nu_e + \mu^+(1-\gamma^5)\nu_\mu \}$$

As one knows the anticommutators (for equal time) of the lepton fields one finds :

$$[Q_W(t), Q_W^+(t)] = 2\, Q_{W3}(t)$$

where :
$$Q_{W3}(t) = \int d^3x \,\{ \nu_e^+(1-\gamma_5)\nu_e - e^+(1-\gamma_5)e + $$
$$+ \nu_\mu^+(1-\gamma_5)\nu_\mu - \mu^+(1-\gamma_5)\mu \}$$

Setting : $Q_W(t) = 2K_+$, $Q_W^+(t) = 2K_-$, $Q_{W3}(t) = 4K_3$

one has : $[K_+,K_-] = 2K_3$, $[K_3, K_+] = K_+$, $[K_3,K_-] = - K_-$

or $$[K_a, K_b] = i\, \epsilon_{abc}\, K_c$$

where :
$$K_+ = K_1 + iK_2, \quad K_- = K_1 - iK_2$$

Now if one introduces :

$$K_a^V = \sum_{\ell=e,\mu} \int d^3x\, \psi_\ell^+ \frac{\tau_a}{2} \psi_\ell\,, \qquad \psi_\ell = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$K_a^A = \sum_\ell \int d^3x\, \psi_\ell^+ \gamma_5 \frac{\tau_a}{2} \psi_\ell$$

so that : $$K_a = K_a^V - K_a^A$$

then one finds :

$$[K_a^V, K_b^V] = [K_a^A, K_b^A] = i\, \epsilon_{abc}\, K_c^V$$

$$[K_a^V, K_b^A] = i\, \epsilon_{abc}\, K_c^A$$

These were the commutations rules that Gell-Mann postulated for I_a, I_a^5, I_a replacing K_a^V, I_a^5 replacing K_i^A.

If one introduces the left and right-handed operators I_a^R, I_a^L,

$$I_a^R = \frac{I_a + I_a^5}{2}$$

$$I_a^L = \frac{I_a - I_a^5}{2}$$

then :

$$[I_a^R, I_b^R] = i\,\varepsilon_{abc}\, I_c^R$$

$$[I_a^L, I_b^L] = i\,\varepsilon_{abc}\, I_a^L$$

$$[I_a^R, I_b^L] = 0$$

The last group of commutation rules shows that one has two commuting sets of operators I_a^R, I_a^L, each one spanning the algebra of SU(2). The six generators determine the algebra of a group which is the direct product of two SU(2) groups : it is the SU(2) ⊗ SU(2) algebra or chiral algebra.

Let us now recall the equation (V,2) :

$$\langle 0 | \partial_\mu A_+^\mu(0) | \pi^- \rangle = f_\pi m_\pi^2$$

As we have assumed that in the limit of the coupling constant $\lambda = 0$

$$\partial_\mu A^\mu(0) = 0$$

we see that in this limit

$$f_\pi m_\pi^2 = 0 \qquad \text{in the limit } \lambda \to 0$$

As we cannot have $f_\pi = 0$ we are led to accept that in the limit $\lambda \to 0$, that is, in the limit of the SU(2) ⊗ SU(2) symmetry, $m_\pi \to 0$.
Consider the dispersion relation for $D(q^2)$:

$$D(q^2) = \frac{\sqrt{2}\, g_{\pi N} f_\pi m_\pi^2}{m_\pi^2 - q^2} + \int_{(3m_\pi)^2}^{\infty} \frac{\sigma(t)\,dt}{t - q^2}$$

As $D(q^2)$ occurs in the matrix element of $\gamma_\lambda g'_4$ for neutron-proton transition we see that $D(q^2)$ results from the term λL_1 in the Lagrangian and is of order

$$D(q^2) \sim O(\lambda)$$

for $q^2 \gg m_\pi^2$. Now for these values of q^2 the pole term is proportional to m_π^2 hence it is also of order $O(\lambda)$. Hence $\sigma(t) \sim O(\lambda)$ for $q^2 \gg m_\pi^2$.
Now for $q^2 \simeq m_\pi^2$ the denominator of the pole term is of order λ so that this term is of order zero and thus we understand the dominance of the first term.

The Adler-Weisberger sum rule.

The experimental value of $\frac{G_A}{G_V} = f_A^{(1)}(0) \equiv f_A$ suggests that the axial coupling constant is renormalized by the strong interactions.

However, if the axial current were conserved it would not necessarily imply that there would be no renormalization of G_A.

The problem of calculating f_A was a challenge to the theory, an important but difficult problem. The question is to know by theoretical arguments why $f_A - 1$ is small, ~ 0.23, and positive.

Adler and Weisberger established a sum rule relating the value of $f_A - 1$ to the pion-nucleon dynamics. And this sum rule is established by assuming the SU ⊗ SU(2) algebra for the operators I_a, I_a^5 and also PCAC.

[illegible] algebra :

$$[I_a, I_b] = i\,\epsilon_{abc}\, I_c$$

$$[I_a^5, I_b^5] = i\,\epsilon_{abc}\, I_c$$

$$[I_a, I_b^5] = i\,\epsilon_{abc}\, I_c^5$$

We have for [illegible]

$$I_+^5 = I_1^5 + i I_2^5 \quad , \qquad I_-^5 = I_1^5 - i I_2^5$$

$$[I_+^5, I_-^5] = 2\, I_3$$

Take the matrix element of both sides between two proton states with the same spin z-component :

$$\langle p'|[I_+^5, I_-^5]|p\rangle = \langle p'|2I_3|p\rangle = \langle p'|p\rangle$$

Has our normalization is :

$$\langle p.|p'\rangle = 2p^0\, \delta^3(p - p').$$

we have

$$\langle p'|[I_+^5, I_-^5]|p\rangle = 2p^0 \delta^3(p - p')$$

Now :

$$\langle p'|[I_+^5, I_-^5]|p\rangle = \langle p'|I_+^5 I_-^5|p\rangle - \langle p'|I_-^5 I_+^5|p\rangle$$

$$= \sum_n \{\langle p'|I_+^5|n\rangle \langle n|I_-^5|p\rangle - \langle p'|I_-^5|n\rangle \langle n|I_+^5|p\rangle\}$$

In the sum over all possible intermediate states n, we consider separately the one nucleon states.

We have :

$$\langle p'|I_+^5|k\rangle = \langle p'|\int g_+^o(x) d^3x|k\rangle = (2\pi)^3 \delta^3(p'-k).$$

$$\langle p'|g_+^o(0)|k\rangle = \delta^3(p'-k)\ \bar{u}_p(p')\gamma^o\gamma^5 u_N(k) f_A$$

where f_A results from the delta function.

Therefore :

$$\sum_{\substack{\text{one nucleon}\\ \text{states}}} \langle p'|I_+^5|k\rangle \langle k|I_-^5|p\rangle$$

$$= \sum_{\text{spin}} \int \frac{d^3k}{2k_o} f_A^2\ \delta^3(p'-k)\delta^3(k-p)\bar{u}_p(p)\gamma^o\gamma^5 u_N(k).$$

$$\times\ \bar{u}_N(p)\gamma^o\gamma^5 u_p(p) =$$

$$\frac{\ldots}{\ldots} \Gamma_A^2 \ldots \bar{u}_p(p)\gamma^0\gamma^5(\not p + m_N)\gamma^0\gamma^5 u_p(p)$$

Now

$$\ldots \gamma^0\gamma^5(\not p + m_N)\gamma^0\gamma^5 u(p) =$$

$$\bar{u}(p)\gamma^0(\not p - m_N)\gamma^0 u(p) = \bar{u}(p)(p^0\gamma^0 + \vec{p}\cdot\vec{\gamma} - m_N)u(p)$$

$$\ldots (2p^0\gamma^0 \ldots m_N)u(p) = 4\left((p^0)^2 - m_N^2\right)$$

[illegible] $\bar{u}(p)u(p) = 2m_N$, $u^+(p)u(p) = 2p^0$

Hence :

$$\sum_{\substack{\text{one nucleon}\\ \text{states}}} \langle p'|I_+^5|k\rangle\langle k|I_-^5|p\rangle = \frac{1}{2p^0}\,\Gamma_A^2\,\delta^3(p'-p)\,4(p^{o2} - m_N^2)$$

$$= 2p^0\delta^3(p'-p)\Gamma_A^2\left(1 - \frac{m_N^2}{p_o^2}\right)$$

As to the other intermediate states we use the following indentities :

$$\langle p'|I_+^5|n\rangle = \frac{1}{i(p'-p_{no})}\langle p'|\dot{I}_+^5|n\rangle = \frac{1}{i(p_o'-p_{no})}\langle p'|\int d^3x\,\partial_\mu g^\mu(x)|n\rangle$$

$$= \frac{(2\pi)^3\delta^3(p'-p_n)}{i(p_o'-p_{no})}\langle p'|\mathscr{D}_+(0)|n\rangle$$

where

$$\mathscr{D}_\pm(0) = \partial_\mu g_\pm^\mu(0).$$

So :

$$\langle n|I_-^5|p\rangle = \frac{(2\pi)^3\,\delta^3(p-p_n)}{-i(p_o-p_{no})}\,\langle n|\mathcal{D}_-(0)|p\rangle$$

Therefore these states contribute to the sum as :

$$\delta^3(p'-p)\sum_n{}'(2\pi)^6\,\frac{\delta^3(p-p_n)}{(p_o-p_{no})^2}\Big\{\langle p'|\mathcal{D}_+(0)|n\rangle\langle n|\mathcal{D}_-(0)|p\rangle -$$

$$-\langle p'|\mathcal{D}_-(0)|n\rangle\langle n|\mathcal{D}_+(0)|p\rangle\Big\}$$

where $\sum_n{}'$ means the sum over all states except the one neutron intermediate state. Next we write :

$$\sum_n{}' = \sum_{\substack{n\\ \text{internal}\\ \text{numbers}}}{}' \int \frac{d^3p_n}{2p_{no}} = \sum_{\substack{n\\ \text{int}}}{}'\int\frac{d^3p_n}{2p_{no}}\int_{m_N+m} dE\,\delta(E-M_n)$$

where M_n is the invariant mass of the nth state :

$M_n = \sqrt{p_{on}^2-\vec{p}_n^{\,2}}$; the last integral is 1 since $M_n > m_N + m$. After computing the integration over p_n one gets :

$$(2\pi)^6\delta^3(p'-p)\int_{m_N+m} dE\sum_{\substack{n\\ \text{int}}}{}'\frac{\delta(E-M_n)}{2p_{no}(p_o-p_{no})^2}\;\cdot$$

$$\cdot\Big\{\langle p|\mathcal{D}_+(0)|n\rangle\langle n|\mathcal{D}_-(0)|p\rangle - \langle p|\mathcal{D}_-(0)|n\rangle\langle n|\mathcal{D}_+(0)|p\rangle\Big\}_{\vec{p}_n=\vec{p}}$$

Hence equating this computation of $\langle p'|[I_+^5,\,I_-^5]|p\rangle$ over intermediate states to $2p^o\,\delta^3(p'-p)$ we find :

$$2p^o\,\delta^3(p'-p) = 2p^o\,\delta^3(p'-p)\,f_A^2\,\Big(1-\frac{m_N^2}{p_o^2}\Big)+$$

$$[\text{illegible}] \quad \frac{[\text{illegible}] \; M_n)}{4p_{no}(p_o - p_{no})^2} \; .$$

$$[\text{illegible}] \; \mathcal{D}(0) \; [\text{illegible}] \; \mathcal{D}(0) \; [\text{illegible}] \; \mathcal{D}(0) \; [\text{illegible}] \; \mathcal{D}(0) \, p \; \Big|_{\vec p_n = \vec p}$$

[illegible]

$$[\text{illegible}] \; (2\pi)^{n} \; [\text{illegible}] \; dE \; \sum_{\text{int}} \; \frac{[\text{illegible}] \; M_n)}{4p_o p_{no}(p_o - p_{no})^2} \; .$$

$$[\text{illegible}] \; \mathcal{D}(0) \; [\text{illegible}] \; \mathcal{D}(0) \, p \; [\text{illegible}] \; \mathcal{D}(0) \; [\text{illegible}] \; \mathcal{D}(0) \, p \; \Big|_{\vec p_n = \vec p}$$

[illegible] sum, on the right hand side, over the internal quantum numbers [illegible] invariant mass $M_n = E$ and then one integrates over all [illegible] invariant masses E.

Now take the limit $p_o \to \infty$ and assume it is possible to commute [illegible] the integration and summation above.

Since (remember the delta function $\delta^3(p - p_n)$):

$$p_{no} = \sqrt{\vec p_n^{\,2} + M_n^2} = \sqrt{\vec p^{\,2} + M_n^2} = \sqrt{p_o^2 - m_N^2 + M_n^2}$$

$$\frac{[\text{illegible}]}{(p_o - p_{no})^2} = \frac{(p_o + p_{no})^2}{(p_o^2 - p_{no}^2)^2} = \frac{(p_o + p_{no})^2}{(m_N^2 - M_n^2)^2}$$

we have, for fixed M_n [illegible] :

$$\lim_{p_o\to\infty} \frac{1}{4p_o p_{no}(p_o - p_{no})^2} = \frac{4p_o^2}{(E^2 - m_N^2)^2\, 4p_o^2} = \frac{1}{(E^2 - m_N^2)^2}$$

Therefore :

$$f_A^2 = (2\pi)^6 \int_{m_N+m_\pi}^{\infty} dE \, \frac{1}{(E^2 - m_N^2)^2} \lim_{p_o\to\infty} \sum_{\substack{n\\ int}}' \delta(E - M_n) \cdot$$

$$\cdot \left\{ \langle p|\mathcal{D}_+|n\rangle\langle n|\mathcal{D}_-|p\rangle - \langle p|\mathcal{D}_-|n\rangle\langle n|\mathcal{D}_+|p\rangle \right\}_{\vec{p}_n = \vec{p}}$$

The sums

$$(2\pi)^6 \sum_{\substack{n\\ int}}' \delta(E - M_n) \langle p|\mathcal{D}_+|n\rangle\langle n|\mathcal{D}_-|p\rangle$$

and

$$(2\pi)^6 \sum_{\substack{n\\ int}}' \delta(E - M_n) \langle p|\mathcal{D}_-|n\rangle\langle n|\mathcal{D}_+|p\rangle$$

are Lorentz invariants and can depend only on $p_n^2 = E^2$ (the invariant mass of state n) and
$(p - p_n)^2 = (p_o - p_{no})^2 = (p_o - \sqrt{p_o^2 - m_N^2 + M_n^2})^2$. Thus we denote these sums by $F^-(E,q^2)$ and $F^+(E,q^2)$ where $q^2 = (p - p_n)^2$: in the limit $p_o \to \infty$ one will have :

$$f_A^2 = \int_{m_N+m_\pi}^{\infty} dE \, \frac{1}{(E^2 - m_N^2)^2} \left\{ F^-(E,0) - F^+(E,0) \right\}$$

(note that as $p_o \to \infty$, $p_{no} \to p_o$ and so $q^2 = (p_o - p_{no})^2 \to 0$).

What is $F^{\mp}$? Consider Gell-Mann-Lévy equation :

$$\partial_\lambda g_i^\lambda(x) = \sqrt{2}\, f_\pi m_\pi^2\, \varphi_i$$

[illegible]

[illegible]

[illegible] . Then :

[illegible]

[illegible]

[illegible]

Now

$$[\text{illegible}]_{q^2=0} = T^{(0)}(\pi^- + p \to n)$$

is the invariant amplitude for the process $\pi^- + p \to n$ with π^- a zero-mass pion. Thus :

$$F^{\mp}(E,0) = 2 \sum_{\substack{n \\ int}}{}' \, \delta(E - M_n) \, |T(\pi^{\mp} + p \to n)|^2 f_\pi^2$$

The total cross section for

$$\pi^{\pm} + p \to \text{everything}$$

is

$$\sigma^{\pm}(E) \times \text{flux} = (2\pi)^4 \sum_n{}' \, \delta^4(p + k - p_n) \, \frac{|T(\pi^{\pm} + p \to n)|^2}{2p^0 \, 2k_0} \, \frac{1}{(2\pi)^3}$$

$$= (2\pi)^4 \sum_{\substack{n \\ \text{int}}} \int \frac{d^3 p_n}{2p_{no}} \frac{|T(\pi^- + p \to n)|^2}{2p^0\, 2k^0} \frac{\delta^4(p + k - p_n)}{(2\pi)^3}$$

$$= (2\pi) \sum_{\substack{n \\ \text{int}}} \frac{|T(\pi^- + p \to n)|^2}{2p_{no} 2p_o 2k_o} \delta\left(p_o + k_o - \sqrt{(\vec{p} + \vec{k})^2 + M_n^2}\right)$$

If one chooses the center of mass system

$$\vec{p} + \vec{k} = 0 \quad , \qquad p_o + k_o = E$$

$$p_{no} = \sqrt{(\vec{p} + \vec{k})^2 + M_n^2} = M_n$$

the flux is :

$$\text{flux} = \vec{v}_\pi - \vec{v}_p = \frac{|\vec{k}|}{k_o} + \frac{|\vec{k}|}{p_o} = \frac{|\vec{k}|(p_o + k_o)}{p_o k_o} = \frac{E|\vec{k}|}{p_o k_o} = \frac{E}{p_o}$$

Also :

$$E^2 = (p_o + k_o)^2 = p_o^2 + k_o^2 + 2k_o p_o = \vec{p}^2 + m_N^2 + \vec{k}^2 + 2k_o p_o$$

and as :

$$\vec{p} + \vec{k} = 0$$

one has :

$$\vec{p}^2 + \vec{k}^2 = -2\vec{k}.\vec{p}$$

so

$$E^2 = m_N^2 + 2k_o p_o - 2\vec{k}.\vec{p} = m_N^2 + 2k_o p_o + 2\vec{k}^2 =$$

$$= m_N^2 + 2k_o p_o + 2k_o^2 = m_N^2 + 2k_o E$$

[illegible] zero mass), So :

$$k_0 = \frac{E^2 - m_N^2}{2E}$$

Therefore :

$$\sigma^{\pm} = \frac{k_0}{E}\,(2\pi)^{4} \sum_{\substack{n \\ int}} \delta(E - M_n)\,\frac{|T(\pi^{\pm} + p \rightarrow n)|^2}{2p_0\,2E(E^2 - m_N^2)}\Big|_E =$$

$$= \frac{(2\pi)^{4}}{4}\,\frac{1}{E(E^2 - m_N^2)} \sum_{\substack{n \\ int}} |T(\pi^{\pm} + p \rightarrow n)|^2\,\delta(E - M_n)$$

Thus :

$$\frac{1}{f_\pi^2}\,\frac{1}{2}\,F^{\pm}(E,0) = \frac{4}{(2\pi)}\,E(E^2 - m_N^2)\,\sigma^{\pm}$$

$$F^{\pm}(E,0) = \frac{1}{(2\pi)}\;4f_\pi^2\;2E(E^2 - m_N^2)\;\sigma^{\pm}(E)$$

Therefore :

$$1 - f_A^2 = \frac{4}{\pi}\;f_\pi^2 \int_{m_N + m_\pi}^{\infty} dE\;\frac{E}{E^2 - m_N^2}\;(\sigma^{-}(E) - \sigma^{+}(E))$$

Now from the G-T relation :

$$f_\pi = \frac{\sqrt{2}\;m_N\;f_A}{g_{\pi N}}$$

we get :

$$1 - f_A^2 = \frac{8}{\pi}\,\frac{m_N^2\,f_A^2}{g_{\pi N}^2} \int_{m_N + m_\pi}^{\infty} dE\;\frac{E}{E^2 - m_N^2}\;(\sigma^{-}(E) - \sigma^{+}(E))$$

$$1 - \frac{1}{f_A^2} = \frac{2}{\pi}\,\frac{m_N^2}{g_{\pi N}^2}\int \frac{E\,dE}{E^2 - m_N^2}\left[\sigma_0^+(E) - \sigma_0^-(E)\right]$$

Here $f_A = \frac{G_A}{G_V}$, $\sigma^{\pm}(E)$ is the total cross section for scattering of $\pi^{\pm}$ on proton at center of mass energy E, $g_{\pi N}$ is the π-nucleon coupling constant at zero momentum transfer.

Adler and Weisberger idea was to utilise information on total cross-section for pion-nucleon scattering and thus compute f_A. However in the above sum rule it is the cross-section for zero-mass pion that occurs and so it is necessary to extrapolate from $m_\pi = 0$ to the physical value of $\sigma^{\pm}$ for $m_\pi \neq 0$. So here intervenes the PCAC hypothesis.

[illegible]

[illegible] stated the CVC hypothesis for the $\Delta S = 0$ [illegible] postulate for the $\Delta S =$ [illegible] axial current. What [illegible] such currents exist and are associated [illegible] in the strangeness such as :

[illegible]

[illegible] evidence of suppression of reactions with $\Delta S = 2$.

[illegible] have seen that the conservation of $V^{\lambda}_{(o)}(x)$, for $\Delta S = 0$, is based on the assumption of invariance of the lagrangean under the SU(2) group. One has then identified this current essentially with the isospin current :

$$V^{\lambda}_{(o)}(x) = \frac{G_V}{\mathcal{G}_F}\, j^{\lambda}_{+}(x) \; ,$$

$$V^{\lambda +}_{(o)}(x) = \frac{G_V}{\mathcal{G}_F}\, j^{\lambda}_{-}(x)$$

the isospin generators being :

$$I_+ = \int j^{o}_{+}(x)\, d^3x$$

$$I_- = \int j^{o}_{-}(x)\, d^3x$$

$$\left[I_+, I_-\right] = 2I_3 \,,$$

$$\left[I_3, I_\pm\right] = \pm I_\pm$$

$$I_a = \int j_a^\lambda (x)\, d^3x, \qquad a = 1, 2, 3$$

Now as the hypercharge Y commutes with the I_a :

$$\left[I_a, Y\right] = 0$$

then the transitions provoked by $j_+^\lambda(x)$ and $j_-^\lambda(x)$cannot change strangeness.

It is thus necessary to assume that a higher symmetry must be associated with the possible conservation of $V_{(1)}^\lambda(x)$, if at all. This symmetry group is the group SU(3) which has eight generators F_a such that :

$$\left[F_a, F_b\right] = i\ f_{abc}\ F_c, \qquad a,b = 1,- - - 8$$

where f_{abc} are the structure constants of this group. This symmetry allows the classification of hadrons in multiplets (octet, decuplets). It is assumed that the hamiltonian of strong interactions is composed of two terms :

$$H_s = H_o + \lambda\ H_1$$

H_o is then assumed to be very strong and invariant under the SU(3) group and H_1, moderately strong, breaks this symmetry. H_1 is invariant only under SU(2) and Y(1). In the limit $\lambda \to 0$ the theory would be exactly invariant under SU(3) and all particles in a given SU(3) multiplet would have the same mass (for instance, all baryons in the octet). For $\lambda \neq 0$, the H_1 term breaks the symmetry and these particles acquire different masses.

[illegible] particles, the quarks, three [illegible] members and that the hadrons [illegible] quarks and anti-quarks. The three quarks are [illegible] complex space

$$\begin{pmatrix} \cdot \\ \cdot \\ n \end{pmatrix}$$

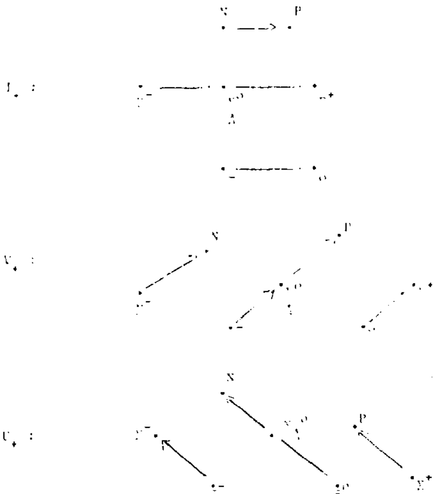
[illegible]

[illegible] one can define the following operators which [illegible] in the charge and strangeness as indicated :

$I_+ = F_1 + iF_2$	$\Delta Q = 1, \ \Delta S = 0$
$I_- = F_1 - iF_2$	$\Delta Q = -1, \ \Delta S = 0$
$I_3 = F_3$	$\Delta Q = 0, \ \Delta S = 0$
$V_+ = F_4 + iF_5$	$\Delta Q = 1, \ \Delta S = 1$
$V_- = F_4 - iF_5$	$\Delta Q = -1, \ \Delta S = -1$
$U_+ = F_6 + iF_7$	$\Delta Q = 0, \ \Delta S = +1$
$U_- = F_6 - iF_7$	$\Delta Q = 0, \ \Delta S = -1$
$Y = \frac{2}{\sqrt{3}} F_8$	$\Delta Q = 0, \ \Delta S = 0$

The I's are identified with the isospin operators, Y with the hypercharge.

If one considers the octet of baryons one sees that the operators I_+, V_+, U_+ act as transitions between these baryons as indicated



To these operators one associates currents such that :

$$F_a = \int d^3x \; j_a^{\,0}(x) \qquad\qquad a = 1, \text{ - - - } 8$$

Now as the weak vector current with $\Delta S = 0$ was identified with the isospin current and since in the table above of generators, there are two and only two corresponding to $\Delta Q = +1$, namely I_+ and V_+, Cabibbo assumed that the weak vector current is a linear combination of the currents associated with I_+ and V_+, namely :

$$V^\lambda(x) = a\,(j_1^{\lambda}(x) + i\;j_2^{\lambda}(x)) + b\,(j_4^{\lambda}(x) + i\;j_5^{\lambda}(x))$$

The first term is the $\Delta S = 0$ vector current. The second term is such that $\Delta S = \Delta Q = 1$, $\Delta I = 1/2$.

Cabibbo further introduced the idea of universality of weak interactions in the following form : the transitions provoked by the hadronic weak vector current must have the same strength as the ones provoked by the leptonic weak vector current.
Now $V^\lambda(x)$ can be written :

$$V^\lambda(x) = (a^2 + b^2)^{1/2} \left\{ \frac{a}{(a^2 + b^2)^{1/2}}\; j^{\lambda}_{1+i2}(x) + \frac{b}{(a^2 + b^2)^{1/2}}\; j^{\lambda}_{4+i5}(x) \right\}$$

In the quark model

$$j_1^\lambda(x) + i\;j_2^\lambda(x) = \bar{p}(x)\gamma^\lambda n(x)$$

$$j_4^\lambda(x) + i\;j_2^\lambda(x) = \bar{p}(x)\gamma^\lambda\;\lambda(x)$$

so that :

$$V^\mu(x) = (a^2 + b^2)^{1/2}(\bar{p}(x)\gamma^\mu n_c(x))$$

where

$$n_c(x) = \frac{a\,n(x) + b\lambda(x)}{(a^2 + b^2)^{1/2}}$$

Comparing with the lepton current

$$l_v^{\ \mu}(x) = \bar{\nu}_e(x)\gamma^\mu e(x)$$

one must impose, for universality

$$a^2 + b^2 = 1$$

and so we can set :

$$a = \cos\theta, \ b = \sin\theta$$

Thus :

$$V^{\mu}(x) = \cos\theta \ (j_1^{\mu}(x) + i \ j_2^{\mu}(x)) + \sin\theta \ (j_4^{\mu}(x) + i \ j_5^{\mu}(x))$$

or in the quark model :

$$V^{\mu}(x) = \bar{p}(x)\gamma^{\mu} \ n_c(x), \qquad n_c(x) = n(x) \cos\theta + \lambda(x) \sin\theta$$

We also see by our previous study of CVC, that :

$$\cos\theta = \frac{G_V}{\mathcal{G}_F} \quad .$$

Cabibbo's assumption is equivalent to saying that the <u>hadronic weak vector current is a member of a SU(3) octet</u>.

Now what are the conditions for conservation of $V^{\mu}_{(1)}(x) = \sin\theta \ (j_4^{\mu}(x) + i \ j_5^{\mu}(x))$?

We shall look for inspiration in the quark model afterwards one tries to abstract results from this model and postulate them for the currents. Assume the masses of the quarks are different :

Let : $\qquad m_p \neq m_n \neq m_\lambda$

$$q = \begin{pmatrix} p \\ n \\ \lambda \end{pmatrix}$$

and $$M = \begin{pmatrix} m_p & 0 & 0 \\ 0 & m_n & 0 \\ 0 & 0 & m_\lambda \end{pmatrix}$$

The Lagrangean for quarks will be

$$L = L_0 + L'$$

where

$$L_0 = \bar{q}\ (i\gamma^\mu \partial_\mu - M)q$$

L' is not known but if it does not contain fields derivatives the results will not depend on the form of L'. L' could be for instance :

$$L' = g\ \bar{q}(x)\gamma^\mu q(x) \mathcal{G}_\mu(x)$$

with $\mathcal{G}_\mu(x)$ a vector field in interaction with the current.

Now given a lagragian built up out of fields $\phi_a(x)$, $\partial^\mu\phi_a(x)$:

$$L = L\ (\phi_a(x),\ \partial^\mu\phi_a(x))$$

the current associated to a gauge transformation :

$$\phi_a(x) \to \phi'_a(x) = \phi_a(x) + i\ \Lambda_\alpha(x)\Omega^\alpha_a(\phi)$$

is :

$$j^\alpha_\mu(x) = -\ i\ \frac{\partial L}{\partial(\partial^\mu\phi_a)}\ \Omega^\alpha_a(\phi)$$

where $\Lambda_\alpha(x)$ are the gauge functions and $\Omega^\alpha_a(\phi)$ depends on structure constants of the transformation and on the fields ϕ.

Indeed, the above transformation on ϕ induces a change in L :

$$\delta L = \frac{\partial L}{\partial \phi_a} \delta\phi_a + \frac{\partial L}{\partial(\partial_\mu \phi_a)} \delta(\partial_\mu \phi_a) =$$

$$= i \frac{\partial L}{\partial \phi_a} \Lambda_\alpha \Omega_a^\alpha + i \frac{\partial L}{\partial(\partial_\mu \phi_a)} \partial_\mu (\Lambda_\alpha \Omega_a^\alpha)$$

Now the field equations are :

$$\frac{\partial L}{\partial \phi_a} = \partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi_a)}$$

so that :

$$\delta L = i \left(\partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi_a)}\right) \Lambda_\alpha \Omega_a^\alpha + i \frac{\partial L}{\partial(\partial_\mu \phi_a)} \partial_\mu (\Lambda_\alpha \Omega_a^\alpha)$$

or :

$$\delta L = \Lambda_\alpha(x) \partial_\mu \left\{ i \frac{\partial L}{\partial(\partial_\mu \phi_a)} \Omega_a^\alpha(\phi) \right\} + \partial^\mu \Lambda_\alpha \left\{ i \frac{\partial L}{\partial(\partial_\mu \phi_a)} \Omega_a^\alpha \right\}$$

thus if one defines :

$$j_\mu^\alpha(x) = - i \frac{\partial L}{\partial(\partial_\mu \phi_a)} \Omega_a^\alpha(\phi)$$

one will have :

$$\delta L = - \Lambda_\alpha(x) \partial^\mu j_\mu^\alpha(x) - \partial^\mu \Lambda_\alpha(x) j_\mu^\alpha(x)$$

$$= - \partial^\mu \left\{ \Lambda_\alpha(x) j_\mu^\alpha(x) \right\}$$

One sees also that :

$$j_\mu^\alpha(x) = - \frac{\partial}{\partial(\partial^\mu \Lambda_\alpha(x))} \quad (\delta L)$$

$$\partial^\mu j_\mu^\alpha(x) = - \frac{\partial}{\partial \Lambda_\alpha(x)} \quad (\delta L)$$

The current will be conserved if the lagrangian is invariant under gauge transformations of the 1st kind.

Now in our case of the quark lagrangian consider the transformation :

$$q(x) \to q'(x) = e^{i\varepsilon_\alpha \frac{1}{2} \lambda_\alpha} q(x) \simeq (1 + i\varepsilon_\alpha \frac{1}{2} \lambda_\alpha) q(x)$$

where ε_α are the eight parameters of the transformation (Λ_α above supposed constant) and $\frac{1}{2} \lambda_\alpha$ are the generators of SU(3) in the fundamental representation. Then you will find :

$$\delta L = i \bar{q} \left[M, \frac{1}{2} \lambda_\alpha \right] q \, \varepsilon_\alpha$$

$$j_\mu^\alpha(x) = \bar{q}(x) \gamma_\mu \frac{1}{2} \lambda^\alpha q(x)$$

$$\partial^\mu j_\mu^\alpha(x) = i \bar{q} \left[M, \frac{1}{2} \lambda_\alpha \right] q$$

Now :

$$\bar{q} \left[M, \frac{1}{2} \lambda_\alpha \right] \varepsilon_\alpha q = \varepsilon_\alpha \bar{q}_a (m_a - m_b) \frac{1}{2} (\lambda_\alpha)_{bc} q_c$$

Let us form the currents for $\Delta S = 0$ and for $\Delta S = 1$:

$$J_1^\mu + i j_2^\mu = \bar{q} \gamma^\mu \frac{1}{2} (\lambda_1 + i\lambda_2) q = \bar{q} \gamma^\mu I_+ q = \bar{p} \gamma^\mu n$$

$$j_4^\mu + i j_5^\mu = \bar{q} \gamma^\mu \frac{1}{2} (\lambda_4 + i\lambda_5) q = \bar{q} \gamma^\mu V_+ q = \bar{p} \gamma^\mu \lambda$$

and :

$$\partial_\mu (j_1^\mu + i\ j_2^\mu) = i\ (m_p - m_n)\ (\bar{p}n)$$

$$\partial_\mu (j_4^\mu + i\ j_5^\mu) = i\ (m_p - m_\lambda)\ (\bar{p}\lambda)$$

We see that :

the vector current $\Delta Q = 1$, $\Delta S = 0$ is conserved if $m_p = m_n$, that is, in the limit of exact SU(2) symmetry; the vector current $\Delta Q = 1$, $\Delta S = 1$ is conserved if $m_n = m_p = m_\lambda$, that is, in the limit of exact SU(3) symmetry.

Cabibbo introduced the axial current with $\Delta S = 1$ by imposing universality for the V-A weak interaction. As the lepton current has the form :

$$l^\lambda(x) = \bar{\nu}_e \gamma^\lambda (1 - \gamma^5) e + \bar{\nu}_\mu \gamma^\lambda (1 - \gamma^5) \mu =$$

$$\equiv l_V^\lambda(x) - l_A^\lambda(x)$$

then Cabibbo postulated

$$h^\lambda(x) = \cos\theta \left[(j_1^\lambda(x) + i\ j_2^\lambda(x)) - (g_1^\lambda(x) + i\ g_2^\lambda(x)) \right]$$

$$+ \sin\theta \left[(j_4^\lambda(x) + i\ j_5^\lambda(x)) - (g_4^\lambda(x) + i\ g_5^\lambda(x)) \right]$$

where

$$j_\alpha^\lambda(x) = \bar{q}(x) \gamma^\lambda \frac{1}{2} \lambda_\alpha\ q(x)$$

is the vector current in the quark model ; and

$$g_\alpha^\lambda(x) = \bar{q}(x) \gamma^\lambda \gamma^5 \frac{1}{2} \lambda_\alpha q(x)$$

is the axial current in this model.

One therefore assumes that the hadronic axial current is also a member of a SU(3) octet.

In the quark model one can write :

$$h^{\lambda}(x) = \bar{p}\gamma^{\lambda}(1 - \gamma^5)n_c$$

where :

$$n_c = n \cos \theta + \lambda \sin \theta$$

The experiment has confirmed Cabibbo's theory. Thus the ratio of decay rates for the reactions

$$K \to \mu + \nu$$

$$\pi \to \mu + \nu$$

is :

$$\frac{\Lambda(K \to \mu + \nu)}{\Lambda(\pi \to \mu + \nu)} = \frac{f_K^2}{f_\mu^2} \ tg^2 \theta \ \frac{m_K}{m_\pi} \ \frac{\left(1 - \frac{m_e^2}{m_K^2}\right)^2}{\left(1 - \frac{m_e^2}{m_\pi^2}\right)^2}$$

From the experimental value of this ratio one finds $\frac{f_K}{f_\pi}$ tg θ. In the limit of exact SU(3) symmetry $f_K = f_\pi$ and one obtains the value of θ.

Experimental data agree whith the value

$$\theta = 0.22$$

The SU(2) ⊗ SU(2) chiral algebra seen in previous chapter is extended into a SU(3) ⊗ SU(3) chiral algebra, a natural symmetry for massless quarks, defined by the charges :

$$F_a(t) = \int d^3x \; j_a^o(x)$$

$$F_{5a}(t) = \int d^3x \; g_a^o(x)$$

and the equat-time commutators :

$$\left[F_a, F_b \right] = i \; f_{abc} \; F_c = \left[F_{5a}, F_{5b} \right]$$

$$\left[F_a, F_{5b} \right] = i \; f_{abc} \; F_{5c}$$

From this algebra it is possible to deduce the condition of universality which gives rise to Cabibbo's angle.

The PCAC hypothesis can be extended to the $\Delta S = 1$ axial current which means to say that the matrix elements of this current are dominated by the K-meson pole in the range $0 \leq q^2 \leq m_K^2$, an extrapolation much bigger than in the $\Delta S = 0$ case. The Adler-Weisberger sum rule can also be obtained for the $\Delta S = 1$ axial current and one starts from the commutator :

$$\left[F_4^{\,5} + i \; F_5^{\,5}, \; F_4^{\,5} - i \; F_5^{\,5} \right] = F_3 + \sqrt{3} \; F_8$$

instead of the one involving $I_+^{\,5}$, $I_-^{\,5}$ as prevously done for $\Delta S = 0$.

VII

NEUTRAL WEAK CURRENTS AND THE CHARM MODEL

Lepton neutral currents

From the time of the definite establishment of the form V-A for the weak interaction it was thought that these interactions would be mediated by an intermediate charged vector boson.

If g is the coupling constant of the interaction between a vector boson field $W^{\mu}(x)$ with mass m_W and weak currents

$$L = gW^{\mu}(x)\ j_{\mu}(x) + \text{h.c.}$$

the amplitude for the neutron-proton decay will be proportional to :

$$M \propto g^2 < p|h^{\alpha}|n > \frac{g_{\alpha\beta} - \frac{k_{\alpha}k_{\beta}}{m_W^2}}{k^2 - m_W^2} (\bar{e}\gamma^{\beta}(1-\gamma^5)\nu_e)$$

whereas in the current-current theory this is proportional to :

$$M \propto - \frac{\mathscr{G}_F}{\sqrt{2}} < p|h^{\alpha}|n >(\bar{e}\ \gamma_{\alpha}(1-\gamma^5)\nu_e).$$

If the momentum transfer is very small $k^2 \ll m_W^2$, one gets :

$$\frac{g^2}{m_W^2} = \frac{\mathscr{G}_F}{\sqrt{2}}$$

If one assumes that $g \approx e$, then the mass of the vector boson is of the order of

$$m_W \approx \left(\frac{\alpha}{\mathcal{G}_F}\right)^{\frac{1}{2}} \approx \left(\frac{1}{137} \times 10^5\right)^{\frac{1}{2}} m_p \cong 30\ m_p$$

This theory of intermediate massive vector bosons, as well as the current-current theory, was however, not renormalizable and thus could serve only as an effective model, its Lagrangian serving to calculate only lowest order processes.

The discovery of the gauge models of weak and electromagnetic interactions justified the theoretical assumption of heavy charged vector bosons. They also led, however, to the prediction of new phenomena such as :

a) interactions due to neutral currents

b) the existence of new, charmed particles

c) the possible existence, although not necessary for the theory if neutral mesons exist, of heavy leptons.

In the Weinberg-Salam model the relation between the mass m_W and g is :

$$\frac{g^2}{8m_W^2} = \frac{\mathcal{G}_F}{\sqrt{2}}$$

and g is related to the electric charge e by

$$g = \frac{e}{\sin\theta_W}$$

θ_W, the Weinberg angle, is a parameter of the model and its value as indicated by the present experiments on neutral current events is :

$$\sin^2\theta_W \sim 0.4$$

so that

$$m_W \sim 60 \text{ GeV}$$

Besides the charged bosons $W^\pm$, the gauge models introduce neutral bosons Z, *the exchange of which between currents with* $\Delta Q = 0$ *give rise to* the so called neutral current weak interactions. The mass of the neutral bosons is given by :

$$m_Z = \frac{m_W}{|\cos\theta_W|}$$

so that

$$m_Z \sim 80 \text{ GeV} \quad \text{for} \quad \sin^2\theta_W \sim 0.4.$$

The Lagrangian for weak interactions between leptons mediated by the W and Z boson fields is, in the Salam-Weinberg model :

$$L_W = \frac{\sqrt{2}}{4} g(\bar{\nu}_e \gamma^\mu (1 - \gamma^5)e)W_\mu +$$

$$+ \frac{\sqrt{2}}{4} g(\bar{\nu}_\mu \gamma^\lambda (1 - \gamma^5)\mu)W_\lambda +$$

$$+ \frac{1}{4}(g^2 + g'^2)^{\frac{1}{2}} \left\{ (\bar{\nu}_e \gamma^\lambda (1 - \gamma^5)\nu_e) + \right.$$

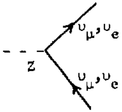
$$+ (\bar{\nu}_\mu \gamma^\lambda (1 - \gamma^5)\nu_\mu) + (\bar{e}\gamma^\lambda \gamma^5 e) + (\bar{\mu}\gamma^\lambda \gamma^5 \mu) +$$

$$\left. + \frac{3g'^2 - g^2}{g^2 + g'^2} [(\bar{e}\gamma^\lambda e) + (\bar{\mu}\gamma^\lambda \mu)] \right\} Z_\lambda + \text{h.c.}$$

The Weinberg angle is defined by :

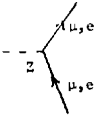
$$\cos\theta = \frac{g}{(g^2+g'^2)^{1/2}}\ , \qquad \sin\theta = \frac{g'}{(g^2+g'^2)^{1/2}}$$

So that the interaction between ν_μ or ν_e and the Z-field is :



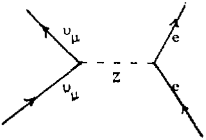
$$L_{Z\nu} = \frac{g}{4\cos\theta_W}\, \bar{\nu}_\mu \gamma^\lambda (1-\gamma^5)\nu_\mu\, Z_\lambda$$

and the interaction between the Z-field and muons or electrons :



$$L_{Z\mu} = \frac{g}{4\cos\theta_W}\ \bar{\mu}\, \gamma^\lambda\ \{\gamma^5 + (4\sin^2\theta_W - 1)\}\ \mu Z_\lambda$$

Thus the scattering of ν_μ by electrons occurs in first order only through exchange of a virtual Z-meson :



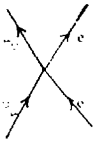
The amplitude for this graph is :

$$M_{\nu_\mu e} = \frac{g^2}{16\cos^2\theta_W}\ \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}}{m_Z^2 - q^2} (\bar{\nu}_\mu \gamma^\mu (1-\gamma^5)\nu_\mu)(\bar{e}\gamma^\nu\ \{\gamma^5 + (4\sin^2\theta_W - 1)\}\ e)$$

For small momentum transfer one gets :

$$\mathcal{M}_{\nu_\mu e}(q^2 \approx 0) = \frac{g^2}{m_Z^2 \cos^2\theta_W} (\bar{\nu}_\mu \gamma^\lambda (1 - \gamma^5) \nu_\mu)(\bar{e} \gamma_\lambda \{ \gamma^5 + (4\sin^2\theta_W - 1) \} e)$$

Given the Salam-Weinberg relationship between g and $\mathcal{G}_F$ one sees that the effective current-current Lagrangian for the interaction $\nu_\mu - e$ is :



$$L_{eff}(\nu_\mu - e) = \frac{\mathcal{G}_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\lambda (1 - \gamma^5) \nu_\mu)(\bar{e} \; \{ g_V \gamma_\lambda - g_A \gamma_\lambda \gamma_5 \} \; e)$$

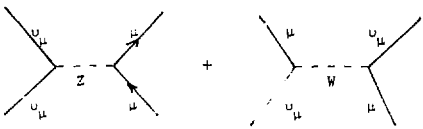
with :

$$g_V = 2\sin^2\theta_W - \frac{1}{2}$$

$$g_A = -\frac{1}{2}$$

We see that in <u>the Salam-Weinberg model the electron neutral current is not V - A</u> having the coefficients g_V and g_A. It will be important when experimental data will allow us to know the form of g_V and g_A, whether it agrees or not with the Salam-Weinberg model.

On the other hand the scattering of ν_μ by muons (or of ν_e by electrons) has in first order the following graphs :



the exchange graph being due to a virtual W.

One finds that after one makes a Fierz rearrangement of the exchange amplitude, the effective Lagrangian for $\nu_\mu - \mu$ interaction at low momentum transfers is :

$$L_{eff}(\nu_\mu - \mu) = \frac{\mathcal{G}_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\lambda (1 - \gamma^5) \nu_\mu)(\bar{\mu} \; \{ g'_V \gamma_\lambda - g'_A \gamma_\lambda \gamma^5 \} \; \mu)$$

where :

$$g_V^! = 2\sin^2\theta_W + \frac{1}{2}$$

$$g_A^! = \frac{1}{2}$$

Neutral hadronic current and the charm-model

In the SU(3) quark model the resultant interaction of the charged vector boson field with the charged quark current is of the form :

$$\frac{\sqrt{2}}{4}\, g(\bar{p}\gamma^\lambda(1-\gamma^5)n_c)W_\lambda + h.c.$$

where

$$n_c = n\cos\theta_c + \lambda\sin\theta_c$$

The gauge model would give rise to a neutral current of the form :

$$\bar{n}_c\gamma^\mu(1-\gamma^5)\, n_c$$

and thus there would occur in this model a neutral hadronic current with $\Delta S = 0$

$$\bar{n}\gamma^\alpha(1-\gamma^5)n\cos^2\theta_c + \bar{\lambda}\gamma^\alpha(1-\gamma^5)\lambda\sin^2\theta_c \;; \quad \Delta S = 0$$

but also a neutral current with $\Delta S = 1$:

$$\{\bar{n}\gamma^\alpha(1-\gamma^5)\lambda + \bar{\lambda}\gamma^\alpha(1-\gamma^5)n\}\ \cos\theta_c\ \sin\theta_c \;; \quad \Delta S = 1$$

This would predict reactions with $\Delta S = 1$ such as $K_o \to \mu^+ + \mu^-$

$$K_+ \to \pi^+ + \nu + \bar{\nu}$$

with amplitudes of the same order of magnitude as those of

$K^+ \to \mu^+ + [illegible]_\mu$

$K^+ \to \pi^0 + e^+ + [illegible]_e$

Now as experiments have so far ruled out such reactions with $Q = 0$, S [illegible] this theory is not acceptable.

The way out of this difficulty has been proposed by Glashow, Illiopoulos and Maiani and is based on a SU(4) quark-model. A fourth quark, p', is introduced with zero isospin but charge $\frac{2}{3}$. The quantum numbers are given in the table below.

	I	I_3	Q	Y	B	S	C
p'	0	0	2/3	+1/3	1/3	0	1
p	1/2	1/2	2/3	+1/3	1/3	0	0
n	1/2	-1/2	-1/3	+1/3	1/3	0	0
λ	0	0	-1/3	-2/3	1/3	-1	0

In fact both in the SU(3)-quark model as in the SU(4)-quark model one assumes three types of triplets for the SU(3) and three types of quadruplets for the SU(4) model. Each type is distinguished by a colour quantum number so that :

p_1	p_2	p_3	
n_1	n_2	n_3	[SU(3) ⊗ SU(3)'
λ_1	λ_2	λ_3	SU(3)' is the colour group]
(colour 1)	(colour 2)	(colour 3)	

in SU(3) and :

p'_1	p'_2	p'_3	
p_1	p_2	p_3	[SU(4) ⊗ SU(3)']
n_1	n_2	n_3	
λ_1	λ_2	λ_3	
(colour 1)	(colour 2)	(colour 3)	

in SU(4). Thus if we call :

$$(q^{\alpha a}) = \begin{pmatrix} p'_1 & p'_2 & p'_3 \\ p_1 & p_2 & p_3 \\ n_1 & n_2 & n_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix}$$

where $\alpha = 1,2,3,4$: $a = 1,2,3$, α is the SU(4) index, a is the SU(3)' colour index we have

$$\text{SU(4) transf :} \qquad q'^{\alpha a} = \mathcal{U}^{\alpha}{}_{\beta}\, q^{\beta a}\,, \quad \text{sum over } \beta = 1,2,3,4$$

and

$$\text{SU(3)' transf :} \qquad q'^{\alpha a} = \mathcal{U}'^{a}{}_{b} q^{\alpha b}\,, \quad \text{sum over } b = 1,2,3$$

To the SU(4) quantum numbers I, I_3, Y, C there are now added three colour quantum numbers I', I'_3, Y' which are conserved by strong interactions. Hadrons are composed of quarks of all colours, the usual, colourless hadrons, being colour singlets ($I' = I'_3 = Y' = 0$). For instance a π^+ is

$$\pi^+ = \frac{1}{\sqrt{3}} (p_1\bar{n}_1 + p_2\bar{n}_2 + p_3\bar{n}_3)$$

instead of $p\bar{n}$ in the simple SU(3)-quark model. There would of course be new states of hadrons with colour quantum numbers different from zero.

If we forget colour for the moment we see that the SU(4)-quark model gives rise to hadrons with charm $C = 0$, the usual hadrons, and to charmed hadrons $C \neq 0$.

The Gell-Mann-Nishijima formula is extended to :

$$Q = I_3 + \frac{1}{2}(Y + C)$$

where C is the charm and

$$Y = B + S$$

The electromagnetic current is a colour singlet :

$$J^{\mu}_{(\gamma)} = \sum_a \frac{2}{3}(\bar{p}'_a\gamma^{\mu}p'_a + \bar{p}_a\gamma^{\mu}p_a) - \frac{1}{3}(\bar{n}_a\gamma^{\mu}n_a +$$

$$+ \bar{\lambda}_a\gamma^{\mu}\lambda_a)$$

sum over colour index a.

So :

$$Q = \frac{2}{3}(n_p + n_{p'}) - \frac{1}{3}(n_n + n_\lambda)$$

$$I_3 = \frac{1}{2}(n_p - n_n)$$

$$S = -n_\lambda$$

$$C = n_{p'}$$

$$B = \frac{1}{3}(n_{p'} + n_p + n_n + n_\lambda)$$

$$Y = \frac{1}{3}(n_{p'} + n_p + n_n) - \frac{2}{3} n_\lambda$$

where :

$n_{p'}$ = number of p' - number of anti-p'
etc.

In the Glashow-Iliopoulos-Maiani model the <u>charged hadronic weak</u> current is the sum :

$$\bar{p}\gamma^{\mu}(1-\gamma^5)n_c + \bar{p}'\gamma^{\mu}(1-\gamma^5)\lambda_c$$

where

$$n_c = n\cos\theta + \lambda\sin\theta$$

$$\lambda_c = \lambda\cos\theta - n\sin\theta$$

(sum over colour is supposed in the above expression)

The first term is the Cabibbo current, the second term is the G.I.M. term and couples p' with λ_c.

The selection rules are :

a) current $\bar{p}\gamma^\mu(1-\gamma^5)n\cos\theta$: $\Delta Q = 1$, $\Delta S = 0$

b) current $\bar{p}\gamma^\mu(1-\gamma^5)\lambda\sin\theta$: $\Delta S = \Delta Q = 1$, $\Delta I = \frac{1}{2}$

c) current $\bar{p}'\gamma^\mu(1-\gamma^5)\lambda\cos\theta$: $\Delta Q = \Delta S = \Delta C$

d) current $-\bar{p}'\gamma^\mu(1-\gamma^5)\, n.\sin\theta$: $\Delta Q = \Delta C$, $\Delta S = 0$

As θ is small we see that charmed particles decay prédominantly into strange particles.

The neutral hadronic weak current is , in the G.I.M. model (sum over colour understood) :

$$\bar{p}'\gamma^\mu(1-\gamma^5)p' + \bar{p}\,\gamma^\mu(1-\gamma^5)p - (\bar{n}_c\gamma^\mu(1-\gamma^5)n_c + \bar{\lambda}_c\gamma^\mu(1-\gamma^5)\lambda_c) =$$

$$= \bar{p}'\gamma^\mu(1-\gamma^5)p' + \bar{p}\gamma^\mu(1-\gamma^5)p - (\bar{n}\gamma^\mu(1-\gamma^5)n + \bar{\lambda}\gamma^\mu(1-\gamma^5)\lambda)$$

and contains no term with $\Delta S = 1$.

SU(4) symmetry is broken by the mass difference between p' and the other quarks. An estimate of this mass difference is obtained from the branching ratio :

$$\frac{K_L \to \mu^+ + \mu^-}{K_L \to \text{all}} \sim 10^{-9}$$

where

$$n_c = n\cos\theta + \lambda\sin\theta$$

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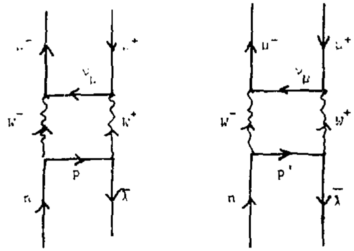
Theoretically the two mu decays would come from the diagram



The first has amplitude proportional to

$$A_1 \sim \frac{g^4}{m_W^2} \cos\theta \sin\theta \sim G_F\alpha \cos\theta \sin\theta$$

The second to

$$A_2 \sim - \frac{g^4}{m_W^2} \cos\theta \sin\theta \sim - G_F \alpha\sin\theta \cos\theta$$

and the sum would vanish if $m_{p'} = m_p$. Otherwise one gets for the sum

$$A_1 + A_2 \sim G_F \alpha \cos\theta \sin\theta \frac{m_p^2 - m_{p'}^2}{m_W^2}$$

To have the experimental branching ratio one must have

$$\frac{m_p - m_{p'}}{m_W} \sim 10^{-1}$$

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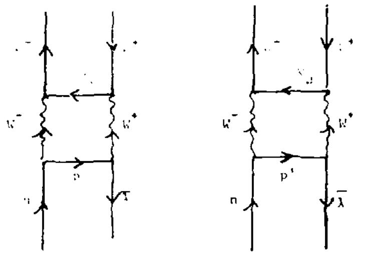

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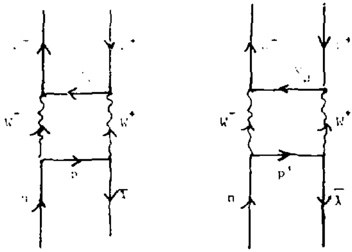

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To have the experimental branching ratio one must have

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WEAK CURRENTS

Lepton current	charged : $\sum_{\ell = e,\mu} (\bar{\nu}_\ell \gamma^\alpha (1-\gamma^5)\ell)$ Neutral (Salam-Weinberg model) : a) $\bar{\nu}_\mu \gamma^\alpha (1-\gamma^5)\nu_\mu$ b) $\bar{\nu}_e \gamma^\alpha (1-\gamma^5)\nu_e$ c) $\bar{e}\{ g_V \gamma^\alpha - g_A \gamma^\alpha (1-\gamma^5)\} e$ (exchange of Z-mesons only) d) $\bar{\mu}\{ g_V \gamma^\alpha - g_A \gamma^\alpha (1-\gamma^5)\} \mu$ (exchange of Z-mesons only) e) $\bar{e}\{ g_V' \gamma^\alpha - g_A' \gamma^\alpha (1-\gamma^5) \} e$ (exchange of Z-mesons and W-mesons and Fierz rearrangement to give above effective current in current-current Lagrangian) f) $\bar{\mu}\{ g_V' \gamma^\alpha - g_A' \gamma^\alpha (1-\gamma^5)\} \mu$ (idem)
Hadronic current (Cabibbo and Glashow-Iliopoulos-Maiani models)	Charged : $\bar{p}\gamma^\alpha (1-\gamma^5) n_c + \bar{p}' \gamma^\alpha (1-\gamma^5)\lambda_c$ Neutral : $-(\bar{n}_c \gamma^\alpha (1-\gamma^5) n_c + \bar{\lambda}_c \gamma^\alpha (1-\gamma^5)\lambda_c) + $ $+ \bar{p}' \gamma^\alpha (1-\gamma^5) p' + \bar{p}\gamma^\alpha (1-\gamma^5) p$

$$g_V = 2\sin^2\theta_W - \frac{1}{2}, \qquad g_A = -\frac{1}{2}$$

$$g_V' = 2\sin^2\theta_W + \frac{1}{2}, \qquad g_A' = \frac{1}{2}$$

$$n_c = n\cos\theta_c + \lambda\sin\theta_c$$

$$\lambda_c = \lambda\cos\theta_c - n\sin\theta_c$$

APPENDIX

THE SU(4) GENERATORS IN THE BASIC REPRESENTATION

The simplest way to obtain the form of the generators of the SU(2),SU(3), SU(4) groups in the fundamental representation is to work à la Lipkin with the operators of creation and annihilation of the fundamental objects.

In SU(2) these objects are denoted p and n and the annihilation and creation operators a_p, a_p^+, a_n, a_n^+ obey the commutation rules :

$$\left[a_p, a_p^+ \right] = \left[a_n, a_n^+ \right] = 1,$$

$$\left[a_p, a_n \right] = \left[a_p, a_p \right] = - - - = 0 \qquad (A, 1)$$

There are four basic operators which do not change the number of objects (either p or n) :

$$a_p^+ a_p, \quad a_n^+ a_n, \quad a_p^+ a_n, \quad a_n^+ a_p \qquad (A, 2)$$

Call N the operator for the total number of particles :

$$N = a_p^+ a_p + a_n^+ a_n \qquad (A, 3)$$

The reader will show that N will commute with the above operators which do not change the number of particles

$$\left[N, a_p^+ a_p \right] = \left[N, a_n^+ a_n \right] = \left[N, a_p^+ a_n \right] = \left[N, a_n^+ a_p \right] = 0 \qquad (A, 4)$$

Instead of the operators (A, 2) we can define an equivalent set of four operators ;

$$\tau_+ = a_p^+ a_n$$

$$\tau_- = {}^+_n a_p = (\tau_+)^+$$

$$\tau_o \equiv \frac{1}{2} \tau_3 = \frac{1}{2} (a_p^+ a_p - a_n^+ a_n)$$

$$N = a_p^+ a_p + a_n^+ a_n$$

The commutation rules for these are

$$\left[\tau_o, \tau_+ \right] = \tau_+$$

$$\left[\tau_o, \tau_- \right] = - \tau_-$$

$$\left[\tau_+, \tau_- \right] = 2\tau_o$$

If we call

$$N_p = a_p^+ a_p, \quad N_n = a_n^+ a_n$$

one has :

$$\left[a_p, N_p \right] a_p, \quad \left[N_p, a_p^+ \right] = a_p^+$$

and if

$$N_p \mid n_p > = n_p \mid n_p >$$

then $a_p \mid n_p >$ and $a_p^+ \mid n_p >$ belong to the eigenvalues $(n_p - 1)$ and $(n_p + 1)$ of N_p respectively

From :

$$a_p \mid 0 > = 0 , \quad a_p^+ \mid 0 > = \mid p > \equiv \mid 1, 0 >$$

$$a_n \mid 0 > = 0 \quad a_n^+ \mid 0 > = \mid n > \equiv \mid 0, 1 >$$

We get :

$$a_p \mid n_p > = \sqrt{n_p} \mid n_p - 1 >$$

$$a_p^+ \mid n_p > = \sqrt{n_p + 1} \mid n_p + 1 >$$

as everybody knows.

In the representation

$$\Omega = \begin{pmatrix} < 10|\Omega|10 > & < 10|\Omega|01 > \\ < 01|\Omega|10 > & < 01|\Omega|01 > \end{pmatrix}$$

where $| 1,0 >$ means $| n_p = 1, n_n = 0$ and $| 0,1 >$ are basic states we get :

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau_3 = 2\tau_o = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Note that :

$$Q = a_p^+ a_p = \tau_o + \frac{1}{2} N.$$

<u>In SU(3)</u> the fundamental objects are denoted p, n, λ. Corresponding to (A,1) we have :

$$\left[a_p^+, a_p \right] = \left[a_n^+, a_n \right] = \left[a_\lambda^+, a_\lambda \right] = 1,$$

all other commutators are zero

There are then nine operators corresponding to [illegible]

$$
\begin{array}{ccc}
a_p^+ a_p & a_n^+ a_p & a_\lambda^+ a_p \\
a_p^+ a_n & a_n^+ a_n & a_\lambda^+ a_n \\
a_p^+ a_\lambda & a_n^+ a_\lambda & a_\lambda^+ a_\lambda
\end{array}
$$

Considering the basic states $|1,0,0\rangle$, $|0,1,0\rangle$, $|0,0,1\rangle$ [illegible] matrix representations

$$\Omega = \begin{pmatrix} 100 \; 100 \; [illegible] \\ 010 \; 100 \; [illegible] \\ [illegible] \end{pmatrix}$$

for the above operators : $a_p^+ a_p = \begin{pmatrix} 100 \\ 000 \\ 000 \end{pmatrix}$, $a_n^+ a_n = \begin{pmatrix} 000 \\ 010 \\ 000 \end{pmatrix}$, [illegible] $\begin{pmatrix} [illegible] \end{pmatrix}$,

$$a_p^+ a_n = \begin{pmatrix} 010 \\ 000 \\ 000 \end{pmatrix} , \qquad a_n^+ a_p = \begin{pmatrix} 000 \\ 100 \\ 000 \end{pmatrix}$$

$$a_p^+ a_\lambda = \begin{pmatrix} 001 \\ 000 \\ 000 \end{pmatrix} , \qquad a_\lambda^+ a_p = \begin{pmatrix} 000 \\ 000 \\ 100 \end{pmatrix}$$

$$a_n^+ a_\lambda = \begin{pmatrix} 000 \\ 001 \\ 000 \end{pmatrix} , \qquad a_\lambda^+ a_n = \begin{pmatrix} 000 \\ 000 \\ 010 \end{pmatrix}$$

The following set of operators is usually chosen in place of the above nine :

$$\lambda_1 = a_p^+ a_n + a_n^+ a_p = \begin{pmatrix} 010 \\ 100 \\ 000 \end{pmatrix} = \begin{pmatrix} \tau_1 & 0 \\ & 0 \\ 000 & \end{pmatrix}$$

$$\lambda_2 = i(a_p a_n^+ - a_n a_p^+) = \begin{pmatrix} \tau_2 & 0 \\ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_3 = a_p^+ a_p - a_n^+ a_n = \begin{pmatrix} \tau_3 & 0 \\ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = a_p^+ a_\lambda + a_\lambda^+ a_p = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = - i \ (a_p^+ a_\lambda - a_\lambda^+ a_p) = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = a_n^+ a_\lambda + a_\lambda^+ a_n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = - i \ (a_n^+ a_\lambda - a_\lambda^+ a_n) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \ (a_p^+ a_p + a_n^+ a_n - 2 \ a_\lambda^+ a_\lambda) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$B = \frac{i}{3} \ (a_p^+ a_p + a_n^+ a_n + a_\lambda^+ a_\lambda)$$

There are now besides B, two diagonal operators λ_3 and λ_8. Note that the hypercharge is defined as :

$$Y = \frac{1}{\sqrt{3}} \lambda_8$$

and the charge :

$$0 = \frac{\lambda 3}{2} + \frac{Y}{2} = \begin{pmatrix} 2/3 & & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

One finds :

$$[illegible]$$

where $f_{\alpha\beta\gamma}$ and $d_{\alpha\beta\gamma}$ are numerical coefficients. In the case of SU(4) the four objects are denote p,n,λ,p'. Instead of sixteen operators $a^+_\alpha a_\beta$, α,β = p,n,λ,p' one chooses the following linear combinations :

$$\eta_1 = a_p^+ a_n + a_n^+ a_p = \begin{pmatrix} \lambda_1 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_2 = -i\,(a_p^+ a_n - a_n^+ a_p) = \begin{pmatrix} \lambda_2 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_3 = a_p^+ a_p - a_n^+ a_n = \begin{pmatrix} \lambda_3 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_4 = a_p^+ a_\lambda + a_\lambda^+ a_p = \begin{pmatrix} \lambda_4 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_5 = -i\,(a_p^+ a_\lambda - a_\lambda^+ a_p) = \begin{pmatrix} \lambda_5 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_6 = a_n^+ a_\lambda + a_\lambda^+ a_n = \begin{pmatrix} \lambda_6 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_7 = -i\,(a_n^+ a_\lambda - a_\lambda^+ a_n) = \begin{pmatrix} \lambda_7 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_8 = \frac{1}{\sqrt{3}}\,(a_p^+ a_p + a_n^+ a_n - 2a_\lambda^+ a_\lambda) = \begin{pmatrix} \lambda_8 & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 0 \end{pmatrix}$$

$$\eta_9 = a_p^+ a_{p'} + a_{p'}^+ a_p = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\eta_{10} = -i\,(a_p^+ a_{p'} - a_{p'}^+ a_p) = \begin{pmatrix} 0&0&0&-i \\ 0&0&0&0 \\ 0&0&0&0 \\ i&0&0&0 \end{pmatrix}$$

$$\eta_{11} = a_n^+ a_\lambda + a_\lambda^+ a_n = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&1&0&0 \end{pmatrix}$$

$$\eta_{12} = -i\,(a_n^+ a_\lambda - a_\lambda^+ a_n) = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&-i \\ 0&0&0&0 \\ 0&i&0&0 \end{pmatrix}$$

$$\eta_{13} = a_\lambda^+ a_{p'} + a_{p'}^+ a_\lambda) = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix}$$

$$\eta_{14} = -i\,(a_\lambda^+ a_{p'} - a_{p'}^+ a_\lambda) = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&-i \\ 0&0&i&0 \end{pmatrix}$$

$$\eta_{15} = \frac{1}{\sqrt{6}}\,(a_p^+ a_p + a_n^+ a_n + a_\lambda^+ a_\lambda - 3\, a_{p'}^+ a_{p'}) = \frac{1}{\sqrt{6}} \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&-3 \end{pmatrix}$$

$$B = \frac{1}{3}\,(a_p^+ a_p + a_n^+ a_n + a_\lambda^+ a_\lambda + a_{p'}^+ a_{p'})$$

the latter is the baryonic number.
SU(4) has rank 3 so there are three diagonal commutators, η_3, η_8, η_{15}, besides B.
The physical quantities defined are <u>the hypercharge</u> :

$$Y = \frac{1}{\sqrt{3}}\,\eta_8 + \frac{1}{4}\,(B - \frac{\sqrt{6}}{3}\,\eta_{15}) =$$

$$\eta_{10} = -i\,(a_p^+ a_{p'} - a_{p'}^+ a_p) = \begin{pmatrix} 0&0&0&-i \\ 0&0&0&0 \\ 0&0&0&0 \\ i&0&0&0 \end{pmatrix}$$

$$\eta_{11} = a_n^+ a_\lambda + a_\lambda^+ a_n = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&1&0&0 \end{pmatrix}$$

$$\eta_{12} = -i\,(a_n^+ a_\lambda - a_\lambda^+ a_n) = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&-i \\ 0&0&0&0 \\ 0&i&0&0 \end{pmatrix}$$

$$\eta_{13} = a_\lambda^+ a_{p'} + a_{p'}^+ a_\lambda) = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix}$$

$$\eta_{14} = -i\,(a_\lambda^+ a_{p'} - a_{p'}^+ a_\lambda) = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&-i \\ 0&0&i&0 \end{pmatrix}$$

$$\eta_{15} = \frac{1}{\sqrt{6}}\,(a_p^+ a_p + a_n^+ a_n + a_\lambda^+ a_\lambda - 3\,a_{p'}^+ a_{p'}) = \frac{1}{\sqrt{6}} \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&-3 \end{pmatrix}$$

$$B = \frac{1}{3}\,(a_p^+ a_p + a_n^+ a_n + a_\lambda^+ a_\lambda + a_{p'}^+ a_{p'})$$

the latter is the baryonic number.
SU(4) has rank 3 so there are three diagonal commutators, η_3, η_8, η_{15}, besides B.
The physical quantities defined are <u>the hypercharge</u> :

$$Y = \frac{1}{\sqrt{3}}\,\eta_8 + \frac{1}{4}\,(B - \frac{\sqrt{6}}{3}\,\eta_{15}) =$$

the charm :

$$C = a^{+}_{p'} \, a_{p'} = \frac{3}{4} \left(B - \frac{\sqrt{6}}{3} \, \eta_{15} \right)$$

so that :

$$Y = \frac{1}{\sqrt{3}} \left\{ \eta_8 + \frac{1}{\sqrt{3}} \, C \right\}$$

<u>and the charge</u> :

$$Q = \frac{1}{2} \eta_3 + \frac{1}{2} (Y + C)$$

Note that :

$$\left[\eta_{15} , F_k \right] = 0 , \quad k = 1, 2, \dots 8 .$$

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There are a large number of [illegible] review articles and lecture notes on recent developments in weak interaction theory. I will mention only :

1) S.B. Treiman, R. Jackiw and D.J. Gross, Lectures on current algebra and its applications, Princeton University Press (1972).

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6) L. Maiani, Weak interactions of charmed particles, in La Physique du Neutrino à haute énergie, Centre National de la Recherche Scientifique, Paris (1975).

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