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G. KOSÁLY

INVESTIGATION OF THE LOCAL COMPONENT OF POWER-REACTOR NOISE VIA DIFFUSION THEORY

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ABSTRACT

In recent years several experimental workers succeeded in measuring steam velocity in a BWR, by correlating the signals of axially placed incore neutron detectors. This success indicates that the field of neutron noise contains a strong component which is driven by local disturbances.

The existence of such a local term was one of the main assumptions of a recent phenomenological theory of the noise field in a BWR core. In the present paper the theoretical background of the phenomenological theory is given by solving the two-group diffusion equations satisfied by the neutron noise driven by a propagating disturbance of moderator density. We identify one of the terms in the solution as the local term and discuss its characteristic features and its ratio to other terms.

АННОТАЦИЯ

Многочисленные измерения,проведенные за последние годы в зоне кипящего реактора продемонстрировали, что путем корреляции шума нейтронных детекторов, размещенных аксиально, можно измерить скорость распространения пара. Этот результат указывает на то, что в нейтронном шуме имеется такой компонент, на который влияют, в первую очередь, локальные источники шума.

Существование такого локального компонента было основной гипотезой той феноменологической теории, которая была разработана в прошлом году, для объяснения шума кипящих реакторов. В настоящей статье исследуется эта основная гипотеза с помощью теории диффузии двух групп.

KIVONAT

Az utóbbi években több, forralóvizes reaktorok zónájában végzett mérés demonstrálta, hogy axiálisan elhelyezett neutrondetektorok zajának korrelálatása révén meg lehet mérni a gőz terjedési sebességét. Ez az eredmény arra utal, hogy a neutronzajban van egy olyan komponens, amelyet elsősorban a lokális zajforrások befolyásolnak.

Egy ilyen lokális komponens léte volt az alapfeltevése annak a fenomenologikus elméletnek, amelyet a forralóvizes reaktorok zajának magyarázatára konstruáltak az elmult évben. Jelen cikkben ezt az alapfeltevést vizsgáljuk meg a két-csoport diffuziós elmélet segitségével.

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Introduction

Following the early work of Thie /1967, 1968/ several papers have been published in recent years reporting on the investigation of the noise-field in a BWR by using axially placed incore neutron detectors.

It turned out, that the noise-field in a large BWR core is strongly space dependent /Thie, 1967, 1968, Seifritz, 1972/ moreover there is a component in the noise which acts in a local manner, that is its behaviour is influenced mainly by the disturbances which are close to the detector. It is this local component of the noise which has been recently used to determine the velocity of steam bubbles using incore neutron detectors. /Seifritz, 1972, Stegemann, Gebureck, Mikulski and Seifritz, 1973, Seifritz and Cioli, 1973, Wach, 1973, Nomura, 1974, Ando, Naito, Tanabe and Kitamura, 1974, Blomberg and Akerhielm, 1974/

In the wake of the this experimental work the neutron noise in a BWR has been visualized in several papers as the sum of a local contribution driven by the bubbling in the vicinity of the detector, plus a global background driven by fluctuations of reactivity. /Wach and Kosály, 1974, Kosály, Maróti and Meskó, 1974, Nomura, 1974, Ando, Naito, Tanabe, and Kitamura, 1974, Blomberg and Akerhielm, 1974/

Using this physical picture a model was constructed recently /Wach and Kosály, 1974, Kosály, Maróti, and Meskó, 1974/ and it was successfully applied to understand some features of the noise spectra in a BWR. For example, using this model a relation was established between the amplitude of neutron noise and local steam void fraction. /Kosály, Maróti, and Meskó, 1974/. The relation was successfully tested by comparing it to the experimental results of Seifritz /1972/.

The model is a phenomenological one, that is it was constructed in such a way, that besides being as simple as possible, and besides being supported by "physical feeling" it should be instrumental in the interpretation of incore neutron-noise measurements in a large BWR.

Further justification of the model can be sought primarily via further experimental work on light water cooled BWR's. On the other hand in the present work we intend to give some theoretical support to the model, by trying to relate the model to diffusion theory.

To this end in Chapter 1. we shortly review the main concepts of the model. In Chapter 2. one-group diffusion theory is used to calculate the response of the neutron field to a propagating disturbance of moderator density. As in one-group theory no localised behaviour is found, in Chapter 3. we turn to two-group theory and identify one term in the solution with the local component of the noise field. In the Appendix we investigate whether this local component can be a dominating term in the solution or not.

1. Survey of a phenomenological model of the neutron noise in a BWR

The neutron noise in a power reactor is induced by several perturbations. In the theory of reactor kinetics /Bell-Glasstone, 197o/ one usually considers the neutron response induced by each perturbation as composed of two parts: the global part, and the space-dependent part. By the global part one means the noise driven by the fluctuations of reactivity. The space dependent part can be defined simply as the difference of the total fluctuation of the flux $\delta\phi$ and of the global contribution. Considering a simple axial dependent model we write accordingly:

$$\delta\phi(t,z) = \delta\phi^{\text{space-dep.}}(t,z) + \delta\phi^{\text{global}}(t,z) \qquad /1.1/$$

It is a fundamental result of reactor kinetics /Weinberg and Schweinler, 1948, Loewe, 1965/ that perturbations which are homogeneous in the core do not result in space-dependent terms, that is they act exclusively as global noise sources. Inhomogeneous perturbations on the other hand contribute both to the global and to the space-dependent terms.

In view of BWR applications we consider now a case when several types of perturbations are acting simultaneously as global noise sources but in the spatial region where our detectors are positioned only a single inhomogeneous perturbation is active, this single noise source being the local fluctuation of the steam volume $\delta V_{st}(t,z)$. The fluctuation of the steam-volume results in local fluctuations of the density

of the moderator inducing both space dependent and global fluctuations of the flux.

As regards the exclusively global /homogeneous/ noise sources it is just for the sake of example, that we refer to the fluctuation of recirculation flow /Thie, 1967, Blomberg and Akerhielm, 1974/ as one of the possible sources of reactivity noise in a BWR. The global term is kept completely general in the present treatment.

The main assumption of the model concerns the space dependent term of the fluctuation of the flux. It is assumed that this term is so much space dependent that it is responding to its driving source mainly in a local manner, that is the space-dependent term of the neutron noise at a particular space point is driven by the fluctuation of the steam-content at the very same space point. It is in the above sense that in the present model we refer to the space-dependent term as the <u>local term</u> and to the fluctuation $\delta V_{st}(t,z)$ as the <u>local driving source</u> of the fluctuation of the flux. /Wach and Kosály, 1974, Kosály, Maróti and Meskó, 1974/.

Obviously even the local fluctuations of the flux are influenced by occurences inside a finite volume /volume of sensitivity/, that is to assume a strictly local response would be rather non-physical. It will become clear from the derivations of the present work, that a finite /but small!/ volume of sensitivity is compatible with the model.

With the above ideas in mind we write that

$$NCPSD^{\phi}_{z_1,z_2}(\omega) = H(\omega)\, CPSD^{st}_{z_1,z_2}(\omega) + \qquad /1.2/$$

$$+ |G_0(\omega)|^2\, APSD^{\rho}(\omega)$$

In Eq./1.2/ the following notations have been used:

$NCPSD^{\phi}_{z_1,z_2}(\omega)$ = normalized cross-spectral density of the stochastic variables $\delta\phi(t,z_1)$ and $\delta\phi(t,z_2)$.

$CPSD^{st}_{z_1,z_2}(\omega)$ = cross-spectral density of the stochastic variables $\delta V_{st}(t,z_1)$ and $\delta V_{st}(t,z_2)$.

$G_0(\omega)$ = zero-power reactivity transfer function

$APSD^{\rho}(\omega)$ = auto-spectral density of the fluctuations of reactivity.

According to Eq./1.2/ the local contribution "follows" in space the fluctuation of the steam content.

In previous papers /Wach and Kosály, 1974, Kosály, Maróti, and Meskó, 1974/ the frequency dependent factor before the CPSD of the steam content was considered as the square of the gain of a transfer function ($H_{\ell}(\omega)$) relating the local noise source to the fluctuation of the flux.

While in principle such a factor should be certainly present, its break point must be at high frequencies not influencing the measurements to which the present model refers. On the other hand it was pointed out very recently /Wach, 1974, Fuge, 1975/ that a factor accounting for the finite volume of sensitivity is to be included. According to this point of view /Wach, 1974, Fuge, 1975/ the function $H(\omega)$ is band limited with a break frequency related to the finite time while the disturbance propagates through the volume of sensitivity of the local effect.

It is to be emphasized at this point, that the above volume is finite even if a point-like detector is considered. In this case we speak about the volume of "neutron-physical" sensitivity. To determine the radius of this volume is one of the aims of the present paper.

Consider now the steam flowing upwards with an average velocity $V(z)$, and neglect any fluctuations of this velocity. Then because of the conservation of steam the relation

$$CPSD^{st}_{z_1,z_2}(\omega) = e^{-i\omega\tau_{12}} \frac{V(z_1)}{V(z_2)} APSD^{st}_{z_1}(\omega) \qquad /1.3/$$

$$z_2 \geq z_1$$

holds.

In the above equation τ_{12} is the mean transit time of steam between the positions z_1 and z_2 . The equation accounts for the steam generated between these two positions, but the condensation of steam is neglected.

Eqs./1.2/, /1.3/ are the equations to be used in the interpretation of correlation measurements with axially placed detectors /Wach and Kosály, 1974, Kosály, Maróti and Meskó, 1974/.

The normalised-auto-spectral density of the fluctuation of the flux at the position z can be obtained by substituting $z_1 = z_2 = z$ in the cross-spectrum.

$$NAPSD_z^{\phi}(\omega) = H(\omega)\, APSD_z^{St}(\omega) + + |G_0(\omega)|^2 APSD^{\rho}(\omega) \qquad /1.4/$$

It was shown recently by Kosály, Maróti and Meskó /1974/ that the function $APSD_z^{St}(\omega)$ is white, and its axial dependence is dictated by ratio of the average local steam content $\alpha(z)$ and the local velocity of steam, that is

$$APSD_z^{St}(\omega) = (\text{const.})\ \frac{\alpha(z)}{V(z)} \qquad /1.5/$$

In the interpretation of auto-spectral measurements Eqs./1.4/ and /1.5/ are relevant. /Kosály, Maróti and Meskó, 1974/

It is a general tendency of the measured $NAPSD$ -functions /Thie, 1967, 1968, Wach 1973, Nomura 1974, Ando, Naito, Tanabe, Kitamura 1974, Blomberg and Akerhielm 1974/ that the curves fit together for $f < 1\,Hz$, but more noise is experienced for the upper ion-chambers, than for the lower ones in the region $1\,Hz < f < 10\,Hz$.

We account for this phenomenon by assuming that the break-frequency of the function $H(\omega)$ is much higher $(\sim 10\,Hz)$ than the break-frequency of reactivity fluctuations. This choice makes the local contribution gradually dominant for $f > 1\,Hz$ resulting in a space dependence dictated by Eq. /1.5/.

Inspection of Eq./1.2/ suggests, that for $f < 1\,Hz$ the global term will dominate in the cross-spectrum as well. In the region between 1 Hz and lo Hz the local term becomes gradually more and more important, that is the phase of the cross-spectrum follows approximately the line $-\omega\tau_{12}$ and the gain is independent of the frequency. This predictions agree again with experimental findings. /Stegemann, Gebureck, Mikulski, Seifritz 1973, Seifritz and Cioli 1973, Wach 1973/ Deviations from the above behaviour /oscillation of the phase around the line, dips in the gain/ have been interpreted successfully by taking into account that the influence of the reactivity fluctuations decreases but gradually, that is in several cases they still contribute to the noise field in the frequency region considered. /Wach and Kosály, 1974/

The most crucial point of the above model is the assumption that the space dependent part of the neutron noise has a local character expressed by Eqs./1.2/ and /1.4/ and by the smallness of the volume of neutron-physical sensitivity.

To clarify this point we will investigate the neutron noise induced by a propagating disturbance of the moderator density. The reactor will be considered as homogeneous and infinite. Because of the assumption of infinite core the reactivity fluctuations induced by fluctuations of moderator density vanish, that is one obtains directly the space dependent part of the neutron noise from the equations.

The aim of the paper is to investigate the conditions under which this space dependent solution can be described by the first terms on the right hand sides of Eqs./1.2/ and /1.4/. A theoretical derivation of the local fluctuations will necessarily result in an analytical expression of $H(\omega)$, that is it will result in a theoretical estimate of the volume of the neutron-physical sensitivity.

In our considerations we will keep to light water cooled reactors exclusively. In spite of that in our conclusions we will have a remark concerning localised behaviour in a heavy-water moderated BWR.

As mentioned above we are considering a homogeneous reactor model throughout the present paper. This is certainly a poor approximation in a BWR, where because of the increase of steam content the cross-sections change rather strongly along the axis. It is to be considered in this context that the aim of the present work is to provide a general theoretical background rather, than to derive quantitatively correct end results.

Before concluding the present Chapter we point out that the basic concepts of the phenomenological model are not new in reactor kinetics. In their early paper on the theory of oscillating absorber in a reactor Weinberg and Schweinler /1948/ clearly distinguish between the "local response" and the "overall response" which in our case corresponds to local an global fluctuations respectively. The recent trend of trying to interpret the noise field in a large BWR core by using the concept of local and global fluctuations, is the application of a very traditional way of thinking to BWR problems.

Investigation of the neutron noise induced by a propagating disturbance of moderator density.

2. Calculation of the neutron noise in one-group diffusion theory.

Let us denote the density of the moderator by $R(t,z)$ and separate the average value and the fluctuation as

$$R(t,z) = R + \delta R(t,z) \qquad /2.1/$$

Assume now that the disturbance $\delta R(t,z)$ propagates upwards with the velocity V, that is

$$CPSD^{R}_{z',z''}(\omega) = e^{-\frac{i\omega}{V}(z''-z')} APSD^{R}_{z'}(\omega) \qquad /2.2/$$

$$z'' \geq z'$$

In Eq./2.2/ the cross-spectral-density of the stochastic variables $\delta R(t,z')$ and $\delta R(t,z'')$, $(z' < z'')$ is related to the auto-spectral-density of $\delta R(t,z')$ by a "dead-time term" describing the propagation of the disturbance.

Using the general relation

$$\left[CPSD^{R}_{z',z''}(\omega)\right]^{*} = CPSD^{R}_{z'',z'}(\omega) \qquad /2.3/$$

one obtains from Eq./2.2/, that

$$CPSD^{R}_{z',z''}(\omega) = e^{-i\frac{\omega}{V}(z''-z')} \begin{cases} APSD^{R}_{z'}(\omega), & z' < z'' \\ APSD^{R}_{z''}(\omega), & z' > z'' \end{cases} \qquad /2.4/$$

Eq./2.4/ was derived to specify the physical behaviour of the source of neutron noise. To calculate the neutron response to this excitation we assume, that the unperturbed reactor is critical, that is the critical equation

$$D\frac{\partial^2 \phi(z)}{\partial z^2} - \Sigma_a \phi(z) + \nu\Sigma_f \phi(z) = 0 \qquad /2.5/$$

holds.

The fluctuation of the moderator density results in fluctuating group-constants $(D+\delta D(t,z)),\ \Sigma_a+\delta\Sigma_a(t,z),\ \Sigma_f+\delta\Sigma_f(t,z))$ which in turn induce fluctuations of the neutron flux. The time dependent neutron flux obeys the one-group diffusion equation.

$$\frac{1}{v}\frac{\partial\phi(t,z)}{\partial t} = \frac{\partial}{\partial z}\left[(D+\delta D(t,z))\frac{\partial\phi(t,z)}{\partial z}\right] +$$

$$+\left[(\nu\Sigma_f+\nu\delta\Sigma_f(t,z))(1-\beta_{eff}) - (\Sigma_a+\delta\Sigma_a(t,z))\right]\phi(t,z) + \qquad /2.6/$$

$$+\beta_{eff}(\nu\Sigma_f+\nu\delta\Sigma_f(t,z))\int_{-\infty}^{t} e^{-\lambda(t-t')}\phi(t',z)\,dt'$$

Writing Eq./2.6/ only one delayed-neutron group has been considered. /Delayed neutron parameters: β_{eff}, λ /.

Separating the static and the fluctuating part of the flux as

$$\phi(t,z) = \phi(z) + \delta\phi(t,z) \qquad /2.7/$$

and using Eq./2.5/ in Eq./2.6/ one gets after linearization the following equation:

$$D\frac{\partial^2\delta\phi(\omega,z)}{\partial z^2} - \left(\Sigma_a + \frac{i\omega}{v}\right)\delta\phi(\omega,z) +$$

$$+\nu\Sigma_f\left[1-h(\omega)\right]\delta\phi(\omega,z) = S(\omega,z) \qquad /2.8/$$

$$h(\omega) = \frac{i\omega\beta_{eff}}{\lambda+i\omega}$$

Eq./2.8/ is written in the frequency domain, that is the notation

$$\delta\phi(\omega,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\omega t}\delta\phi(t,z)\,dt \qquad /2.9/$$

is used.

Eg./2.8/ has the usual form of a Langevin-equation /Williams,1974/. In the linear approximation the driving noise source on the rhs.

of the equation can be given as

$$S(\omega,z) = -\frac{\partial}{\partial z}\left[\delta D(\omega,z)\ \frac{\partial \phi(z)}{\partial z}\right] + \left[\delta\Sigma_a(\omega,z) - \nu\,\delta\Sigma_f(\omega,z)\,(1-h(\omega))\right]\phi(z) \qquad /2.1o/$$

Using the relation /Bell-Glasstone, 197o/

$$\frac{1}{G_0(\omega)} = i\omega\left[\Lambda + \frac{\beta_{eff}}{\lambda + i\omega}\right] \qquad /2.11/$$

as the definition of the reactivity transfer function and remembering that in one-group theory

$$\Lambda = \frac{1}{\nu\Sigma_f v} \qquad /2.12/$$

Eq./2.8/ can be rearranged to give

$$D\frac{\partial^2\delta\phi(\omega,z)}{\partial z^2} + \left[\nu\Sigma_f - \Sigma_a - \frac{\nu\Sigma_f}{G_0(\omega)}\right]\delta\phi(\omega,z) = S(\omega,z) \qquad /2.13/$$

Consider now an infinite system, (by) assuming that

$$K_\infty = \frac{\nu\Sigma_f}{\Sigma_a} = 1 \quad ; \quad \phi(z) = \phi = \text{const.} \qquad /2.14/$$

Using the above assumptions in Eqs./2.1o/, /2.13/ one obtains, that

$$\frac{\partial^2\delta\phi(\omega,z)}{\partial z^2} - \gamma^2\delta\phi(\omega,z) = \frac{1}{D}S(\omega,z) \qquad /2.15/$$

$$\gamma^2 = \frac{1}{M^2 G_0(\omega)} \quad ; \quad M^2 = \frac{D}{\Sigma_a}$$

$$S(\omega,z) = \left[\delta\Sigma_a(\omega,z) - \nu\,\delta\Sigma_f(\omega,z)\,(1-\beta_{eff})\right]\phi \qquad /2.16/$$

In writing Eq./2.16/ ultra-low frequencies has been excluded that is the approximation

$$h(\omega) \simeq \beta_{eff} \qquad /2.17/$$

has been made.

In the above equations we have not specified the physical mechanism resulting in fluctuations of the cross-sections. We indicate now the connection between cross-section fluctuations and fluctuation of the density by writing

$$S(\omega,z) = S.\ \phi.\ \delta R(\omega,z) \qquad /2.18/$$

Here ϕ stands for the constant critical flux, S is a constant of proportionality.

Eqs./2.15/, /2.18/ can be easily solved. One obtains, that

$$\frac{\delta\Phi(\omega,z)}{\Phi} = -\frac{S\,G_0(\omega)}{\nu\,\Sigma_f}\,R_\gamma(\omega,z) \qquad /2.19/$$

$$R_\gamma(\omega,z) = \frac{\int_{-\infty}^{+\infty} e^{-\gamma|z-z'|}\,\delta R(\omega,z')\,dz'}{\int_{-\infty}^{\infty} e^{-\gamma|z-z'|}\,dz'} \qquad /2.2o/$$

$$\gamma = \frac{1}{M\sqrt{G_0(\omega)}} \qquad /2.21/$$

According to the above results the fluctuation of the flux at the space point z , is influenced by the fluctuations of the density in the region

$$z-\frac{1}{\gamma} \le z' \le z+\frac{1}{\gamma}$$

that is in one-group diffusion approximation the distance characterising the "locality" of the neutron response to density fluctuations is $1/\gamma$. /cf. radius of the volume of the neutron physical sensitivity./

As reviewed in the previous chapter in recent experimental work on boiling water reactors a rather localised neutron-response has been found in the frequency region between 1 Hz and 1o Hz. In this region for light water cooled reactors

$$\frac{1}{G_0(\omega)} \simeq \beta_{eff} \qquad /2.22/$$

that is

$$\frac{1}{\gamma} \simeq \frac{M}{\sqrt{\beta_{eff}}}\ ;\ M=\sqrt{L^2+\tau}\ ;\ \beta_{eff} \cong 8.10^{-3} \qquad /2.23/$$

As $L^2 \ll \tau$, and $\tau \simeq \tau_{H_2O}$ are certainly realistic approximations for a light water moderated core one obtains, that

$$\frac{1}{\gamma} \simeq 65\,cm \qquad /2.24/$$

Obviously the above value is far too big to account for the localised behaviour found in experimental work. For frequencies higher than the break frequency of $G_0(\omega)$ the $\frac{1}{\gamma}$ -value becomes rather small but this does not explain the experimental findings in the frequency region below 1o Hz.

The results of this chapter correspond very closely to an earlier result of Kostic and Seifritz /1971/ who calculated the "macroscale of the spatial coherence function of the neutron field" and found this quantity to be rather big. Later in the paper we will refer in more detail to their activity.

Let us turn now to two-group diffusion theory where one has two spatial relaxation lengths. Because of the structure of two-group equations we anticipate one of them composed of β_{eff} and M as found in one-group theory, and the cther beeing related basically to the spatial relaxation in the thermal group(L) which is rather fast in a light water moderated core.

3. Calculation of the neutron noise in two-group diffusion theory

We again start from the assumption, that the unperturbed reactor is critical, that is the critical equation

$$D_1 \frac{\partial^2 \phi_1(z)}{\partial z^2} - \Sigma_1 \phi_1(z) + \nu \Sigma_f \phi_2(z) = 0$$

$$D_2 \frac{\partial^2 \phi_2(z)}{\partial z^2} - \Sigma_2 \phi_2(z) + \Sigma_R \phi_1(z) = 0 \qquad /3.1/$$

holds.

In Eq./3.1/ all symbols have their usual meaning. Fast fission have been neglected.

As we did in the previous Chapter we again consider fluctuations of the moderator density resulting in fluctuations of the group constants. Assuming, that the core is infinite, that is

$$K_\infty = \frac{\nu \Sigma_f \Sigma_R}{\Sigma_1 \Sigma_2} = 1 \quad ; \quad \phi_1(z) = \phi_1 = \text{const.}$$

$$\phi_2(z) = \phi_2 = \text{const.} \quad ; \quad \frac{\phi_2}{\phi_1} = \frac{\Sigma_R}{\Sigma_2} \qquad /3.2/$$

we obtain after linearization the system of Langevin-equations /Williams, 1974/ satisfied by the fluctuations of the fast and thermal flux respectively.

$$D_1 \frac{\partial^2 \delta\phi_1(\omega,z)}{\partial z^2} + \nu \Sigma_f [1-h(\omega)] \, \delta\phi_2(\omega,z) -$$

$$- \left(\Sigma_1 + \frac{i\omega}{v_1}\right) \delta\phi_1(\omega,z) = S_1(\omega,z) \qquad /3.3/$$

$$D_2 \frac{\partial^2 \delta\phi_2(\omega,z)}{\partial z^2} + \Sigma_R \, \delta\phi_1(\omega,z) -$$

$$- \left(\Sigma_2 + \frac{i\omega}{v_2}\right) \delta\phi_2(\omega,z) = S_2(\omega,z)$$

The driving noise sources on the rhs. of the equations can be given as

$$S_1(\omega,z) = \delta\Sigma_1(\omega,z)\,\phi_1 - \nu\,\delta\Sigma_f(\omega,z)\,[1-h(\omega)]\,\phi_2 \qquad /3.4/$$

$$S_2(\omega,z) = \delta\Sigma_2(\omega,z)\,\phi_2 - \delta\Sigma_R(\omega,z)\,\phi_1$$

For the sake of getting lucid end results we exclude very low $(\omega \ll \lambda)$ and very high $(\omega > \alpha_{cr})$ frequencies, that is we restrict the treatment to the region where the relations

$$\Sigma_1 \gg \frac{i\omega}{v_1} \quad ; \quad \Sigma_2 \gg \frac{i\omega}{v_2}$$

$$h(\omega) \simeq \beta_{eff} \quad ; \quad \frac{1}{G_0(\omega)} \simeq \beta_{eff} \qquad /3.5/$$

hold.

We again indicate the connection between cross-section fluctuations and fluctuations of the density by writing

$$S_1(\omega, z) = S_1 \phi_1 \, \delta R(\omega, z)$$
$$S_2(\omega, z) = S_2 \phi_1 \, \delta R(\omega, z) \qquad /3.6/$$

Here S_1 and S_2 are constants of proportionality, ϕ_1 is the constant flux in the fast group being related to the thermal flux ϕ_2 by Eq./3.2/.

Using the approximations indicated in Eq./3.5/, Eqs./3.3/, /3.6/ can be solved readily. One obtains the following result:

$$\frac{\delta\Phi_1(\omega, z)}{\Phi_1} = A_1 R_\alpha(\omega, z) + G_1 R_\gamma(\omega, z) \qquad /3.7a/$$

$$\frac{\delta\Phi_2(\omega, z)}{\Phi_2} = A_2 R_\alpha(\omega, z) + G_2 R_\gamma(\omega, z) \qquad /3.7b/$$

$$R_\mu(\omega, z) = \frac{\int_{-\infty}^{+\infty} e^{-\mu|z-z'|} \delta R(\omega, z') \, dz'}{\int_{-\infty}^{+\infty} e^{-\mu|z-z'|} \, dz'} \qquad /3.8/$$

$$\mu = \alpha, \gamma$$

$$A_1 = \frac{1}{D_1(\alpha^2-\gamma^2)} \left[S_1 \left(\frac{1}{L^2\alpha^2} - 1 \right) + S_2 \frac{\nu\Sigma_f(1-\beta_{eff})}{D_2\alpha^2} \right] \qquad /3.9a/$$

$$G_1 = \frac{1}{D_1(\alpha^2-\gamma^2)} \left[S_1 \left(1 - \frac{1}{L^2\gamma^2} \right) - \frac{S_2 \nu\Sigma_f(1-\beta_{eff})}{D_2\gamma^2} \right] \qquad /3.9b/$$

$$A_2 = \frac{1}{D_2(\alpha^2-\gamma^2)} \left[S_1 \frac{\Sigma_2}{D_1\alpha^2} + S_2 \frac{\Sigma_2}{\Sigma_R} \left(\frac{1}{\tau\alpha^2} - 1 \right) \right] \qquad /3.9c/$$

$$G_2 = \frac{1}{D_2(\alpha^2-\gamma^2)}\left[-S_1\frac{\Sigma_2}{D_1\gamma^2} + S_2\frac{\Sigma_2}{\Sigma_R}\left(1-\frac{1}{\tau\gamma^2}\right)\right] \quad /3.9d/$$

The two eigenvalues determining spatial relaxation are determined by the expressions:

$$\alpha^2 = \frac{\tau+L^2}{2\tau L^2}\left[1+\sqrt{1-\frac{4\tau L^2\beta_{eff}}{(\tau+L^2)^2}}\right] \quad /3.10a/$$

$$\gamma^2 = \frac{\tau+L^2}{2\tau L^2}\left[1-\sqrt{1-\frac{4\tau L^2\beta_{eff}}{(\tau+L^2)^2}}\right] \quad /3.10b/$$

The reason for the appearance of two eigenvalues instead of one is obvious. In fact α^2 and γ^2 correspond to the famous γ^2 and $-\mu^2$ roots well known in static two group theory /Glasstone-Edlund, 1952/.

As

$$\frac{4\tau L^2\beta_{eff}}{(\tau+L^2)^2} \ll 1$$

we may write approximately, that

$$\alpha^2 \simeq \frac{1}{L^2}+\frac{1}{\tau} \quad /3.11a/$$

$$\gamma^2 \simeq \frac{\beta_{eff}}{\tau+L^2} \quad /3.11b/$$

According to Eq./3.11b/ the relaxation length $1/\gamma$ is approximately equal to the corresponding quantity of one-group theory. The quantity $1/\alpha$ on the other hand is a new candidate for a characteristic length describing localised behaviour of the neutron noise.

As $L^2 \ll \tau$ the quantity $1/\alpha$ is approximately equal to the diffusion length in the core. Considering a typical value of $L \simeq 1\,cm$ it is obvious, that in Eqs./3.7a, b/ the α -terms do describe a local response of both the thermal and the fast flux to variations of density.

It seems to be rather certain from the above derivation that it is the α -term of two-group diffusion theory which corresponds to the local term of the phenomenological model. To find and prove this correspondence is the main aim of the present paper.

By substituting $S_1\phi_1 = 1$; $S_2 = 0$ and $\delta R(\omega, z) = \delta(z-z')$ in Eqs./3.7/, /3.8/, /3.9/ one gets back the result of Kostic and Seifritz /1971/ who calculated the response of the thermal flux to the injection of a fast neutron. In their result the γ -term dominates over the α -contribution, that is no localised behaviour can be predicted.

Obviously it ~~[illegible]~~ would not make sense to speak about any correspondence between the localised behaviour found in the experiments and the α -terms appearing in the formulae unless the latter dominate the solution.

In the Appendix we will demonstrate certain tendencies which result in the dominance of the α -contribution in many cases when light water moderated reactor cores are considered and thermal neutron detectors are used.

The considerations of the Appendix refer specifically to the case when the noise is induced by a density perturbation characterized by Eq./2.4/, that is, they do not contradict to the result of Kostic and Seifritz /1971/ who considered a different noise-source.

Let us investigate now the α -term exclusively.

To this end we ignore the γ -term in Eq./3.7b/ and calculate the normalised-cross-spectral-density of the stochastic variables $\delta\Phi_2(t,z_1)$ and $\delta\Phi_2(t,z_2)$.

One obtains, that

$$NCPSD^{\Phi_2}_{z_1 z_2}(\omega) =$$

$$= (A_2)^2 \frac{\alpha^2}{4} \int_{-\infty}^{+\infty} dz' \int_{-\infty}^{+\infty} dz'' \, e^{-\alpha|z_1 - z'|} \, e^{-\alpha|z_2 - z''|} \, CPSD^{R}_{z',z''}(\omega) \qquad /3.12/$$

If one considers now the case when

$$z_2 - z_1 \gg \frac{1}{\alpha}$$

then the major contribution to the above integral comes from the region where $z' < z''$, that is for the cross-spectrum

of density fluctuations Eq./2.2/ can be used. If one assumes furthermore, that in the region $z_1 - \frac{1}{\alpha} < z' < z_1 + \frac{1}{\alpha}$ the function $APSD^{R}_{z'}(\omega)$ changes but slightly one obtains that

$$NCPSD^{\Phi_2}_{z_1, z_2}(\omega) \simeq$$

$$\simeq (A_2)^2 \frac{\alpha^2}{4} APSD^{R}_{z_1}(\omega) \int_{-\infty}^{+\infty} dz' \int_{-\infty}^{+\infty} dz'' \, e^{-\alpha|z_1 - z'|} \times \qquad /3.13/$$

$$\times \, e^{-\alpha|z_2 - z''|} \, e^{-i\frac{\omega}{V}(z'' - z')}$$

Performing the integral we get the result:

$$NCPSD^{\Phi_2}_{z_1, z_2}(\omega) \simeq H(\omega) \, APSD^{R}_{z_1}(\omega) \, e^{-i\frac{\omega}{V}(z_2 - z_1)} \qquad /3.14/$$

In Eq./3.14/ the notation

$$H(\omega) = \frac{1}{\left[1 + \left(\frac{\omega}{\omega_\alpha}\right)^2\right]^2} \qquad /3.15/$$

$$\omega_\alpha = \alpha V$$

has been used.

To calculate the normalised-auto-spectrum we write $z_1 = z_2 = z$ in Eq. /3.12/ and assume again that the auto-spectrum of the density fluctuations changes but slightly arond z . One obtains, that

$$NAPSD_{2}^{\phi_2}(\omega) \simeq H(\omega)\, APSD_{2}^{R}(\omega) \qquad /3.16/$$

Comparing Eqs./3.14/, /3.15/ and /3.16/ with Eqs./1.2/, /1.3/ and /1.4/ of the phenomenological model one finds that the local terms of the phenomenological equations are really similar to the results of the present Chapter, that is two-group theory accounts for the localised character of the space dependent response of the neutron field to the density-excitation.

According to the above derivation the radius of the volume of neutron physical sensitivity is $1/\alpha$. The break frequency of $H(\omega)$ is just αV as it should be according to the arguments of Wach /1974/ and Fuge /1975/.

As regards the actual value of the break frequency, considering typical values of 1 cm^{-1} for α, and 100 cm/sec - 600 cm/sec for the velocity one gets, that ω_α is between 16 cps and 96 cps, which values are both much higher, than a typical break frequency of reactivity fluctuations (~ 1 Hz).

With this result for the break frequency we have completed the derivation of local fluctuations from two group diffusion theory.

Before concluding the present Chapter let us remark, that the above estimates of the break frequency ω_α seem to be somewhat exaggareted. The actual value of the break frequency is possibly lowered both by the presence of steam which increases the diffusion length,x/ that is, lowers the value of α,

----

x/ = The value $L \simeq 1$cm is a typical PWR data. In a BWR diffusion length is certainly bigger.

and by the finite dimensions of the neutron detector not considered in the present paper. On the other hand the tendency that the break frequency increases with increasing values of the velocity coincides with recent experimental results of Ando, Naito, Tanabe and Kitamura /1974/ who found that the width of the transit time peak of their cross-correlation function decreases with increasing velocity of the coolant flow.

4. Conclusions

The aim of the paper was to provide a theoretical background for the phenomenological model which postulates, that there is a local component in the neutron noise of a light water cooled boiling water reactor.

Using two-group diffusion model we succeeded in finding a term in the response to a propagating disturbance of density which results in a small volume of neutron physical sensitivity around the point of observation. The volume is small in the sense, that the space dependence of some relevant static parameters /average local steam-content, average steam-velocity/ can be neglected inside the volume. It is just this feature of the local component which enabled several workers to determine steam-velocity by neutron-noise measurements, and which might enable the determination of local steam content as well.

The smallness of the volume of sensitivity results in a relatively high break-frequency of the local contribution to neutron-noise spectra.

Effects caused by the finite dimensions of the detector were not considered in the paper. To account for this effect one has to modify the function $H(\omega)$.

As mentioned several times the derivations of the paper refer specifically to light water moderated reactors. It is to remembered though, that in an earlier work of Seifritz /1972/ a case has been demonstrated when in a heavy water moderated boiling water reactor (HBWR) the phase of the CPSD could be related to the transit time of the bubbles travelling between the two incore detectors. This experimental finding indicates that a local component could be present in heavy water moderated systems as well.

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Appendix

In Chapter 3. the neutron response driven by a propagating disturbance of moderator density has been calculated. It turned out that the response is composed of two-terms: the α -term being responsible for the local effect, and a background term, called γ -term.

In the present Appendix we investigate the ratio of α -terms and the corresponding γ -terms.

To this end we calculate the normalised-cross-spectral-density of the stochastic variables $\delta\phi_i(t,z_1)$ and $\delta\phi_i(t,z_2), (i=1,2)$. Using Eqs./3.7a/, /3.7b/, /3.8/ for the fluctuation of the flux, and Eq./2.4/ to describe the fluctuation of the density and assuming for the sake of simplicity that the auto-spectral-density of the density fluctuation is independent of the space variable one obtains by straightforward calculation, that

$$NCPSD^{\phi_i}_{z_1 z_2}(\omega) = e^{-i\frac{\omega}{V}(z_2-z_1)}\, NAPSD^{\phi_i}(\omega) \qquad /A.1/$$

$(i=1,2)$

$$NAPSD^{\phi_i}(\omega) = NAPSD^{R}(\omega)\left\{\frac{A_i^2}{\left[1+\left(\frac{\omega}{\omega_\alpha}\right)^2\right]^2} + \frac{2G_iA_i}{\left[1+\left(\frac{\omega}{\omega_\alpha}\right)^2\right]\left[1+\left(\frac{\omega}{\omega_\gamma}\right)^2\right]} + \frac{G_i^2}{\left[1+\left(\frac{\omega}{\omega_\gamma}\right)^2\right]^2}\right\} \qquad /A.2/$$

In the above equation $NAPSD^{\phi_i}(\omega)$ and $NAPSD^{R}(\omega)$ are the normalized-auto-spectral densities of the fluctuation of the neutron-flux and moderator density, respectively. As regards the characteristic frequencies the notations

$$\omega_\alpha = \alpha V \quad ; \quad \omega_\gamma = \gamma V \qquad /A.3/$$

have been used. The constants A_1, G_1, A_2, G_2 are given by Eqs./3.9a - 3.9d/.

It is the assumption of the space-independent auto-spectrum of the density fluctuations which resulted in space-independent auto-spectrum of the fluctuations of the flux both in the fast $(i=1)$, and in the thermal group $(i=2)$.

According to the estimates of α and γ given in the present paper, in a light water cooled system

$$\alpha \simeq 65\,\gamma \qquad /A.4/$$

that is a rather broad range of frequencies exist where the inequality

$$\omega_\gamma \ll \omega \ll \omega_\alpha \qquad /A.5/$$

holds.

Considering this, one may argue, that it is just the above region where in Eq./A.2/ the α -factors are still unity, but the γ -factors have rolled off already, that is, it is this region where one anticipates the existence of local effects.

One should not forget of course that the ratio of the α -terms and γ -terms is influenced by the magnitude of the constants A_i and G_i as well. /By α -term we mean the "pure" α -term in this Appendix. In Eq./A.2/ both the "pure" γ - and the mixed term are named as γ -term/ In fact it is the quantity

$$\frac{G_i}{A_i} \; \frac{1 + \left(\frac{\omega}{\omega_\alpha}\right)^2}{1 + \left(\frac{\omega}{\omega_\gamma}\right)^2} \qquad /A.6/$$

which rules the ratio of the two contributions.

Using Eqs./3.9a - 3.9d/ are obtains by straightforward calculation, that

$$\frac{G_i}{A_i} = - \frac{\omega_\alpha^2}{\omega_\gamma^2} X_i \qquad (i = 1, 2) \qquad /A.7a/$$

$$X_1 = \frac{1 + \frac{S_1}{S_2} \frac{\Sigma_R}{D_1} \frac{\tau L^2}{1-\beta_{eff}} \left(\frac{1}{L^2} - \alpha^2\right)}{1 + \frac{S_1}{S_2} \frac{\Sigma_R}{D_1} \frac{\tau L^2}{1-\beta_{eff}} \left(\frac{1}{L^2} - \gamma^2\right)} \simeq \qquad /A.7b/$$

$$\simeq \frac{1 - \frac{S_1}{S_2} \frac{L^2}{\tau}}{1 + \frac{S_1}{S_2}}$$

$$X_2 = \frac{1 + \frac{S_2}{S_1} \frac{D_1}{\Sigma_R} \left(\frac{1}{\tau} - \gamma^2\right)}{1 + \frac{S_2}{S_1} \frac{D_1}{\Sigma_R} \left(\frac{1}{\tau} - \alpha^2\right)} \simeq$$

/A.7c/

$$\simeq \frac{1 + \frac{S_2}{S_1}}{1 - \frac{S_2}{S_1} \frac{\tau}{L^2}}$$

In deriving the approximate versions of the formulae giving X_1 and X_2 the following approximations have been used:

$$\Sigma_1 \simeq \Sigma_R \quad ; \quad \tau = \frac{D_1}{\Sigma_1} \simeq \frac{D_1}{\Sigma_R}$$

$$\frac{1}{L^2} - \alpha^2 \simeq -\frac{1}{\tau} \quad ; \quad 1 - \beta_{eff} \simeq 1$$

$$\frac{1}{L^2} - \gamma^2 \simeq \frac{1}{L^2} \qquad /A.8/$$

$$\frac{1}{\tau} - \gamma^2 \simeq \frac{1}{\tau}$$

Consider now for the sake of example as a frequency which is still inside the region indicated by Eq./A.5/, the value

$\omega = 0,5\,\omega_\alpha$. At this frequency the factor defined by Eq./A.6/ can be given as

$$\frac{G_i}{A_i}\left[\frac{1+\left(\frac{\omega}{\omega_\alpha}\right)^2}{1+\left(\frac{\omega}{\omega_\gamma}\right)^2}\right]_{\omega=0,5\,\omega_\alpha} = \qquad /A.9/$$

$$= -\frac{\omega_\alpha^2}{\omega_\gamma^2}\;\frac{1+0,25}{1+0,25\,\frac{\omega_\alpha^2}{\omega_\gamma^2}}\;X_i \simeq -5\,X_i$$

The above estimation shows that the value which is decisive concerning α -dominance is greatly influenced by the values of X_1 and X_2 which values according to Eq./A.7b/, /A.7c/ are strongly related to the magnitude of the ratio S_1/S_2 . To estimate this ratio we start from Eqs./3.4/ and /3.6/, and consider that the cross-sections appearing in the homogeneous-core approach are, as a matter of course, cell-averaged ones /Glosstone-Edlund, 1952/, that is considering unit disadvantage factor we may write, that

$$\Sigma = \frac{V_m}{V_m+V_f}\,\Sigma^m + \frac{V_f}{V_m+V_f}\,\Sigma^f \qquad /A.1o/$$

In eq./A.1o/ the volume ratios per unit cell V_m/V_m+V_f V_f/V_m+V_f and the cross-sections Σ^m, Σ^f refer to moderator and fuel respectively. According to Eq./A.1o/

$$\delta\Sigma = \frac{V_m}{V_m+V_f}\,\delta\Sigma^m + \frac{V_f}{V_m+V_f}\,\delta\Sigma^f \qquad /A.11/$$

It is to be pointed out now, that the fluctuation $\delta R(\omega,z)$ is by its definition the change in moderator density, that is it drives primarily moderator cross-sections and influences fuel-cross-sections via the spectral effect only. Neglecting this latter effect as a secondary one and using the obvious relation

$$\delta\Sigma^m(\omega,z) = \Sigma^m\,\frac{\delta R(\omega,z)}{R} \qquad /A.12/$$

one obtains from Eqs./3.4/, /3.6/, that

$$S_1(\omega,z) = S_1\,\phi_1\,\delta R(\omega,z) \simeq \Sigma_1^m\,\phi_1\,\frac{\delta R(\omega,z)}{R} \qquad /A.13a/$$

$$S_2(\omega,z) = S_2\,\phi_2\,\delta R(\omega,z) \simeq$$

$$\simeq \left(\Sigma_2^m\,\frac{\phi_2}{\phi_1} - \Sigma_R^m\right)\phi_1\,\frac{\delta R(\omega,z)}{R} \qquad /A.13b/$$

that is

$$\frac{S_2}{S_1} \simeq -\frac{\Sigma_R^m}{\Sigma_1^m} + \frac{\Sigma_2^m}{\Sigma_R^m}\,\frac{\phi_2}{\phi_1} \simeq -1+\varepsilon \qquad /A.14/$$

In Eq./A.14/, the approximation

$$\Sigma_1^m \simeq \Sigma_R^m$$

was made and the notation

$$\varepsilon = \frac{\Sigma_2^m}{\Sigma_R^m} \; \frac{\phi_2}{\phi_1} \qquad /A.15/$$

was used.

Both Σ_2^m / Σ_R^m and ϕ_2 / ϕ_1 are smaller than unity, resulting in a rather small value of ε, typical values being between o.ol and o.l.

Substituting Eq./A14/ into Eqs./A.7b/ and /A.7c/ and considering that ε is a small quantity we obtain, that

$$X_1 \cong - \frac{1}{\varepsilon}\left(1 + \frac{L^2}{\tau} - \varepsilon\right) \approx - \frac{1}{\varepsilon}\left(1 + \frac{L^2}{\tau}\right) \qquad /A.16a/$$

$$X_2 \cong \frac{\varepsilon}{1 + \frac{\tau}{L^2}(1-\varepsilon)} \approx \frac{L^2}{\tau}\,\varepsilon \qquad /A.16b/$$

As in a light water cooled reactor $L^2 \ll \tau$, according to the above equations the quantity X_2 tends to be rather small, but X_1 might become rather big.

Because of the many approximations made in the above derivation it would be risky to draw any definite conclusions concerning the ratio of α - and γ - contributions but a few tendencies can be pointed out.

Firstly, the fact, that $\omega_\gamma \ll \omega_\alpha$, works in the direction of the dominance of the α -term, as a rather broad frequency region can be defined where the pure α -term is still "active", but the γ -term and the mixed term has rolled off already.

Secondly, as the quantity ε is small, and $L^2 \ll \tau$, the quantity X_2 tends to be small, which certainly helps the pure α -term to dominate. On the other hand the same reasons which make X_2 small might result in rather big values of X_1 working againts α -dominance, therefore from the point of view of the dominance of the local term the situation is more favourable in the thermal group, than in the fast group.

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