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THEORIES FOR CONVECTION IN STELLAR ATMOSPHERES

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THEORIES FOR CONVECTION IN STELLAR ATMOSPHERES

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ABSTRACT

A summary and discussion is given in connection with the two papers "On convection in stellar atmospheres" (Nordlund, 1974; henceforth referred to as Paper I), and "A two-component representation of stellar atmospheres with convection" (Nordlund, 1976; Paper II). A discussion of the fundamental differences between laboratory convection and convection in a stellar atmosphere is presented. The shortcomings of laterally homogeneous model atmospheres are analysed, and the extent to which these shortcomings are avoided in the two-component representation is discussed. Finally, a qualitative discussion on the scaling properties of stellar granulation is presented.

1. INTRODUCTION

In this short discussion of convection in stellar atmospheres, the Sun will be frequently used as an illustrative example, providing us with a few order of magnitude data needed for the discussion and, more important, providing the comparatively most detailed observational information of relevance to the discussion.

Most of the visible light emitted from the Sun comes from a layer of a few hundred kilometres thickness, the solar photosphere. The energy produced in the solar interior is emitted as radiation on the surface. This requires an overall surface temperature of the order of 6000 K. (The effective temperature, $T_{eff}^{sun} \approx 5800$ K, is the required surface temperature, assuming a black body radiation.)

On the large scale, the run of temperature and pressure with height is given by conditions of equilibrium; the gas at a given level is at rest, the pressure is given by the total weight of the gas above that level, and the temperature is such that the amount of radiation emitted compensates for the net heating from the layers below and above. However, on a smaller scale, comparable to the thickness of the photosphere, conditions are non-stationary. Among other phenomena, convection contributes to the small scale fluctuations of temperature, velocity, etc. Convection is visible as granulation; that is, as a non-stationary pattern of brightness fluctuations on a horizontal scale of $\approx$ 1000 km, and with a characteristic time of evolution of the order of 10 minutes. According to recent observations, the brightness fluctuations are well correlated with the vertical velocity of the gas, with upward velocity corresponding to excess brightness. The solar granulation, thus, is one (and the only directly observable) example of convection in stellar atmospheres.

Before proceeding on the discussion of this subject, a short discussion of the fundamental differences between stellar convection and convection as it can be studied in a physicists laboratory, is appropriate.

2. LABORATORY CONVECTION AND CONVECTION IN STELLAR ATMOSPHERES

In a laboratory experiment, a convective system is, apart from the configuration of the experiment, characterised by two dimensionless numbers: the Rayleigh number Ra, and the Prandtl number Pr. A third dimensionless number, the Reynolds number Re, characterises the degree of turbulence of the resulting motion. In a typical experiment, the configuration is fairly simple, with homogeneous conditions and well defined dimensionless numbers Ra and Pr. One might say that most of the information about the system lies in the dimensionless numbers; two flows with the same Ra and Pr are <u>similar</u> and differ by a simple scaling of dimensions only.

Convection in the solar photosphere, and in stellar atmospheres in general, is, in a sense, <u>qualitatively different</u> from laboratory convection. This is essentially due to two facts: 1) the linear dimensions of the system are much larger, and 2) the system is strongly non-homogeneous, with pressure, temperature, etc., varying with orders of magnitude over the height of the system.

The large linear dimensions have consequences for the Reynolds and Rayleigh numbers. The Reynolds number, which essentially gives the ratio of the size of the system to the scale of motion where viscous dissipation becomes important, becomes very large ($\approx 10^{10}$). Large Reynolds numbers correspond to turbulent flows; that is, highly irregular and time dependent flows, However, the exact value of Re is unimportant, as long as it is much larger than a critical value ($\approx 10^3$). The Rayleigh number, Ra, which characterizes the strength of convective instability (instability starting at $Ra = Ra_{crit}$, increasing with Ra), also scales with the linear size of the system (as ℓ^4). Ra contains, as a factor, the temperature gradient, or in the case of a stratified gas, the temperature gradient difference $(\nabla - \nabla_{ad})$ (cf. Paper I, pages 408-409). With a very large multiplicative factor from the linear size of the system, the Rayleigh number is either very large and negative, or very large and positive, depending on the sign of $(\nabla - \nabla_{ad})$. Thus, either Ra indicates stability or, as with Re, Ra is large enough for the actual value to be unimportant. The condition $\nabla > \nabla_{ad}$ for convection to occur is usually called the Schwarzschild criterion.

The strong variation of physical conditions over the system makes it impossible to assign values to Ra and Pr, that will hold for the system as a whole. The thermal conductivity, for example, which enters into the definition of both Ra and Pr, is essentially due to radiation and is, of course, much larger in the upper, transparent layers, than in the lower, opaque layers of the photosphere. Thus, for example, the importance of radiative heat transfer relative to convective heat transfer (as measured by the combination Pr · Re) changes over the system.

In conclusion: in contrast to laboratory convection, convection in a stellar atmosphere is governed mainly by the "configuration"; that is, by the detailed structure of the stellar atmosphere. Estimates of the dimensionless numbers give qualitative information only: the Rayleigh number indicates whether there is convectional instability or not, according to the Schwarzschild criterion; the Reynolds number indicates turbulent conditions; and a comparison of the Prandtl number with the Reynolds number indicates that radiative heat "conduction" is important in the transparent layers.

In order to describe approximately convection in a stellar atmosphere; that is, to estimate such things as the transport of energy by convection, the velocities associated with the convection, and the resulting temperature structure, one has to (explicitly or implicitly) adopt some kind of model of the actual situation. The successes and shortcomings of the description are conveniently discussed in terms of the assumptions of the adopted model.

3. LATERALLY HOMOGENEOUS MODEL ATMOSPHERES WITH CONVECTION (PAPER I)

An assumption common to the theories discussed in Paper I is that the essential structural variation of the atmosphere is the variation of physical properties with height. In other words, one assumes that it is meaningful to speak of a mean model, in which temperature, pressure, etc. are functions of height alone. The horizontal fluctuations associated with convection are then regarded as perturbations on the mean model.

The customary way to describe the mixing length model of convection is the following: Superimposed on the mean model are temperature fluctuations in the form of bubbles. Due to its excess temperature, a bubble is accelerated upwards and, due to its upward velocity, its excess temperature is increased. After travelling a distance comparable to its linear dimension ($\approx \ell$, the mixing length), a bubble "dissolves" and delivers its excess energy to the mean atmosphere. Since the process is self-amplifying before the dissolution of the bubble, it is obvious that the resulting energy transport estimate will be strongly dependent on the assumed mixing length (the dependence is actually quadratic in ℓ).

An alternative description, leading to the same numerical estimates, but somewhat more realistic (and therefore easier to generalize) is the following: Temperature fluctuations, of unspecified shape, but with typical linear extent ℓ, are sustained by the convection. Being convective in origin, the temperature fluctuations are correlated with the vertical velocity, and we may consider the thermal energy and vertical momentum balances, following the mean motion of a fluctuation. Since the velocity field is turbulent, the fluctuation is constantly exchanging thermal energy and vertical momentum with its surroundings. Taking ℓ as a characteristic length for this exchange, the resulting estimates of convective energy flux and velocity come out the same as in the "bubble-picture".

Obviously, the common point is a continuous "disorganization" of the motion, competing with the continuous "organization" of the motion, introduced by the convective instability.

The local mixing length estimates are obtained by requiring the sums of gains and losses (of thermal energy and vertical momentum) to be exactly zero at each level. If the balances of gains and losses thus are exact, obviously the fluctuations can change neither velocity nor excess energy. On the other hand, the estimates resulting from this balance requirement may well be changing rapidly, following the strong variation of physical conditions with height in the stellar atmosphere. Consequently, there is an internal inconsistency in the local estimate, and this inconsistency is the more severe the more rapidly the resulting estimate is changing with height. One example of this is the vanishing of the velocity estimate above the zone of convective instability. In a solar model, the local velocity estimate drops from $\simeq 2\ \mathrm{kms}^{-1}$ down to zero in a few tens om kilometres, although the size of a convective element is approximately 1000 km.

Another inconsistency also arises as a consequence of rapid height variation: The concepts of "mean velocity" and "mean excess energy" of a fluctuation of linear extent $\simeq \ell$ no longer make sense when the resulting estimates change considerably over a distance short compared with ℓ.

Obviously, consistent estimates of convective flux and velocity have to be non-local in character. Several non-local estimates for convective flux and/or velocity have been given, in attempts to obtain more realistic results for convection in stellar atmospheres. Three such non-local treatments, due to Parsons (1969), Ulrich (1970ab), and Travis & Matsushima (1973ab), are analysed in Paper I.

The treatment due to Parsons is based on a very simple argument: When estimating the convective velocity, it is reasonable to take an average of the buoyancy force over a distance ℓ, instead of using the local buoyancy force only. In this way, the velocity estimate does not vanish immediately above the convectively unstable zone, but extends for a distance $\simeq \ell/2$ above that zone. The temperature fluctuations are estimated in the same way as in the local theory, given the velocity from the non-local expression.

The treatment due to Travis & Matsushima is unrealistic, in that the radiative exchange of energy is not taken account of appropriately. This results in a relative overestimation of the importance of convection in transparent layers (cf. section IIe, Paper I).

The treatment given by Ulrich is based on approximate differential equations for the balances of excess energy and vertical momentum. As discussed in Paper I, section IIc, the results given by Ulrich on the basis of these equations are erroneous. This is due to the use of an invalid approximation in the handling of the equations. However, the basic differential equations given by Ulrich provide a convenient tool for the subsequent analyses carried out in Paper I.

A discussion justifying an estimate of the convective flux, quite similar to the estimate given by Parsons, is given in section IId for Paper I. In sections III and IV it is shown how the expressions for convective flux and velocity can be combined with radiative transfer and pressure equations, and how the resulting system of equations can be solved numerically to yield model stellar atmospheres.

The differential equation governing the ratio of excess energy to vertical velocity (Eq. (52) of Paper I) is rediscussed in section V of Paper I. Using a fixed model atmosphere, the solutions representing rising and sinking gas are compared to the approximate solution used in calculating the model atmosphere (cf. Fig. 6, Paper I). The conclusions concerning the non-local behaviour of the solutions may be stated as follows: As previously discussed in this summary, there is an inconsistency in using an exact balance of gains and losses to find the desired estimate. In the differential equation formulation, such a balance corresponds to neglecting the differential term itself, and requiring the other terms to cancel ut. Now, in the differential equation, deviations from this balance are governed by the term corresponding to the physical process responsible for the balance. Thus, for example, if radiative exchange (radiative cooling) is the process balancing the heating, and the characteristic time for radiative exchange is 10 seconds, say, then a deviation from heat balance is cancelled out in approximately 10 seconds. This "cooling rate" is typical for the uppermost layers of the solar convection zone (cf. Table 1, Paper I). With convective velocities of the order 1-2 kms^{-1}, this time corresponds to a distance of the order of a few tens of kilometres. Thus, in the uppermost parts of the solar convection zone, where conditions are changing very rapidly, this is the order of distance over which deviations from heat balance are important, _if_ the estimates of the rate of radiative exchange are realistic. This point leads us to the discussion of an other important inconsistency in the treatments of convection with the methods of Paper I:

As shown in section IId of Paper I (cf. Table 1 and the discussion on the same page), the radiative cooling rate q and the specific heat capacity C_p are very temperature sensitive in these layers of the photosphere. For example, the temperature of the rising gas corresponds to a substantially lower cooling rate q than the temperature of the mean model. Thus it is clear that the cooling of the rising gas is overestimated when the cooling rate is calculated at the mean model temperature. (A similar inconsistency is present for the specific heat capacity C_p.) Actually, from the fact that the estimated cooling rates are very different in the solutions representing rising and sinking material at the same level, it is clear that the conditions for radiative transfer may vary drastically with horizontal position.

The problem discussed in the last paragraph is obviously one of the shortcomings of methods in which fluctuations are regarded as small perturbations on a mean model only. Both in the radiative transfer and in the thermodynamics of the gas, conditions are changing drastically with horizontal position (as well as with height) in a real stellar atmosphere.

To sum up, we have discerned the following shortcomings of the methods analysed in Paper I:

_i_: The "following-the-motion-of-a-single-cell" picture is not consistent when the physical "input" as well as the resulting estimates are varying drastically over distances small compared with the size of a "cell".

_ii_: Estimates resulting from a detailed balancing of "gains and losses" (of thermal energy, for example) are not consistent with a rapid height variation of the same estimates.

iii: The use of a laterally homogeneous mean model atmosphere for the radiative transfer and for the calculation of thermal properties of the gas is not consistent with the resulting estimates of the horizontal temperature fluctuations.

iv: The velocity estimates have been made qualitatively more realistic, but this has been achieved using rather arbitrary averaging procedures.

v: The resulting estimates are rather strongly dependent on parameters, and especially on the parameter specifying the typical linear extent of the dominant fluctuations.

These shortcomings were the starting point for the work resulting in Paper II. In this short summary, Paper II is discussed mainly on the basis of these five points. A more detailed summary can be found in the ABSTRACT, INTRODUCTION, and CONCLUSIONS of Paper II.

4. A TWO-COMPONENT REPRESENTATION OF STELLAR ATMOSPHERES WITH CONVECTION (PAPER II)

The inconsistency of the "single-cell" picture (point i) is circumvented in Paper II by applying the balancing arguments to thin horizontal layers, instead of to vertically extended "cells". The horizontal extent of the layer is supposed to be very large, thus intersecting a large number of fluctuations in temperature and vertical velocity. The horizontal layer is split into two parts, or "components", according to the sign of the vertical velocity. (The two components are referred to as the + and - components, respectively, the sign being the sign of the vertical velocity.) In the convection zone, this split will obviously result in, on the average, the collection of the hot material into the + component, and of the cool material into the - component. Since, by assumption, the convection is stationary in the large scale properties (like surface averaged convective flux, etc.), the components contain stationary amounts of energy, vertical momentum, etc. These amounts, or "component means", pertain to the thin layer, which has a well defined height position in the atmosphere. The use of these averages as dependent variables thus circumvents point i.

A conservation principle, conservation of thermal + radiative energy, say, applied to the two components of the thin horizontal layer, results in the following requirement: A net inflow of thermal energy into the volume making up one component must correspond to a conversion of this thermal energy into radiative energy and a subsequent net outflow of the same amount of radiative energy from the volume. The net inflow is the difference between the inflow through the bottom surface and the outflow through the top surface of the volume, plus the inflow from the side surfaces; that is, from the other component. This corresponds to the following type of equation:

$$\frac{d}{dz}\left\{\begin{matrix}\text{vertical}\\ \text{flow}\end{matrix}\right\} + \left\{\begin{matrix}\text{exchange with the}\\ \text{other component}\end{matrix}\right\} = \left\{\begin{matrix}\text{total conversion}\\ \text{inside component}\end{matrix}\right\}$$

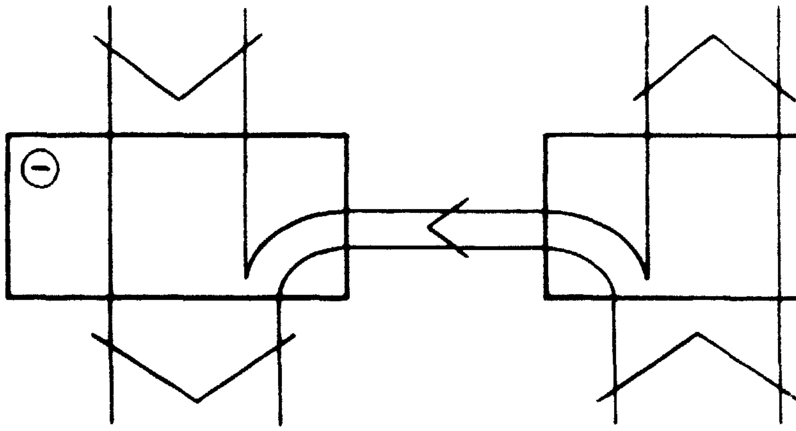
Note that the xchange term will appear in the corresponding equation for the other component as well, but with the opposite sign.

Local estimates would result if the differential terms were neglected. By keeping these terms in the numerical scheme, point ii on the list is circumvented.

The resulting equations are differential equations in the vertical flux of the considered quantity; that is, in the convective flux, in the case of thermal energy. Similar equations can be derived for the radiative flux (using radiative transfer at a large number of wavelengths), and for the vertical momentum (see further discussion below). The system of equations can be incorporated into a numerical scheme and solved simultaneously (cf. section V, Paper II).

As mentioned above, the dependent variables of the analysis are different averages over the two components. However, the averages are of several different types; surface averages, averages weighted by vertical mass flux, and averages weighted by the intensity of the radiation field. To obtain a closed system of equations, these averages have to be related to each other in an approximate way. In Paper II this is done using one representative value of the temperature for each component (the "component temperatures"). The other variables are related to these temperatures using the common total pressure and the equations of state. This avoids the use of a homogeneous "mean model" and allows the modelling of strongly non-homogeneous situations, — for example, the situation mentioned earlier, where one component is opaque and the other one transparent (cf. point iii). However, it must be stressed that the use of a single temperature to represent each component is a crude method, employed for reasons of simplicity. A more realistic relation between the averages would require the use of assumed distributions of temperature, velocity, etc., over the components. Without a much more refined analysis, such distributions would have to be taken ad hoc.

A simple consequence of applying the principle of conservation of mass to the two components is the following: When the average mass flux in each component is decreasing with height, this requires a net flow of gas from the + component to the - component (cf. illustration).



This is what might be called a kinetic requirement (pertaining to the motion). A plausible dynamic requirement (pertaining to the force causing the motion) corresponding to this situation is that the pressure in the + component should be larger than the pressure in the - component, and that this difference should be proportional to the net flow between the components.

If this pressure difference is taken into account in the vertical momentum balances for each component, there results a non-local, second order differential term in the equation for the mass flux. The equation is of the "diffusion type", and the solutions of this equation extend outside the convectively unstable layers (cf. point iv on the list). The typical scale height for this "overshooting" is of the order of ℓ, where ℓ is a characteristic scale for the velocity fluctuations, entering in the estimate of the pressure difference mentioned above.

This characteristic scale for the convective motions enters into all the estimates of exchange between the two components, with the exchange rates being proportional to $1/\ell$ (except for the radiative energy exchange, which is proportional to $1/\ell^2$ in the limit of opaque fluctuations). Obviously, ℓ plays the role of a mixing-length; that is, a length characteristic of the "disorganization" of the motion.

This brings us, finally, back to the question of the choice of value for the mixing length (point v). As discussed in the CONCLUSIONS of Paper II, the penetrating convection in the solar photosphere can be rather successfully modelled, using values for ℓ estimated from the observed solar granulation. However, for the construction of non-solar model atmospheres, the question arises: how does this typical size of atmospheric convection scale from one stellar atmosphere to another? Or, to put the question differently, what is the reason for this size being approximately 1000 km in the solar case?

A tentative answer to this question may be argued for in the following way:

Consider the solar photosphere, using one of the reasonable solar model atmospheres to estimate approximate temperatures, densities, etc. From observations, we know that the dominant convective modes have sizes of approximately 1000 km. Now, can we understand why much larger and much smaller modes do not give significant contributions in this (given) environment? For the much smaller models, the turbulent losses as well as the radiative losses will increase and thus make these modes less efficient. (This is equivalent to diminishing the mixing-length in the two-component scheme which, we know, results in less efficient convection.) Why then, doesn't this line of reasoning work the other way around? One would, by the same arguments as above, expect larger-than-average modes to be more efficient, and these would then dominate the convective transfer.

Consider a mode considerably larger than average, with a diameter of 10 000 km, say, in the environment of average-scale modes. A few hundred kilometres down in the atmosphere, the temperature of the environment is rather well defined (cf. Fig. 3, Paper II), even if fluctuations in the temperature are allowed for. The upward moving part of the large-scale mode will rise practically adiabatically, until the gas starts becoming transparent in the vertical direction. However, this is the case already for the average-scale modes (cf. Fig. 4, Paper II); most of the energy loss in the rising ags is due to the vertical loss near the surface, the motion further down being almost adiabatic. Thus, the temperature of the large-scale mode is not much higher than the temperature of the average-scale mode. Therefore, in order to compete, the large-scale mode has to have a comparable vertical velocity; that is, approximately 1-2 kms^{-1}. Now, consider the conservation of mass for the large-scale mode. For simplicity, we suppose the mode to be cylindrical, with base area $\pi d^2/4$, and circumference area $\pi d \Delta z$. The density drops with a factor two from $z = 0$ to $z \approx + 100$ km (cf. Fig. 2, Paper II). The mean horizontal velocity out of the circumference area then would have to be $(1/2) \cdot (\pi d^2/4)/(\pi d \Delta z) = d/(8 \Delta z) > 10$ times greater than the vertical velocity, in order to conserve the mass inside the cylinder. This would correspond to more than 15 kms^{-1}, which is approximately twice the local sound speed. This is not a very plausible situation and, in fact, one might expect the atmosphere to "resist" this very anisotropic situation already at horizontal velocities well below the sound velocity. Half the sound velocity would, by this rough argument, correspond to something like d = 2000 km.

The decisive factor in the above reasoning is the minimum density scale height in the region of interest. In the hydrogen ionization zone, the density decreases more slowly than the pressure (or even increases, cf. Fig. 2. Paper II), but above the hydrogen ionization zone, the density scale height is approximately equal to the pressure scale height.

It thus appears that the customary choice of a mixing-length proportional to the pressure scale height does have some relevance. However, it should be noted, firstly, that the value of ℓ is most important in a comparatively thin layer near the surface. The size of the "average-scale" modes, thus, may well be increasing with depth below these layers. Secondly, other things, like the ratio of convective velocity to sound velocity, and the "resistance" of the atmosphere to velocity anisotropy, also enter into the argument. Thus, to turn the qualitative reasoning presented in the preceeding paragraphs into quantitative estimates of the scale of stellar granulation requires a careful analysis. Observational estimates of the scale of stellar granulation, using careful analysis of spectral line shifts and asymmetries, are needed to check such estimates.

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