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Mixing Angle θ and Magnetic Monopole in Weinberg's Unified Gauge Theory*

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Gauge symmetry admits a local unit isovector and leads to the magnetic monopoles in Weinberg's unified theory. We predict $\sin^2\theta = 1/2$ for the mixing angle θ on the basis of Dirac's condition for charge quantization. This interesting result should be tested experimentally.

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The magnetic monopole has been shown to exist in certain non-Abelian gauge theories, excluding Weinberg's unified theory.[1,2] The magnetic charge follows from the topological structure of three Higgs scalar fields in a three-dimensional space.[3]

Here, we wish to point out that one can always introduce a local unit isovector in a non-Abelian gauge theory to substitute the role played by three Higgs scalars, so far as the magnetic monopole is concerned. Thus, the requirement that the electromagnetic group be a subgroup of a larger group with a compact covering in Ref.1 is not necessary for the monopole solution.[2] In such a formulation of the monopoles, the magnetic charge and its conservation have nothing to do with the topology of Higgs fields and the dynamics of gauge fields. They are simply consequences of the local isospin gauge symmetry. We get exact solutions for the vector gauge fields and show the presence of a stable monopole with the magnetic charge $e_m = -\sin\theta/g$ in Weinberg's unified theory. Based on the Dirac condition for charge quantization, the theory predicts $\sin^2\theta = 1/2$ for the mixing angle θ, which can be tested experimentally.

In Weinberg's theory, the equations for the classical fields $\vec{A}_\mu$, B_μ, φ, $\varphi^\dagger$ are[4,5]

$$\partial_\mu \vec{A}^{\mu\nu} - g\vec{A}^{\mu\nu} \times \vec{A}_\mu + ig\Phi^{\dagger\nu}\vec{t}\varphi - ig\varphi^\dagger\vec{t}\Phi^\nu = 0, \tag{1}$$

$$\partial_\mu B^{\mu\nu} - \frac{1}{2}ig'\varphi^\dagger\Phi^\nu + \frac{1}{2}ig'\Phi^{\dagger\nu}\varphi = 0, \tag{2}$$

$$\partial_\mu \Phi^\mu - M_1^2 \varphi + 2h\varphi^\dagger\varphi\varphi - (ig\vec{A}_\mu\cdot\vec{t} + \tfrac{1}{2}ig'B_\mu)\Phi^\mu = 0, \tag{3}$$

$$\varphi = \begin{pmatrix} \varphi^+ \\ (\varphi_1^0 + \sqrt{2}\lambda + i\varphi_2^0)/\sqrt{2} \end{pmatrix},$$

$$\vec{A}_{\mu\nu} \equiv \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu, \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$\Phi^\mu \equiv \partial^\mu \varphi - ig\vec{A}^\mu\cdot\vec{t}\,\varphi - \tfrac{1}{2}ig'B^\mu\varphi,$$

where we have neglected the leptons for simplicity. The photon field A_μ and the neutral massive vector field Z_μ are given by

$$A^\mu = A_3^\mu \sin\theta + B^\mu\cos\theta, \qquad Z^\mu = A_3^\mu\cos\theta - B^\mu\sin\theta. \tag{4}$$

The mixing angle θ and the charge e are given by

$$\tan\theta = g'/g, \quad \text{and} \quad e = -g\sin\theta. \tag{5}$$

We are interested in the nontrivial solutions for the vector fields A_a^μ and B^μ. The scalar fields have obviously the following trivial solutions:

$$\varphi^\pm = 0, \qquad \varphi_2^0 = 0, \qquad \varphi_1^0 = -\sqrt{2}\lambda = -2\sqrt{2}M_W/g, \tag{6}$$

where M_W is the mass of $W^{\pm\mu} = (A_1^\mu \mp iA_2^\mu)/2^{1/2}$. We look for the static spherically symmetry solution of the form[6]

$$A_0^a = v^a A_0(r), \quad A_i^a = \varepsilon_{iab}v^b A(r), \quad v^b = r^b/r, \qquad i,a,b = 1,2,3, \tag{7}$$

$$B^0 = 0, \quad B^i = v^i B(r), \qquad i = 1,2,3, \tag{8}$$

where v^b is a local unit isovector. Equations (2) and (3) are satisfied by the solution (6) and (8) with arbitrary B(r). Equation (1) reduces to

$$r^2 d^2A/dr^2 + 2r\,dA/dr - A(1+grA)(2+grA) + grA_0^2(1+grA) = 0,$$
$$r^2 d^2A_0/dr^2 + 2r\,dA_0/dr - 2A_0(1+grA)^2 = 0. \tag{9}$$

The special 'particlelike' solution to (9) is

$$A(r) = F/(gr), \quad A_0 = 0, \qquad F = -1, -2, \tag{10}$$

which have singularities of the Coulomb form.

We also have the following singularity-free solution for SU(2) gauge field,

$$A(r) = (R - \sinh R)/(gr \sinh R), \quad R = \beta r, \quad \beta \text{ real},$$
$$A_c(r) = i(R\cosh R - \sinh R)/(gr \sinh R). \tag{10a}$$

We note that if β is complex with $\mathrm{Re}\,\beta \neq 0$ (10a) is also a solution.

To understand the meaning of the classical solutions, we define a generalized electromagnetic field tensor $\bar{F}_{\mu\nu}$ with the help of a local unit isovector $v^a(x_\mu)$:

$$\bar{F}_{\mu\nu} = v^a A^a_{\mu\nu} \sin\theta + B_{\mu\nu}\cos\theta - (\sin\theta/g)\varepsilon^{abc} v^a (D_\mu v^b)(D_\nu v^c), \tag{11}$$
$$(D_\mu v^b) = \partial_\mu v^b + g\varepsilon^{bce} A^c_\mu v^e, \qquad v^a(x_\mu) v^a(x_\mu) = 1.$$

As usual, the definition (11) is invariant under SU(2)XU(1) gauge transformation and $\bar{F}_{\mu\nu}$ becomes the usual electromagnetic field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $A^\mu = A^\mu_3 \sin\theta + B^\mu \cos\theta$, when $v^a = (0,0,1)$.[1] Note that the unit isovector v^a in (11) can be a function of space-time in general because of the local isospin gauge symmetry.[7] Since $v^a v^a = 1$, we can rewrite (11) as

$$\bar{F}_{\mu\nu} = [\partial_\mu (v^a A^a_\nu) - \partial_\nu (v^a A^a_\mu)]\sin\theta + B_{\mu\nu}\cos\theta$$
$$- (\sin\theta/g)\,\varepsilon^{abc} v^a \partial_\mu v^b \partial_\nu v^c. \tag{12}$$

The electric and the magnetic fields, E_j and H_k, are given by

$$H_k = \tfrac{1}{2}\varepsilon_{kij}\bar{F}_{ij}, \qquad E_j = \bar{F}_{j0}. \tag{13}$$

It follows from (7), (8), (10), (12) and (13) that

$$\vec{H} = -\vec{r}\sin\theta/(gr^3), \quad \vec{E} = 0. \tag{14}$$

The total magnetic flux is $-4\pi\sin\theta/g$. Thus we have a stable magnetic monopole at $\vec{r} = 0$ with the magnetic charge e_m,

$$e_m = -\sin\theta/g. \tag{15}$$

From (5) and (15) we obtain

$$ee_m = \sin^2\theta. \tag{16}$$

The Schwinger condition [8] $ee_m = 1$ and (16) give the result $\cos\theta = g = 0$ and therefore, it is incompatible with the theory because we must have $g \neq 0$ and $g' \neq 0$. The only charge-quantization condition compatible with (16) is the Dirac condition[9] $ee_m = 1/2$, which leads to the interesting result,

$$\sin^2\theta = 1/2. \tag{17}$$

This implies a universal coupling, $g = g'$, for the vector fields A_a^μ and B^μ in Weinberg's theory. Moreover, (17) leads to $M_W^2 = M_Z^2/2 = e^2/(2\sqrt{2}G_W)$ and the total effective e-ν interaction $(G_W/\sqrt{2})\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\,\bar{e}\gamma^\mu(\frac{1}{2}+\frac{3}{2}\gamma_5)e$. Thus, arbitrary features in Weinberg's theory are greatly reduced. Our prediction (17) is consistent with the average value of various experimental results.[10] The prediction should be further tested. We stress that these unambiguous predictions in Weinberg's theory are made on the basis of simplicity and beauty in equations derived from the concepts of local gauge symmetry and charge quantization. In view of the present technical difficulty[10] in testing (17), one should not allow oneself to be too discouraged simply because there is not complete agreement between (17) and some of experiments, e.g. reactor experiment $\bar{\nu}_e + e \rightarrow e + \bar{\nu}_e$.[10]

The magnetic current j_λ^m and the electric current j_λ^e are related to $\bar{F}_{\mu\nu}$ by

$$j_\lambda^m = \frac{1}{2}\,\varepsilon_{\lambda\rho\mu\nu}\,\partial^\rho\,\bar{F}^{\mu\nu} \tag{18}$$

and

$$j_\lambda^e = \partial^\rho\,\bar{F}_{\lambda\rho}\ , \tag{19}$$

which are obviously conserved: $\partial^\lambda j_\lambda^m = \partial^\lambda j_\lambda^e = 0$. When the vector fields A_a^μ and B^μ are free from line singularity, we have[3]

$$\varepsilon^{\lambda\rho\mu\nu}\,\partial_\rho\left[\,\partial_\mu(v^a A_\nu^a) - \partial_\nu(v^a A_\mu^a) + \partial_\mu B_\nu - \partial_\nu B_\mu\right] = 0 \tag{20}$$

and the magnetic current j_λ^m takes the form

$$j_\lambda^m = -\left(\sin\theta/2g\right)\varepsilon^{abc}\,\varepsilon_{\lambda\rho\mu\nu}(\partial^\rho v^a)(\partial^\mu v^b)(\partial^\nu v^c). \tag{21}$$

We see that the magnetic charge $(1/4\pi)\int j_0^m d^3r$ and its conservation are direct consequences of the local isospin gauge symmetry[7,11] which admits the local unit isovector $v^a(x_\lambda)$ in the theory.[12] We may remark that, in general, (21) with a time-independent $v^a(\vec{r})$ implies that the magnetic charge must be integer in units of $-\sin\theta/g$.[3] The general Dirac condition $ee_m = n/2$ is satisfied if and only if (17) holds.

The magnetic monopole mass M_m is of course very important for experiment. The solution (6), (8) and (10) leads to the energy E or the monopole mass,

$$M_m = E = -\int \mathcal{L}\, d^3r = \begin{cases} 0\ , & F = -2\ , \quad (22a) \\ \infty\ , & F = -1\ , \quad (22b) \end{cases}$$

for the static monopole system. The physical monopole probably could have a non-zero mass due to quantum corrections to (22a). The singularity-free solution (10a) does not have a simple physical interpretation because A_0 is imaginary. Yet the exact solution (10a) with β real is interesting for it leads to a finite energy :

$$E = \frac{4\pi}{g^2}\int_0^\infty dr \frac{d}{dr}\left[\frac{\beta R^2 \cosh R}{\sinh^3 R} - \frac{\beta \cosh R}{\sinh R} - \frac{\beta R}{\sinh^2 R} + \frac{\beta}{R}\right]$$

$$= (4\pi/g^2)|\beta| , \qquad R = \beta r, \quad A_0 \neq 0, \tag{23}$$

where the arbitrary constant β has the dimension of a mass. According to variational principle, we expect the finite energy solution to exist even if $A_0 = 0$:[13] Let us consider the simple case where $A_0 = B_\mu = M_1 = h = 0$, while M_1^2/h may not be zero. With the help of an arbitrary parameter m with the dimension of a mass, we may write E as

$$E = \frac{4\pi}{g^2} m \int_0^\infty dx \left[\left(\frac{d\bar{A}}{dx}\right)^2 + \frac{(\bar{A}^2 + 2\bar{A})^2}{2x^2} + \frac{(x\, d\bar{\varphi}/dx - \bar{\varphi})^2}{2x^2} + \frac{\bar{\varphi}^2(\bar{A}+2)^2}{4x^2} \right] \tag{24a}$$

$$= (4\pi/g^2) m I , \tag{24b}$$

where the dimensionless quantities x, $\bar{A}$, and $\bar{\varphi}$ are given by $x = mr$, $A_i^a = \epsilon^{iab}(r^b/r)m\bar{A}/(xg)$ and $\varphi^a = (r^a/r)m\bar{\varphi}/(xg)$. The quantity I is the minimum value of (24a) and can be found by computer calculations, using trial functions and adjusting their parameters. The numerical value of I is not important physically because m in (24b) is arbitrary and therefore E cannot be determined at the classical level.

To conclude, in contrast to 't Hooft's formalism[1] we have given a formalism in which the local gauge symmetry admits a local unit isovector and leads to the magnetic monopole with a finite mass in Weinberg's unified theory. In general, the properties of the monopole in SU(2)XU(1) theory are not necessary exactly the same as those of the monopole in U(1) theory or SU(2) theory. For example, the quantized monopole strength $e_m = 1/e$, derived from the SU(2) symmetry group, does not necessarily apply to Weinberg's theory.[14] The possible existence of the monopole should be searched experimentally without preconception, especially if the prediction (17) is confirmed.[15]

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11. It was stressed in Ref. 7 that local gauge symmetry leads to gauge fields and completely determined their interaction dynamics. In a similar sense, the magnetic monopole is a consequence of local gauge symmetry. See also J. P. Hsu, Lett. Nuovo Cimento 11, 525 (1974); 12, 128 (1975) and J. P. Hsu and J. A. Underwood, Phys. Rev. D12, 620 (1975).

12. Both the unit isovector v^a and the ratio $\phi^a/|\phi|$ for Higgs' fields ϕ^a (see Ref. 1) have nothing to do with dynamics at the classical level. For example, one looks for the solution ϕ^a of the form $\phi^a = r^a \xi(r)$, where $\xi(r)$ is to be determined dynamically; the ratio $\phi^a/|\phi|$ does not involve $\xi(r)$ and hence has nothing to do with dynamics.

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14. This is because $e = -g\sin\theta$ and $A^\mu = A_3^\mu \sin\theta + B^\mu \cos\theta$ (as shown in equations (5) and (4)) and, therefore, the assumption of the Dirac condition $e_m e = 1/2$ does not lead to contradiction in Weinberg's theory. Note that in a trivial $SU(2)\times U(1)$ theory in which $A^\mu = A_3^\mu$ and $g = -e$, one can derive the condition $e_m e = 1$ so that it is inconsistent to assume $e_m e = 1/2$. Also, in SU(2) theory one must have $e_m e = 1$; see Tai Tsun Wu and Chen Ning Yang, Phys. Rev. D12, 3845 (1975).

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