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Ronald H. Cohen

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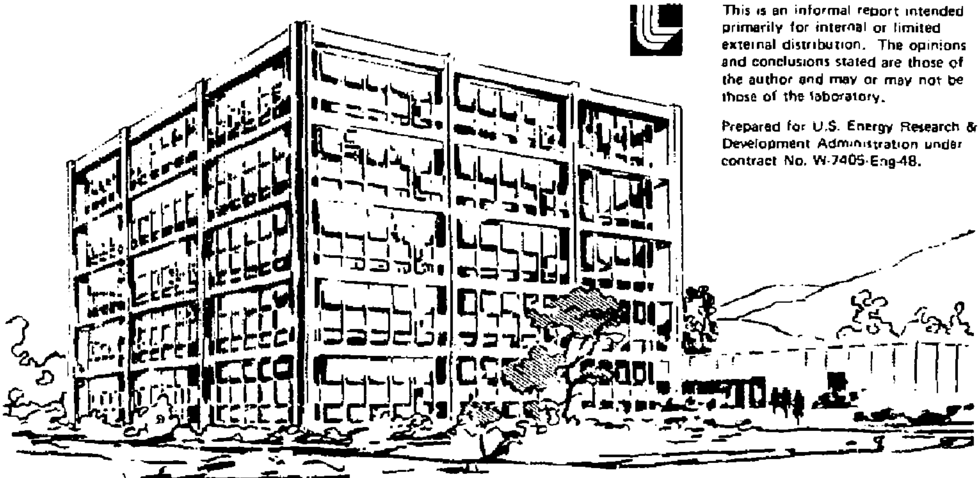

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Connection Between Adiabaticity and the Mirror Mode

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Abstract

The size of magnetic moment jumps of a particle in a long, thin equilibrium magnetic mirror field is shown to be related to the complex zeroes of the mirror mode parameter $B + 4\pi dP_\perp/dB$. A consequence is that adiabaticity places a lower limit on β than does the mirror mode.

* * * * * *

In a recent communication,[1] Hall noted a relationship between the mirror mode stability paramet·r for a long, thin equilibrium and the adiabatic energy limit formula of Cohen, et. al.[2] The purpose of the present report is to point out a more fundamental connection between the mirror mode parameter and adiabaticity theory in a long, thin equilibrium, which exists independent of the approximations involved in the application of the energy limit formula.

Hall's observation is that the scale length $L_\parallel = \left(2^{-1}B^{-1}d^2B/ds^2\right)^{-\frac{1}{2}}$ which appears in the adiabatic energy limit formula $(W \propto L_\parallel^2)$ is, in the long thin approximation, proportional to the square root of the mirror mode parameter $h_0' \equiv (B + 4\pi dP_\perp/dB)_0$. Hence adiabaticity is a more restrictive limit than the mirror mode, since the adiabatic criterion is that h_0' be greater than some positive number while mirror stability requires only $h_0' > 0$.

The notion that $W \propto B^2L_\parallel^2$ as some parameter such as β is varied is true only if the shape of $B(s)$ between bounce points of a typical particle remains fixed (changing only in scale length and magnitude)(and provided that the ass med $v_\perp/v$ for an average particle is held fixed) as the parameter is varied; otherwise the parameter κ *which appears in the energy limit formula* (see appendix) is also β-dependent. Since the shape is not, *in practice*, constant, Hall's treatment cannot be considered formal; in particular, the quantitative estimates of h' at the adiabatic limit are only as good as a quadratic well fit to the finite-β field. But the conclusion that adiabaticity places a *lower limit on* β than does the mirror mode applies to a much more general class of fields, as the following discussion shows.

The fundamental parameter involved in the $\Delta\mu$ *expression is not* $L_{\parallel}$, but rather $\kappa L_{\parallel}$, which is a particular integral of B in the complex plane. In fact the form of the mirror mode parameter h', viewed as a function of B or S, and in particular the location of the complex zeroes of h', plays a key role in *determining* $\kappa L_{\parallel}$. $\Delta\mu$ is determined by an integral along a field line,

$$\Delta\mu = R\sqrt{\frac{\mu}{2}}\, v^2 \int ds\, \frac{\rho_\perp}{B^{1/2}}\, e^{i\psi} \qquad (1)$$

where $\frac{d\psi}{d\tau} \cong \omega_{ci}$ and $\rho_\perp$ is the curvature.

The integral (1) is most simply evaluated by deforming the contour *into the* complex s plane to pass near the singularities of $\rho_\perp/B^{1/2}$ and the stationary points of ψ, where $d\psi/ds = 0$. The stationary points are in fact the zeroes of B. If these singularities are either well-separated or close enough that *they may be considered* merged, it follows that $\Delta\mu \propto \exp(-\kappa/\epsilon)$, where

$$\frac{\kappa}{\epsilon} = \text{Im} \int_{s=0}^{s_*} \frac{ds}{v_{\parallel}}\, \omega_{ci} \qquad (2)$$

and s_* is the position of the isolated singularity or merged cluster of singularities closest to the real axis.

The cases considered in previous work have all been of the type where the most important singularities are the zeroes of B; hence s_* is the position of a zero or the center of a merged cluster of zeroes.

For a long, thin equilibrium with P(B) of the form

$$P_\perp(B) = B^{-m}g(B) \tag{3}$$

with $m > 0$ and g(0) finite, we have

$$B_v^2 = B^2 + \beta B^{-m}g \tag{4}$$

so that there are no zeroes of B at finite B_v. For such a field a new class of singularities becomes important, namely the zeroes of dB_v^2/dB (i.e. the singularities of h'B in the long thin approximation). Note that in the limit $\beta \to 0$ these singularities become the zeroes of B_v.

We illustrate by example. Consider

$$P_\perp = C/B^2 \tag{5}$$

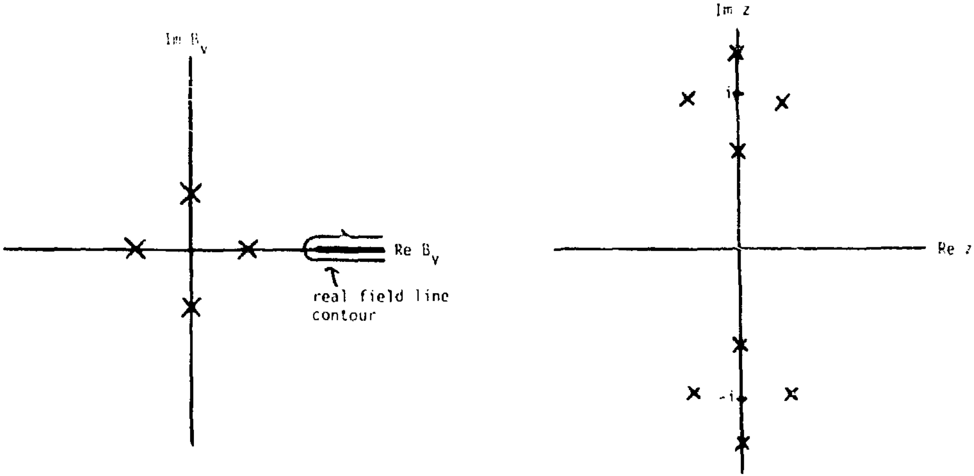
For this model, we may explicitly invert eq. (2) to find $B(B_v)$,

$$B^2 = \frac{1}{2}\left(B_v^2 + \sqrt{B_v^4 - 32\beta C}\right) \tag{6}$$

From Eq. (6) we note that: (a) B has no zeroes if $C \neq 0$; (b) In the complex B_v *plane, there are four branch points at* $B_v^4 = 32\beta C$. $4B_v = 1 + z^2$, there are 8 branch points in the complex z plane, four in each half plane arranged about $z = \pm i$ (see Fig. 1); (c) At these branch points, $dB_v^2/dB = 0$; (d) In the limit $\beta \to 0$, the branch points merge into the zero of B_v; (e) As β increases the branch points move further apart, until at β = mirror mode limit, the branch point s_m nearest the real axis touches the axis; (f) When

Figure 1

Branch Points of B for $P_\perp = C/B^2$



is large enough that the branch points may be considered well-separated (formally, [illegible] ≫ 1, where $[\text{illegible}]^{-1} = \Omega_{ci}(0)\, L/v$) then [illegible] is given by (2) with $s_* = s_m$. It is clear that as [illegible] approaches the mirror mode limit [illegible] decreases to zero, so, as Hall concluded, for some [illegible] less than the mirror mode limit [illegible] is such that *nonadiabatic losses are appreciable.*

For more complicated forms of $P(B)$, it is difficult to invert Eq. (2) to find $B(s)$. It is more convenient to use B, rather than s (or z) as the variable of integration in the [illegible] integral (1). In the complex B plane, the points $dB_v^2/dB = 0$ appear as stationary points of [illegible], since

$$\frac{d\,\cdot}{dB} = \frac{d\,\cdot}{dt}\left(\frac{ds}{dt}\right)^{-1}\left(\frac{dB_v}{ds}\right)^{-1}\frac{dB_v}{dB}$$

or $$\frac{d\,\cdot}{dB} = \frac{q}{mc\,v}\,\frac{B}{B_v}\left(\frac{dB_v}{ds}\right)^{-1}\left(\frac{1}{2}\frac{dB_v^2}{dB}\right) \qquad (7)$$

Exceptional cases arise when the zero of dB_v^2/dB coincides with a zero *of B_v or dB_v/ds. These cases must be examined individually to determine if a* stationary phase point exists. Apart from these cases, the exponential factor [illegible] is given by

$$\mathrm{Im}\,\frac{q}{mc}\int_0^{s_*} ds\,\frac{B}{v_\parallel} = \mathrm{Im}\,\frac{q}{mc}\int_{B_0}^{B_*}\frac{dB}{v_\parallel}\,\frac{B}{B_v}\left(\frac{1}{2}\frac{dB_v^2}{dB}\right)\left(\frac{dB_v}{ds}\right)^{-1} \qquad (8)$$

where B_* is the solution of (7) which gives the smallest [illegible] *(the stationary* point closest to the real field line).

We note the following properties of (7) and (8): (a) For $P_\perp$ of form (3), the only stationary points (in the long, thin approximation) are the points where the mirror mode parameter $h' = B^{-1}2^{-1}dB_v^2/dB$ vanishes; $B = 0$ is not a stationary point; (b) In the limit $\beta = 0$, we have $B_v = B$; the vanishing of dB_v^2/dB is thus *equivalent to the more standard stationary phase condition* $B_v = 0$; (c) There is a solution B_m to $dB_v^2/dB = 0$ which is real and which satisfies $0 \le B_m \le B_o$, where B_o is the minimum value of B on the real field line. B_m is the root of $dB_v^2/dB = 0$ which is to be used in eq. (8). As β is increased, B_m moves from 0 toward B_o (and B_o decreases) until $B_m = B_o$ at the mirror stability limit. For some β less than the mirror limit, B_m and B_o are close enough and hence Δ/L is small enough, that nonadiabatic losses are important; (d) When $P_\perp$ is a simple function of z or B_v instead of z, then $B^2 = B_v^2 - 8\pi P_\perp\left(B_v^2\right)$, and $dB_v^2/dB \propto B$. Hence the stationary points of ϕ are just the zeroes of B, which now happen to be also the zeroes of the mirror mode parameter. It may be noted that for such a $P_\perp$, the mirror mode stability limit occurs for $\beta \equiv 8\pi P_\perp/B_v^2 = 1$ or equivalently, at $B_o = 0$.

If the long thin approximation is dropped, then (apart from the exceptional cases noted earlier) the zeroes of dB_v^2/dB are still stationary points of ϕ, but these are not necessarily the points where the mirror mode parameter $h' = 1 + 4\pi dP_\perp/dB$ *vanishes. Finally, we note the importance of determining* all singularities of the ΔW integrand--both stationary points of ϕ and singularities in the pre-exponential coefficient--once a variable of integration has been chosen. In the preceding examples, it is clearly insufficient to look only for zeroes of B; furthermore, if s is used as the variable of integration rather than B, $dB_v^2/dB = 0$ points are singular points of B but are not stationary points of ϕ.

ΔW expressions for some particular forms of $P_\perp(B)$ will be given in a forthcoming report.

APPENDIX

We review here the adiabatic energy limit formula given in ref. 2:

$$W = 1.17 \cdot 10^{-3} \; \frac{\kappa^2 B_o^2 L_\parallel^2 Z^2/A}{(1-.039\ell n\Lambda)^2} \qquad \text{(A-1)}$$

where

$$\Lambda = \frac{A}{2}\left(\frac{50 \text{ kev}}{W}\right)\left(\frac{L_p}{340 \text{ cm}} \; \frac{\langle v_\parallel\rangle/v}{.5} \; \frac{10 \text{ ms}}{\tau}\right)^2 \left[\frac{5}{A_o}\,(3\Delta)\,\frac{(v_\perp/v)_{rms}}{.59}\;\frac{1}{\sqrt{2}(\cos\psi)_{rms}}\right]^4 \qquad \text{(A-2)}$$

Here, B_o is the mid-machine field strength, $L_\parallel = \left(2^{-1}B^{-1}d^2B/ds^2\right)_o^{-\frac{1}{2}}$, Z and A are ionic charge and mass, L_p is the mean bounce-to-bounce guiding center path length of an average particle in the process of diffusing from its initial pitch angle into the loss cone, $\langle v_\parallel\rangle$ is its initial mean parallel speed, τ is the particle's lifetime against all other loss processes, Δ is the total fractional change in μ required (in time τ) for a typical particle to be considered lost from the distribution, ψ is the gyro phase, and A_c is the assumed pre-exponential coefficient in $\Delta\mu/\mu$: $\Delta\mu/\mu = A_o(v/v_\perp)\cos\psi\,\exp(-\kappa/\varepsilon)$. A_c is typically of order 2-5, with the smaller values applying to machines with radial plasma and field scale lengths of order a few gyroradii, and large values applying to machines with larger radial scale lengths. κ is given by:

$$\kappa = \text{Im}\int_{s=0}^{s_*} \frac{d(s/L_\parallel)B/B_o}{(1-\lambda^2 B)^{1/2}} \qquad \text{(A-3)}$$

with $s_* = s(B=0)$ and $\lambda = (v_\perp/v)_{s=0}$. For a quadratic well,

$$\cdot = \frac{1}{2\lambda^2}\left(\frac{1+\lambda^2}{2\lambda}\ln\frac{1+\lambda}{1-\lambda} - 1\right) . \qquad \text{(A-4)}$$

In the long, thin approximation, we note that $L_\parallel$ is related to its value in vacuum as

$$L_\parallel^2 = L_{\parallel vac}^2 \, (1-\beta)^{\frac{1}{2}} \; \frac{B + 4\pi P_\perp/dB}{B_v} . \qquad \text{(A-5)}$$

REFERENCES

1. Hall, L. S., Lawrence Livermore Laboratory unpublished internal memorandum, April 20, 1976.

2. Cohen, R. H., Rowlands, G., and Foote, J. H., "Nonadiabaticity in Mirror Machines", Lawrence Livermore Laboratory Report (in preparation).

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