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Lawrence Livermore Laboratory

ZONE - A FINITE ELEMENT MESH GENERATOR

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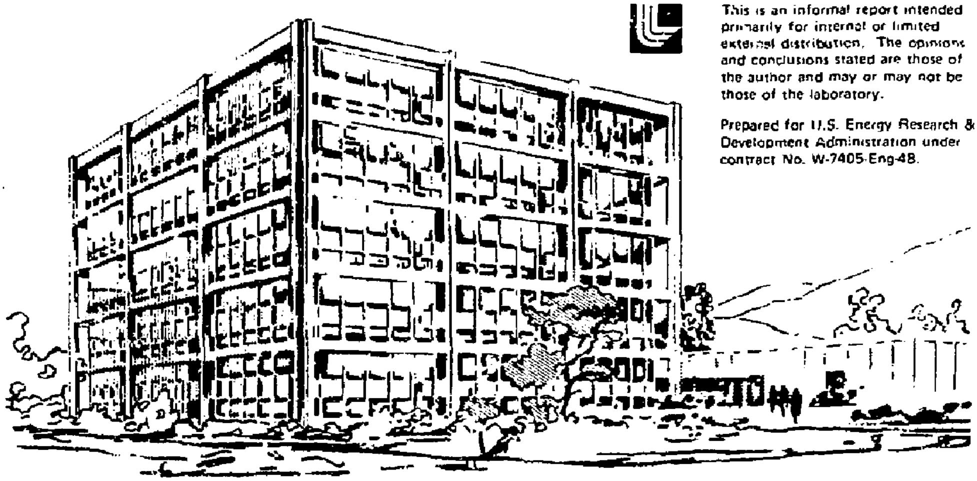

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ZONE — A FINITE ELEMENT MESH GENERATOR

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I. INTRODUCTION

The ZONE computer program is a finite element mesh generator which produces the nodes and element description of any two dimensional geometry. The geometry is subdivided into a mesh of quadrilateral and triangular zones arranged sequentially in an ordered march through the geometry. The order of march can be chosen so that the minimum bandwidth is obtained.

The node points are defined in terms of the x and y coordinates in a global rectangular coordinate system. The zones generated are quadrilaterals or triangles defined by four node points in a counter clockwise sequence. Node points defining the outside boundary are generated to describe pressure boundary conditions. The mesh that is generated can be used as input to any two dimensional as well as any axisymmetrical structure program.

The output from ZONE is essentially the input file to NAOS, HONDO, and other axisymmetric finite element programs. Two additional files generated are the input to the PLOT program, which is a post processor for ZONE used to generate the graphical display of the geometry for TMDS (TV Monitor Display System) viewing or for hard copy.

II. PROGRAM CONCEPT AND CHARACTERISTICS

The basic concept used in the program is the definition of a two dimensional structure by the intersection of two sets of lines which describe material and geometric boundaries. The geometry of the structure is established in a global x-y "right handed" rectangular coordinate system and must lie in the positive x quadrants for axisymmetric problems. Non-axisymmetric geometries can lie anywhere in the x-y space. The y axis is the axis of symmetry for axisymmetric geometries. A set of lines called *meridians* are defined, which describe the geometric and material boundaries and run more-or-less in the same direction. Another set of lines called *rays* which intersect the meridians are also defined at material and geometric boundaries. The order in which the meridians are defined is arbitrary. The rays, however, must be defined in sequences as one proceeds from one end of the structure to the other. These rays and meridians are defined by a sequence of coordinate points along the line in the global system forming a set of linear line segments.

In general the choice of lines to be designated as rays and those to be designated as meridians is arbitrary. The meridians and rays in Figure 1, for

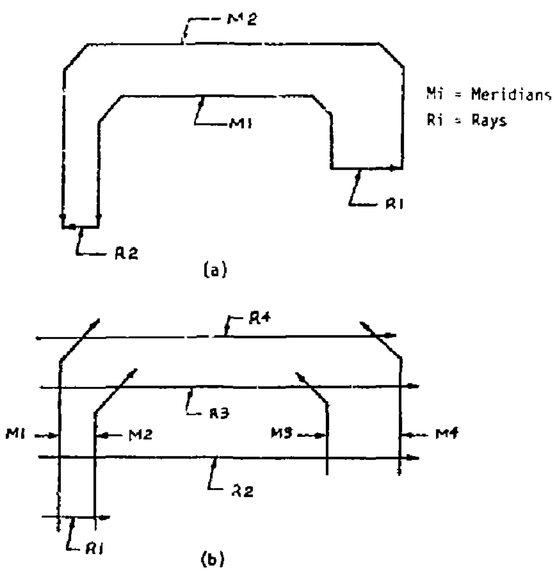


Figure 1. An example illustrating the relationship between rays and meridians. The same structure can be described two different ways. Note that the ray and meridian end points do not have to correspond to the intersection points as in (b)

example, can be interchanged if the positive direction of the ray is *reversed*. Nodes are numbered in sequence along the rays. The zones are similarly numbered, i.e., numbering begins at the *meridian first* encountered along a ray and continues to the last *meridian* encountered along the ray.

Associated with every point along the meridian is a local "right handed" x'-y' rectangular coordinate system. The y' axis corresponds to the tangent and the x' axis corresponds to the normal to the meridian at the point. The positive y' direction coincides with the positive direction along the meridian as shown in Figure 2. The sequence of coordinate points defining the meridian establishes the positive direction along the meridian. The positive x' direction is then specified. The ray *must pass through the entire structure* in the positive x' direction so that a sequence of ray-meridian intersections is formed. This concept is illustrated in Figure 1, which shows two correct ways to model the same structure. In Figure 1a only two meridians (M1 and M2) and two rays (R1 and R2) are required to completely *define the* structure. The positive direction and the termination of the rays and meridians are marked

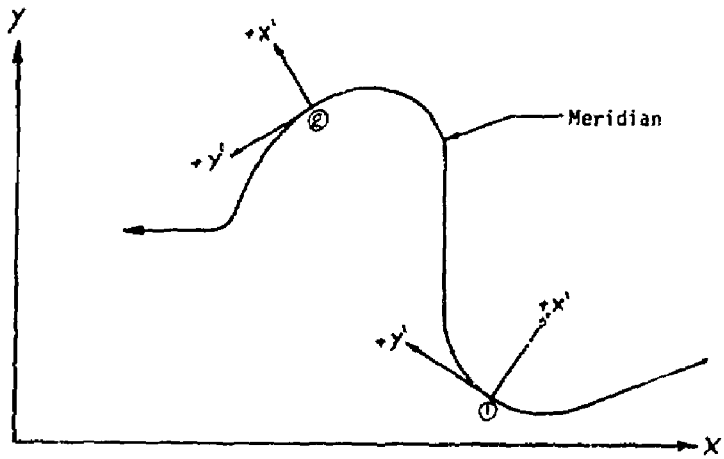


Figure 2. An example of the relationship of the local x'-y' coordinate system and the x-y global coordinate system in which the meridian is *defined*.

by the arrows. In Figure 1b four meridians and four rays are needed to define the same structure. Note that ray R2 as well as R3 and R4 pass through the entire structure. Also ray R1, the first ray, is at the beginning of the structure, i.e., the first ray encountered with a meridian as one proceeds along the meridian in the positive direction from the meridian origin.

III. REGIONS

The section of the structure between successive rays is called a region, as illustrated in Figure 3, for example, and cannot have material or geometric discontinuities in the meridional direction, i.e., meridians cannot be discontinuous (begin or end) within a region. This means that a meridian must begin or terminate on a ray. Or conversely, a ray must pass through the beginning or end of every meridian. What is always implied here by the beginning or end point is that an intended ray-meridian intersection be found. The be-

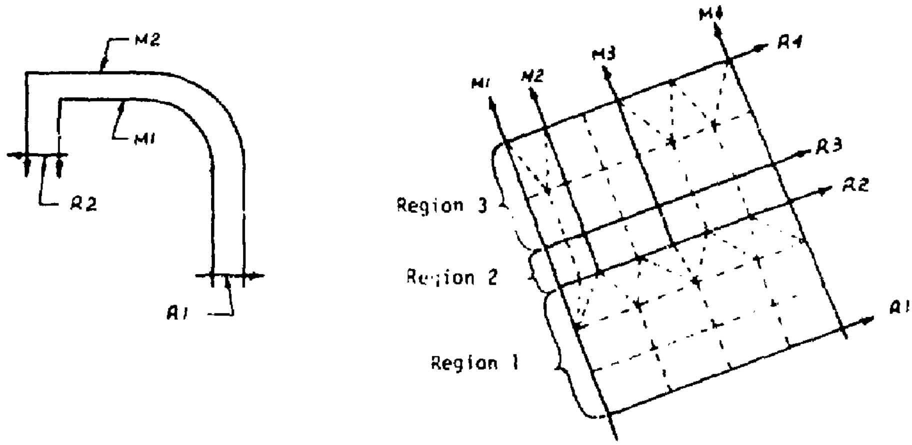


Figure 3. Examples of regions. In (a) the entire structure has only one region between R1 and R2. In (b) region 2 may be a very narrow strip between R2 and R3. Regions 1 and 3 have transition zones automatically generated.

ginning and terminal points of the ray and meridians may, indeed, extend beyond the intersection to insure that the intersection is found as is shown in Figure 1b. A line may also fall short of an intersection to within an error. Each region is in turn divided into a number of evenly spaced or nonlinearly spaced zones in the meridional direction by passing additional intermediate rays through the region. Care must be taken not to extend the meridians to the point that an unintended intersection is found by other rays. In Figure 4, for example, meridian M3 can extend into region 2 and will be ignored unless it is within an ε distance from R3 or intersects it.

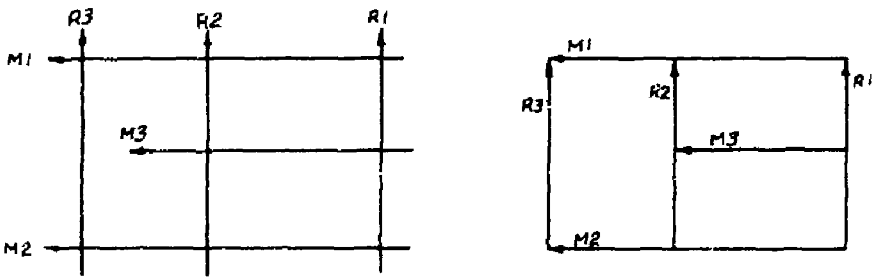


Figure 4. The geometry in (a) is interpreted as shown in (b). Meridian M3 is ignored in region 2 since it is not within an ε distance from R3.

A transition zone is automatically generated with triangular zones at the end of each region when the number of zones on the ray at the beginning of the region is not equal to the number of zones on the ray at the end of the region. In region 1 (Figure 3) the transition zone is an expansion from four zones in region 1 to seven zones in region 2.

A nonlinear zoning option in the meridian direction generates geometric progression of zone spacing between the beginning and end rays in each region. The three options available are illustrated in Figure 5.

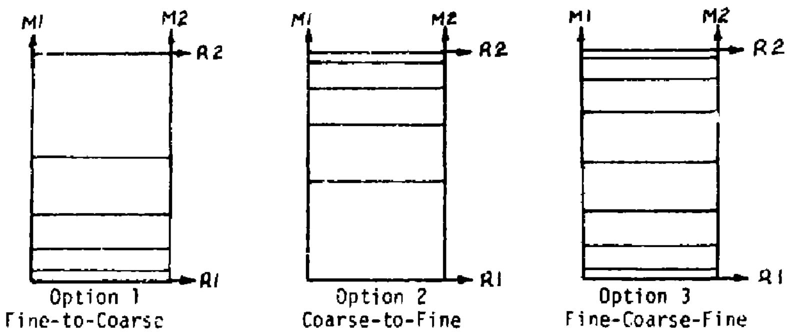


Figure 5. Nonlinear zoning in the meridian direction.

IV. LAYERS

The ray segment between any two consecutive ray-meridian intersections is called a *layer* and is described as passing through, or bounding a material or void area in the structure. By definition, a layer is the segment along the ray between two consecutive ray-meridian intersections. Voids are considered layers. The merging point of two meridians is also considered a layer. The number of zones dividing the two sides of a layer (the subdivision of the ray segments) need not be the same.

When they are not the same a transition zone is generated at the end of the region as explained in Section III. Each layer in the region is treated independently and may have different transition zones. In Figure 3, for example, the two outer layers in region 3 each have different number of zones (in the ray direction) at ray R3 and at ray R4.

V. MERIDIANS

Meridians are defined by a series of up to 100 coordinate points in the global x-y coordinate system and may, therefore, be multivalued functions. Options can be used to generate arcs, straight lines, and offset meridians and ellipses. There is no limit to the number of times that any option may be used in defining a meridian, except to satisfy the 100 point limit. The total number of meridians is limited to 40 which can easily be expanded if necessary.

Meridians may be any length and may be discontinuous within the structure, as opposed to rays which must pass through the entire structure. However, a ray must pass through every meridian discontinuity. A discontinuity means the beginning or ending of a line. The meridians cannot cross each other but any number can have a common beginning or end point. The common point may be a coordinate point anywhere along another meridian, as illustrated in Figure 6, in which meridians M3 and M5 have a common beginning point on M4, and M2 has a common point on M1.

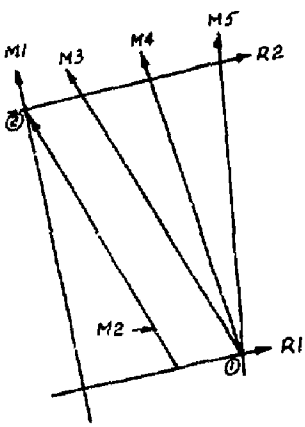


Figure 6. An example of meridians with a common point. Meridians M3, M4, and M5 have a common point at (1). Meridians M1 and M2 have a common point at (2).

The sequence in which meridians are defined is not important except when meridians meet at a common point. In this case they must be defined in the sequence that the ray, slightly away from the common point, would intersect them.

1. Offset

A meridian may be defined entirely or in part as an offset from any other meridian which has previously been defined. There is no limit to the number of times the offset option may be used to define a meridian, except to satisfy the 100 point limit.

Two coordinate points define the beginning and end point of any offset as shown in Figure 7, where part of meridian M2 is an offset ($+\delta_1$) from M1 in the +x' direction between points ① and ②. The number of intermediate points on M2 are those found on M1 between the normals to points ① and ②.

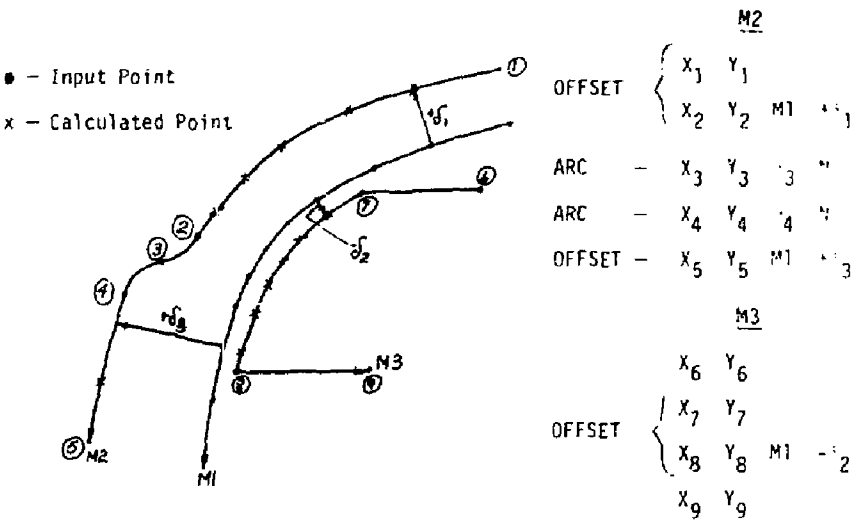


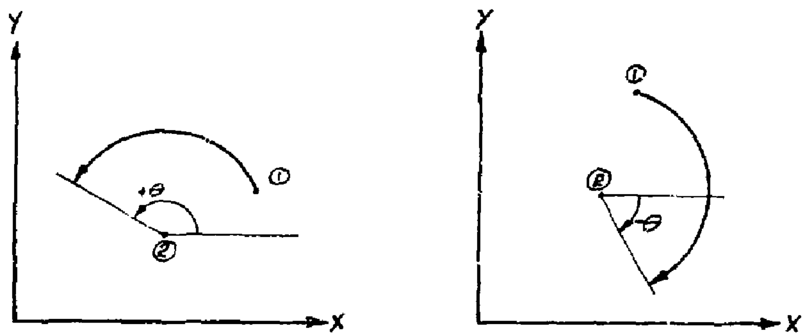
Figure 7. Example of the offset option.

Part of meridian M3 is also offset ($-\delta_2$) from M1 in the -x' direction between points (7) and (P).

2. Circular Arc

A meridian may contain a number of circular arc segments which may sweep an arc of up to 360°. There is no limit to the number of times that the arc option may be used to define a meridian except to satisfy the 100 point limit. The arc may be defined two different ways, either of which require only two cards for definition.

a) The angle option — defines an arc which begins at a given point and sweeps to an end point at an absolute angle theta (θ) with a radius (R). Theta is measured from the x global axis and is positive counter clockwise and negative clockwise. The radius is defined from the center of rotation which is also given. The arc is swept in the positive or negative direction depending on the sign of θ. Two examples of angle arc option are shown in Figure 8 in which the beginning point is at point (1), and the center of rotation is at point (2). The radius (R) is the distance between (1) and (2).

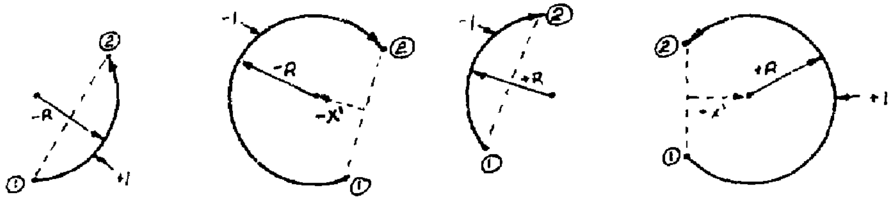


Card 1 — x_1 y_1
Card 2 — x_2 y_2 $+\theta$ N

Card 1 — x_1 y_1
Card 2 — x_2 y_2 $-\theta$ N

Figure 8. Examples of the angle arc option with the two cards needed for their definition. N is the number of points along the arc.

b) The radius option defines an arc which begins at the first given point and ends with a second given point. An arc with a given radius (R) is swept in the counter clockwise direction for a +1 flag and in the clockwise direction for a -1 flag. If the center of rotation of the arc has a -x' coordinate as measured from the line connecting points (1) and (2), a negative radius (-R) is specified. If the center of rotation has a positive coordinate, (+x) then a positive radius (+R) is specified. The examples in Figure 9 illustrate all four radius arc option possibilities.



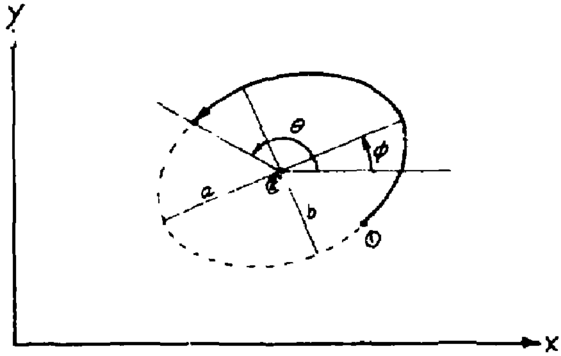
Card 1 – x_1 y_1 x_1 y_1 x_1 y_1 x_1 y_1

Card 2 – x_2 y_2 -R N +1 x_2 y_2 -R N -1 x_2 y_2 -R N +1 x_2 y_2 +R N +1

Figure 9. Example of the radius arc option with the two cards needed for its definition. N is the number of points along the arc.

3. Ellipse

A meridian may contain a number of elliptical segments which may sweep up to 360 degrees as measured by an absolute positive or negative angle with 0 degrees on the global x axis and the center at the ellipse center. The sweep convention is the same as for the arc. Two points are used to define the ellipse -- one per card. The first point (point (1) in the example shown in Figure 10) is the beginning of the ellipse. Point (2), the center of the ellipse, is on the second card. Also on the second card are the sweep angle θ, the tilt angle of the major axis [illegible], which is measured just like [illegible], and half the major and minor ellipse axis (a and b) as shown in Figure 10.



Card 1 - x_1 y_1

Card 2 - x_2 y_2 ·· ℵ a b :

Figure 10. Example of a segment of an ellipse beginning at ① and ending at ·· with center at ② and tilt angle ¢.

A blank card follows each meridian.

VI. RAYS

Rays are lines which must pass through the *entire geometry intersecting* the meridians in the *positive* x' direction as defined in Figure 2.

<u>IMPORTANT</u> - A ray cannot begin or terminate, like a meridian, arbitrarily within the structure. A ray *must pass through the entire structure.*

This requirement is imposed to insure the minimum bandwidth for every problem. Problems should, therefore be modeled so that the ray proceeds in the direction of the smallest geometry dimension if bandwidth is critical. Rays are also defined by a series of up to 100 coordinate points in the global x-y coordinate system and may be multivalued functions. There is no limit to the number of rays that may be defined, but they must be defined sequentially beginning at one end of the geometry and proceeding in the positive direction along the meridians. Otherwise the ray geometry is defined in the same way as the meridian using the arc, offset, free form, and ellipse options described in Section V. The offset option for the ray, however, can only be used with an offset from the previous ray. In addition to the geometry, however, each ray is preceeded by a control card and a set of cards describing the material number and the number of zones in the layer. Each ray, therefore, is comprised of three groups of cards, assembled in the following order:

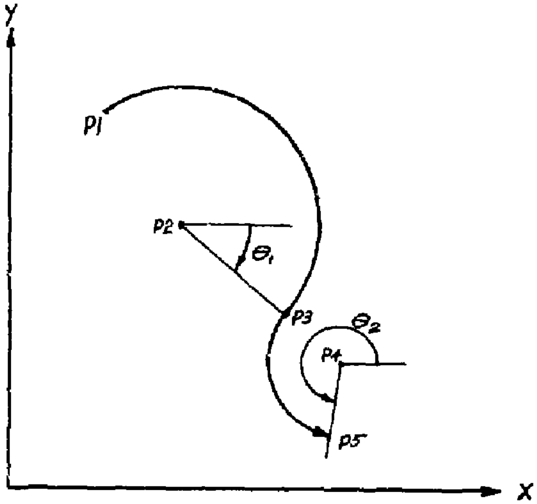
1) one control card,
2) one card for each layer in the ray describing the material number and the number of zones in the layer,
3) a set of cards describing the geometry of the ray as for a meridian.

Except for the last one, each ray is followed by a blank card.

VII. COMMON CHARACTERISTICS — RAYS AND MERIDIANS

a) The beginning point (P2) in Figure 11 need not be specified in the arc, offset or ellipse options if it has already been generated as the termination point of a previously defined arc, offset, ellipse, or free form. The two consecutive arcs in Figure 11, for example, are defined by the three cards shown. The same thing holds for ellipses and offsets or a combination of any two options.

b) Ray and meridian end points need not lie on a geometric or material boundary but they must be defined so as to insure ray-meridian intersections. End points may be defined short of a ray-meridian intersection within a distance ϵ.



Card Coordinate

P1 – x_1 y_1

P2 – x_2 y_2 $-\theta_1$

P4 – x_4 y_4 θ_2

Figure 11. Example of the definition of consecutive arcs.

c) The beginning point of the sequence of points defining a meridian or ray may be at either end but all meridian and ray definitions must be defined in a consistent direction.

d) The positive direction along a ray or meridian coincides with the increasing sequence of x-y coordinate points defining it.

VIII. ZONE INPUT DESCRIPTION

The input file to ZONE is ZIN in which every card except the title and control card is written with a 7F10.4 format.

ZOUT — The node and zone output file

FILE1 — A file with node points only (input to PLOT)

FILE2 — A file with zone connectivities only (input to PLOT)

NOTE: A blank card delimits the meridians and rays, therefore, a blank in columns 1-10 should not be used for a zero. No blank card should follow the last ray. The problem terminates upon encountering the END-OF-FILE mark.

A. IDENTIFICATION (7A10)

Problem identification.

B. PROBLEM CONTROL CARD (7F10.4, 2F5.2)

col 1-10	NLINE	Number of meridians.
col 11-20	SHORT = 1.	Long form — every node and zone is listed.
	SHORT = Ø.	Short form — truncated listing of nodes and zones.
col 21-30	EPS	ϵ tolerance of ray-meridian mismatch (default ϵ = .0001).
col 31-40	XCENT	X-coordinate of center of zoom area.
col 41-50	YCENT	Y-coordinate of center of zoom area.
col 51-60	DXX	ΔX of zoom area.
col 61-70	DYY	ΔY of zoom area.
col 71-75	NOD	The zero node number. Every node number, N, is renumbered NOD+N. This is useful when modeling a large complex structure as several smaller substructure problems which are later assembled into one input to an analysis program.

col 76-80 IZ The zero zone number. Every zone number, I, is renumbered IZ+I.

C. MERIDIAN DEFINITION (7F10.4)

A maximum of 40 meridians and 100 points per meridian are allowed. Any combination of the options below may be used and as often as needed.

1. Line Segment Option (2 Cards)

Card 1

col 1-10	X_1	Coordinate of first point.
col 11-20	Y_1	Coordinate of first point.

Card 2

col 1-10	X_2	Coordinate of second point.
col 11-20	Y_2	Coordinate of second point. Any number of point may be used to define a piece-wise linear curve.

2. Arc – Angle Option (2 Cards)

Card 1

col 1-10	X_1	Coordinate of beginning of arc.
col 11-20	Y_1	Coordinate of beginning of arc.

Card 2

col 1-10	X_2	Coordinate of center of rotation of arc.
col 11-20	Y_2	Coordinate of center of rotation of arc.
col 21-30		Absolute angle θ, in degrees, as measured from the x axis up to $\pm 360°$. A positive angle indicates a counter clockwise arc sweep starting at the last point defined on the line (card 1 or the last point defined on previous option) and ending at θ. A negative angle indicates a clockwise sweep. Since $\theta = -0.$ is indistinguishable from $\theta = +0.$ a small angle ($\pm 1. \times 10^{-6}$) must be used in place of a value $\theta = \pm 0.0$.

col 31-40	N	The number of points used internally in the program to define the arc. N should be large enough to define the arc properly. The range is $10 < N \leq 100$. If left blank, N = 20.

3. Arc — Radius Option (2 Cards)

Card 1

col 1-10	X_1	Coordinate of the beginning of arc.
col 11-20	Y_1	Coordinate of the beginning of arc.

Card 2

col 1-10	X_2	Coordinate of the end of the arc.
col 11-20	Y_2	Coordinate of the end of the arc.
col 21-30	R	Radius of arc. A positive radius (+R) indicates a center of rotation on the +x' side of the line connecting the arc end points. A negative radius (-R) indicates a center of rotation on the -x' side (see Figure 9).
col 31-40	N	The number of points used internally in the program to define the arc. N should be large enough to properly define the arc. The range is $1 \leq N \leq 100$. If left blank N = 20.
col 41-50	XN1	Flag indicating the direction of arc sweep. A +XN1 indicates a counter clockwise sweep. A -XN1 indicates a clockwise sweep (see Figure 9).

4. Ellipse Option (2 Cards)

Card 1

col 1-10	X_1	Coordinate of the beginning of the ellipse.
col 11-20	Y_1	Coordinate of the beginning of the ellipse.

Card 2

col 1-10	X_2	Coordinate of the center of the ellipse.
col 11-20	Y_2	Coordinate of the center of the ellipse.

col 21-30		Absolute angle θ, in degrees, as measured from the x axis up to ±360°. A positive θ indicates a counter clockwise ellipse is swept starting at the last point defined on the line (card 1 or the last point defined in the previous option) and ending at θ. A negative θ indicates a clockwise sweep. For θ = 0. (see "Arc – Angle Option").
col 31-40	N	The number of points used internally in the program to define the ellipse. N should be large enough to define the ellipse properly. The range is $1 \leq N \leq 100$. If left blank, N = 20.
col 41-50	A	The half-length of the major axis of the ellipse.
col 51-60	B	The half-length of the minor axis of the ellipse.
col 61-70		The tilt angle φ of the major axis measured in degrees the same way as θ (see Figure 10).

5. Offset Option (2 Cards)

Card 1

col 1-10	X_1	Coordinate of the beginning of the offset.
col 11-20	Y_1	Coordinate of the beginning of the offset.

Card 2

col 1-10	X_2	Coordinate of the last point of the offset.
col 11-20	Y_2	Coordinate of the last point of the offset
col 21-30		Meridian number, M, of a previously defined meridian from which the offset distance is measured.
col 31-40		Offset distance ±δ. A +δ is measured in the +x' direction from meridian M. A -δ is measured in the -x' direction. A point is generated a normal distance ±δ from every point encountered in meridian M between the beginning and end point defined on these two cards (see Figure 7).

NOTE: 1. The beginning and end lie on the new meridian and not on meridian M.

2. The offset distance δ must be smaller than any radius of curveture encountered in meridian M.

3. The maximum offset distance δ is ±10.

D. RAY DEFINITION (7F10.4)

The maximum number of points defining a ray is 100. There is [illegible] to the number of rays that may be used. Any combinations of the options below may be used, and as often as needed.

a. Control Card

col 1-10	NL	Number of layers intersected by this ray. By definition this is the number of ray-meridian intersections minus one.
col 11-20	NR	Number of zones dividing the region between this ray and the next (meridian direction).
col 21-30	NMER	Nonlinear zone spacing option (see Figure 5).
	NMER=1	Fine spacing to course spacing.
	NMER=2	Course spacing to fine spacing.
	NMER=3	Fine-course-fine spacirg.

b. Layer Description (NL Cards) (NL ≤ 40)

col 1-10	MN	Material number of *succeeding* material — the material between this ray and the next. The material number can be any number between Ø and 40. A zero material number indicates the following layer is a void space.
col 11-20	NE	Number of zones *along the ray segment* defined by this layer.

NOTE: The number of zones must be completely interpreted as the number of divisions that the *line itself* (ray segment) is subdivided. It must not be interpreted as the number of zones in the previous or succeeding layers because this will lead the user into interpretation difficulties. See, for example, ray R2 in Figure 4.

The number of zones can be one or more. A value for NE = Ø indicates that the meridians bounding this layer converge to a common point on this ray.

col 21-30 SWITCH An option to change the node point distribution within a layer. ZONE will, at times, produce meshes like the one shown in Figure 12a. Setting SWITCH = 1. will produce the mesh in Figure 12b.

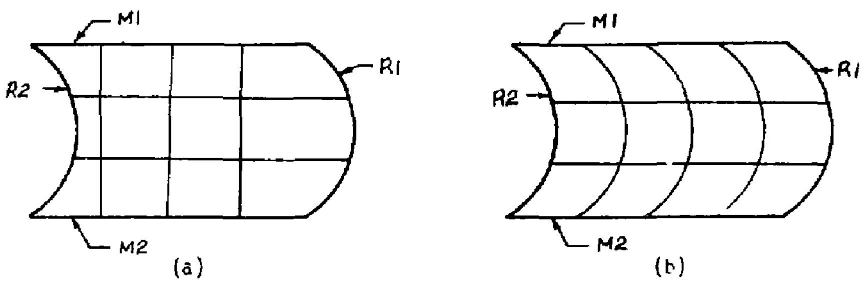


Figure 12. An example of the SWITCH option. ZONE generates the mesh in (a). Setting SWITCH = 1. produces the mesh in (b).

c. <u>Ray Description</u> (Maximum — 100 Points per Ray)

Any combination of the options below may be used to define the geometry of the ray.

1. Line Segment Option

   (Same as VIII C.1)

2. Arc – Angle Option

   (Same as VIII C.2)

3. Arc – Radius Option

   (Same as VIII C.3)

4. Ellipse Option

   (Same as VIII C.4)

5. Offset Option

   The offset option for rays is the same as for meridians in VIII C.5 except that the meridian number (M_i) from which the offset is measured is always set to $M_i = 1$. The offset distance, δ, is input as a positive number and the offset ray will always be generated a positive δ distance from the preceeding ray as measured in the positive meridian direction.

6. Vector Option (1 Card)

col 1-10	X	Coordinate of the beginning of a vector.
col 11-20	Y	Coordinate of the beginning of a vector.
col 21-30	θ	Angle, in degrees, of a line (vector) as measured from the global x axis as in the arc option (VIII C.2). A straight line is generated from the X,Y coordinates at an angle θ and with a length of 5000.

IX OUTPUT

ZONE produces 7 output files as follows:

1. ZOUT – The complete BCD output file to be used as input in structure programs. It includes the X-Y coordinates of the nodes, the

node connectivities of the zones, the boundary pressure cards along those meridians which are the first encountered by the rays, and the areas, volumes and centroids of each different material in the geometry.

2. FILE1 — A BCD file containing the node point coordinates for input to PLOT — a plotting program.
3. FILE2 — A BCD file containing the node connectivities of the zones for input to PLOT.
4. SAVE — A "cleaned up" copy of the input file, ZIN, which can subsequently be used as the input to ZONE.
5. DUMP — Contains the meridian and ray coordinat:: as they are interpreted by ZONE from the various input options. It also contains the X-Y coordinates of the ray-meridian intersections found along each ray — very useful when debugging the input.
6. BUFF — An empty file used for decoding only.
7. PRESFIL — A scratch file.

X EXAMPLES

A. Example 1.

This example illustrates the use of the ellipse option, arc option, and ray vector option. The dimensions of the geometry are shown in Figure 13a and the resulting mesh is shown in Figure 13b. The listing of the input that follows is the SAVE file generated by ZONE.

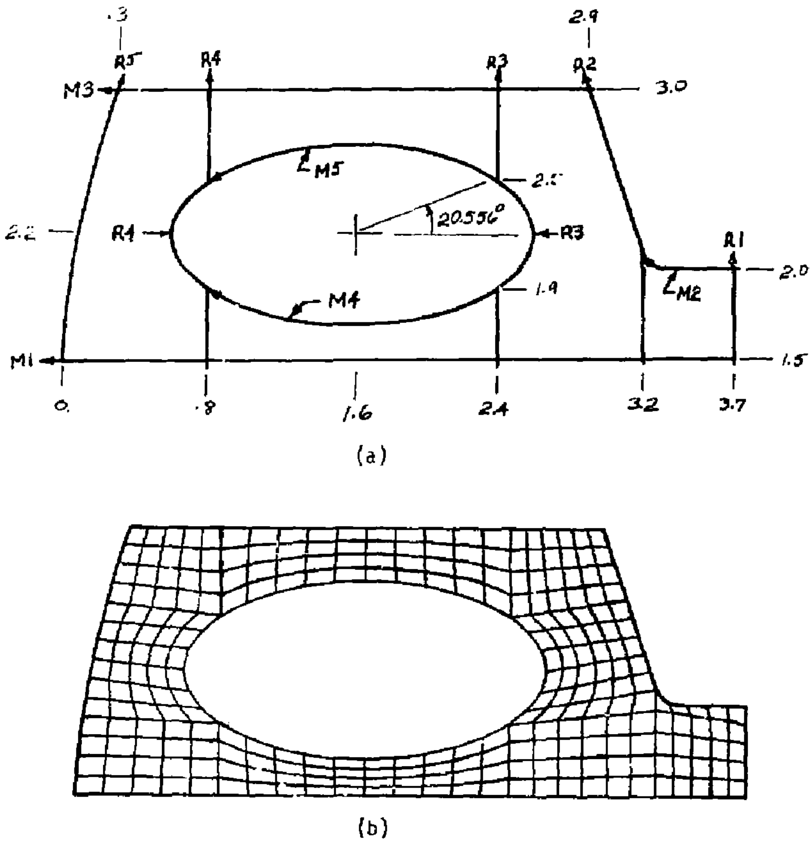


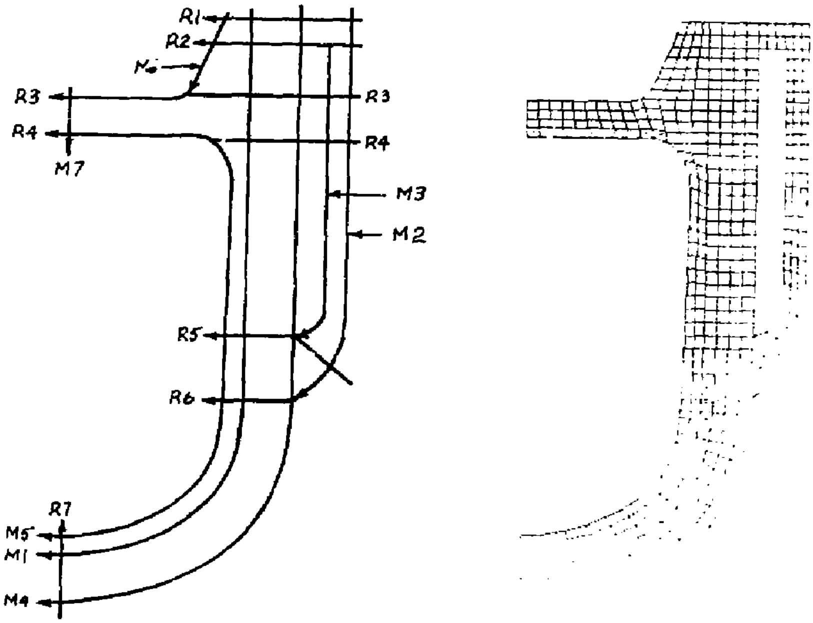
Figure 13. (a) geometry for Example 1. (b) The mesh generated by ZONE.

LISTING OF EXAMPLE 1

```
                              .0    -1.0000

                 70.0000    1.0000      .5000       .0

                 70.0000     .0000      .5000       .0

                              -.0

                              -.0
                              -.0

                            1.0000      .5000

                 70.0000    1.0000      .5000       .0

                  40.0     -1.0000
```

Example 2

The mesh for the pressure vessel in Figure 14a is shown in Figure 14b. The input listing for this example is also the SAVE file.



(a) (b)

Figure 14. (a) Example 2 pressure vessel geometry. All the rays and meridians are free forms. M4 and M5 are offset from M1. There is a common point on M4 with M2 and M3. (b) Mesh generated by ZONE.

LISTING OF EXAMPLE 2

```
PRESSURE VESSEL
    7.0000                  .001       1.967       2.549       .3        01
    1.5450    5.2880
    1.5450    2.5490
    1.5260    1.9760
    1.5140    1.8260
    1.4900    1.6500
    1.4570    1.5000
    1.4000    1.3550
    1.3270    1.2260
    1.2270    1.1040
    1.0760     .9690
     .8700     .8470
     .6210     .7460
     .4170     .6950
     .2260     .6560
     .1160     .6400
    -.0210     .6430
                    MERIDIAN $ 2
    2.4120    5.2960
    2.4030    2.7550
    2.3950    2.6160
    2.3710    2.5020
    2.3320    2.4040
    2.2930    2.3280
    2.2210    2.2230
    2.1300    2.1360
    2.0130    2.0400
    1.9250    1.9810
                    MERIDIAN $ 3
    2.2140    5.0820
    2.2200    2.8550
    2.2190    2.7450
    2.1650    2.6810
    2.1370    2.6170
    2.0790    2.5780
    1.9675    2.5490
                    MERIDIAN $ 4
    1.9670    5.2800
    1.9672    2.5490
    -.0050     .2200     1.0000     -.4220
                    MERIDIAN $ 5
    1.1830    4.2490
    1.1830    4.2330
    1.2550    4.1930
    1.3040    4.1460
    1.3610    4.0680
    1.3950    3.9850
    1.4100    3.8970
     .0060     .8100     1.0000      .1670
                    MERIDIAN $ 6
    1.3220    5.2730
    1.0540    4.6850
    1.0230    4.5470
     .9820    4.6170
     .9800    4.3920
                    MERIDIAN $ 7
        .0    4.6050
        .0    4.2380
                    RAY    # 1
    3.0000    2.0000
    1.0000    6.0000
    2.0000    4.0000
    3.0000    4.0000
    2.4360    5.2620
    1.2910    5.2620
                    RAY    # 2
    4.0000    5.0000
    1.0000    3.0000
        .0    3.0000
    2.0000    4.0000
    3.0000    4.0000
    2.4350    5.0590
    1.1890    5.0590
                    RAY    # 3
    5.0000    4.0000
    1.0000    3.0000
        .0    1.0000
    2.0000    4.0000
    3.0000    4.0000
    3.0000   10.0000
    2.4290    4.5990
     .9780    4.6120
     .9360    4.5890
     .8810    4.5810
    -.0030    4.5810
```

```
                                RAY     # 4
 5.0000        15.0000
 1.0000         3.0000
     .0         1.0000
 2.0000         4.0000
 3.0000         2.0000
     .0        10.0000
 2.4190         4.2290
 1.1850         4.2290
 1.1310         4.2510
 1.0500         4.2650
 -.0090         4.2650
                                RAY     # 5
 4.0000         5.0000
 1.0000         3.0000
 2.0000             .0
 2.0000         4.0000
 3.0000         2.0000
 2.3060         2.3180
 1.9670         2.5490
 1.2000         2.5490
                                RAY     # 6
 3.0000        18.0000      1.0000            -.0
     .0             .0
 2.0000         4.0000
 3.0000         2.0000
 1.9990         2.0030
 1.2000         2.0030
                                RAY     # 7
 2.0000            -.0
 1.0000         4.0000
 1.0000         2.0000
     .0          .1780
     .0          .8540
```

XI PROGRAM AVAILABILITY

The executable object program can be executed from the public library MJBLIB on the CDC 7600's by typing:

X MJBLIB ZONE / T V

The FORTRAN IV source listing is included (microfiche on back cover).

/mr