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ON THE SUPPRESSION OF REGGE CUT CONTRIBUTIONS *

Swee-Ping Chia **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

It is shown that contributions of reggeon-pomeron cuts are suppressed in amplitudes with opposite naturality to the reggeon. This suppression grows logarithmically with energy. The suppression in the πP cut is, however, found to be weak. Consequence on conspiracy is discussed.

MIRAMARE - TRIESTE
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** On leave of absence from Physics Department, University of Malaya, Kuala Lumpur, Malaysia.

It is well known that Regge cut arising from the simultaneous exchange of two reggeons in the t channel, has no definite parities. Recently, Jones and Landshoff [1] *) have shown that the contributions of such a Regge cut are suppressed in amplitudes which have naturalities opposite to the product of the reggeon naturalities. Their argument, though based on general considerations, is rather vague. It is therefore necessary to examine the existence of such a suppression in an explicit model of Regge cut.

Previously, we have presented a model for calculating the pion-pomeron cut based on the Mandelstam diagram [2]. We shall show that, in this model, the suppression as suggested by JL does exist for reggeon-pomeron cuts. The suppression, however, depends not only on ℓns, but also on m_R^2, where m_R is the mass of the lowest-lying recurrence on the reggeon trajectory. Because of the smallness of m_π, the suppression in the πP cut is weak at laboratory accessible energies.

Our model utilizes explicitly the Mandelstam diagram as shown in Fig.1. The diagram is treated as a Feynman graph. The reggeon exchange is taken to be of the following form:

$$F^{(R)}_{\{\lambda\}}(s,\tau) = \frac{B_{\{\lambda\}}}{m_R^2+\tau}\, e^{-A_R(m_R^2+\tau)}\, (-is)^{\varphi_R(\tau)-j_R} \tag{1}$$

with trajectory function $\varphi_R(\tau) = j_R - \varphi_R'(m_R^2+\tau)$. Throughout this communication, the variable $\tau = -t$ is used instead of t. The propagator-like denominator $(m_R^2+\tau)^{-1}$ is due to the long-range force of the lowest-lying recurrence (of mass m_R and spin j_R) on the trajectory. The helicity structure of the exchange is contained in the Born coupling $B_{\{\lambda\}}$, which also gives an additional factor of s^{j_R}. As shown in Fig.1, the pomeron exchange is introduced through the scattering of "σ" particles. Here, we adopt the picture that hadrons emit these isoscalar-scalar "σ" particles before they themselves interact via reggeon exchange. The pomeron exchange is assumed to be

$$F^{(P)}(s,\tau) = -\gamma_P^2\, e^{-A_P\tau}\, (-is)^{\varphi_P(\tau)} \tag{2}$$

with trajectory function $\varphi_P(\tau) = 1 - \varphi_P'\tau$.

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*) Hereafter, it shall be referred to as JL.

Omitting details [3], we can write down the cut amplitudes as

$$F^{(cut)}_{\{\lambda\}}(s,\tau) = -\frac{1}{2}\int \frac{d^2\underline{k}}{(2\pi)^2} \frac{e^{-A_R(m_R^2+\tau_1)}\, e^{-A_P\tau_2}}{m_R^2+\tau_1} (-is)^{\varphi_R(\tau_1)+\varphi_P(\tau_2)-j_R-1}\, N^L_{\lambda_3\lambda_1}\cdot N^R_{\lambda_4\lambda_2} \tag{3}$$

where $\tau_1 = \underline{k}^2$, $\tau_2 = (\underline{k}-\underline{q})^2$, and $\underline{k}$ and $\underline{q}$ are the transverse momentum transfer carried by the reggeon and the total transverse momentum transfer, respectively. The functions N^L and N^R are the cross structure functions [2] describing particle-reggeon scattering

$$N^L_{\lambda_3\lambda_1} = \int_0^1 d\beta_1\cdot \beta_1^{\varphi_R(\tau_1)-j_R}(1-\beta_1)^{\varphi_P(\tau_2)}\, \eta^L_{\lambda_3\lambda_1}(\underline{k},\underline{q};\beta_1). \tag{4}$$

The variable β_1 is the fraction of p_{1+} ($p_{1+}\simeq\sqrt{s}$) which flows through the hadron line k_1 (cf. Fig.1). The structure function on the right is similarly given by an integral over another variables α_2 which is k_{2-}/p_{2-}.

At this point, we make one simplifying assumption [4]. It is evident from the diagram that when a hadron emits a "σ" particle, its large momentum component (p_{1+} on the left and p_{2-} on the right) must be shared by the two fragments. The assumption made is simply that this large momentum component should be shared equally by the two fragments. The plausibility of this assumption lies in the fact that we are concerned only with extracting the Regge cut behaviour from the Mandelstam diagram. To do so, it is necessary that both the variables β_1 and α_2 should be far away from the end points 0 and 1, for at these end points the internal exchanges do not have Regge behaviour. With this assumption, we can rewrite (3) as:

$$F^{(cut)}_{\{\lambda\}}(s,\tau) = -\frac{1}{8}\int \frac{d^2\underline{k}}{(2\pi)^2} \frac{e^{-\lambda_R(m_R^2+\tau_1)}\, e^{-\lambda_P\tau_2}}{m_R^2+\tau_1} K_{\{\lambda\}}, \tag{5}$$

where $\lambda_R = A_R + \varphi_R'(\ln\frac{s}{4} - \frac{i\pi}{2})$, $\lambda_P = A_P + \varphi_P'(\ln\frac{s}{4}-\frac{i\pi}{2})$, and
$K_{\{\lambda\}} = \eta^L_{\lambda_3\lambda_1}(\underline{k},\underline{q};\frac{1}{2})\cdot\eta^R_{\lambda_4\lambda_2}(\underline{k},\underline{q};\frac{1}{2})$.

After examining a large class of reactions associated with four different exchanges, the pseudoscalar, the vector, the axial-vector and the tensor mesons, we arrive at the following properties concerning the cross structure functions:

1. $N_{\lambda_3\lambda_1} = \eta_R\eta_1\eta_3(-1)^{\lambda_3-\lambda_1}\, N^*_{-\lambda_3-\lambda_1}$, where η_R, η_1 and η_3 are the naturalities of the reggeon, particle 1 and particle 3, respectively.
2. The combinations $N_{\lambda_3\lambda_1} \pm \eta_1\eta_3(-1)^{\lambda_3-\lambda_1} N_{-\lambda_3-\lambda_1}$ have naturalities $\pm$ in the t channel.
3. $\mathrm{Im}\, N_{\lambda_3\lambda_1}$ is proportional to k_y and is an odd function of k_y, whereas $\mathrm{Re}\, N_{\lambda_3\lambda_1}$ is an even function of k_y.

Combining the first two properties, we see that the real and imaginary parts of the structure functions are of naturalities $\pm\eta_R$, respectively. It then follows from the third property that structure functions of naturality $-\eta_R$ are proportional to, and are odd in k_y; whereas those of naturality η_R are even in k_y. This result was previously noted in Ref.2, but as shown by JL, it holds generally and is independent of this particular model. Because of this, the cut contributes to amplitudes of naturality $-\eta_R$ with a factor of k_y^2 in $\eta_{\{\lambda\}}$. But for amplitudes of naturality η_R there need not be any factor of k_y at all.

In order to compare these two contributions of the Regge cut, we must look at the helicity structure of the amplitudes. Of most interest is the case where there is a unit helicity flip at both sides, such as the amplitudes $F_{\pm 1\frac{1}{2};0-\frac{1}{2}}$ in vector meson production processes. In this case, the combinations

$$F^{(\pm)} = F_{1\frac{1}{2};0-\frac{1}{2}} \pm \eta_R\eta_1\eta_3\, F_{-1\frac{1}{2};0-\frac{1}{2}}$$

have naturalities $\pm\eta_R$, respectively. Expressions for cut contributions to $F^{(\pm)}$ are similar to Eq.(5), with $K^{(+)} = \mathrm{Re}\,\eta^L\cdot\mathrm{Re}\,\eta^R$ and $K^{(-)} = -\mathrm{Im}\,\eta^L\cdot\mathrm{Im}\,\eta^R$. In general, the structure functions can be expressed in the following form [4]:

$$\eta^L(\underline{k},\underline{q};\tfrac{1}{2}) = \sum_i P_i(\underline{k},\underline{q})\, Z_i(\tau'),$$

where P_i are some finite polynomials in $\underline{k}$ and $\underline{q}$, and Z_i are slowly varying functions of the variable $\tau' = (\underline{k}-\frac{1}{2}\underline{q})^2$. In practice, we can set $Z_i(\tau')\simeq Z_i(0)$ without loss of generality. It is found that for $F^{(-)}$, the corresponding $K^{(-)}$ has a factor of k_y^2 as expected. But for $F^{(+)}$, the corresponding $K^{(+)}$ has three terms, each proportional to k_x^2, q_xk_x, and q_x^2, respectively. Defining the following integrals:

$$I_{ij} = \int \frac{d^2 k}{(2\pi)^2} \frac{e^{-\lambda_R (m_R^2+\tau_1)} e^{-\lambda_P \tau_2}}{m_R^2+\tau_1} \left(\frac{k_x}{m_R}\right)^i \left(\frac{k_y}{m_R}\right)^j , \tag{6}$$

we should then compare I_{02} to I_{20}, $(\sqrt{\tau}/m_R)\, I_{10}$ and $(\tau/m_R^2)\, I_{00}$.

Let us first look at I_{00}. The integration can be performed explicitly to give

$$I_{00} = \frac{1}{4\pi z} e^{-\lambda_R m_R^2} e^{-\frac{\lambda_R \lambda_P}{\lambda_R + \lambda_P}\tau}\, G(z, x) \tag{7}$$

where

$$z = (\lambda_R + \lambda_P)\, m_R^2 \tag{8}$$

$$x = \left(\frac{\lambda_P}{\lambda_R + \lambda_P}\right)^2 \frac{\tau}{m_R^2} \tag{9}$$

and $G(z,x)$ is given by a descending series in z,

$$G(z,x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{1}{z}\right)^n \left(\frac{d}{dx}\right)^n x^n \left(\frac{d}{dx}\right)^n \frac{x^n}{1+x} . \tag{10}$$

In arriving at (7) and the series expansion (10), we have explicitly assumed that z is large and that $|x| < 1$. With I_{00} given by (7), it is a simple matter to show that

$$I_{10} = \sqrt{x} \left(1 + \frac{1}{zG} \frac{\partial G}{\partial x}\right) I_{00} , \tag{11}$$

$$I_{20} = \left\{ x + \frac{1}{2z} + \left(\frac{2x}{zG} + \frac{1}{2z^2 G}\right) \frac{\partial G}{\partial x} + \frac{x}{z^2 G} \frac{\partial^2 G}{\partial x^2} \right\} I_{00} , \tag{12}$$

$$I_{02} = \left(\frac{1}{2z} + \frac{1}{2z^2 G} \frac{\partial G}{\partial x}\right) I_{00} . \tag{13}$$

At large z, the ratio of I_{02} to I_{20} is therefore given by

$$\frac{I_{02}}{I_{20}} \cong \left\{ 2xz + \frac{1-4x-x^2}{(1+x)^2} \right\}^{-1} . \tag{14}$$

The ratios of I_{02} to $(\sqrt{\tau}/m_R)\, I_{10}$ and $(\tau/m_R^2)\, I_{00}$ are similar to (14). It is evident from (14) that at $\tau \sim m_R^2$, suppression of the type suggested by JL is present, provided z is sufficiently large. As z depends linearly on $\ell n s$, this suppression grows logarithmically with energy.

There are, however, several points on which we differ from JL. First, we note that the result of JL is based on comparing $m_R^2 I_{02}$ to I_{00}

$$m_R^2 I_{02} / I_{00} \simeq \frac{1}{2(\lambda_R + \lambda_P)} . \tag{15}$$

They conjecture, on the basis of (15), that the strength of the suppression is characterized by the quantity $(\lambda_R + \lambda_P)s_0$, with $s_0 \approx 1$ GeV^2. Our result, on the other hand, indicates that at $\tau \sim m_R^2$, the suppression strength is characterized by $z = (\lambda_R + \lambda_P)\, m_R^2$, i.e. there is a factor of m_R^2 instead of s_0. The origin of this m_R^2 factor comes from the explicit long-range propagator $(m_R^2 + \tau)^{-1}$ of the reggeon exchange. The result, however, should not depend too critically on singling out this longest range part of the reggeon exchange. The reason is that in the usual signature factor we have the denominator $\{\sin \frac{1}{2}\pi(\alpha_R - J_R)\}^{-1}$, which contains a series of poles, corresponding to the entire family of the Regge recurrence. The longest range force comes from the lowest-lying recurrence, which, as it is closest to the physical region, is more dominant than the other recurrences.

The second point concerns the τ-dependence of the suppression. It is evident from Eq.(14) that, at $\tau = 0$, I_{02} and I_{20} are equal, i.e. the cut contributes equally to both $F^{(+)}$ and $F^{(-)}$. As we move away from $\tau = 0$, I_{20} increases initially before bending down, whereas I_{02} is increasingly suppressed. From $\tau \sim \frac{1}{2} m_R^2$ onward, the suppression increases almost linearly as τ increases. Fig.2(a) clearly illustrates this characteristic τ-dependence of the suppression in the case of the ρP cut. The suppression of I_{02} with respect to $(\sqrt{\tau}/m_R)\, I_{10}$ and $(\tau/m_R^2)\, I_{00}$ has a stronger τ-dependence. Fig.2(b) shows the actual contributions of the ρP cut to $F^{(+)}$ and $F^{(-)}$ in the reaction $\pi^- p \to \omega^0 n$.

On account of the smallness of the pion mass, the πP cut is different from the other reggeon-pomeron cuts. For ρP-, BP-, and A_2P-cuts, we expect the suppression to be significant at $s >> 30$ GeV^2. For example, at $s = 100$ GeV^2, it is estimated that *) $\mathrm{Re}\, z = 5.09$ for the ρP cut. For the πP cut, on the other hand, the suppression at $\tau \sim m_\pi^2$, according to (14), will not be significant until $s \gg 10^{23}$ GeV^2 ! Strictly speaking, (14) is not applicable to the πP cut, because its derivation utilizes explicitly the expansion (10), which is valid only at large z. For the πP cut, Re z ranges only from 0.23 at $s = 100$ GeV^2 to 0.50 at $s = 10^6$ GeV^2. *) We can, however, approximate I_{00} by the formula

$$I_{00} \simeq A\, e^{-\lambda\tau} \tag{16}$$

for $\tau < 10\, m_\pi^2$. From (16), it is easily shown that

$$I_{02}/I_{20} \simeq \left[1 + 2(\lambda_P - \lambda)\tau\right]^{-1} \tag{17}$$

Comparing this to Eq.(14), we see a similarity, with the quantity $m_\pi^2(\lambda_P-\lambda)\,(\lambda_R+\lambda_P)^2/\lambda_P^2$ playing the role of z here. At $s = 100$ GeV^2, it is estimated that $\mathrm{Re}(\lambda_P - \lambda) = 0.021\ m_\pi^{-2}$, *) so that even at $\tau = 10\, m_\pi^2$, suppression will not be strong. In fact, the suppression remains weak even when s is increased to 10^6 GeV^2. This is clearly indicated in Figs.3a and 3b. The suppression is expected to appear at higher values of τ. For example, at $\tau = 100\, m_\pi^2$, the quantity Re $2(\lambda_P-\lambda)\tau$ is already 4.2, indicating that suppression is beginning to show up. However, it is generally believed that π exchange is not important beyond $\tau \sim 10\, m_\pi^2$, due to the presence of the other exchange, the A_2, which is a higher lying trajectory. Fig.3(a) shows the actual contributions of the πP cut to $F^{(+)}$ and $F^{(-)}$ in the reaction $\pi^- p \to \rho^0 n$. There is, in fact, a sizeable suppression even at $\tau \sim 10\, m_\pi^2$. This is actually due to the presence in $F^{(+)}$ of two other terms, $(\sqrt{\tau}/m_R)I_{10}$ and $(\tau/m_R^2)I_{00}$, in addition to I_{20}. The ratio of I_{02} to $(\tau/m_R^2)I_{00}$, for example, is $\{2\lambda_P^2\tau/(\lambda_P-\lambda)\}^{-1}$, which is $\simeq 0.1$ at $\tau = 10\, m_\pi^2$. This suppresion in the πP cut is much weaker than the corresponding one in, say, the ρP cut, and it does not seem to improve with increasing energy.

Concerning conspiracy, we find that all reggeon-pomeron cuts are self-conspiratorial. The conspiracy relation, we recall, relates amplitudes of opposite naturalities at $t = 0$. In terms of s-channel amplitudes, it is simply

*) Values of parameters used for such estimates are: $\alpha'_P = 0.5$, $A_P = 3.5$, $\alpha'_\rho = 0.83$, $A_\rho = 0.8$, $\alpha'_\pi = 1.0$ and $A_\pi = 3.69$ (all in units of GeV^{-2}).

$$F^{(+)}(s,0) = F^{(-)}(s,0) = 0 \tag{18}$$

Contrary to the suggestion of JL, all reggeon-pomeron cuts satisfy the above relation non-trivially. Away from $\tau = 0$, all cuts except the πP cut have suppressed contributions to $F^{(-)}$. The contribution to $F^{(+)}$ is brought to conform with (18) by suppressing itself at $\tau = 0$. The cut therefore appears to be evasive. But, as mentioned above, suppression is significant only at $s \gg 30$ GeV^2. Conspiratorial effect of the cut should therefore be observable at $s < 30$ GeV^2. Unfortunately, the "natural" choice, the reaction $\pi^- p \to \omega^0 n$, for observing such an effect is marred by the presence of the other exhange, the B pole, and its associated cut. There is, nevertheless, hope in the reaction $\pi^- p \to B^0 n$, where the only allowed exchange is the A_2 pole, together with its associated cut.

For the πP cut, the suppression is not strong near the forward direction at all energies. Here the contributions to both $F^{(+)}$ and $F^{(-)}$ are strong. The destructive interference [2] with the rapidly varying evasive pion pole then allows this conspiratorial effect to be readily observed as the well-known forward spike in $np \to pn$, $\gamma p \to \pi^+ n$, and in $\rho_{11}^{(H)} d\sigma/dt$ of $\pi^- p \to \rho^0 n$.

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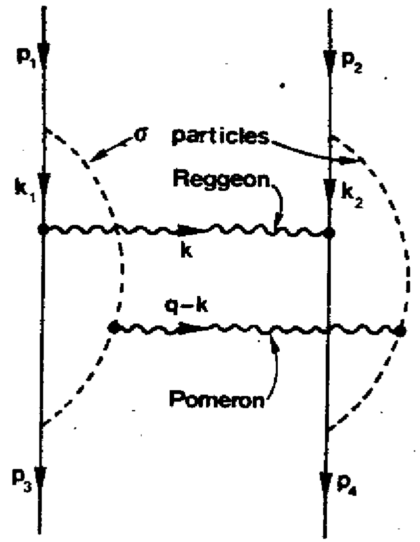


FIG. 1

Mandelstam diagram.

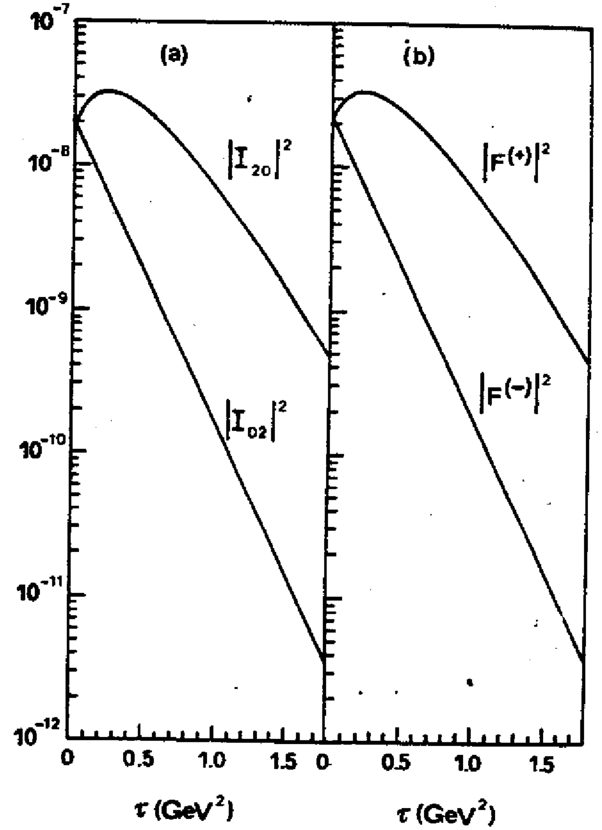
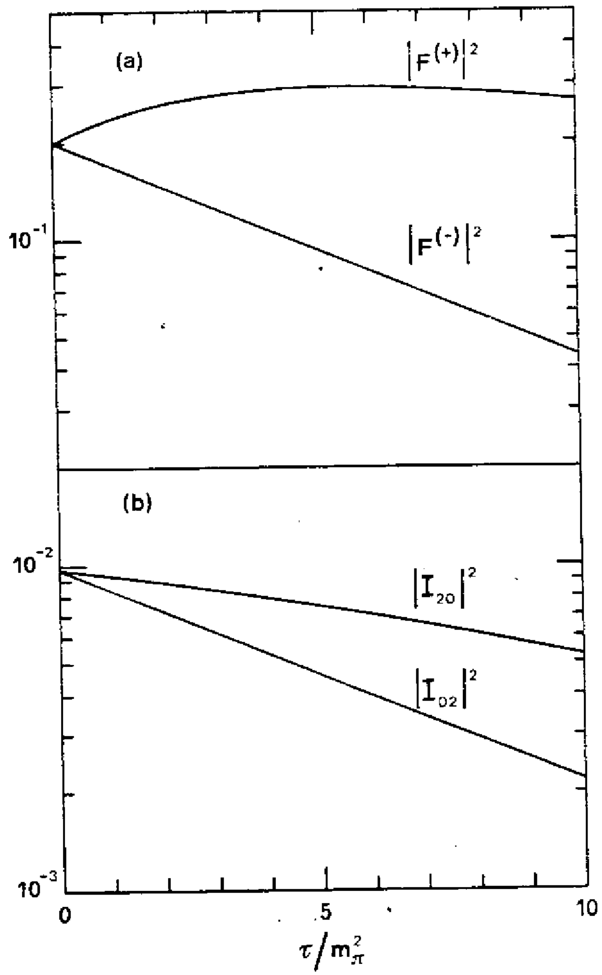


FIG. 2

ρP cut contributions at $s = 100$ GeV^2:
(a) $|I_{20}|^2$ and $|I_{02}|^2$;
(b) $|F^{(+)}|^2$ and $|F^{(-)}|^2$ in $\pi^- p \to \omega^0 n$.



FIG

πP cut contributions at $s = 100\ GeV^2$;
(a) $|F^{(+)}|^2$ and $|F^{(-)}|^2$ in $\pi^- p \to \rho^0 n$;
(b) $|I_{20}|^2$ and $|I_{02}|^2$.