

CMIS

von Karman Institute for Fluid Dynamics

Lecture Series

62

Two-phase Flows with Application to Nuclear

Reactor Design Problems

Pod. St. Genese (Belgium)

December 9 - 13, 1974

CEA-CONF--3538

FR7603078

Constitutive Equations for Two-phase Flows.

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NOMENCLATURE

A	Cross section area
$A' = \frac{dA}{dz}$	Dérivative of the cross-section area with respect to z.
α	Surface
C	Contour
D	Equivalent diameter
e	Specific internal energy
\vec{F}	External Force
$g \cos \theta$	Projection on -oz of the gravity acceleration.
\vec{h}	Specific enthalpy
\vec{J}	Heat flux density.
\vec{j}	Volumetric flux.
\vec{n}	Unit vector on oz.
p	Pressure
q	Heating power per unit of volume
\vec{r}	Position vector of a point.
t	Time
\vec{T}	Stress tensor.
\vec{v}	Velocity.
φ	Volume
W	Projection on oz of velocity.
x	Quality
z	Abscissa
Re	Reynolds number
λ	Void fraction
δ	Slip ratio
Γ_0	Phase change mass transfer, per unit of volume and per unit of time.
λ	Friction factor
μ	Viscosity
ρ	Density
σ	Surface tension
τ	Friction pressure drop per unit of length
τ	Deviatoric stress tensor.

SUBSCRIPTS :

C Related to contour.
G " to gas-phase.
i,I " to interface.
K " to phase K.
L " to liquid-phase.
LG Difference between the gas and the liquid values
sat On the saturation line.

1.- INTRODUCTION

The mathematical model of a system involving a fluid consists of several kinds of equations, complemented by boundary and initial conditions.

The equations of the first kind result from the application to the system of the fundamental conservation laws (mass, momentum, energy.)

The equations of the second kind characterize the fluid itself, i.e. its intrinsic properties and in particular its mechanical and thermodynamical behaviors. They are the mathematical model of the particular fluid under consideration. For this reason, the laws which are expressed by these equations are called the constitutive laws of the fluid (Truesdell, 1969, Ishii, 1975.)

The remaining equations are the relevant thermodynamic relationships and definitions.

If the model is consistent, the number of constitutive laws is just sufficient to effect closure of the set of equations, i.e. to enable a solution to be calculated.

In practice, things begin to complicate as soon as a high level of generality is left, and, in particular, when numerical results are sought for. A "practical" set of conservation equations involves, of course, simplifying assumptions, but also considerable mathematical transformations. The engineering variables are time (or statistical) and space averages. The local-instantaneous variables present in the conservation equations are replaced, through averaging procedures, by "engineering" variables. Two consequences follow :

- 1. The tractability of the equations is obtained at the expense of a loss of information,
- 2. The mass, momentum and energy transfer terms at the limit of the space averaging domain combine constitutive quantities and some other system parameters (often, in practice, some boundary conditions). These transfer terms cannot be calculated by means of the constitutive laws, as defined above.

For instance, the practical momentum equation for a steady-state, single-phase flow in a straight tube is obtained by averaging the local momentum equation over a cross-section and projecting it on the axis oz of the tube :

$$\frac{dp}{dz} + \rho w \frac{dw}{dz} + \rho g \cos \theta + \tau = 0, \quad (1)$$

and the expression of τ is :

$$\tau = - \frac{\vec{M}}{A} \int_C \tau \vec{m}_c \frac{dz}{dz} \quad (2)$$

In equation (2), τ is the deviatoric stress tensor, \vec{n}_c the unit vector of the outside normal to the surface Σ of the contour C , and \vec{n} is the unit vector of oz . For ordinary viscous fluids, the constitutive law for τ involves $\nabla \vec{V}$, where \vec{V} is the local fluid velocity. Unfortunately, \vec{V} has disappeared from the conservation equations during the averaging process. Hence, the above constitutive law becomes useless, and it is usually replaced by a correlation, such as

$$\tau = \Lambda \frac{\rho w^2}{2} \frac{1}{D}, \quad (3)$$

$$\text{with} \quad \Lambda = \Lambda(\text{Re}), \quad (4)$$

$$\text{RE} = \frac{\rho w D}{\mu}. \quad (5)$$

Since most constitutive laws are only present in the practical set of equations through the foregoing transfer terms, it is convenient to extend the meaning of the phrase "constitutive law" to include these terms. To keep the possibility of distinguishing, the true constitutive laws may be called intrinsic constitutive laws, while the other ones may be called external constitutive laws.

The intrinsic constitutive laws of interest include the equations of state and are generally well-known for current single phase fluids (tables). The external constitutive laws are often given by empirical correlations.

The foregoing single-phase example may be complemented for illustration purpose. For an adiabatic flow, the practical set of equations is

$$w \frac{d\rho}{dz} + \rho \frac{dw}{dz} + \rho w \frac{\Lambda'}{\Lambda} = 0 \quad (6)$$

$$\frac{dp}{dz} + \rho w \frac{dw}{dz} + \rho g \cos \theta + \tau = 0 \quad (1)$$

$$\rho w \frac{dh}{dz} - w \frac{dp}{dz} - w \tau = 0 \quad (7)$$

$$\rho = \rho(h, p) \quad (8)$$

$$\tau = \Lambda \frac{\rho w^2}{2} \frac{1}{D} \quad (3)$$

$$\Lambda = \Lambda(\text{Re}) \quad (4)$$

$$\text{Re} = \frac{\rho w D}{\mu} \quad (5)$$

$$\mu = \mu(h, p) \quad (9)$$

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Equations (6), (1), (7), express respectively the conservation of mass, momentum and energy. Equations (8) (equation of state), and (9) are intrinsic constitutive laws. Equation (4) is an external constitutive law. Equations (3) and (5) are definitions, respectively of Λ and of the Reynolds number. The complete set involves 8 equations for 8 dependent variables, namely ρ , W , p , h , τ , Λ , Re , μ .

The fig 1 summarizes this introductory section.

2.- CONSERVATIONS EQUATIONS

As shown above, the constitutive laws cannot, in practice, be studied independently of the mathematical model as a whole. It is therefore necessary to discuss briefly the derivation of the conservation equations, which are the common bases of all models.

For two-phase flows, the conservation equations have been studied in detail by Delhaye (1968), Vernier and Delhaye (1968), Ishii (1975), among others. The fundamental conservation laws are expressed by macroscopic balance equations written for a finite volume. They are the balances of mass, linear momentum, angular momentum, and energy. The increase of any of these quantities inside the volume \mathcal{V} is equal to the sum of a production inside \mathcal{V} and of a transfer through the surface α bounding \mathcal{V} .

The volume \mathcal{V} contains in general interfaces i.e: a surface of discontinuity α . Depending on the assumptions made on the interface properties, two cases may be distinguished :

1. Surface tension and other surface mechanical or thermal properties are not taken in account.
2. The surface of discontinuity has physical properties, such as surface tension. Delhaye (1974) has shown that inconsistencies are introduced in the model if surface tension and other material properties of the interface are not taken simultaneously into account. This is not surprising, since deformation of an interface implies that energy is stored on this interface when surface tension exists.

Assuming no surface material properties, the balance equations have the form :

$$\iiint_{\mathcal{V}(t+\Delta t)} Q dV - \iiint_{\mathcal{V}(t)} Q dV =$$

$$\int_t^{t+\Delta t} \left[\iiint_{\mathcal{V}(t)} \mathcal{P} dV - \iint_{\alpha(t)} \mathcal{T} d\alpha \right] dt \quad (10)$$

If \mathcal{V} is, for instance, a material volume, the expressions of Q , \mathcal{P} , \mathcal{T} , are given in the following table :

:	:	:	:	:
: Balance	: Q	: \mathcal{P}	: \mathcal{T}	:
:	:	:	:	:
: Mass	: ρ	: 0	: 0	:
:	:	:	:	:
: Linear Momentum	: $\rho \vec{v}$: $\rho \vec{F}$: $\vec{\pi} \cdot \vec{n}$:
:	:	:	:	:
: Angular Momentum	: $\vec{r} \wedge \rho \vec{v}$: $\vec{r} \wedge \rho \vec{F}$: $\vec{r} \wedge \vec{\pi} \cdot \vec{n}$:
:	:	:	:	:
: Energy	: $\rho \left(e + \frac{1}{2} \vec{v}^2 \right)$: $\rho \vec{F} \cdot \vec{v}$: $(\vec{w} \cdot \vec{v}) \cdot \vec{n} - \vec{J} \cdot \vec{n}$:
:	:	:	:	:

In this table, \vec{F} is the external force per unit of mass, $\vec{\pi}$ the stress tensor, \vec{J} the heat flux and \vec{n} the unit vector of the outside normal to α .

Equation (10) must be valid for any Δt .

Whence :

$$\frac{d}{dt} \iiint_{\mathcal{V}(t)} Q dV - \iiint_{\mathcal{V}(t)} \mathcal{P} dV + \iint_{\alpha(t)} \mathcal{T} d\alpha = 0 \quad (11)$$

The left hand side member of this equation can be transformed by means of mathematical theorems (Delhaye, 1968) into the sum of a volume integral and of a surface integral over the surface of discontinuity α :

$$- \iiint_{\mathcal{V}(t)} \mathcal{F} d\mathcal{V} + \iint_{\alpha(t)} \mathcal{J} d\alpha = 0 \quad (12)$$

Since equation (12) must be valid for any \mathcal{V} , the kernels of the two integrals must be zero.

Equating to zero the kernel of the volume integral yields a partial differential equation.

$$\mathcal{F} = 0, \quad (13)$$

relating local instantaneous values of the dependent variables. This equation is a field conservation equation, valid everywhere except on the surface of discontinuity.

Equating to zero the kernel of the surface integral yields a local instantaneous jump condition,

$$\mathcal{J} = 0, \quad (14)$$

valid on the surface of discontinuity.

By taking into account the linear momentum balance, the angular momentum balance reduces to a relation establishing the symmetry of the stress tensor \mathbb{T} .

If surface material properties are taken into account, three complementary terms appear in equation (11). By analogy with the terms already present, they can be written

$$\frac{d}{dt} \iint_{\alpha(t)} \mathcal{Q}_s d\alpha - \iint_{\alpha(t)} \mathcal{P}_s d\alpha + \int_{\mathcal{L}(t)} \mathcal{C}_s d\mathcal{L}.$$

The quantities Q_s , P_s and τ_s contain surface properties, and in particular the surface tension σ . The line \mathcal{E} is the intersection of the surfaces α_1 and α_2 . The above integrals can be transformed, by means of mathematical theorems (Delhaye, 1974) into a surface integral over the surface of discontinuity:

$$\iint_{\alpha_s(t)} J_s d\alpha$$

Therefore, if surface material properties are taken into account, the field equations (13) remain unchanged, which is logical, while the jump conditions, valid on the surface of discontinuity, take the form

$$J_+ - J_- = 0 \quad (15)$$

The local instantaneous field equations and jump conditions are ultimately transformed, as already explained, to yield a practical set of averaged field equations and jump conditions.

Within the above framework, several philosophies may be adopted :

1. In the "mixture models", the interfaces are ignored, and the mixture is considered as a single fluid, having its own properties. The advantage of this philosophy is the simplicity of the resulting conservation equations : There are no surfaces of discontinuity within the flow field. The conservation equations obtained are identical to the well-known single phase equations. There are only three of them (mass, momentum, energy).

However, it can be expected that the difficulties are only transferred to the constitutive laws. In particular, the intrinsic constitutive laws cannot be found in tables. Moreover, when the transfers between the phases play an essential part in the process, (i.e. when slip and/or departure from thermal-equilibrium are important, and for transients), it seems difficult to ignore the interfaces, where these transfers occur, and to justify the foregoing philosophy.

Historically, the first two-phase flow models were simple mixture models. The use of a mixture model may be considered when one phase is finely dispersed into the other (bubble flow with small bubbles, mist flow.)

2. In the "two-fluid models", the two phases are considered separately. The interfaces are surfaces of discontinuity on which jump and boundary conditions must be written. The conservation equations obtained consist of six partial differential equations (three for each phase, expressing the conservation of mass, momentum and energy) and three interface relationships (expressing the jump conditions for mass, momentum and energy.) The advantage of this philosophy is its comprehensiveness. A two-fluid model may theoretically deal with any two-phase flow.

However the constitutive laws, which include the interfacial transfer terms, have to be much more detailed in a two-fluid model than in a mixture model. The potentialities of the two-fluid models rest on a good knowledge of the external constitutive laws.

3. Other philosophies are possible. For instance, when the flow-pattern in a pipe is annular, the wall film (which may contain small bubbles) and the core (mist flow) may be considered as two distinct flow fields, separated by a surface of discontinuity. A mixture or a two-fluid-model may be developed for each of the two flow fields. Such a philosophy may also be of interest when a part of the flow field, involving large transverse gradients of dependent variables (void fraction, velocity,...), must be distinguished from the remaining part of the flow, which involves only small transverse gradients.

Most of the existing models pertain to the first two philosophies. Moreover, they involve only one space variable. Therefore, the discussion is restricted hereunder to one-dimensional, mixture or two-fluid models.

3:- MIXTURE MODELS

As a consequence of the mixture hypothesis, the state of the fluid is characterized by only three variables, which are functions of z and t (for instance ρ, W, h - see the single phase example of section 1). These three variables must satisfy the three conservation equations (Mass, Momentum, Energy). The remaining quantities of interest result from the constitutive laws of the mixture, which is considered as a single fluid.

Since the existence of two phases and of interfaces is ignored, it must be noticed that it is generally impossible:

1. To deal with small scale phenomena,
2. To interpret the mixture variables (ρ, W, h, p, \dots) in terms of the phasic variables.

Typical "primary" conservation equations, obtained directly from (13) through averaging procedures and by use of conventions and simplifying assumptions which are customary (Bouré and Reocreux, 1972) are :

Mass

$$\frac{\partial}{\partial t} (A\rho) + \frac{\partial}{\partial z} (A\rho W) = 0 \quad (16)$$

Momentum (projection on the oz axis).

$$\frac{\partial}{\partial t} (A\rho W) + \frac{\partial}{\partial z} (A\rho W^2) + A \frac{\partial p}{\partial z} + AZ + \psi + A\rho g \cos \theta = 0, \quad (17)$$

where

$$\tau = - \frac{\vec{m}}{A} \int_C \vec{u} \cdot \vec{m}_c \frac{d\vec{x}}{A_z},$$

$$\psi = - \vec{m} \frac{\partial}{\partial z} (A \vec{u} \cdot \vec{u}).$$

$$\begin{aligned} \text{Energy} \quad & \frac{\partial}{\partial t} \left[A \rho \left(e + \frac{w^2}{2} \right) \right] + \frac{\partial}{\partial z} \left[A \rho w \left(e + \frac{w^2}{2} \right) \right] \\ & + \frac{\partial}{\partial z} (A \rho w) - A q - \Psi + A \rho g w \omega \theta = 0 \end{aligned} \quad (18)$$

$$\text{where} \quad q = -\frac{1}{A} \int_c \vec{J} \cdot \vec{m}_c \frac{d\varepsilon}{dz}$$

$$\Psi = -\frac{\partial}{\partial z} (A \vec{J} \cdot \vec{u}) + \frac{\partial}{\partial z} \left[A (\vec{z} \cdot \vec{u}) \cdot \vec{v} \right]$$

(Since the context is subject to no ambiguity, the same symbols are used for local quantities -when under \int - and for averaged quantities - everywhere else.)

In the above equations, the terms τ and q take into account together some of the rheological properties of the fluid and some of the boundary conditions :

τ and q are given by external constitutive laws. In the same way, the terms ψ and Ψ take into account together some of the rheological properties of the fluid and some geometrical conditions : they are also given by external constitutive laws. The forms of all these terms are of primary importance since they affect the mathematical character of the set of equations. In most practical applications, it is often sufficient to assume that τ and q depend only on z , t , and on the values of the other dependent variables, called hereunder main variables to distinguish them (for instance ρ , w , h , p), whereas ψ and Ψ are negligible. It must be emphasized that such assumptions are very restrictive and cannot be substantiated. The expressions of τ and $\frac{\psi}{A}$ on the one hand, of q and $\frac{\Psi}{A}$ on the other hand, may be summed up for convenience. From this point on, τ stands for $\tau + \frac{\psi}{A}$ and q stands for $q + \frac{\Psi}{A}$.

A more practical set of equations may be derived from equations (16) to (18) by expanding and combining the equations. The practical set of conservation equations can be written in terms of four main dependent variables ρ , w , p , h :

$$\text{Mass} \quad \frac{\partial \rho}{\partial t} + w \frac{\partial \rho}{\partial z} + \rho \frac{\partial w}{\partial z} + \rho w \frac{A'}{A} = 0 \quad (19)$$

$$\text{"Momentum"} \quad \rho \left[\frac{\partial w}{\partial t} + w \frac{\partial w}{\partial z} \right] + \frac{\partial p}{\partial z} + \tau + \rho g \cos \theta = 0 \quad (20)$$

$$\text{"Energy"} \quad \rho \left[\frac{\partial h}{\partial t} + w \frac{\partial h}{\partial z} \right] - \left[\frac{\partial p}{\partial t} + w \frac{\partial p}{\partial z} \right] - w \tau - q = 0 \quad (21)$$

To close the set, an intrinsic constitutive law (the equation of state of the mixture) and two external constitutive laws (for τ and q) are needed. As pointed out above for τ and q , the mathematical forms of their expressions are not known a priori. They may involve any of the main dependent variables, which is classical -see equation (4)-, but also, for instance, derivatives of these variables.

In general, the external constitutive laws are given by empirical correlations, as it is the case for single phase flows. Experimental data are available for q . The same is not true in two-phase flows for τ which is deduced from pressure drop experimental data by use of equation (20). Therefore the numerical value of τ depend on the model itself, and in particular on the mixture equation of state : the use of a friction factor correlation is only consistent with the model used to establish it. This fact is often overlooked in practice.

Current correlations include the Jens and Lottes, and the Chen correlations for heat transfer, the Martinelli - Nelson, and the Chisholm correlations for friction.

Several attempts have been made to provide a mixture equation of state. Some of these are briefly discussed hereunder :

3.1.- Homogeneous Model

Assuming no slip, no departure from thermal equilibrium, and no pressure difference between the phases enables the mixture variables ρ , w , p , h , to be interpreted in terms of the phasic variables; w and p are respectively the velocity and the pressure of both phases and,

introducing the quality x and the void fraction α ,

$$h = h_L + x h_{LG} \quad (22)$$

$$\rho = \alpha \rho_G + (1-\alpha) \rho_L \quad (23)$$

Since, in the absence of slip,

$$\frac{\alpha \rho_G}{x} = \frac{(1-\alpha) \rho_L}{1-x} \quad (24)$$

the set of equations is closed by the thermal and caloric equations of state of the two phases on the saturation line :

$$\left. \begin{aligned} \rho_G &= \rho_{G \text{ sat}}(p), & \rho_L &= \rho_{L \text{ sat}}(p) \\ h_L &= h_{L \text{ sat}}(p), & h_{LG} &= h_{LG \text{ sat}}(p) \end{aligned} \right\} (25)$$

By elimination of x and α between equations (22) to (24), the equation of state of the mixture can be written :

$$\frac{\rho_L}{\rho} = 1 + \left(\frac{\rho_L}{\rho_G} - 1 \right) \frac{h - h_L}{h_{LG}} \quad (26)$$

In fact, it is often more convenient to eliminate ρ and h from the set, and to keep as main variables α , W , p , x .

The homogeneous model is the simplest possible two-phase flow model. It results from drastic assumptions. Although it is, in some cases, a satisfactory approximation, the need for more sophisticated models has been recognized for a long time.

3.2. Generalized Homogeneous Models.

In the homogeneous model, ρ is a particular function of p and h , given by equation (26), complemented by equations (25).

A generalization appears therefore possible, by use of another equation of state

$$\rho = \rho(W, p, h), \quad (27)$$

without direct reference to the quality x and the void fraction α . Equation (27) can be fit to any particular purpose. This partly enables the effects of slip and thermal non-equilibriums to be taken into account. For instance, equation (27) may be a partial differential equation.

A simplified form of (27)

$$\rho = \rho_p(h),$$

with the system average pressure as a parameter, has been used by Bouré (1967).

The homogeneous and the generalized homogeneous models are the only true mixture models. Other mixture models are in current use. However, they can only be justified as particular cases of two-fluid models.

3.3. Slip Models. Non-equilibrium Models.

Numerous attempts have been made to re-introduce the two phasic velocities in a mixture model, thus acknowledging the experimental evidence of slip between the phases. Unfortunately, most of these efforts raise consistency problems, since one cannot at the same time consider the mixture as a single fluid and as two fluids.

1. When a single momentum equation is used, the "constitutive law" for the slip ratio γ is given by a correlation, deduced from steady-state experimental data,

$$\gamma = \gamma(\rho, W, p, h). \quad (28)$$

In view of the two phasic momentum equations, it can be argued (Bouré, 1963) that there is definitely no grounds for the form (28). Equation (28) is in fact the second momentum equation, and it should at least involve derivatives of W and/or p .

It can be expected and, using a two-fluid model, it can be shown that the form (28) implies very restrictive

assumptions on the interfacial transfer laws (Bouré, 1975)

2. When the two phasic momentum equations are used, the model is no longer a mixture model. It is a two-fluid model deprived from two phasic equations, one mass conservation equation and one energy conservation equation. This point is further discussed in section 5.

The authors of slip models have been led to the introduction of either several mixture densities (Meyer, 1960), which has been stigmatized by Zuber and Dougherty (1967), or several mixture velocities and enthalpies.

Similar reasoning applies to the non-equilibrium models. However, a particular mention must be made for the so-called "Diffusion-Models".

3.4. Diffusion Models.

The diffusion models have been introduced by Zuber (see for instance Zuber, 1967). The argumentation of section 3.3 can be applied to the diffusion models ; They are not true mixture models and their justification rests on a two-fluid analysis.

In this case, however, the mixture models, their justification, and their conditions of application have been studied in detail by Ishii (1975). Quoting this author : The diffusion model "requires some drastic constitutive assumptions and with them some of the important characteristics of two-phase flow will be lost. Nevertheless it is exactly this simplicity of the diffusion model which makes it very useful in many engineering applications.....information required in engineering problems are often the response of the total mixture and not of each constituent phase ... However, there are considerable questions in applying the diffusion model to the problems of acoustic wave propagations, choking phenomena and high frequency instabilities."

The diffusion-model conservation-equations are equations (19) to(21) , with

$$\begin{aligned}
 \rho &= \alpha \rho_G + (1-\alpha) \rho_L \\
 W &= \frac{\alpha \rho_G W_G + (1-\alpha) \rho_L W_L}{\rho} \\
 h &= \frac{\alpha \rho_G h_G + (1-\alpha) \rho_L h_L}{\rho}
 \end{aligned}
 \tag{29}$$

(It can be noticed that W is the center-of-mass velocity, and that the definition of h is not compatible with equation (22), except in the absence of slip).

The above equations are complemented by a "void diffusion" equation, which is in fact the mass-conservation equation of, say, the gas-phase :

$$\frac{\partial(\alpha \rho_G)}{\partial t} + \frac{\partial(\alpha \rho_G W_G)}{\partial z} = \Gamma_G - \alpha \rho_G W_G \frac{A'}{A}, \tag{30}$$

and by seven constitutive equations :

- a constitutive equation for phase change mass transfer, giving Γ_G ,
- a "kinematic" constitutive equation, which has, in fact, the same role as the slip equation (28).
- a constitutive law for non-equilibrium—often, in practice, as pointed out by Ishii, either $h_G = h_{G \text{ sat}}(p)$ or $h_L = h_{L \text{ sat}}(p)$.
- the equations of state of both phases
- the two external constitutive equations for τ and q :

The set involves 14 equations for 14 dependent variables, namely ρ , W , p , h , τ , q , α , ρ_G , ρ_L , W_G , W_L , h_G , h_L , Γ_G .

The form of the kinematic constitutive equation is more sophisticated than a mere slip ratio correlation. The volumetric flux (or center-of-volume velocity) j , and the drift velocity W_{gj} are first defined by

$$\begin{aligned}
 j &= \alpha W_G + (1-\alpha) W_L, \\
 W_{gj} &= W_G - j
 \end{aligned}
 \tag{31}$$

The kinematic constitutive equation is an equation for $w_{\xi j}$. Unlike δ , $w_{\xi j}$ has a clear physical significance, which may suggest the form of the constitutive equation.

As shown by Ishii, parasitical terms due to slip and thermal non equilibriums appear in Z , q and Γ_G .

4.- TWO-FLUID MODELS

In one-dimensional, two-fluid models, the state of each phase is characterized by three variables, which are functions of z and t (for instance ρ_k, w_k, h_k). These three phasic variables must satisfy the three phasic conservation equations. The averaging procedures introduce another dependent variable, which is distinct from the above phasic variables, the so-called void fraction α_g . This is due to the fact that, at any particular time, a given point is either in the gas, or in the liquid, or on the interface.

Starting from a set of initial conditions, and using a mathematical model based on the local instantaneous conservation equations (which is obviously beyond the practical possibilities), it would in principle be possible to calculate the evolutions of the interfaces and eventually α_g . This information is lost during the averaging procedures, and has to be provided through a particular constitutive law, which may be called interaction law.

The interface material properties, including surface tension, will be ignored in this section. This does not mean any loss of generality, provided the presence, in the interface relationships, of terms corresponding to these properties is not forgot.

Typical "primary" conservation equations and interface relationships, obtained directly from (13) and (14) through averaging procedures and by use of conventions and assumptions which are customary (Bouré and Réocreux, 1972) are, with $\alpha_L = 1 - \alpha_g$:

Mass*Field Equation :*

$$\frac{\partial}{\partial t} (d_k A \rho_k) + \frac{\partial}{\partial z} (d_k A \rho_k w_k) - A M_{ki} = 0, \quad (32_K)$$

$$\text{where } AM_{ki} = - \int_I \rho_k (\vec{v}_k - \vec{v}_I) \cdot \vec{n}_k \frac{dS}{d_z}$$

is the mass flux through the interface I to phase K, per unit length of pipe.

Interface Relationship :

$$\sum_K A M_{ki} = 0 \quad (32_I)$$

Momentum*Field Equation (projection on the oz axis)*

$$\frac{\partial}{\partial t} (d_k A \rho_k w_k) + \frac{\partial}{\partial z} (d_k A \rho_k w_k^2) + d_k A \frac{\partial p_k}{\partial z} - A (MV)_{kz}$$

$$+ A \tau_{kz} + A \tau_{kc} + \psi_k + d_k A \rho_k g \cos \theta = 0, \quad (33_K)$$

$$\text{where } A (MV)_{kz} = - \vec{n} \int_I \left[\rho_k (\vec{v}_k - \vec{v}_I) \cdot \vec{n}_k \right] \vec{v}_k \frac{dS}{d_z}$$

is the momentum flux through the interface to phase K, per unit length of pipe, due to mass transfer,

$$A \tau_{kz} = - \vec{n} \int_I \tau_{kz} \cdot \vec{n}_k \frac{dS}{d_z} \quad \text{and} \quad A \tau_{kc} = - \vec{n} \int_C \tau_{kc} \cdot \vec{n}_k \frac{dS}{d_z}$$

are the projections on the oz axis of the

resultants of the viscous stresses acting on phase K at the interface I and the wall C, per unit length of pipe,

$$\varphi_k = -\vec{n} \frac{\partial}{\partial z} (\rho_k A \vec{\tau}_k \cdot \vec{n})$$

Interface Relationship (projection on the oz axis)

$$\sum_k \left[A (M\dot{V})_{KI} - A \tau_{KI} \right] = 0 \quad (33_I)$$

Energy

Field Equation

$$\begin{aligned} & \frac{\partial}{\partial t} \left[\rho_k A \rho_k \left(e_k + \frac{w_k^2}{2} \right) \right] + \frac{\partial}{\partial z} \left[\rho_k A \rho_k w_k \left(e_k + \frac{w_k^2}{2} \right) \right] \\ & + \frac{\partial}{\partial z} \left[\rho_k A p_k w_k \right] - A (ME)_{KI} - A q_{KI} - A \pi_{cKI} - A \chi_{KI} \\ & - A q_{Kc} - \Psi_k + \rho_k A \rho_k w_k g \cos \theta = 0, \quad (34_k) \end{aligned}$$

where
$$A (ME)_{KI} = - \int_I \rho_k \left(e_k + \frac{v_k^2}{2} \right) (\vec{v}_k \cdot \vec{v}_I) \vec{n}_k \frac{d\xi}{dy}$$

is the total energy flux through the interface to phase K, per unit length of pipe, due to mass transfer,

$$A q_{KI} = - \int_I \vec{j}_k \vec{u}_k \frac{d\xi}{dy} \quad \text{and} \quad A q_{Kc} = - \int_C \vec{j}_k \vec{u}_k \frac{d\xi}{dy}$$

are the heat fluxes to phase K through the interface I and the wall C, per unit length of pipe,

$$A \pi_{cKI} = - \int_I \rho_k \vec{v}_k \cdot \vec{u}_k \frac{d\xi}{dy} \quad \text{and} \quad A \chi_{KI} = \int_I (\vec{u}_k \cdot \vec{m}_k) \vec{v}_k \frac{d\xi}{dy}$$

$$\Psi_k = - \frac{\partial}{\partial z} (\rho_k A \vec{j}_k \cdot \vec{u}) + \frac{\partial}{\partial z} \left[\rho_k A (\vec{u}_k \cdot \vec{u}) \cdot \vec{v}_k \right]$$

Interface Relationship

$$\sum_K \left[A(ME)_{KI} + Aq_{K\tau} + A\eta_{eK\tau} + A\chi_{K\tau} \right] = 0 \quad (34_I)$$

The discussion made in section 3 on the form of τ , q , ψ , Ψ , applies here to $\tau_{K\tau}$, τ_{Kc} , ψ_K , q_{KI} , q_{KC} , Ψ_K and also to M_{KI} , $(MV)_{KI}$, $(ME)_{KI}$, $\eta_{eK\tau}$, $\chi_{K\tau}$.

The expressions of τ_{KC} and $\frac{\psi_K}{A}$ on the one hand, of q_{KC} and $\frac{\Psi_K}{A}$ on the other hand, may be summed up for convenience. From this point on τ_{Kc} stands for $\tau_{Kc} + \frac{\psi_K}{A}$ and q_K stands for $q_{KC} + \frac{\Psi_K}{A}$.

A more practical set of equations may be derived from equations (32) to (34) by expanding and combining the equations. The practical set of conservation equations can be written in terms of nine main dependent variables

$$f_K, w_K, p_K, h_K \text{ and } \alpha_G.$$

Defining

$$A(MH)_{KI} = - \int_{\tau} \rho_K h_K (\vec{v}_K - \vec{v}_\tau) \vec{n}_K \frac{d\xi}{d\tau}$$

enthalpy flux through the interface to phase K, per unit length of pipe, due to mass transfer

$$A(EC)_{KI} = - \int_{\tau} \frac{\rho_K \vec{v}_K^2}{2} (\vec{v}_K - \vec{v}_\tau) \vec{n}_K \frac{d\xi}{d\tau}$$

kinetic energy flux through the interface to phase K, per unit length of pipe, due to mass transfer

$$A \Pi_{hKI} = - \int_{\tau} \rho_K \vec{v}_\tau \vec{n}_K \frac{d\xi}{d\tau},$$

$$M = M_{GI} = -M_{LI} \quad (35)$$

$$(MV) = (MV)_{GI} - \tau_{CI} = - \left[(MV)_{LI} - \tau_{LI} \right] \quad (36)$$

$$\begin{aligned}
 (MH) &= (MH)_{GI} + (EC)_{GI} + q_{GI} + \chi_{GI} \\
 &= - \left[(MH)_{LI} + (EC)_{LI} + q_{LI} + \chi_{LI} \right] - \sum_k \pi_{kxz} \quad , \quad (37)
 \end{aligned}$$

taking into account the fact that with no surface material properties, there is no Δp across the interface and

$$\sum_k \pi_{kxz} = - \int_{\Sigma} (p_c - p_l) \vec{v}_x \vec{m}_c \frac{d\Sigma}{d\Omega} = 0 \quad ,$$

and using

$$- \frac{\partial}{\partial t} (L_k A p_k) - A \pi_{kxz} = - L_k A \frac{\partial p_k}{\partial t}$$

enable the practical set of conservation equation to be written :

Mass

$$\frac{\partial}{\partial t} (L_k \rho_k) + \frac{\partial}{\partial z} (L_k \rho_k w_k) + L_k \rho_k w_k \frac{A'}{A} \mp M = 0 \quad (38_K)$$

"Momentum"

$$L_k \rho_k \left[\frac{\partial w_k}{\partial t} + w_k \frac{\partial w_k}{\partial z} \right] + L_k \frac{\partial p_k}{\partial z} \mp (Mv) \pm M w_k + \tau_k + L_k \rho_k g \cos \theta = 0 \quad (39_K)$$

"Energy"

$$\begin{aligned}
 &L_k \rho_k \left[\frac{\partial h_k}{\partial t} + w_k \frac{\partial h_k}{\partial z} \right] - L_k \left[\frac{\partial p_k}{\partial t} + w_k \frac{\partial p_k}{\partial z} \right] \\
 &\mp (MH) \pm M \left(h_k - \frac{1}{2} w_k^2 \right) \pm (Mv) w_k - w_k \tau_k - q_k = 0 \quad (40_K)
 \end{aligned}$$

In equations (38) to (40), the upper sign is valid for $K = G$, and the lower sign is valid for $K = L$. Other forms of the practical set of conservation equations are used. This one has been chosen here because equations (38_K) to (40_K) are the strict equivalents to equations (19) to (21).

To close the set, two intrinsic constitutive laws (the equations of state of the phases) and eight external constitutive laws are needed. The eight external constitutive laws are :

- the interaction law, discussed above,
- the four laws for heat transfer and friction at the wall $(\tau_c, \tau_L, q_G, q_L)$,

- three interfacial transfer laws -mass, momentum, and energy transfers : M, (MV), (MH).

The number of eight external constitutive laws is to be compared to the single-phase number of two (section 3). This gives an idea of the increase of complexity due to the presence of two phases. It can be noticed that the laws for M, (MV), (MH), are external constitutive laws for the phases, but intrinsic constitutive laws for the mixture. Also, it is emphasized that with the above set of equations, no assumptions are needed on slip and thermal non-equilibriums, which are calculated along with the others variables.

If surface material properties are taken into account, complementary terms appear in equations (32_I) to (34_I) and equations (35) to (37) must be modified. One intrinsic constitutive law (the "equation of state" of the interface, giving σ) and three external constitutive laws must be added to the foregoing.

As pointed out above, the mathematical forms of the external constitutive equations are not known a priori. They may involve any of the main dependent variables, but also, for instance, first order derivatives of these variables. If it is assumed that such is the case, and if moreover the constitutive equations are linear in terms of the above derivatives, the set of partial differential conservation equations (38_K) to (40_K) has the form

$$A(X, z, t) \frac{\partial X}{\partial t} + B(X, z, t) \frac{\partial X}{\partial z} = C(X, z, t) \quad (41)$$

where A and B are square matrixes, and X and C are column vectors.

The practical use of two-fluid models raises three main problems :

1. When α_k is small, the matrixes A and B are ill-conditioned. Since (39_k) and (40_k) must be valid for any α_k , ($0 < \alpha_k \leq 1$), the expressions

$$\bar{z}(MV) \pm MW_K + \tau_K,$$

and
$$\bar{z}(MH) \pm M(h_K - \frac{1}{2} W_K^2) \pm (MV)W_K - q_K - W_K \tau_K$$

must tend to zero when α_k tends to zero. This has to be checked when particular forms of the constitutive equations are tested. The above conditions being fulfilled, equations (39_k) and (40_k) may be divided by α_k and the matrixes are no longer ill-conditioned for small values of α_k .

2. It can be shown (Bouré and Réocreux, 1972, Bouré, 1973) that contradictions are induced by constitutive equations which involve only the main dependent variables but not their derivatives: Since all the transfer terms are on the right hand side of (41), they have no effect on the local values of the critical flow rate and of the propagation velocities of small disturbances. The coupling between the two phases is obtained only through the interaction constitutive law and the right hand side of equation (41). This coupling is weak and the critical flow rates are much larger than the experimental data. Whenever the local conditions are the same, the critical flow rate and propagation velocities are the same, for instance, in air-water and in steam-water flows. On the other hand, it is not possible (Bouré, 1975), by making the appropriate assumptions, to reduce a two-fluid model to the homogeneous model.

On the contrary, encouraging results have been obtained (Wallis, 1969, Lyczkowski, 1974, Bouré et al, 1975) when allowing for the presence of partial derivatives of dependent variables in the constitutive equations. In these cases, the matrixes A and B are affected by the transfer laws, the coupling between the two phases is stronger than above, the critical flow rate predictions are improved, and the homogeneous model appears as a particular case of two-fluid models.

3. Due to their number, it is very difficult to adjust the two-fluid model constitutive equations on experimental data. This is one of the main problems currently under investigation in basic two-phase flow research.

The interaction constitutive equation is often taken as

$$p_G = p_L \quad (42)$$

This form strongly affects the coupling between the phases.

The constitutive equations for τ_G , τ_L , q_G , q_L , are probably analogous to the single phase constitutive equations for τ and q , with only a new variable involved (α_G).

The main problems are with the interfacial transfer laws, and especially with M and (MV). It must be noticed, however, that the same problems exist for the diffusion model, Γ_G and w_{gj} corresponding respectively to M and (MV).

5.- REMARKS ON THE MODELS INVOLVING 4 OR 5

CONSERVATION EQUATIONS.

As pointed out in section 3.3, models involving 4 or 5 conservation equations are numerous. They result from attempts to reintroduce slip and/or thermal non-equilibriums in mixture models. A four- or five-conservation-equation model comprises the three mixture conservation equations (Mass, Momentum, Energy) and one or two phasic conservation equation(s). All the possible combinations of equations can be found in the literature (Bouré and Réocreux, 1972).

The four - or five-conservation -equation models are in fact two-fluid models deprived from two or one phasic equation(s). The missing equation(s) are replaced by assumptions, which are most often two or one of the following :

- no slip
- vapor phase at saturation
- liquid phase at saturation.

For instance, if everywhere and at every time $h_G = h_{G \text{ sat}}(p)$ and $h_L = h_{L \text{ sat}}(p)$ are assumed, the fluid is at thermal equilibrium. Accordingly only four dependent variables are in practice sought for, and only four conservation equations are written. Three dependent variables characterize the mixture, and the fourth characterizes the difference between the velocities of the two phases. The four conservation equations written include logically the three mixture conservation equations and a fourth equation which is in general a gas - or a liquid-momentum conservation equation, or a combination involving one of these momentum equations.

To be consistent, a four- or five- conservation-equation model must appear as a particular case of a two-fluid model (In the above example, $h_G = h_{G \text{ sat}}(p)$ and $h_L = h_{L \text{ sat}}(p)$ must belong to the solution of the complete set). It can be shown that this is possible if, and only if the transfer terms, i.e; the external constitutive laws, fulfill two or one condition(s) (Réocreux, 1974). In general, these conditions cannot be fulfilled by constitutive equations which involve only z , t and the main dependent variables. On the other hand, the presence in the external constitutive equations of partial derivatives of the dependent variables enables the above conditions to be fulfilled (Bouré, 1975). This is another argument in favor of the presence of partial derivatives in the external constitutive equations.

6.- CONCLUSION

It has been shown that, in practice, the constitutive equations which, together with the conservation equations, are the mathematical model of a two-phase flow, cannot be fully stated without reference to these conservation equations.

Two classes of models have been distinguished

In the mixture models, the mixture is considered as a single fluid. Besides the usual friction factor and heat transfer correlations, a single constitutive law, which is the mixture equation of state, is necessary in true mixture models. However, this constitutive law is very complex and, to the author's knowledge, no satisfactory mixture equation of state has been proposed. In the diffusion models, the mixture equation of state is replaced by the phasic equations of state and by three constitutive laws, for phase change mass transfer, drift velocity, and thermal non-equilibrium respectively. The mixture models are relatively simple and their use is often justified, at least as long as two-fluid models are difficult to use.

In the two-fluid models, the two phases are considered separately. In the simplest case (no interface material properties) ten constitutive laws are necessary. They include two phasic equations of state, two friction factor correlations, two heat transfer correlations and four constitutive laws which are very poorly known :

- the interaction law
- the three interfacial transfer laws, which correspond to the three constitutive laws for phase change, drift and non-equilibrium of the diffusion model.

Finding out the interaction law and the three interfacial transfer laws is one of the main problem currently under investigation in basic two-phase flow research.

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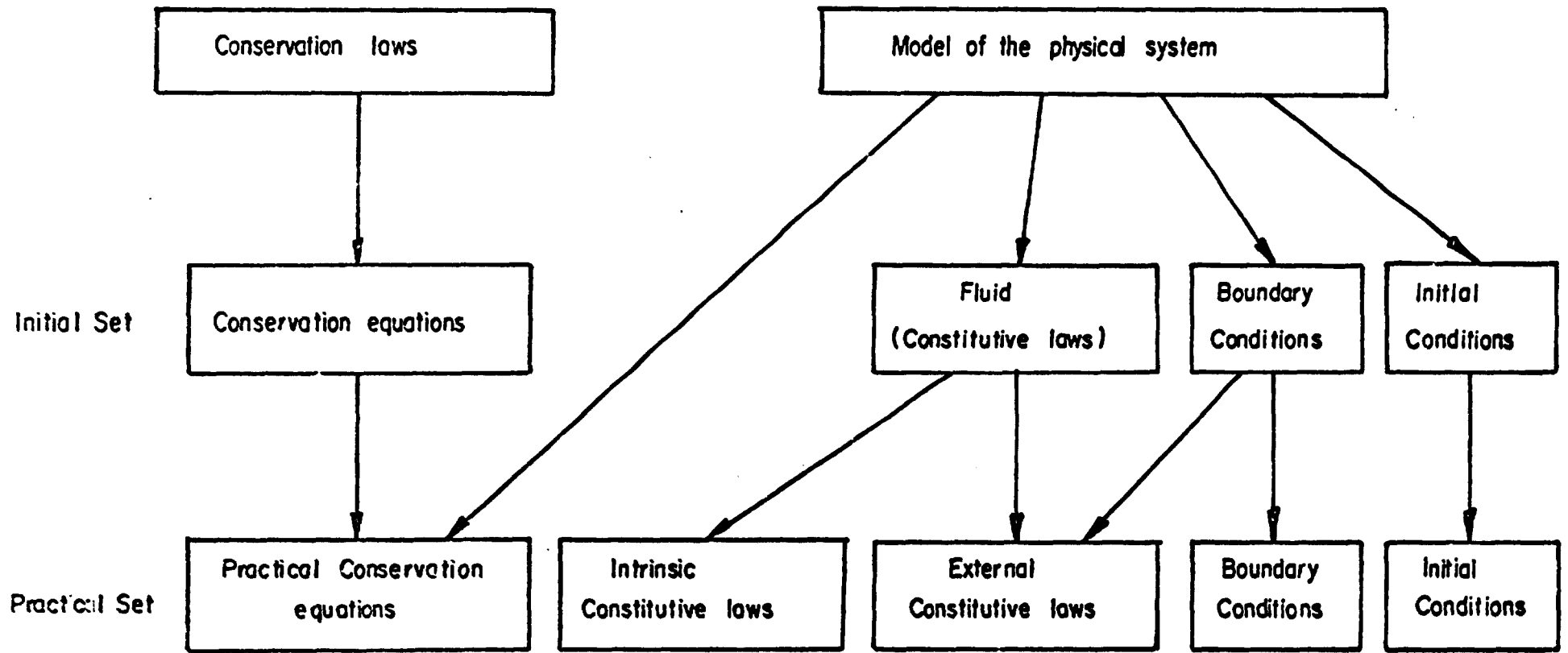


FIG:1 MATHEMATICAL MODELING

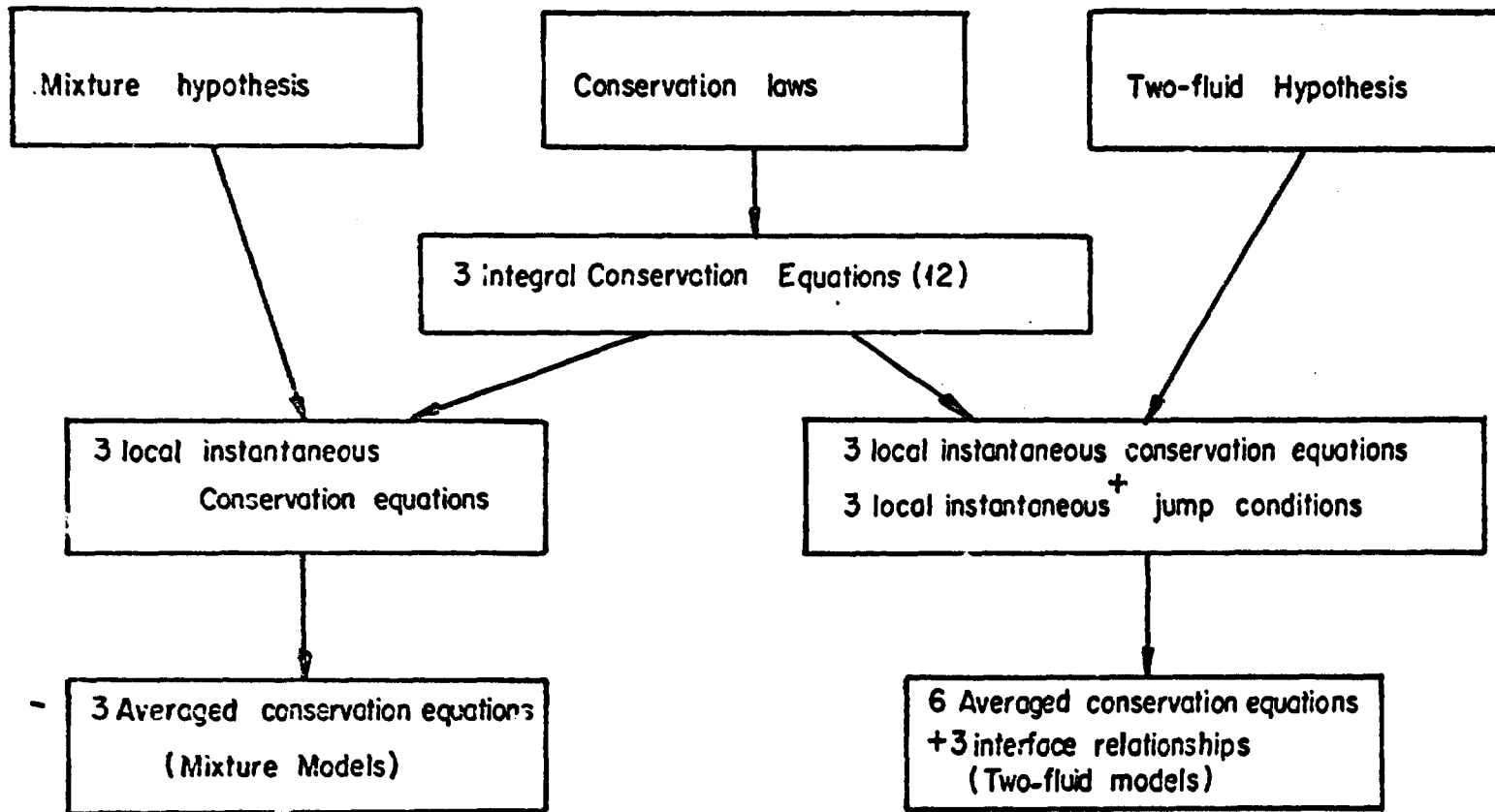


FIG: 2 CONSERVATION EQUATIONS