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AND COVARIANT PERTURBATION
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**DYNAMICAL AFFINE SYMMETRY
AND COVARIANT PERTURBATION
THEORY FOR GRAVITY**

Submitted to TMD

1. Introduction

In the quantization procedure it is conventional to treat gravity as a variant of the gauge field theory^{1,2,3}. In the present note another analogy chiral dynamics, is used for covariant quantization of the gravitational field. As has been shown⁴, the gravitational field theory is the theory of spontaneous breaking of affine and conformal symmetries like that the chiral dynamics is the theory of spontaneous breaking of the chiral symmetry. It allows one to formulate perturbation theory with the most simple reduction properties. In the chiral dynamics, such a perturbation theory^{5,6,7} simplifies considerably the calculation technique and is quite suitable for the use of regularization methods^{8,9} based on summation of certain diagrams.

To make the statement of the problem more clear, consider the simple $\lambda \varphi^4$ theory. Together with the $\lambda \varphi^4$ Lagrangian there also exists a lot of equivalent on the mass shell Lagrangians obtained by the transformations

$$\varphi = \varphi' f(\varphi') \quad ; \quad f(\varphi') = f \quad (1)$$

Constructive methods to fulfil the equivalence theorem are rearrangements of matrix elements¹⁰ called reductions¹⁰ (or contractions⁵). These reductions consist in transferring the vertex derivatives from one line to another and in reducing certain propagators to δ - functions.

Consideration of all possible reductions after the change of variables (1) is equivalent to the inverse transition to the $\lambda\psi^4$ Lagrangian in terms of matrix elements. The latter Lagrangian does not contain derivatives and, accordingly, perturbation theory has the most simple reduction properties.

As has been shown in refs. ^{10,5,6}, an analog of the $\lambda\psi^4$ Lagrangian for nonlinear realization of the chiral symmetry is a Lagrangian to which there correspond the normal coordinates (along geodesics) in the Goldstone field space ¹⁰. Here, to separate the fields into classical (background) and quantum ones the use should be made of the summation of vectors in the given curved space ^{5,6}. For instance, for the chiral $SU(2) \times SU(2)$ theory the Goldstone field space is the space of constant curvature and the summation of vectors is a displacement of the coordinate origin on the sphere, that corresponds to the chiral transformation of quantum fields with parameters of classical fields. For the latter, the coordinates may not be fixed and in this sense the constructed perturbation theory is covariant.

The geometry of the Goldstone field space is determined by the dynamical group algebra. Standard group methods exist for constructing the normal coordinates ¹⁰ given at an arbitrary point of the space ⁶.

In the present note the group methods of constructing Lagrangians with the simplest reduction properties ⁶ are

applied to the gravity theory as the theory of the dynamical affine symmetry. ⁴

In sect. 2 the main results of paper ⁴ are presented and the role of general coordinate transformations is ascertained.

In sect. 3 the covariant perturbation theory is formulated in terms of the Cartan forms.

2. Classical Theory

In paper ⁴ it has been shown that the group of all linear transformations in a four-dimensional space

$$A(4) = P_4 \otimes L(4, R)$$

may be the starting dynamical group of gravity theory. Its algebra consists of generators of the Lorentz group, $L_{\mu\nu}$, generators of the affine transformations $R_{\mu\nu}$, and those of translations P_μ

$$\begin{aligned} \frac{1}{i}[L_{\mu\nu}, L_{\rho\tau}] &= \delta_{\mu\rho} L_{\nu\tau} - \delta_{\mu\tau} L_{\nu\rho} - (\mu \leftrightarrow \nu) \\ \frac{1}{i}[L_{\mu\nu}, R_{\rho\tau}] &= \delta_{\mu\rho} R_{\nu\tau} + \delta_{\mu\tau} R_{\nu\rho} - (\mu \leftrightarrow \nu) \\ \frac{1}{i}[R_{\mu\nu}, R_{\rho\tau}] &= \delta_{\mu\rho} L_{\tau\nu} + \delta_{\mu\tau} L_{\rho\nu} + (\mu \leftrightarrow \nu) \\ \frac{1}{i}[L_{\mu\nu}, P_\rho] &= \delta_{\mu\rho} P_\nu - \delta_{\nu\rho} P_\mu \\ \frac{1}{i}[R_{\mu\nu}, P_\rho] &= \delta_{\mu\rho} P_\nu + \delta_{\nu\rho} P_\mu \end{aligned} \quad (2)$$

We consider the nonlinear transformations in the coset space $A(4)/L$ which parameters are the coordinates x_μ and ten Goldstone fields, $h_{\mu\nu}$ - gravitons. The invariants with respect to the linear transformations with constant parameters are constructed with the help of the Cartan forms:

$$G^{-1} dG = i \left[\omega_\mu^P(d) P_\mu + \frac{i}{2} \omega_{\mu\nu}^R(d) R_{\mu\nu} + \frac{i}{2} \omega_{\mu\nu}^L(d) L_{\mu\nu} \right] \quad (3)$$

$$G = e^{i P x} e^{\frac{i}{2} h_{\alpha\beta} R_{\alpha\beta}} . \quad (4)$$

The form ω^R defines the covariant differential of fields h , and ω^P, ω^L are used to define the covariant differentiation of fields ψ^A transforming by representations of the Lorentz group with matrices generators $L_{\mu\nu}^A$

$$\nabla_\lambda \psi^A = \frac{D \psi^A}{\omega_\lambda^P} ; \quad D \psi^A = \left(d + \frac{i}{2} \omega_{\mu\nu}^L(d) L_{\mu\nu}^A \right) \psi^A . \quad (5)$$

In what follows, the field ψ^A is understood to be a spinor field only. Let us find explicitly the Cartan forms for exponential parametrization (4) that corresponds to the choice of normal coordinates in the ten-dimensional space $h_{\mu\nu}$. Inserting into (3) the parameter t through the change $h \rightarrow th$, differentiating both sides of (4) with respect to t , and using the commutation relations (2) we obtain the equations:

$$\frac{\partial}{\partial t} \omega_{\mu}^{\nu}(\alpha) = h_{\mu\nu} \omega_{\nu}^{\rho}(\alpha) \quad , \quad \omega_{\mu}^{\rho}(\alpha) \Big|_{t=0} = dx_{\mu}$$

$$\frac{\partial}{\partial t} \omega_{\mu\nu}(\alpha) = dh_{\mu\nu} - h_{\mu\lambda} \omega_{\delta\nu}(\alpha) + \omega_{\lambda\delta}(\alpha) h_{\delta\nu} \quad ; \quad \omega_{\mu\nu}(\alpha) \Big|_{t=0} = 0 \quad (6)$$

$$\omega_{\mu\nu}^R = \omega_{[\mu\nu]}(\alpha) = \frac{t}{2} (\omega_{\mu\nu} + \omega_{\nu\mu})$$

$$\omega_{\mu\nu}^L = \omega_{[\mu\nu]}(\alpha) = \frac{t}{2} (\omega_{\mu\nu} - \omega_{\nu\mu}) \quad .$$

Solutions to these equations are the expressions

(at $t=1$):

$$\omega_{\mu}^{\nu}(\alpha) = \zeta_{\mu\nu} dx_{\nu} \quad ; \quad \omega_{\mu\nu}(\alpha) = \zeta_{\mu\lambda}^{-1} d\zeta_{\lambda\nu} \quad (7)$$

$$\zeta_{\mu\nu} = (e^h)_{\mu\nu} \quad ; \quad \zeta_{\mu\nu}^{-1} = (e^{-h})_{\mu\nu} \quad .$$

The invariant elements of length and volume are constructed on the basis of the Cartan forms ω^{μ} . Accordingly, we have ⁴

$$ds^2 = \omega_{\lambda}^{\mu} \omega_{\mu}^{\lambda} = g_{\mu\nu} dx^{\mu} dx^{\nu} \quad , \quad (8)$$

$$g_{\mu\nu} = \zeta_{\mu\lambda} \zeta_{\lambda\nu} \quad ; \quad dV = \det a^{\mu}{}_{\lambda} \quad .$$

The requirement of minimum with respect to the number of derivatives does not fix uniquely the theory because the transformation properties of the covariant derivative (5) do not change if one adds several terms of the same order of derivative with arbitrary coefficients c_1, c_2, c_3

$$\nabla_{\lambda} \psi = \partial_{\lambda} \psi + \frac{c_1}{2} V_{\mu\nu\lambda} (c_1, c_2, c_3) L_{\mu\nu}^{\lambda} \psi$$

$$\begin{aligned} V_{\mu\nu\lambda} = & \omega_{[\mu\nu]}(\partial_{\lambda}) + c_1 [\omega_{(\nu\lambda)}(\partial_{\mu}) - \omega_{(\mu\lambda)}(\partial_{\nu})] + \\ & + c_2 [\delta_{\mu\nu} \omega_{\delta\delta}(\partial_{\lambda}) - \delta_{\nu\lambda} \omega_{\delta\delta}(\partial_{\mu})] + c_3 [\delta_{\mu\lambda} \omega_{\nu\delta}(\partial_{\delta}) - \delta_{\nu\lambda} \omega_{\mu\delta}(\partial_{\delta})] ; \\ & \partial_{\lambda} = \zeta_{\lambda\delta}^{-1} \partial_{\delta} \quad ; \quad \omega_{\lambda\nu}(\partial_{\delta}) = \zeta_{\lambda\delta}^{-1} \partial_{\delta} \zeta_{\delta\nu} \quad . \end{aligned} \quad (9)$$

As has been shown in paper ⁴, the parameters C_1, C_2, C_3 are uniquely defined by the requirement that the theory be simultaneously corresponding to the nonlinear realization of the conformal group

$$C_1 = -1 ; C_2 = C_3 = 0 . \quad (10)$$

This requirement leads to the tensor field theory which equations coincide with the Einstein ones.

In this note we want to indicate that the ambiguity of the theory of nonlinear affine realization may be removed by requiring that in interactions of the tensor Goldstone field there be only the particle with spin two. The interaction of the particle with spin one is completely ruled out by the invariance of Lagrangian with respect to the gauge transformations in the coset space, i.e., with respect to the affine transformations with a parameter being a vector field gradient $C_\mu(x)$

$$e^{i \frac{\hbar_{\mu\nu}}{2} R_{\mu\nu}} \rightarrow e^{i \frac{\partial_\alpha C_\nu(x)}{2} R_{\mu\nu}} \cdot e^{i \frac{\hbar_{\alpha\beta}}{2} R_{\alpha\beta}} . \quad (11)$$

These transformations with $C_\nu(x)$ infinitesimal correspond to the transformations of coordinates

$$x_\mu \rightarrow x_\mu + C_\mu(x) . \quad (12)$$

Provided the quantity $\sqrt{g_{\mu\nu\lambda}}(G_1, G_2)$ is transformed under transformations (11) like under constant transformations, we obtain the values of coefficients C_1, G_1, C_2 in (10) leading to the Einstein theory. Transformations (11), (12) coincide with general coordinate ones which in the given approach acquire a simple physical and geometrical meaning as was mentioned above. A covariant form for the Goldstone fields $h_{\mu\nu}$ may be found by considering the commutator of covariant derivatives for any field $\psi(x)$

$$(\nabla_\lambda \nabla_\rho - \nabla_\rho \nabla_\lambda) \psi = \frac{i}{2} R_{\mu\nu, \lambda\rho} L_{\mu\nu} \psi.$$

The contraction $R = R_{\mu\nu, \mu\nu}$ is the scalar relative to the affine group. To get a full coincidence with standard definitions of gravity theory, one needs to introduce, by means of $\varepsilon_{\mu\nu}$, the linearly transforming quantities with spin integer, for instance:

$$A_\mu = \varepsilon_{\mu\bar{\nu}} \bar{A}^{\bar{\nu}}$$

$$D_\lambda A_\mu = \varepsilon_{\lambda\bar{\lambda}} \varepsilon_{\mu\bar{\mu}} \nabla_{\bar{\lambda}} (\varepsilon_{\bar{\lambda}\bar{\mu}}^{-1} \bar{A}^{\bar{\mu}}) = (\partial_\lambda A_\mu - \Gamma_{\lambda\mu}^\rho A^\rho),$$

where $\Gamma_{\lambda\mu}^\rho$ are the Christoffel symbols. The quantities $g_{\mu\nu}$ (8) $\Gamma_{\lambda\mu}^\rho$ and R are connected with each other by usual formulae of the Einstein gravity theory. The minimal interaction is described by the action

$$S(h, \psi) = \int d^4x \det \varepsilon \left[\mathcal{L}_0(\psi, \nabla_\mu \psi) + \frac{F^2}{4} R \right].$$

(13)

Here $\mathcal{L}_0(\varphi, \partial\varphi)$ is the Lagrangian of free fields, φ ;
 $\epsilon = \sqrt{\kappa 4\pi}^{-1} \sim f_0^{1/2} eV$; κ is the Newton constant. The fields
 $h_{\mu\nu}$ are dimensionless and related to the usual dimensional
 fields h_d : $h_d = Fh$

3. Quantum Theory

We will proceed from the generating functional for the
 Green functions written in the form of the continual integral
 (see the review of Faddeev and Popov ¹)

$$Z(\mathcal{J}, \varphi) = \frac{1}{N} \int \prod_{\mu, \nu, \alpha, \beta, x} dh_{\mu\nu}(x) \prod_x \delta(f(h(x))) \Delta_f(h) \exp \left\{ i \int d^4x \mathcal{L}_0(h, \partial h) + \int d^4x \mathcal{J}_{\mu\nu} h_{\mu\nu} \right\} \quad (14)$$

where N is the normalization, $\mathcal{J}_{\mu\nu}$ - the source, $f(h) = 0$ -
 the equation fixing the gauge, $\Delta_f(h)$ - the Faddeev-Popov
 determinant, $S(h, \varphi)$ is the action function (QD). The
 fields φ can be treated, without loss of generality,
 to be classical. The quantity $\Delta_f(h)$ is calculated by the
 formula ¹

$$\Delta_f(h) \cdot \int \prod_{\mu, \nu, \alpha, \beta, x} d\zeta_{\mu\nu}(x) \delta\left(\int \zeta(h(x))\right) = 1, \quad (15)$$

where \mathcal{C} is the general coordinate transformation (11), (12).

In a quasiclassical expansion of the generating functional
 the following change of integration variables

$$h_{\mu\nu} \rightarrow \varphi_{\mu\nu} + h_{\mu\nu} \quad (16)$$

is conventional. In this way one extracts the classical (background) fields $\varphi_{\mu\nu}$ obeying the classical equations of motion

$$\frac{\delta S'(\varphi)}{\delta \varphi_{\mu\nu}} = -J_{\mu\nu} \quad (17)$$

and the "quantum" fields $h_{\mu\nu}$ over which one integrates.

In paper ⁶, where the nonlinear realizations for dynamical symmetries G of the chiral type were studied, it has been shown that for constructing Lagrangians with the simplest reduction properties it is necessary to separate fields into the classical and quantum ones in the coset space G/H , where H is the subgroup of transformations leaving vacuum invariant. The analogous separation of variables φ and h in the gravity theory

$$e^{i \frac{h_{\alpha\beta}}{2} R_{\alpha\beta}} \rightarrow e^{i \frac{\varphi_{\mu\nu}}{2} R_{\mu\nu}} e^{i \frac{h_{\alpha\beta}}{2} R_{\alpha\beta}} \quad (18)$$

defines the system of normal coordinates in the space of Goldstone fields h with the coordinate origin at the point φ . Transformation (18) is a generalization of the summation of vectors (16) for a curved space

$$h_{\mu\nu} \rightarrow \varphi_{\mu\nu} (+) h_{\mu\nu} .$$

Like in the case (16), the determinant of this transformation equals unity ⁴. Thus, the generating functional (14) takes the form

$$Z(J, \varphi) = \frac{1}{N} \int \prod_{\nu \leq \mu, x} dh_{\mu\nu}(x) \prod_x \delta(\ddagger(\varphi(x), h)) \Delta_{\ddagger}(\varphi(x), h) \cdot \exp \left\{ i S(\varphi(x), h, \varphi) + \int d^4x J_{\mu\nu}(\varphi_{\mu\nu} + h_{\mu\nu}) \right\}.$$

Here it has been considered that on the mass shell the equality ^{x)}

$$J(\varphi(x), h) = J(\varphi + h)$$

holds.

Let us obtain the explicit form of the action $S(\varphi(x), h, \varphi)$. To this end it is sufficient to find new Cartan forms by inserting (18) into (4) and (3).

$$\bar{G}^{-1} d\bar{G} = i \bar{\omega}_{\mu}^P(d) P_{\mu} + \frac{i}{2} \bar{\omega}_{\mu\nu}^R(d) R_{\mu\nu} + \frac{i}{2} \bar{\omega}_{\mu\nu}^L(d) L_{\mu\nu} \quad (19)$$

$$\bar{G} = e^{i\chi P} e^{\frac{i}{2} \varphi_{\alpha\beta} R_{\alpha\beta}} e^{\frac{i}{2} h_{\mu\nu} R_{\mu\nu}}$$

Making the substitution with parameter t $h \rightarrow t h$ and differentiating both sides of eq. (10) with respect to t

^{x)} For renormalization of wave functions in changing variables in the considered generating functional, see ref. ¹¹.

we derive the differential equations for the Cartan forms which are the same as in the classical case (6) but with nonzero boundary conditions which are the Cartan forms of classical fields

$$\begin{aligned} \frac{\partial}{\partial t} \bar{\omega}_{\mu}^P(d) &= h_{\mu\nu} \bar{\omega}_{\nu}^P(d) ; & \bar{\omega}_{\mu}^P(d) \Big|_{t=0} &= \omega_{\mu}^P(d) \equiv \zeta_{\mu\nu}(\varphi) dx_{\nu} \\ \frac{\partial}{\partial t} \bar{\omega}_{\mu\nu}(d) &= dh_{\mu\nu} - h_{\mu\delta} \bar{\omega}_{\delta\nu}(d) + \bar{\omega}_{\mu\delta}(d) h_{\delta\nu} ; & \bar{\omega}_{\mu\nu}(d) \Big|_{t=0} &= \zeta_{\mu\delta}^{-1}(\varphi) d\tau_{\delta\nu}(\varphi) \end{aligned} \quad (20)$$

$$\begin{aligned} \bar{\omega}_{\mu\nu}^R &= \bar{\omega}_{[\mu\nu]} \\ \bar{\omega}_{\mu\nu}^L &= \bar{\omega}_{[\mu\nu]} . \end{aligned}$$

Equations (20) describe the parallel displacement of orthogonal moving 10-hedral along geodesics of the space of the Goldstone fields from the point φ to the point h and these are called fundamental ¹².

The solution of these equations at $t=1$ is

$$\begin{aligned} \bar{\omega}_{\mu}^P(d) &= [e^h \tau(\varphi)]_{\mu\nu} dx_{\nu} \\ \bar{\omega}_{\mu\nu}(d) &= [e^{-h} \tau^{-1}(\varphi)]_{\mu\delta} d[\tau(\varphi) e^h]_{\delta\nu} , \end{aligned} \quad (21)$$

where $[AB]_{\mu\nu} = A_{\mu\delta} B_{\delta\nu}$.

In what follows the bar above will stand for the quantities which simultaneously depend on φ and h .

As is shown in paper ⁶, forms (7) and (21) have the simplest reduction properties due to their construction by means of the dynamical group structure constants antisymmetric in lower indices.

From (21) it follows that the transformed metric tensor takes the form

$$\begin{aligned} \bar{g}_{\mu\nu}(\varphi(t), h) &= [\bar{z}(\varphi) e^{-2h} z(\varphi)]_{,\mu\nu} \\ \bar{g}^{\mu\nu}(\varphi(t), h) &= [\bar{z}^{-1}(\varphi) e^{-2h} z^{-1}(\varphi)]_{,\mu\nu}. \end{aligned} \quad (22)$$

Next, we write the action for spinor and gravitational fields

$$\begin{aligned} S(\varphi(t), h, \psi) &= \int d^4x dt e^{3+h} [\bar{R} + \bar{\psi} \cdot \bar{\gamma}_\mu \bar{\nabla}_\mu \psi - M \bar{\psi} \psi]; \\ \bar{R} &= 2 \bar{\partial}_\lambda \bar{V}_{\mu\nu,\nu} + \bar{V}_{\mu\nu,\delta} \bar{V}_{\nu\delta,\mu} - \bar{V}_{\mu\delta,\mu} \bar{V}_{\nu\delta,\nu}; \\ \bar{\nabla}_\lambda \psi &= \bar{\partial}_\lambda \psi + \frac{\xi}{4} \partial_{\mu\nu} \bar{V}_{\mu\nu,\lambda} \psi, \\ \bar{V}_{\mu\nu,\lambda} &= \bar{\omega}_{[\mu\nu]}(\bar{\partial}_\lambda) - \bar{\omega}_{[\nu\lambda]}(\bar{\partial}_\mu) + \bar{\omega}_{[\mu\lambda]}(\bar{\partial}_\nu); \\ \bar{\partial}_\mu &= [e^{-h} z^{-1}(\varphi)]_{,\mu} \partial_\mu; \\ \bar{\omega}_{\mu\nu}(\bar{\partial}_\lambda) &= [e^{-h} z^{-1}(\varphi)]_{,\mu\lambda} \bar{\partial}_\lambda [z(\varphi) e^h]_{,\nu} = \\ &= (e^{-h})_{,\mu\lambda} \bar{\partial}_\lambda (e^h)_{,\nu} + (e^{-h})_{,\mu\nu} [z^{-1}(\varphi) \bar{\partial}_\lambda z(\varphi)]_{,\nu} (e^h)_{,\nu\nu}. \end{aligned} \quad (23)$$

Lagrangians for fields with integer spins are constructed by means of the Christoffel symbols $\bar{\Gamma}_{\mu\nu}^{\rho}$ in a standard way. The curvature \bar{R} and Christoffel symbols $\bar{\Gamma}_{\mu\nu}^{\rho}$ are connected with metric tensor (22) through the usual formulae

$$\bar{f}_{\mu\nu}^2 = \bar{g}^{2\alpha} (\partial_\alpha \bar{g}_{\mu\nu} + \partial_\nu \bar{g}_{\alpha\mu} - \partial_\alpha \bar{g}_{\mu\nu})$$

$$\bar{R} = \bar{g}^{\mu\nu} [\partial_\nu \bar{f}_{\mu\rho}^{\rho} - \partial_\rho \bar{f}_{\mu\nu}^{\rho} + \bar{f}_{\mu\lambda}^{\rho} \bar{f}_{\rho\nu}^{\lambda} - \bar{f}_{\mu\nu}^{\rho} \bar{f}_{\rho\lambda}^{\lambda}] \quad (24)$$

Formulae (22), (23) are the main results of this paper.

To obtain the generating functional directly for the S-matrix elements we use a prescription proposed in papers 13,14.

Let $S_0(\varphi_0)$ be the free action quadratic in φ_0 and the classical fields φ obey the equation of motion

$$\frac{\delta S(\varphi, \varphi)}{\delta \varphi} = \frac{\delta S_0(\varphi_0)}{\delta \varphi_0}$$

Then the generating functional of the S-matrix has the form ¹⁴:

$$Z(\varphi_0, \varphi) = \frac{1}{N} \int \prod_{\mu, \nu, \alpha} d h_{\mu\nu} \int \prod_x \delta^4(\mathcal{D}_\nu(\varphi) g^{\mu\nu}(h) - e^{\mu\nu}) \Delta(h, \varphi) \exp \left\{ i S(\varphi(h), \varphi) \right\}$$

Here $\mathcal{D}_\nu(\varphi) g^{\mu\nu}(h)$ is the covariant derivative of $g^{\mu\nu}(h)$ with the Christoffel symbols dependent on the classical fields φ

$$\Delta(h, \varphi) \cdot \int \prod_{\mu, \nu} d c_{\mu\nu}(x) \int_x \delta^4(\mathcal{D}_\nu(\varphi) g^{\mu\nu}(h) - e^{\mu\nu}) = 1$$

The coefficient functions of the expansion of the functional Z in φ_0 coincide with those of the expansion of the S-matrix over the normal products of asymptotical fields.

Conclusion

We have formulated the perturbation theory for gravity, where the choice of fundamental fields and their separation, in the generating functional, into classical (background) and quantum fields are defined by geometry of the geodesics of the space of gravitational fields.

The choice of normal coordinates of this space and the separation of fields along geodesics lead to the perturbation theory with the simplest reduction properties, therefore the corresponding Lagrangian is an analog of the $\lambda\psi^4$ Lagrangian among all equivalent on the mass shell Lagrangians. For the choice of an arbitrary system of coordinates the consideration of all possible reductions of diagrams on the mass shell is equivalent to the covariant procedure of transition from this coordinate system to the normal one.

For nonlinear realizations of the chiral symmetry such an approach leads to the perturbation theory, which is the most simple for calculations, especially, when one uses the regularizations connected with summation of certain diagrams, the summation being explicitly covariant with respect to the classical fields.

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