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Transition Mixing Among Baryons*

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Abstract

We propose a degenerate perturbation theory model for mass splitting within the $\underline{70},1^-$ baryon multiplet. It is found that dominance of the lowest-lying two-body $\underline{56}\times\underline{35}$ intermediate states produces mixing angles in fair approximation to those previously deduced from $SU(6)_W$ analysis of decay data. We are able to predict the couplings of all hitherto undetected members of the multiplet and to make a number of mass predictions. Our results call into question the nature of $\Lambda(1405)$.

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I. <u>Introduction</u>.

Most members of the $\underline{70},1^-$ baryon multiplet of $SU(6)\times O(3)$ give rise to ambiguity in their group classification. This is because they occur as doublets or triplets of states with the same strong interaction quantum numbers (I,Y,J,P). The mixing angles that pin down the classification of each physical state relative to the $SU(3)\times SU(2)$ basis states of the $\underline{70},1^-$ multiplet can in many cases be determined empirically by fitting $SU(6)\times O(3)$ mass operators to the observed resonance masses[(1)(2)(3)] or by fitting $SU(6)_W\times O(2)_{L_z}$ couplings to observed decay rates[(4)(5)]. This is analogous to determining the ω-ϕ mixing angle using the observed ω, ϕ, K^* and ρ masses on the one hand or their decay rates on the other. Unfortunately, whereas the two methods give nicley consistent results for the mesons, the baryon mixing angles obtained by these two approaches do not in general agree.[(6)]

What is thus needed is a theoretical determination of these mixing angles that does not require the empirical fitting of parameters to experimental data. We have previously noted[(7)] four theoretical models suggested by regularities[(7)(8)(9)] in the decay data that might perform this task. The present paper is an examination of one of these models which will enable us to:

a) predict mixing angles surprisingly close to those indicated by the decay data.

b) predict the mixing angles (and hence approximate branching ratios) of the six unknown Ξ^* resonances and of two unseen $J^P = 1/2^-$ Σ^* states.

c) predict the masses of some as-yet undetected baryons.

d) understand approximately how the phenomenon of ideal mixing[7] among baryons could come about.

II. The Model

Our model is one of degenerate perturbation theory in which states with the same isospin-hypercharge total angular momentum and parity (I,Y,J,P) start off at the same mass. These levels then mix and separate by making virtual transitions into and out of various intermediate states. In the strict symmetry limit states with different (I,Y,J) would also be degenerate but we do not insist on this extra degeneracy here as our immediate interest is in mixing and this only involves transitions among states with the same quantum numbers.

In such a picture one might expect two-body $\underline{56},0^{+}\times\underline{35},0^{-}$ intermediate states to provide the dominant contribution. Moreover, since these exchanged multiplets already exhibit mass splitting within themselves, it is possible that only a few two-body intermediate states provide the major part of the effect. Indeed, this intrinsic symmetry breaking within the $\underline{56},0^{+}$ and $\underline{35},0^{-}$ multiplets is vital for our purposes since if all two-body channels were equivalent we should have to include the complete set and no mixing would ensue. It is thus clear that in order to obtain any results a full dynamical calculation must be watered down to the inclusion of only a few well-chosen intermediate states. The question is therefore how this choice should be made.

III. The First Approximation

We take as a possible clue the pre-quark-era Katz and Lipkin[10] model for ω-ϕ mixing. These authors pointed out that the approximate decoupling of the ϕ-meson from the $\rho\pi$ channel is a natural consequence of $\rho\pi$ being the dominant intermediate state under the hypothesis that two-meson (PP or PV) exchange is the mechanism responsible for mixing. They argued further that there exists a kinematic effect that singles out the $\rho\pi$ channel for this task, namely the peculiarly small (on the scale of the other exchanged masses) pion mass. Applying then this idea to baryon mixing one might picture two basic ingredients to the perturbation. One being the $SU(6)_W$-symmetric part in which all allowed particles are exchanged causing only a uniform shift of all levels but no mixing. The other ingredient would be the symmetry breaking piece related (in some rather imprecise manner) to the empirical fact that even in the absence of mixing not all states (e.g. those having different strangeness) are degenerate. This is the piece we hypothesise to be responsible for mixing, and for which we shall need a mathematical model.

As a first guess we shall assume à la Katz and Lipkin that:

I) There is a kinematical enhancement of the lowest-lying (in mass) intermediate states and that these channels cause the largest part of the mixing phenomenon.

II) We include only the minimum number of intermediate states necessary to completely lift the degeneracy.

III) The couplings to these intermediate states are computed in the $SU(6)_W$ limit.

Although assumptions II & III may appear to be unnecessarily strong they have the dual advantage of (a) leading to zero-parameter starting predictions and (b) being systematically relaxable in the subsequent search for a better approximation.

IV. Computation of Mixing Angles

The simplest mixing situation involves the two pairs of N^* resonances; $N(1535,1/2^-)$ with $N(1700,1/2^-)$ and $N(1520,3/2^-)$ with $N^*(1730,3/2^-)$. Assumptions I & II then require the $N\pi$ channel to be chiefly responsible for this mixing. This is completely analogous to $\rho\pi$ dominance of ω-ϕ mixing and requires in like fashion one physical state to decouple from $N\pi$. This is an encouraging start since among the two $J^P = 1/2^-$ resonances, N(1535) has an anomalously small elasticity (it couples mainly to $N\eta$) whereas N(1700) has one of the largest elasticities of all the πN resonances. Similarly for the $J^P = 3/2^-$ pair, N(1730) is so inelastic as to be hardly visible in $\pi N \to \pi N$ phase shift analyses whereas N(1520) is again one of the most elastic of the known resonances. Assumption III requires a large[†] mixing angle among the $J^P = 1/2^-$ states ($\arctan(2) \approx 63°$) and a small angle for $J^P = 3/2^-$ mixing ($-\arctan(1/\sqrt{40}) \approx -9°$).

For three-level mixtures our first two assumptions require two dominant intermediate states. Taking Λ^* mixing as an example, these two critical channels are $\Sigma\pi$ and $N\bar{K}$. Explicitly, the (ij) contribution to the perturbed mass matrix is proportional to

$$\langle\Lambda_i|\Sigma\pi\rangle\,\langle\Sigma\pi|\Lambda_j\rangle + \langle\Lambda_i|N\bar{K}\rangle\,\langle N\bar{K}|\Lambda_j\rangle$$

where $\langle\Lambda_i|\Sigma\pi\rangle$ is the $SU(6)_W \times O(2)_{Lz}$ Clebsch Gordan coefficient connecting the Λ_i member of the $\underline{70},1^-$ multiplet to the $\Sigma\pi$ channel (with corresponding

[†] see footnote on next page

interpretations for the other bra-kets). In the $J^P=3/2^-$ case, for example, the perturbation is proportional to

	(8,2)	(8,4)	(1,2)
(8,2)	$\frac{7}{9}$	$\frac{\sqrt{10}}{45}$	$\frac{1}{3}$
(8,4)	$\frac{\sqrt{10}}{45}$	$\frac{2}{45}$	$-\frac{\sqrt{10}}{15}$
(1,2)	$\frac{1}{3}$	$-\frac{\sqrt{10}}{15}$	$\frac{5}{3}$

where we have labelled the rows and columns by the SU(3)×SU(2) content of the basis states. The Clebsch Gordan coefficients and phase convention have been taken from ref. (4).

It is then a simple exercise to reduce this matrix to diagonal form. The real orthogonal matrix that effects this diagonalization is

$$\begin{pmatrix} -.15 & .98 & .15 \\ .94 & .19 & -.28 \\ .30 & -.10 & .95 \end{pmatrix}$$

and this is our required mixing matrix. In analogous fashion all the other mixing matrices for the $\underline{70},1^-$ multiplet can be deduced. For Σ^* mixing the two least massive channels which by assumption I are chiefly responsible for mixing are $\Lambda\pi$ and $\Sigma\pi$ and for Ξ^* mixing the relevant channels are $\Xi\pi$ and $\Lambda\bar{K}$. The results are all listed in Tables I & II along side (where known) the corresponding mixing matrices deduced in the fit of ref. (5) to the decay data.

† footnote from previous page

Compared to the S_{11} mixing angle of ref. (5) this is our least satisfactory prediction. We note however that this particular mixing angle is highly controversial, ranging from 0° (ref. 3) through values comparable to our 63° (refs. 11, 12) all the way to 90° (ref. 13).

V. Mass Predictions

Inspection of Table I reveals a remarkable similarity between the mixing matrices derived in our crude (albeit zero-parameter) lowest approximation and those previously deduced from a study of the decay data[5]. The most logical procedure at this stage would be to attempt a less crude approximation but this we defer until the next section for two reasons. First, corrections (subject to caveats to be discussed below) turn out to be small. Secondly there are several mass predictions that can be made whose origin is very transparent at this level of approximation, We shall thus start by making experimental predictions based on our first-approximation mixing matrices and then, in the next section, go on to argue that we expect these predictions to hold substantially true under the inclusion of further intermediate states.

We first note that the eigenvalues of each mass matrix are proportional to the mass shifts brought about by the perturbation. This means that the ratio of the mass spacings for each physical triplet of resonances (to this degree of approximation) is also a zero-parameter prediction of the scheme. These spacing ratios are listed in Table III and lead to several interesting experimental consequences.

1) The Nature of $\Lambda(1405)$ and the Missing S_{01} Resonance

If $\Lambda(1405)$ is a genuine $\underline{70},1^-$ resonance (as is usually assumed to be the case[4][5]) then a hitherto unsuspected S_{01} resonance is predicted at a mass of 1550 MeV. Using the s-wave coupling constant of Hey et al.[5] this resonance should have large partial widths into $N\bar{K}$ and $\Sigma\pi$ (respectively $\sim$40 MeV and $\sim$30 MeV) and could surely not have escaped experimental detection.

It has however been suggested that $\Lambda(1405)$ might not be a "normal" three-quark object *[see ref.(14) for a recent discussion* as to how this question might be settled experimentally]. If such is the case then $\Lambda(1670,1/2^-)$ would most probably be the <u>least</u> massive S_{01} resonance. This is clearly a most crucial issue for our scheme: either $\Lambda(1405)$ is <u>not</u> a normal resonance <u>or</u> there is an undetected S_{01} resonance intermediate in mass between $\Lambda(1405)$ and $\Lambda(1670)$ and having strong $N\bar{K}$ and $\Sigma\pi$ couplings.

2) <u>The Missing D_{03} Resonance</u>

A less exciting prediction. $D_{03}(1520)$ and $D_{03}(1690)$ imply the existence of a $D_{03}[1750]$ which to this level of approximation decouples from $N\bar{K}$ and $\Sigma\pi$. We thus expect such a state not to be readily visible in formation experiments.

3) <u>$\Sigma(1750,1/2^-)$ et al.</u>

If $S_{11}(1750)$ (whose experimental properties are far from clear at present) turns out to have branching ratios substantially in agreement with those discussed in ref. (9) then our scheme requires it to be the most massive or least massive of the three S_{11} resonances belonging to the <u>70</u>,1^- multiplet. Couplings of experimental interest for the two missing S_{11} states are listed in Table IV.

4) <u>$\Sigma(1940,3/2^-)$ Splitting</u>

The well-determined formation properties of $\Sigma(1670,3/2^-)$ are in good agreement with our Σ_c state [See Table I]. The other two D_{13} resonances should then be rather close to one another in mass and <u>both</u> either above or below 1670 MeV. The fit of Hey et al.[5] (whose mixing matrix we approximate quite well) includes $D_{13}(1940)$ and $D_{13}(1580)$ which would contradict our present requirements. We point out however that the properties of $D_{13}(1670)$ and $D_{13}(1940)$ suffice to determine these mixing angles independent of the existence or otherwise of $D_{13}(1580)$. There are moreover two reasons to expect $D_{13}(1940)$ to be the more trustworthy of the two new resonances [i.e. "new" since the fit of ref. (4)]. In the first place it has been observed in a large number of independent experiments whereas $D_{13}(1580)$ has only been suggested once[(15)]. Secondly the D_{13} masses and mixing angles of ref. (5) predict that the unmixed SU(3)-decuplet Σ mass is at 1600 MeV. Although such a state is in itself without physical significance, its mass when taken with that of the unmixable $\Delta(1650,1/2^-)$ leads to the prediction of yet another stable Ω^-, this one being less-massive than the celebrated $\Omega(1672)$.

If we therefore associate $D_{13}(1940)$ with our Σ_a that decouples from $\Lambda\pi$ and $\Sigma\pi$ then Σ_b is predicted to have a mass of 1910 MeV and to couple only weakly to $N\bar{K}$ and $\Sigma\pi$. It is thus clear that these two states could easily have been confused as a single resonance up till now. Table V lists the couplings of experimental interest predicted for the pair.

V) The Ξ^* spectrum

The coupling constants for the six mixed cascade resonances to decay into channels of experimental interest are listed in Table VI. It should be noticed that this lowest order perturbation brought about by dominance of the $\Xi\pi$ and $\Lambda\bar{K}$ channels causes interesting decoupling conditions in the $\Xi(1530)\pi$ channel. In particular only one of the three $J^P=1/2^-$ Ξ^* resonances retains any coupling to this channel. As for the $J^P=3/2^-$ states; one couples only via the s-wave and the other two only via the d-wave. For completeness we also list the couplings of the unmixed $J^P = 5/2^-$ resonance which should probably be associated with $\Xi(1940)$ [4].

Of particular experimental interest is the fact that only one of these seven states has a substantial $\Lambda\bar{K}/\Xi\pi$ branching ratio and a non-vanishing $\Xi(1530)\pi$ coupling. Indeed using the d-wave partial width parametrization and coupling constant of refs. (4) & (5) we may compare the predicted properties of $\Xi_c(1820,3/2^-)$ with the experimental results recently reported from CERN[16]

		Γ_{Theory} (MeV)	Γ_{expt} (MeV) [16]
$\Xi_c(1820,3/2^-) \rightarrow$	$\Xi\pi$	5	$< \sim 3$
	$\Lambda\bar{K}$	11	~ 9
	$\Sigma\bar{K}$	6	~ 2
	$\Xi^*\pi$	1	~ 9
	sum	23	21±7

In view of the experimental uncertainties and the simplicity of our lowest approximation mixing angles this assignment should probably be considered as satisfactory.

VI. Towards a better approximation

Having entered into a number of detailed predictions using the lowest approximation model described above, the big question is clearly: what effect can the inclusion of further channels be expected to have. This is almost a self-defeating question since as emphasized at the start, the more channels that are included in the symmetry limit the closer we get to a situation in which there is no mixing. Thus once again (as was the case for the lowest approximation) we need to have some insight into which additional channels have the largest effect.

One possibility is again to be guided by kinematics. Namely our lowest approximation in each case gives us the starting mass of the unperturbed doublet or triplet of states [i.e. for the $J^P=1/2^-$ levels: N(1535), Λ(1670), Σ(1750) and for the $J^P=3/2^-$ states: N(1710), Λ(1795), Σ(1940)]. Thus a plausible next approximation might be to insert not just the lowest-lying one or two channels but all open channels at these masses, weighted according to phase space. Here, as before, the hope is that all symmetry breaking effects are (somehow) reflected by differences in phase space among the various competing channels. We shall use once again the $J^P=3/2^-$ Λ^* triplet to illustrate what happens.

We recall that our lowest approximation invoked only the $\Sigma\pi$ and $N\bar{K}$ channels and predicted the unperturbed mass of this triplet (and also the mass of the missing D_{03} which decouples from these channels) to be 1795 MeV. The mixing matrix was (as in Table I)

$$\begin{pmatrix} -.15 & .98 & .15 \\ .94 & .19 & -.28 \\ .30 & -.10 & .95 \end{pmatrix}$$

For a starting mass of 1795 MeV the Σ(1385) and Λη channels are also open. We note that the former can make an s-wave in addition to its d-wave contribution to the mass matrix. The other three open channels on the other hand have only d-wave couplings. Let us first include only the four d-wave contributions. Each coupling is now weighted by a factor $k_{cm}^{5/2}$ where k_{cm} denotes the c.m. momentum for a state of mass 1795 MeV to decay into the two-body channel in question. The resulting mixing matrix now turns out to be

$$\begin{pmatrix} -.06 & .99 & .13 \\ .88 & .11 & -.45 \\ .46 & -.09 & .88 \end{pmatrix}$$

which is seen to be comfortably close to our first approximation. Moreover, if we callibrate the corresponding eigenvalues with the physical masses of $\Lambda(1520)$ and $\Lambda(1690)$ we find that instead of the missing D_{03} state being predicted at an unperturbed mass of 1795 MeV, the new unperturbed mass is predicted at 1780 MeV and the missing state is shifted to 1765 MeV. The latter state will of course no longer precisely decouple from $N\bar{K}$ and $\Sigma\pi$.

This $\sim$2% mass shift and the accompanying small change in the mixing angles are encouraging but a major problem arises when we try to include the hitherto neglected s-wave $\Sigma(1385)\pi$ contribution. The reason is that it is now established fact[17] that nature does not respect the $SU(6)_W$ relationship predicted between the s-wave and d-wave couplings. Thus an undesirable model dependence necessarily enters if we try to include the effects of both s- and d-waves. We have attempted several different ways of including the effect of the s-wave $\Sigma(1385)\pi$ coupling but in each case the perturbation was enormous i.e. large enough to destroy any resemblence of the D_{03} mixing matrix to that obtained in ref. (5). We note in particular that if s-waves and d-waves are parametrized independently and in the same manner as was used in refs. (4) and (5) to describe partial widths, "satisfactory" mixing angles can only be obtained if the s-wave coupling-squared of ref. (4) is reduced by at least a factor of 10.

Similar results are found for the other mixing matrices; namely the inclusion of all open channels in the same orbital state as the lowest channel produce additional mass shifts of only one or two percent and small changes in the corresponding mixing matrices; whereas the inclusion of both orbital states produces unrealistic mixing angles.

In summary therefore, we find that, provided we do not encroach upon the realm where we know $SU(6)_W$ is violated, the effect of including further channels that are energetically open is to produce only small changes in the mass predictions and mixing angles obtained via our lowest approximation.

VII. Summary and Outlook

We have assumed that mixing among $\underline{70},1^-$ baryons having the same quantum numbers is brought about via transitions to intermediate states. As a first approximation we have neglected all contributions other than the lowest lying two-body ones, and of these, only the minimum number needed to break the hypothesised initial degeneracy have been included. $SU(6)_W$ couplings are assumed. This prescription is found to produce mixing angles qualitatively similar to those previously deduced from the decay properties of the observed resonances. We emphasize however that to this degree of approximation our present mixing angles are deduced without utilizing any physical property of any known resonance.

In an attempt to improve upon this first approximation we have included further energetically open two-body channels, weighted according to phase space, and found that these cause only a small change in our first approximation results (typically 1% mass shifts).

We were not, however, able to find any satisfactory way of investigating the additional perturbation to be expected from the inclusion of both s-wave and d-wave intermediate states in the same channel. We have therefore simply left out any intermediate states that involve the necessity of "ℓ-breaking" the $SU(6)_W$ symmetry. We have also not considered the effects of any two-body channels other than $\underline{35}\times\underline{56}$ or any multiparticle intermediate states. Empirically the goodness of our lowest approximation and its stability under the inclusion of intermediate states that we can handle suggest that the combined effect of these neglected channels is small.

There is clearly an unsatisfactory element of arbitrariness in the manner we have chosen to break the symmetry in order to achieve mixing. We were motivated in this choice by the Katz-Lipkin model[10] for ω-ϕ mixing on the one hand and by a previous empirical observation we had made[7] regarding an apparent connexion between baryon mixing angles and certain decoupling conditions. It will be observed that our presently deduced mixing matrices are similar (but not identical) to a previously suggested "ideal" set. In spite of this arbitrariness (which is under further investigation) we have found this same symmetry-breaking prescription remarkably successful for "understanding" the systematics of the even-parity resonances that abound in the 2 GeV mass region. There the situation is considerably complicated by the existence of more resonances than can be accommodated into a single $SU(6)\times O(3)$ multiplet. We shall report on the details elsewhere[18].

In conclusion, our lowest approximation model may be regarded as successful in that it establishes a definite link between mixing angles and coupled channels and thereby constitutes a first step in the direction of a unified description of baryon masses and couplings. Weaknesses of the model in its present form are the degree of arbitrariness with which we have inserted intermediate states and our failure to consider anything other than the narrow resonance approximation (real mass matrix). Both of these features require further study.

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Table I: A classification of 70,1$^-$ resonances according to the $SU(6)_W$ analysis of decay rates from ref. (5) compared with B our lowest approximation transition mixing matrices

A	B
$\begin{pmatrix} N_a(1535, \frac{1}{2}^-) \\ N_b(1700, \frac{1}{2}^-) \end{pmatrix} = \begin{pmatrix} .85 & -.53 \\ .53 & .85 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \end{pmatrix}$	$\begin{pmatrix} \sqrt{\frac{1}{5}} & -\sqrt{\frac{4}{5}} \\ .89 & .45 \end{pmatrix}$
$\begin{pmatrix} \Lambda_a(1670, \frac{1}{2}^-) \\ \Lambda_b(\quad, \frac{1}{2}^-) \\ \Lambda_c(1405, \frac{1}{2}^-) \end{pmatrix} = \begin{pmatrix} -.04 & .95 & .30 \\ .89 & .17 & -.43 \\ .46 & -.25 & .85 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (1,2) \end{pmatrix}$	$\begin{pmatrix} -\sqrt{\frac{1}{6}} & \sqrt{\frac{4}{6}} & \sqrt{\frac{1}{6}} \\ .89 & .45 & 0 \\ .18 & -.37 & .91 \end{pmatrix}$
$\begin{pmatrix} \Sigma_a(1750, \frac{1}{2}^-) \\ \Sigma_b(\quad, \frac{1}{2}^-) \\ \Sigma_c(\quad, \frac{1}{2}^-) \end{pmatrix} = \begin{pmatrix} -.19 & .45 & .87 \\ & \text{unknown} & \\ & \text{unknown} & \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (10,2) \end{pmatrix}$	$\begin{pmatrix} 0 & \sqrt{\frac{1}{5}} & \sqrt{\frac{4}{5}} \\ .41 & .82 & -.41 \\ .91 & -.37 & .18 \end{pmatrix}$
$\begin{pmatrix} N_a(1710, \frac{3}{2}^-) \\ N_b(1520, \frac{3}{2}^-) \end{pmatrix} = \begin{pmatrix} .18 & .98 \\ .98 & -.18 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \end{pmatrix}$	$\begin{pmatrix} -\sqrt{\frac{1}{41}} & \sqrt{\frac{40}{41}} \\ .99 & .16 \end{pmatrix}$
$\begin{pmatrix} \Lambda_a(\quad, \frac{3}{2}^-) \\ \Lambda_b(1690, \frac{3}{2}^-) \\ \Lambda_c(1520, \frac{3}{2}^-) \end{pmatrix} = \begin{pmatrix} .01 & 1.00 & .04 \\ .92 & .00 & -.39 \\ .39 & -.04 & .92 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (1,2) \end{pmatrix}$	$\begin{pmatrix} -\sqrt{\frac{1}{42}} & \sqrt{\frac{40}{42}} & \sqrt{\frac{1}{42}} \\ .94 & .19 & -.28 \\ .30 & -.10 & .95 \end{pmatrix}$
$\begin{pmatrix} \Sigma_a(1940, \frac{3}{2}^-) \\ \Sigma_b(1580, \frac{3}{2}^-) \\ \Sigma_c(1670, \frac{3}{2}^-) \end{pmatrix} = \begin{pmatrix} .35 & .92 & .18 \\ .31 & .07 & -.95 \\ .89 & -.38 & .26 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (10,2) \end{pmatrix}$	$\begin{pmatrix} 0 & \sqrt{\frac{5}{7}} & \sqrt{\frac{2}{7}} \\ .17 & .53 & -.83 \\ .99 & -.05 & .15 \end{pmatrix}$

We have altered the phase convention of ref. (5) to coincide with our own[4]. States are ordered monotonically with increasing eigenvalue of our mass matrix. The top rows (a-states) correspond to zero eigenvalue and represent states that decouple from the lowest threshold channel(s) in this approximation.

Table II. Lowest approximation cascade mixing matrices

$$J^P = 1/2^- \quad \begin{pmatrix} \Xi_a \\ \Xi_b \\ \Xi_c \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ 0.63 & 0.22 & -0.74 \\ 0.39 & -0.92 & 0.07 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (10,2) \end{pmatrix}$$

$$J^P = 3/2^- \quad \begin{pmatrix} \Xi_a \\ \Xi_b \\ \Xi_c \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{9}} & \sqrt{\frac{5}{9}} & \sqrt{\frac{2}{9}} \\ 0.13 & 0.47 & -0.87 \\ 0.87 & 0.47 & 0.13 \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (10,2) \end{pmatrix}$$

Table III. Lowest approximation mass splitting predicted among resonances with the same quantum numbers

	$J^P = 1/2^-$	$J^P = 3/2^-$
$(\Lambda_a - \Lambda_b) : (\Lambda_b - \Lambda_c)$	0.89 : 1.11	0.69 : 1.10
$(\Sigma_a - \Sigma_b) : (\Sigma_b - \Sigma_c)$	0.22 : 0.52	0.07 : 0.60
$(\Xi_a - \Xi_b) : (\Xi_b - \Xi_c)$	0.13 : 0.75	0.09 : 0.15

Table IV. Predicted couplings (in lowest approximation) for the two missing $J^P = 1/2^- \ \Sigma^*$ states and $\Sigma_a(1750)$.

channel	$\Sigma_a(1750)$ [9]	Σ_b coupling	Σ_c coupling
$\Lambda\pi$	0	0.47 f	0
$\Sigma\pi$	0	0	-0.86 f
$N\bar{K}$	-0.42 f	-0.38 f	-0.34 f
$\Sigma(1385)\pi$	1.79 g	0	-0.97 g
$\Delta(1232)\bar{K}$	1.19 g	-2.18 g	0.97 g
$\Sigma\eta$	0.34 f	0.31 f	-0.07 f
ΞK	0	-0.58 f	0.43 f

If partial widths are parametrized as $\Gamma_\ell \sim k^{2\ell+1} \ M_N/M_{\Sigma^*}$ then the most recent values of the coupling constants are[5] $g^2 \gtrsim 12 \ \text{GeV}^{-4}$, $f^2 \gtrsim 0.5$. Note that $fg < 0$.

Table V. Predicted couplings (in lowest approximation) for the two $J^P = 3/2^- \Sigma^*$ states at $\sim$1900 MeV and $D_{13}(1670)$

channel	Σ_a coupling	Σ_b coupling	$\Sigma_c(1670)$ coupling
$\Lambda\pi$	0	0.26 g	0.15 g
$\Sigma\pi$	0	-0.05 g	0.81 g
$N\bar{K}$	-0.25 g	0.05 g	0.15 g
$\Sigma(1385)\pi_s$	-0.59 f	-0.05 f	-0.31 f
$\Sigma(1385)\pi_d$	0	-0.68 g	0.44 g
$\Delta(1232)\bar{K}_s$	0.67 f	0.89 f	
$\Delta(1232)\bar{K}_d$	1.01 g	-0.21 g	
$\Sigma\eta$	0.21 g	-0.06 g	
ΞK	0	-0.29 g	

For definitions of f and g see footnote to Table IV.

Table VI. Predicted squares of couplings for the seven cascade resonances in the $\underline{70},1^-$ multiplet.

J^P	Name	$\Xi\pi$	$\Lambda\bar{K}$	$\Sigma\bar{K}$	$\Xi\eta$	$\Xi(1530)\pi_s$	$\Xi(1530)\pi_d$
$1/2^-$	Ξ_a	0	0	$0.59f^2$	$0.07f^2$	-	$0.67g^2$
	Ξ_b	$0.04f^2$	$0.09f^2$	$0.18f^2$	$0.26f^2$	-	0
	Ξ_c	$0.67f^2$	$0.24f^2$	~ 0	$0.05f^2$	-	0
$3/2^-$	Ξ_a	0	0	$0.30g^2$	$0.03g^2$	$0.67f^2$	0
	Ξ_b	$0.07g^2$	$0.03g^2$	~ 0	$0.06g^2$	0	$0.27g^2$
	Ξ_c	$0.07g^2$	$0.17g^2$	$0.35g^2$	$0.28g^2$	0	$0.27g^2$
$5/2^-$	Ξ	$0.36g^2$	$0.09g^2$	$0.09g^2$	0	-	$0.31g^2$

For definitions of f^2 and g^2 see footnote to Table IV.

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