PREPRINT UCRL- 78411

CONF-760832--6

Lawrence Livermore Laboratory

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D. C. Thompson

July 21, 1976

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This paper was prepared for submission to

Advances in Precision Machining of Optics 20th Technical Symposium
Society of Photo-Optical Instrumentation Engineers - August, 1976, San Diego, CA

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THEORETICAL TOOL MOVEMENT REQUIRED TO DIAMOND TURN AN OFF-AXIS PARABOLOID ON AXIS

D. C. Thompson *
Lawrence Livermore Laboratory, Univc ity of California
Livermore, California 94 ɔ0

Abstract

Current techniques for manufacturing off-axis paraboloids are both expensive and insufficiently accurate. An alternative method, turning the workpiece about its axis on a diamond-turning machine, is presented, and the equations describing the necessary tool movement are derived. A discussion of a particular case suggests that the proposed technique is feasible.

Introduction

High-quality, off-axis parabolic reflectors, required by the CTR an laser-fusion programs at Lawrence Livermore Laboratory (LLL) and other ERDA laboratories, are currently :-nufactured oy hand. There are several drawbacks to this method, including lead times of up to a year osts of up to $50,000 for a small reflector, and unsatisfactory limits to the tolerances obtainable. This tuation has led to a search for cheaper and more accurate methods of manufacturing off-axis paraboloids.

One technique under consideration would involve mounting the workpiece at its normal position relative to the paraboloid's axis of rotation and cutting the reflector on a diamond-turning machine. Although this method should generate a reflector to the desired tolerances, the size of the machine required can be excessive. The Los Alamos Laboratory, for instance, is considering the design of a 50-in.-diameter, off-axis paraboloid, which would require a turning machine with a 20-ft swing. In addition, there is no guarantee that the size requirements will not continue to grow.

As an alternative to the technique outlined above, it has been suggested that optical quality off-axis paraboloids could be cut directly on a diamond-turning machine. The central axis of the workpiece (the off-axis paraboloid) would be mounted concentric with the axis of rotation of the machine. A numerically controlled (NC) tool slide would vary the depth of cut in coordination with rotational position of the workpiece and radial distance from the axis of rotation (Fig. 1).

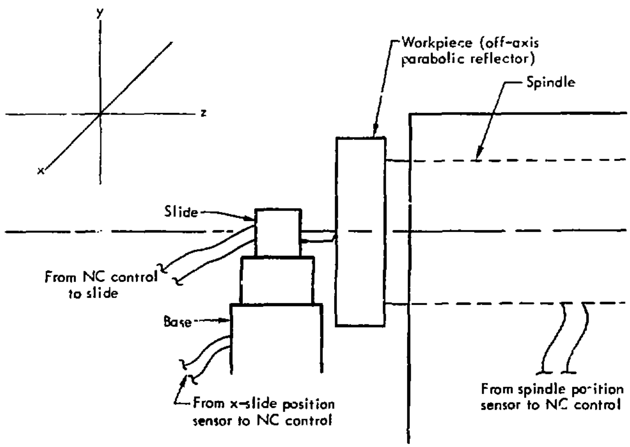

Fig. 1. Conceptual diagram of the proposed machining method. The base is numerically controlled to move in the x and z directions. The slide gives an additional movement in the z direction in coordination with the spindle position and the position of the tool in the x direction.

* Work performed under the auspices of the U.S. Energy Research & Development Adm. under contract W-7405-Eng-48.

Because of the "potato chip" shape of an off-axis parabolic reflector there is a maximum and minimum z for any given radius. The difference, Δz, between z_{max} and z_{min} is greatest at the outer perimeter of the workpiece. The maximum displacement, Δz_{max}, determines the maximum distance traveled by the additional slide.

In the following discussion the equations necessary to implement such a system are derived, and a particular case is analyzed.

Derivation of Equations

Off-Axis-Paraboloid Surface

To derive an equation describing the surface of an off-axis paraboloid with respect to its central axis we first write the general equation for a paraboloid:

$$z = \frac{x^2 + y^2}{c} \tag{1}$$

where c is a constant. Figure 2 graphically displays a parabolic reflector superimposed on an x, y, z coordinate system.

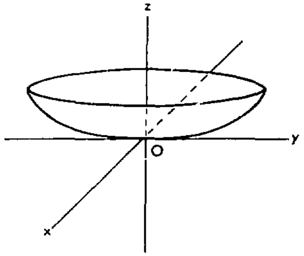

Fig. 2. The relationship of the parabolic reflector to the coordinate axes.

The desired equation can be obtained by performing two linear transforms on Eq. (1) - one translational and the other rotational. Figure 3 shows the axes translated to a new origin O' and rotated through θ.

Translation Assume that the point of origin of the transformed set of axes lies at a distance S_y from the original point of origin along the original y axis. Further assume that there is no transformation along the original x axis. The necessary transformation in the z direction, then, is

$$S_z = \frac{x^2 + y^2}{c} = \frac{(0)^2 + S_y^2}{c} = \frac{S_y^2}{c} . \tag{2}$$

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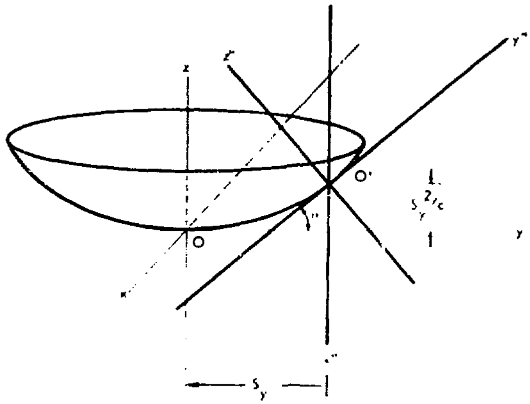

Fig. 3. The coordinate axes translated to the new origin, O', and rotated through θ to make z" perpendicular to the reflector surface.

Thus,

$$x = x' ,$$
$$y = y' + S_y , \tag{3}$$
$$z = z' + \frac{S_y^2}{c},$$

where x', y', and z' represent the transformed axes.

Using Eqs. (1) and (3), the equation describing the surface may then be written as

$$z' + \frac{S_y^2}{c} = \frac{x'^2 + (y' + S_y)^2}{c} . \tag{4}$$

which reduces to

$$z' = \frac{x'^2 + y'^2 + 2y' S_y}{c} . \tag{5}$$

<u>Rotation</u> Next we must determine the angle θ through which the axes must be rotated to make the z' axis normal to the surface at the new point of origin. We know that

$$\tan \theta = \text{slope} = \frac{dz}{dy} \tag{6}$$

and, from Eq. (1), that

$$\frac{dz}{dy} = \frac{d}{dy}\left(\frac{x^2 + y^2}{c}\right). \qquad (7)$$

Since $x = 0$ along the y-axis,

$$\frac{dz}{dy} = \frac{2y}{c} . \qquad (8)$$

Translating to the new origin, 0', we get

$$y = S_y , \qquad (9)$$

and, from Eqs. (8) and (9),

$$\frac{dz}{dy} = \frac{2S_y}{c} . \qquad (10)$$

Using Eqs. (6) and (10) we get

$$\tan \theta = \frac{2S_y}{c} , \qquad (11)$$

and

$$\theta = \arctan \frac{2S_y}{c} . \qquad (12)$$

For rotation around the z' axis,

$$x' = x" ,$$

$$y' = y" \cos \theta - z" \sin \theta , \qquad (13)$$

$$z' = z" \cos \theta + y" \sin \theta ,$$

where $\theta = \arctan 2S_y/c$.

Using Eqs. (5) and (13) we get

$$z" \cos \theta + y" \sin \theta = \frac{x"^2 + (y" \cos \theta - z" \sin \theta)^2 + 2S_y (y" \cos \theta - z" \sin \theta)}{c} \qquad (14)$$

which reduces to the quadratic equation

$$0 = \left(\frac{4S_y^2 \cos \theta}{c^3}\right) z"^2 - \left(1 + \frac{4S_y^2}{c^2} + \frac{4S_y}{c^2} y" \cos \theta\right) z" + \left(\frac{y"^2 \cos \theta}{c} + \frac{x"^2}{c \cos \theta}\right) \qquad (15)$$

Surface Equation. Only the lower set of values of z" is of interest. Thus, the solution set for the equation of the form

$$0 = ax"^2 + bx" + c \qquad (16)$$

will be described by the quadratic equation

$$z" = \frac{-b - (b^2 - 4ac)^{1/2}}{2a} . \qquad (17)$$

Using Eqs. (15) and (17), the equation describing the surface of an off-axis paraboloid can be written as

$$z'' = \frac{\left\{1 + \frac{4S_y^2}{c^2} + \frac{4S_y}{c^2} y'' \cos\theta\right\} - \left\{\frac{4S_y}{c^2}\left[y''\left(\frac{c^2}{2S_y} + 2S_y\right)\cos\theta + \frac{c^2}{2} + \frac{c^4}{16S_y^2} + S_y^2 - x''^2\right]^{1/2}\right\}}{\frac{8S_y^2}{c^3}\cos\theta} \quad (18)$$

In polar coordinates the solution may be written as

$$z'' = \frac{\left[1 + \frac{4S_y^2}{c^2} + \frac{4S_y}{c^2} r\cos\phi\cos\theta\right] - \left\{\frac{4S_y}{c^2}\left[\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\phi\cos\theta + \frac{c^2}{2} + \frac{c^4}{16S_y^2} + S_y^2 - r^2\sin^2\phi\right]^{1/2}\right\}}{\frac{8S_y^2}{c^3}\cos\theta} \quad (19)$$

where $y'' = r\cos\phi$, $x'' = r\sin\phi$, r is the radius, and ϕ is the rotation about the z axis with the y^+ axis as the origin.

Velocity in the z Direction

The velocity in the z direction with respect to the position ϕ may be obtained by taking the derivative of z with respect to ϕ:

$$\frac{dz}{d\phi} = \frac{c^3}{8S_y^2\cos\theta}\left[\left[-\frac{4S_y}{c^2} r\cos\theta\sin\phi\right] - \frac{2S_y}{c^2}\left[\left[\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\theta\cos\phi + \frac{c^2}{2} + \frac{c^4}{16S_y^2} + S_y^2 - r^2\sin^2\phi\right]^{-1/2}\left[-\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\theta\sin\phi - 2r^2\sin\phi\cos\phi\right]\right]\right] \quad (20)$$

Acceleration in the z Direction

The instantaneous acceleration in the z direction with respect to the position ϕ may be obtained by taking the second derivative of z with respect to ϕ:

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$$\frac{d^2z}{d\phi^2} = \frac{c^3}{8S_y^2 \cos\theta}\left(\left[-\frac{4S_y}{c^2} r\cos\theta\cos\phi\right] - \frac{2S_y}{c^2}\left\{\left[\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\theta\cos\phi + \frac{c^2}{2} + \frac{c^4}{16S_y^2} + S_y^2 - r^2\sin^2\phi\right]^{-1/2}\left[-\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\theta\cos\phi - 2r^2\cos^2\phi + 2r^2\sin^2\phi\right] + \left[-\frac{1}{2}\right]\left[-\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\theta\sin\phi - 2r^2\sin\phi\cos\phi\right]^2\left[\left(\frac{c^2}{2S_y} + 2S_y\right) r\cos\theta\cos\phi + \frac{c^2}{2} + \frac{c^4}{16S_y^2} + S_y^2 - r^2\sin^2\phi\right]^{-3/2}\right\}\right) . \qquad (21)$$

EXAMPLE

To determine whether or not cutting an off-axis parabolic reflector by the suggested method would be feasible, a particular situation was analyzed. The reflector used as an example is required by the LLL CTR program and has the following parameters:

Focal length = 24.818 in.
S_y = 23.972 in.
c = 36.7887
Reflector radius, r = 6.571 in.
ϕ = 52.50°

The displacement, Δz_{max}, determining the maximum travel required of the additional slide can be obtained from Eq. (18).

$$\Delta z_{max} = z_{max} - z_{min}$$
$$= 0.723 \text{ in.} - 0.240 \text{ in.}$$
$$= 0.483 \text{ in.}$$

The maximum displacement required of the variable position tool slide, then, would be less than 1/2 in.

Using Eq. (20) the maximum velocity, which occurs at the outer perimeter, is found to be

$$\left(\frac{dz}{d\phi}\right)_{max} = .457 \text{ in/rad}$$

This figure may be converted to in./s by multiplying by

$$\left(2\pi \ \frac{\text{rad}}{\text{rev}} \cdot \text{rpm} \cdot \frac{1 \text{ min}}{60 \text{ S}}\right)$$

Thus, for example, the maximum tool velocity required for a cutting speed of 100 rpm would be 4.79 in./s. For a cutting speed of 10 rpm this figure would be reduced to .48 in./s.

Using Eq. (21), the maximum acceleration, which also occurs at the outer perimeter, is found to be

$$\left(\frac{d^2z}{d\phi^2}\right)_{max} = 0.977 \text{ in./rad}^2 .$$

This figure may be converted to in./s^2 by multiplying by

$$\left(2\pi \frac{\text{rad}}{\text{rev}} \cdot \text{rpm} \cdot \frac{1 \text{ min}}{60 \text{ s}}\right)^2 .$$

Thus, the acceleration of the tool required for a cutting speed of 100 rpm would be 107.14 in./s^2. This figure is roughly one-fourth of the value of the acceleration of gravity.

CONCLUSION

We can conclude from the preceding analysis and discussion that it would be possible to turn off-axis paraboloids on one of the LLL diamond-turning machines. This method would permit satisfactory diamond-turning-machine tolerances and surface finish, would require a relatively small machine, and would take less time than the hand method. The only apparent limitation would be the possible need for low cutting speeds to minimize the tool velocity requirement.

To implement this technique, several modifications would have to be made to one of LLL's two diamond-turning machines. These would include the addition of a fast-response tool slide, a control system to be integrated with the existing NC unit, and position sensors on the slides and spindle.

These modifications would give the diamond-turning machine much greater versatility. The manufacture of complex shapes other than off-axis paraboloids would also be possible.

The cost of manufacturing off-axis paraboloids by the proposed method will depend on the cost of the required modifications and the number of reflectors to be machined. However, because of the present high manufacturing cost of and anticipated increase in demand for off-axis parabolic reflectors, it is probable that the manufacturing technique outlined here will become cost competitive.