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COMPUTER SIMULATION OF A STAGING SYSTEM FOR A THETA-PINCH REACTOR (RTPR)*

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MAST

NOMENCLATURE

(by order of appearance)

Length Calculations (cm)

LIHC	Length of IH coil around plasma
RIH	Mean radius of IH coil
LIHCM	Length of Marshall IH coil
LIH	Length of IH coil segment parallel to plasma
α_{IH}	Angle of Marshall coil to plasma core
Vs	Voltage supplied to IH coil
V	Total voltage of the IH coil in θ direction
LIHMC	Length of Marshall coil segment
n_MCS	Number of Marshall coil segments
DELIHC	Thickness of IH coil conductor
DELRIH	Thickness of IH coil

Skin Depth Calculations

τ_f	Hold time, fast IH circuit
ρ_{nb}	Resistivity of niobium
δSWF	Skin depth, fast IH rise
τ_s	Hold time, slow IH circuit
δSWS	Skin depth, slow IH rise
δSWWH	Skin depth, window hold time
τ_{wh}	Hold time, window
δRWR	Skin depth, ramp window rise
τ_{wr}	Rise time, window

Area Calculations (cm^2)

APFWIH	Area between plasma and first wall at implosion heating
XIH	Plasma radius after implosion heating divided by first wall radius
B	Radius of first wall
APIH	Area plasma at implosion heating
ABLKT	Area blanket
RBO	Outer radius of blanket
AIHL	Current carrying area of IH coil leads
AINS	Area insulator, thermal, between blanket and IH coil
AIHC	Area implosion heating coil (cross sectional, current carrying)
AANN	Area annulus between implosion and compression coil
RCI	Inner radius of compression coil
NRINGS	Number of rings in blanket
SEGS	Number of segments around the torus
SAISE	Thickness of electrical insulator
AISE	Area insulator, electrical, around blanket segments
ABLKTFF	Area blanket penetrated by fast implosion field
ABLKTFS	Area blanket penetrated by slow implosion field

Resistance Calculations (ohms)

REIHL	Resistance of IH coil leads
ρ_{cu}	Resistivity of copper
LIHL	Length of implosion coil leads to power supply
REIHC	Resistance of either Marshall or segmented coil
REIHF	Resistance of IH circuit, fast
REIHSF	Resistance of IH switch, fast

Resistance Calculations (ohms) (Contd.)

REIHS	Resistance of IH circuit, slow
REIHSS	Resistance of IH switch, slow
RESGF	Resistance of staging circuit, fast
RESGSF	Resistance of staging circuit switch, fast
RESGS	Resistance of staging circuit, slow
RESGSS	Resistance of staging circuit switch, slow
REW	Resistance of window circuit
REWS	Resistance of circuit window switch
REBF	Resistance, eddy current in blanket, fast

Magnetic Field Calculations

Bs	Magnetic field implosion heating
XO	Ratio of plasma radius to first wall radius after maximum compress
Bo	Magnetic field, maximum compression field
Bf	Applied magnetic field in gauss
L	Length of plasma chamber
BRFF	Magnetic field, return flux in annulus, fast
BRFS	Magnetic field, return flux in annulus, slow
BRFSG	Magnetic field, return flux in annulus, staging
BPW	Magnetic field before window opens
BAW	Magnetic field during annulus window
BIHF	Magnetic field, immediately after window closed. (Assumes XIH = 1 at start of second bounce.)
BANIHW	Magnetic field, return flux from negative IH spike, window

Timing

τ_{hold}	Time for compression field to rise to Bs
τ_r	Time for compression field to rise to BO
τSGF	Hold time, fast staging circuit
WI2RSGS	Joule loss in slow staging circuit
τSGS	Hold time, slow staging circuit

Currents (amp)

IEB	Current, eddy in blanket
IW	Current in implosion heating coil, window
Iθ	Current in Marshall coil, θ direction
IZ	Current in Marshall coil, Z direction
P_MARSHALL	Equivalent magnetic pressure within the Marshall coil, atmosphere
B_COMP_ALPHA	Field generated by total current in conductor
I **Alpha**	Total current in conductor
IIH	Current in IH coil

Energy Calculations

(megajoule per meter of plasma length)

I^2R Terms

QI2R	Maximum joule loss in IH coil (single stage)
WI2RIHF	Joule loss in IH fast circuit
WI2RIHS	Joule loss in slow IH circuit
WI2RSGF	Joule loss in fast staging circuit
WI2RIHEBF	Joule loss, eddy current in blanket, fast IH

Energy Calculations (Contd.)

WI2RIHEBS Joule loss, eddy current in blanket, slow IH

REBS Resistance, eddy current in blanket, slow

WI2RW Joule loss in window circuit

Plasma Terms

WPIH Work done on plasma by IH field

Magnetic Energy Terms

WBIHI Magnetic energy in thermal insulator from IH field

WBIHIS Magnetic energy in electrical insulators during IH

WBIHF Magnetic energy in blanket skin depth from fast IH

WBIHB Magnetic energy in blanket from IH field

WBIHM Magnetic energy in IH coil

WBIHS Magnetic energy in blanket skin depth from slow IH

WBIHPW Magnetic energy between plasma and first wall during IH

WBW Magnetic energy in window field

WBRFIHF Magnetic energy in annulus during fast IH

WBRFIHS Magnetic energy in annulus during slow IH

WBRFSGF Magnetic energy in annulus from fast staging

Magnetic Energy Conservation

~RFR Efficiency of conversion of return flux energy to stored electri energy

WDC2 Direct conversion energy from field in annulus, collected by isolation ring

Energy Calculations (Contd.)

Heating Terms

QIBE	Heat dissipated in conducting portion of blanket from eddy current during IH and staging or window
QEHBW	Heat dissipated in eddy heating of blanket during window rise and hold time
QIC	Heat dissipated in compression coil from IH, staging and window circuits
QICE	Heat generated by eddy currents in IH coil
QICL	Heat transferred from IH coil to compression coil
QBI	Thermal conduction from blanket to IH coil

Energy Summations

QTW	Heat from staging system rejected to cooling water
QT[illegible]S	Eddy current heating of blanket
WSGS	Staging energy, slow
WSGF	Staging energy, fast
WIHF	IH energy, fast
WIHS	IH energy, slow
WWIN	Total energy supplied from window staging circuit
WIH	Total IH energy
WSG	Total staging energy
QPIH	Heat to plasma on implosion heating

COMPUTER SIMULATION OF A STAGING SYSTEM

FOR A THETA-PINCH REACTOR

by

Philip G. Crnkovich

ABSTRACT

To reduce excessive energy requirements for the implosion heating system of a theta-pinch reactor, two staging methods, the brute force and bucking field options, were proposed. A Marshall coil and a segmented coil were also considered. Calculations involved in coding these coil designs and staging options into a PL/I subprogram are described. A marked savings in the energy required for the IH system is realized with the bucking option and others.

I. INTRODUCTION

The possibility of employing a pulsed theta-pinch fusion reactor for central-station power generation was considered in a cooperative LASL-ANL study on the Reference Theta-Pinch Reactor (RTPR).[1] This work formalized the identification of the theta-pinch subsystems according to their functions and provided the necessary background information for the study of the overall system performance of a Theta-Pinch Fusion Power Plant (TPFPP now proceeding at ANL.[2] The design subsequently was translated into the form of a mathematical model that is programmed on the digital computer.[3] With this model, energy requirements for the implosion heating system were calculated and found to be substantial.

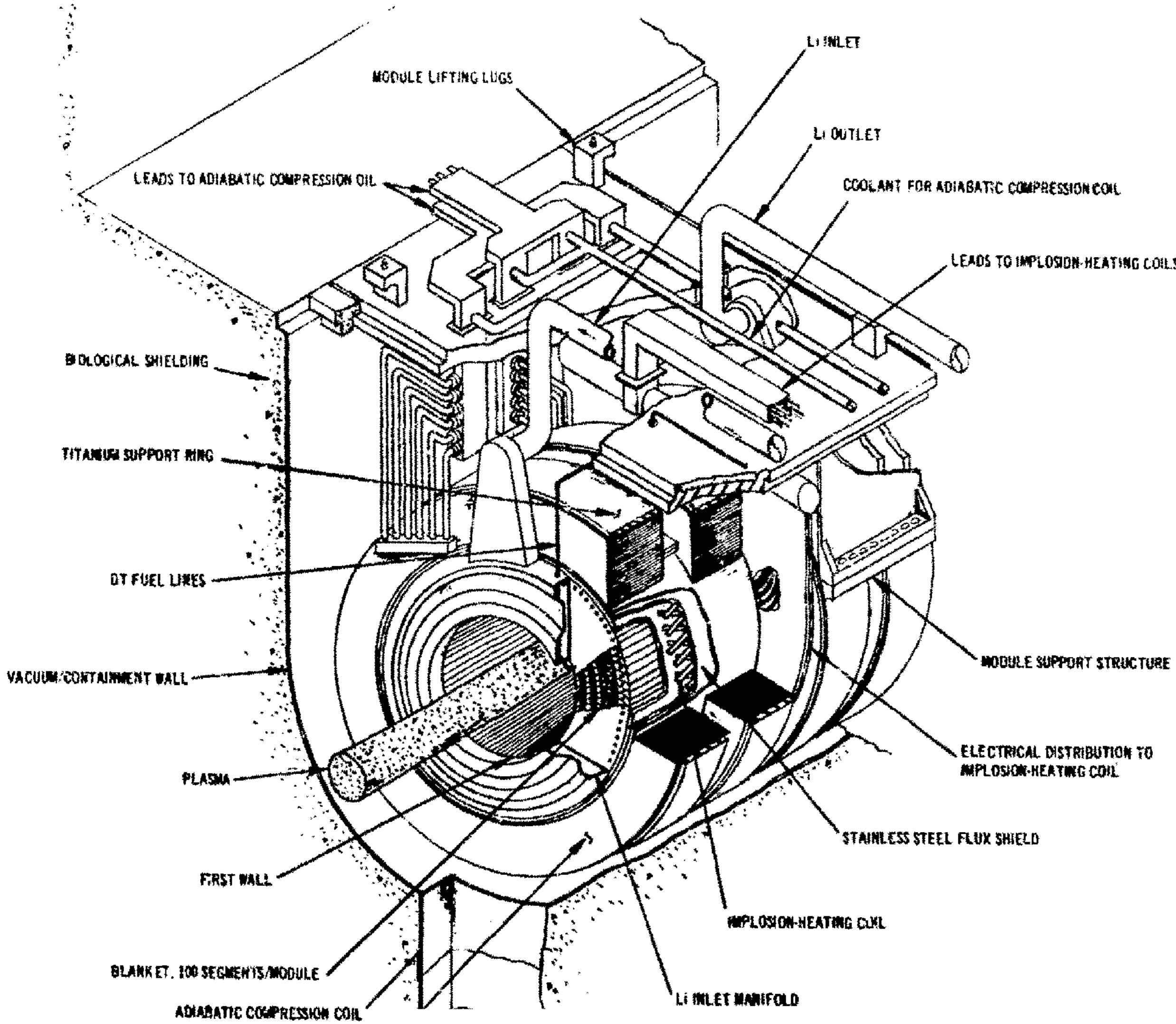
Two different staging options were suggested to correct this problem (see Sec. 1.1.2); they, in turn, were coded into a PL/I subprogram which is presented in the succeeding sections of this report.

All major power plant subsystems are described mathematically, giving the performance of each subsystem as a function of the subsystem operational parameters. Not included, however, are critical constraints such as stress and temperature limitations, corrosion resistance, electrical properties, shielding requirements, and material lifetimes that relate to reactor safety and lifetime. These will be added as the model is developed.

The resulting computer code can be used to explore the compatibility of the design parameters of the individual subsystems, to identify and study critical subsystem interfaces, and to establish the sensitivity of the system performance to the adjustment of the design parameters. Thus, as the work progresses, the sophistication of the design and of the mathematical models evolves from a rather low level from which only the basic system compatibility can be studied to a rather high level that ultimately will yield meaningful economic and environmental impact information.

1.1 Summary of RTPR Design Features

The plasma chamber of the RTPR is a toroid with a major radius of 76 m and a minor radius of 50 cm. The toroid consists of one hundred 200 cm-long segments assembled into modules. Only one of the 200 cm-long modules (Fig. 1.1-1) needs to be modeled since the relationship between variables is dependent of the number of modules used in the reactor. Each module is nearly independent of the other modules since each has its own fuel delivery and ash removal system and other support devices. As a



Li INLET
MODULE LIFTING LUGS
Li OUTLET
LEADS TO ADIABATIC COMPRESSION OIL
COOLANT FOR ADIABATIC COMPRESSION COIL
LEADS TO IMPLOSION-HEATING COILS
BIOLOGICAL SHIELDING
TITANIUM SUPPORT RING
DT FUEL LINES
MODULE SUPPORT STRUCTURE
VACUUM/CONTAINMENT WALL
ELECTRICAL DISTRIBUTION TO
IMPLOSION-HEATING COIL
PLASMA
STAINLESS STEEL FLUX SHIELD
FIRST WALL
IMPLOSION-HEATING COIL
BLANKET, 100 SEGMENTS/MODULE
Li INLET MANIFOLD
ADIABATIC COMPRESSION COIL

Fig. 1.1-1

result, the units used in the model are in terms of one meter of length, and the total for the plant is easily calculated from the total length (major circumference) of the torus.

1.1.1 Implosion Heating System

The theta-pinch power plant operates on a pulsed power cycle[3] with the thermal power being produced in short bursts like a single piston engine (Fig. 1.1-2). At the start of the cycle, a small quantity of the 50/50 deuterium-tritium fuel is introduced into the plasma chamber and ionized. The temperature is then increased by implosion heating to approximately 10^7 K. The implosion heating is accomplished by two very rapid compressions each with a rise time of approximately 0.1 μsec. The advancing magnetic field front of the rapid compression accelerates the charged particles with a consequent increase in the temperature.

Before the implosion, charged particles are assumed to be at rest. During the implosion, the magnetic wall containing the plasma moves inward radially at a velocity U_s. When a particle collides with the moving magnetic wall, it rebounds with velocity $2U_s$ and moves toward the other containing wall (Fig. 1.1-3). The particles begin to strike the magnetic wall when it has travelled to one-third of the tube radius.

1.1.2 Implosion Coil

The segmented coil is a sheet of copper that is wrapped around the plasma radially. If more than one segment is required, then each segment wraps only part way around the plasma. The total voltage in the coil is the sum of the voltage differences encountered on the circumference of the coil.

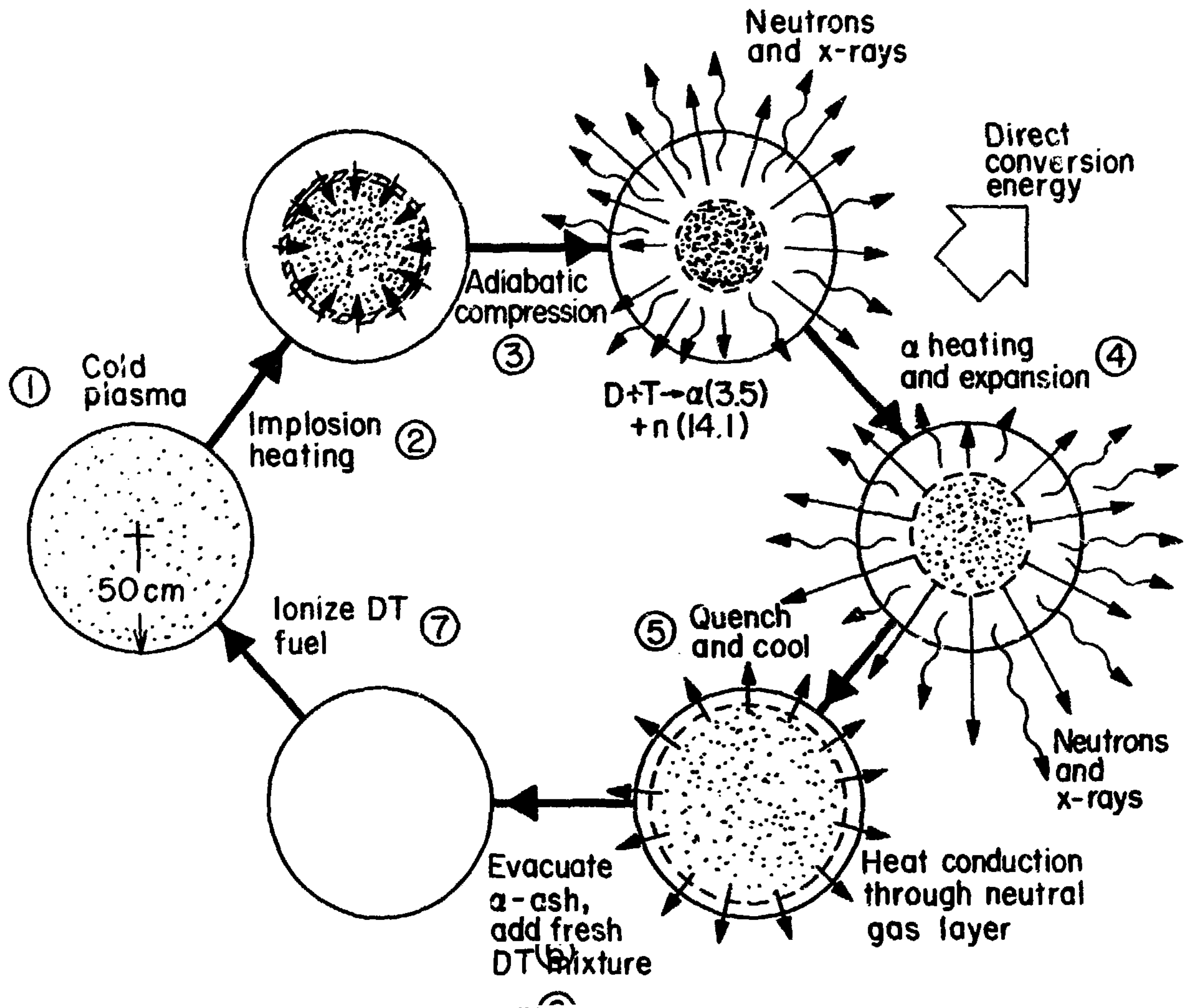


Fig. 1.1-2
RTPR Power Cycle

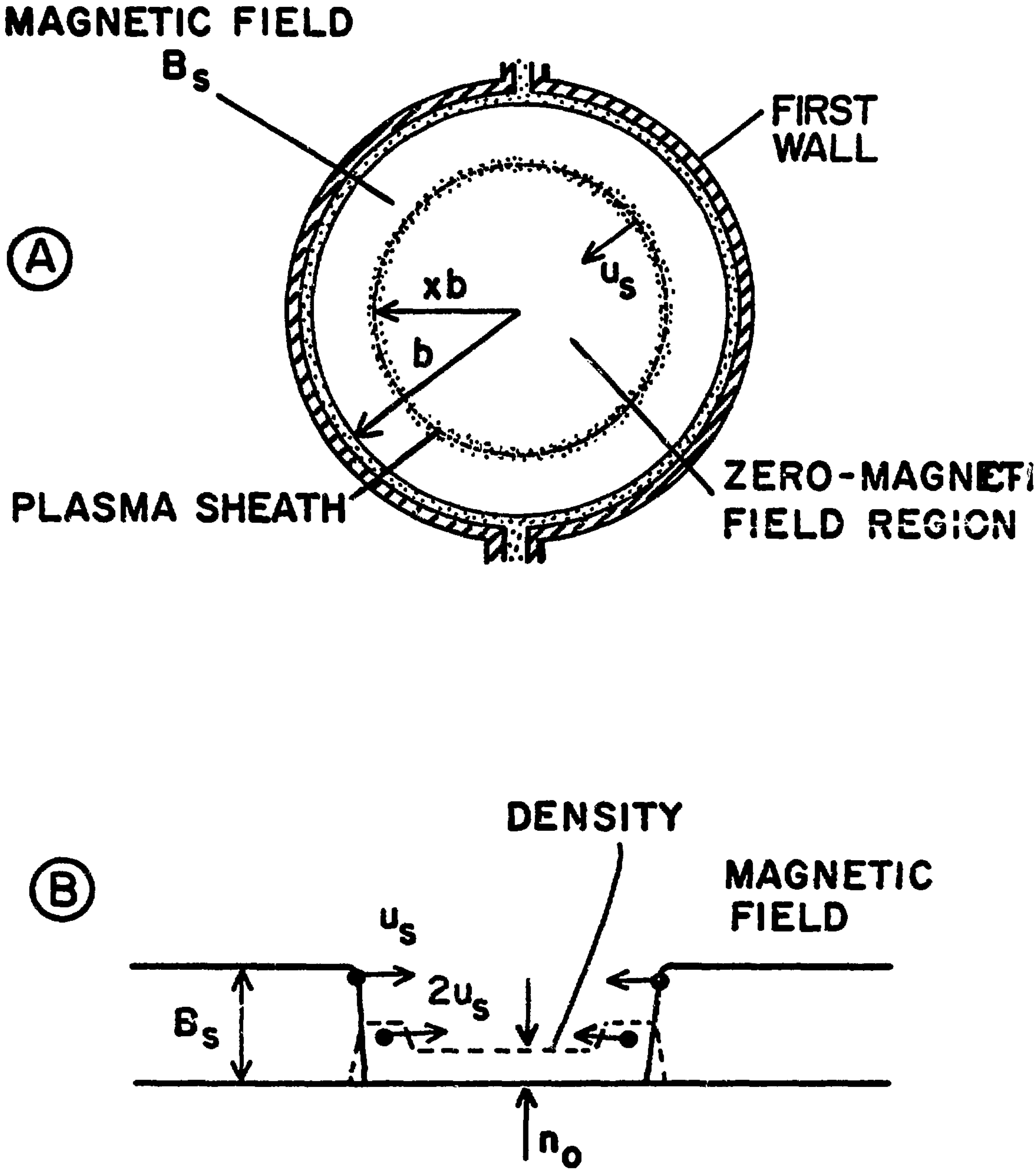


Fig. 1.1-3

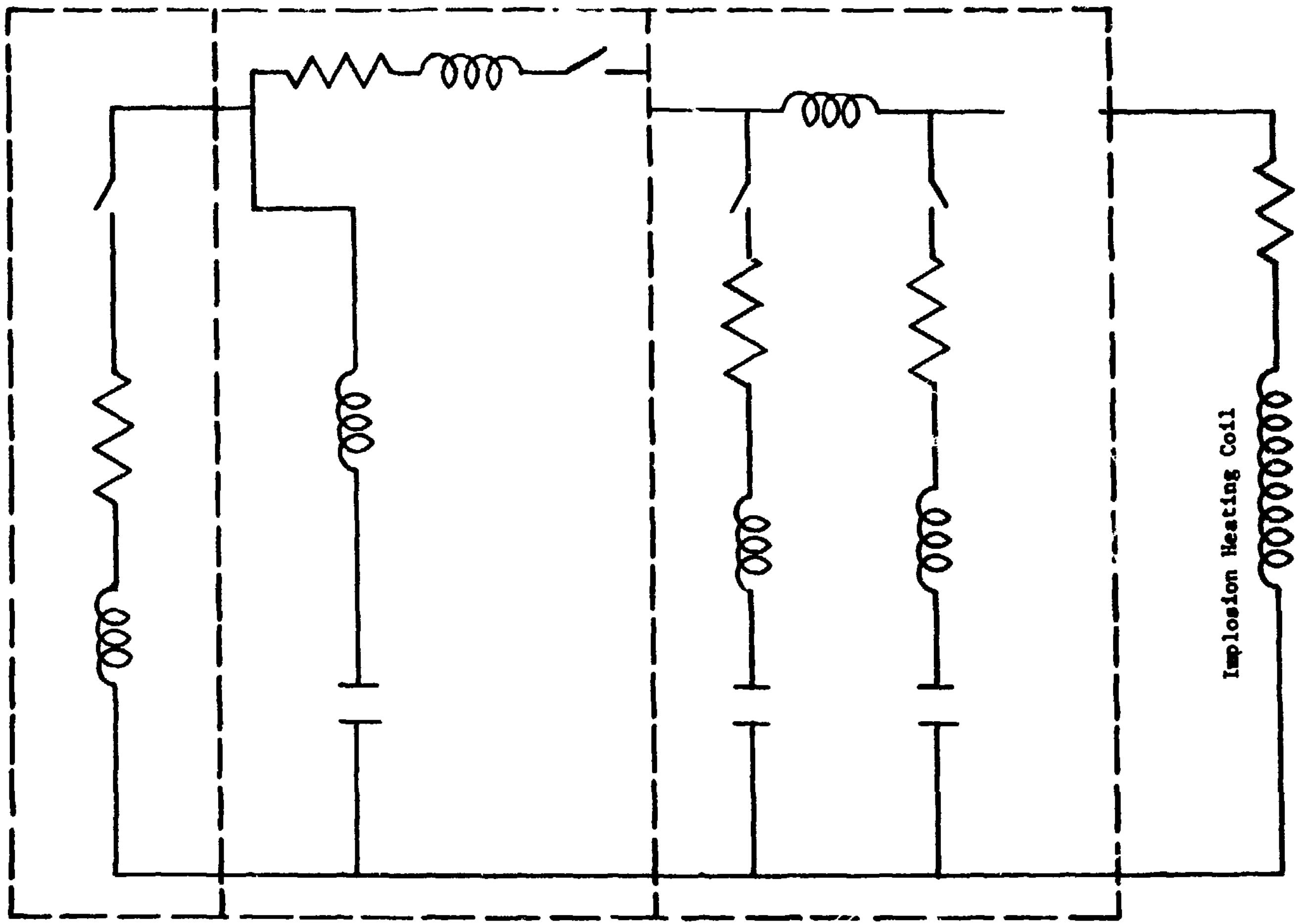
Processes involved in an ideal implosion heating of the preionized plasma

The IH coil is separated from the blanket by a thermal insulator of alumina and metal foil. It is powered from a sophisticated capacitor bank whose leads are attached to each of two 100 cm-long coils at the center of each module.

Early calculations showed that with this design the IH system required more energy than the reactor delivered. To alleviate this effect, two different staging systems were proposed in the spring of 1974. The first method, or the "brute force" option, consists of four separate circuits: the fast and slow IH heating circuits, which cause the two rapid compressions, and the fast and slow staging circuits, which hold the plasma compressed until the compression field has sufficient strength to continue the compression (Fig. 1.1-4).

An alternative to the direct staging method is proposed for the FERF. In this method, called the "window" or "bucking" method, the compression coil is first energized. When the magnetic field in the chamber is near the desired implosion heating field, Bs, a bucking or negative current is allowed to flow in the implosion heating coil, to return the field in the chamber to near zero. Then the double bounce implosion heating occurs, and the compression field is allowed to contain the plasma.

A circuit diagram schematic for the window method is shown in Fig. 1.1-5. Shown in Fig. 1.1-6 is a schematic representation of the time variation fields in the plasma chamber (solid line) and in the annulus between the



Crowbar Staging Circuit Implosion Heating Pulsing Forming Network

Implosion Heating and Staging Circuit

Fig. 1.1-4

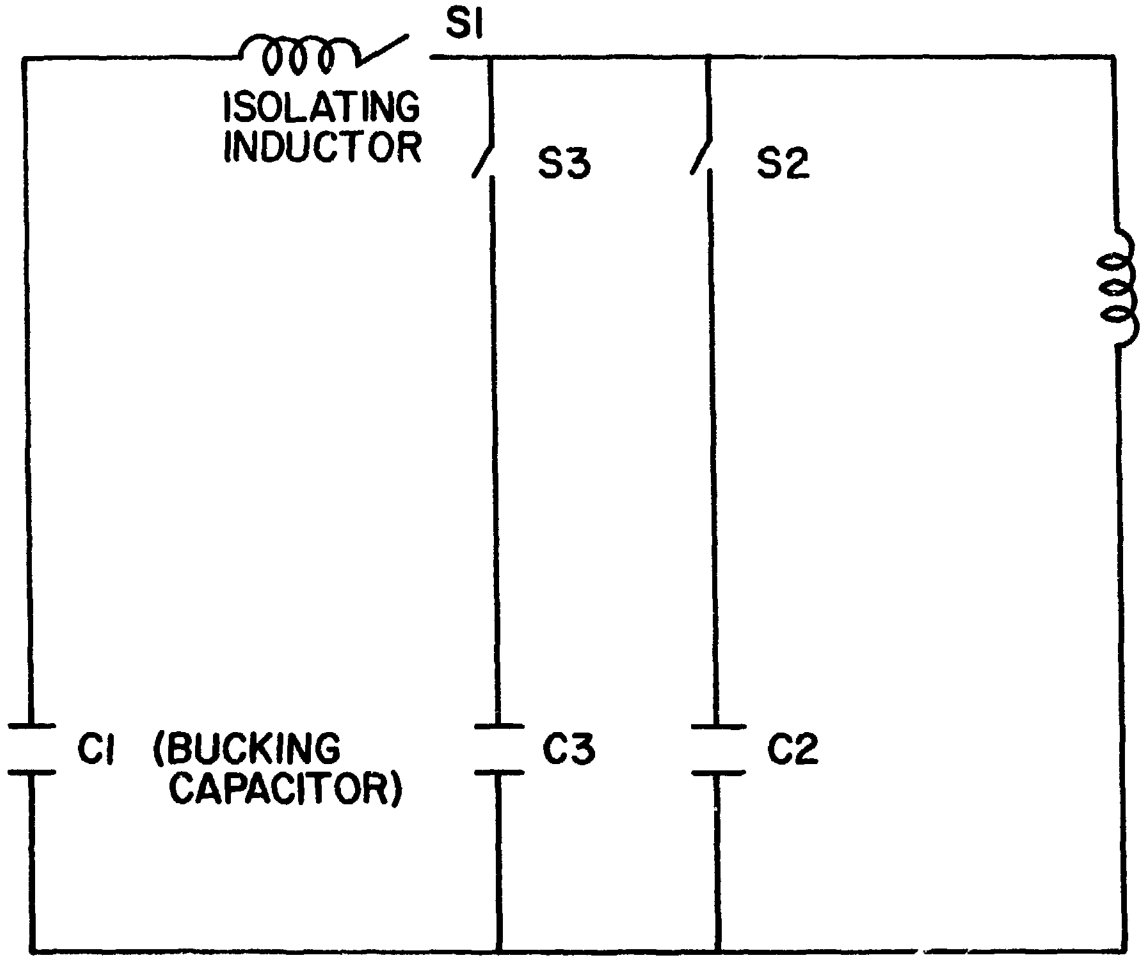


Fig. 1.1-5

Circuit Diagram for Window Implosion Heating Method

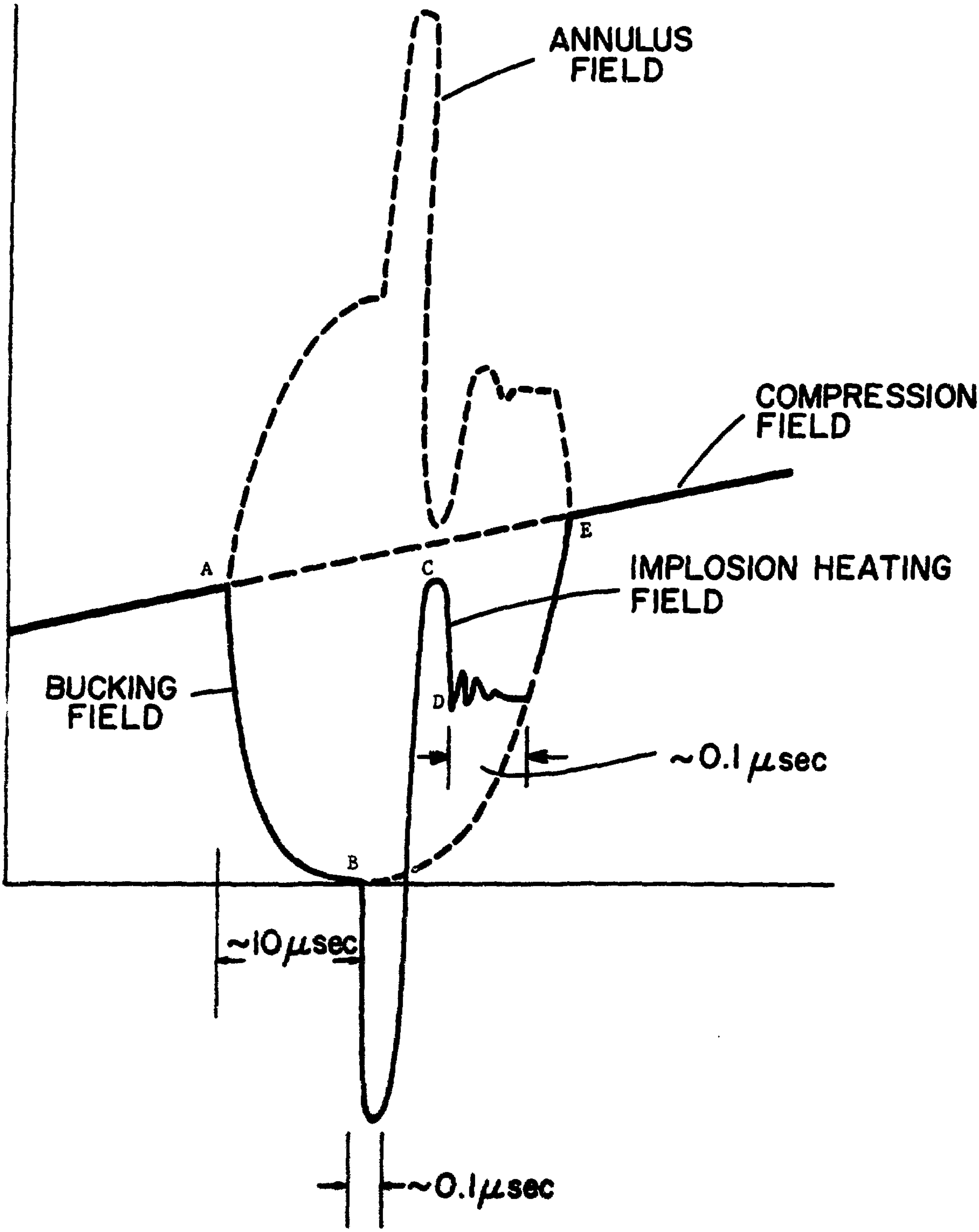


Fig. 1.1-6

Schematic of Magnetic Fields in Window Implosion Heating Method

Time			
0			S_2 closed
0	A	A	Close S_1, current flows opposite that in compression coil
10 μsec	B	B	$B \cong 0$ in chamber
12 μsec		B	Pre-ionize
12.001 μsec		B	Close S_3, buck some opposing current
12.005 μsec		C	$B \cong \sqrt{2}$ Bs in chamber
		D	$B \cong 0$ in chmber, ringing C_3
		D^+	Close S_4, bucking all opposite current; I goes to zero in load
		E	Open S_2

implosion and compression coil (dotted line). A proposed series of events would be as follows:

At time A, when the compression field is nearing the desired implosion heating field, Switch S-1 is closed, and a current is slowly (tens of microseconds) fed to the load (implosion heating coil) in a direction so as to null the magnetic field already existing inside the implosion coil and to create a window in the compression field. At time B, when that field is near zero, the D-T gas in the chamber is pre-ionized. Soon thereafter Switch S-3 is closed to give a current from C-3 opposite to that initially given from C-1. This allows the field within the implosion coil to rise to approximately $\sqrt{2}$ Bs. This circuit then rings down to zero current through Switch S-3, allowing the field inside the implosion coil to return to near zero.

At time D, Switch S-4 is closed, allowing current from C-4 to buck that from C-1 and bring the field within the implosion heating coil to the desired Bs. At this time there is essentially zero current at Switch S-2, and the switch is opened. This method appears practical, but it has yet to be thoroughly analyzed with a dynamic program such as NET II.[4] It is expected that the very fast opening time required from Switch S-2 may be very difficult to obtain with good reliability. However, a switch of this type is currently under development for inductive energy storage systems.[5]

Should the development of a switch to fulfill the function of S-2 prove too formidable, a less attractive (from an energy use standpoint) method could be as shown by the dotted line in the circuit diagram (Fig. 1.1-5). Switch 5 would close at time E and prevent circuit ringing in the coil.

With the bucking method there is no need for crowbarring the implosion coil since the plasma is immediately contained by the already present compression field. Although this method appears feasible, it is clear that considerably more work is required on the definition of values of XIH which can be attained, possible large energy losses in switches required, and particularly the interaction of the plasma processes with the electrical circuitry. Further, the switch technology, reliability of the entire circuit, fabrication methods required for attachment of the leads to the implosion heating coil, and other factors will require experimental investigation and development.

The coil design and staging options have been coded into a PL/I sub-program which is called by TPFPP, the code being used to simulate the entire Theta-Pinch Fusion Power Plant. The calculations involved at each tep will now be discussed.

2. COMPUTER CODING

2.1 Length Calculations

The resistance of a conductor of length ℓ can be expressed as $\frac{\rho \ell}{A}$, where ρ is the resistivity of the material, A is the cross-sectional area, and ℓ is the length of the conductor. Two coil options are included here. For the first, a segmented coil, the length of the implosion heating coil around the plasma (LIHC) is

$$\text{LIHC} = 2\pi \text{RIH}, \qquad (2.1.1)$$

where RIH is the mean radius of the coil.

The second option, a Marshall coil, is wound at an angle α_m with respect to the major radius of the toroid. The conductor is bent back on itself at one end so that the leads are both connected at the same end. The length of a single turn of the coil per segment, LIHCM, is

$$\text{LIHCM} = 2\text{LIH}/\cos(\alpha_m), \qquad (2.1.2)$$

where LIH is the length of the coil segment parallel to the plasma and α_m is the wrap angle of the coil (see Fig. 2.1-1):

$$\alpha_m = \text{Tan}^{-1}(\text{LIHC}/\text{LIH}) \cdot (V/V_s) \,.$$

(2.1.3)

The quantity V_s/V is the number of conductors needed to obtain the total IH voltage V_s in the coil with V on each wrap from the capacitor storage bank.

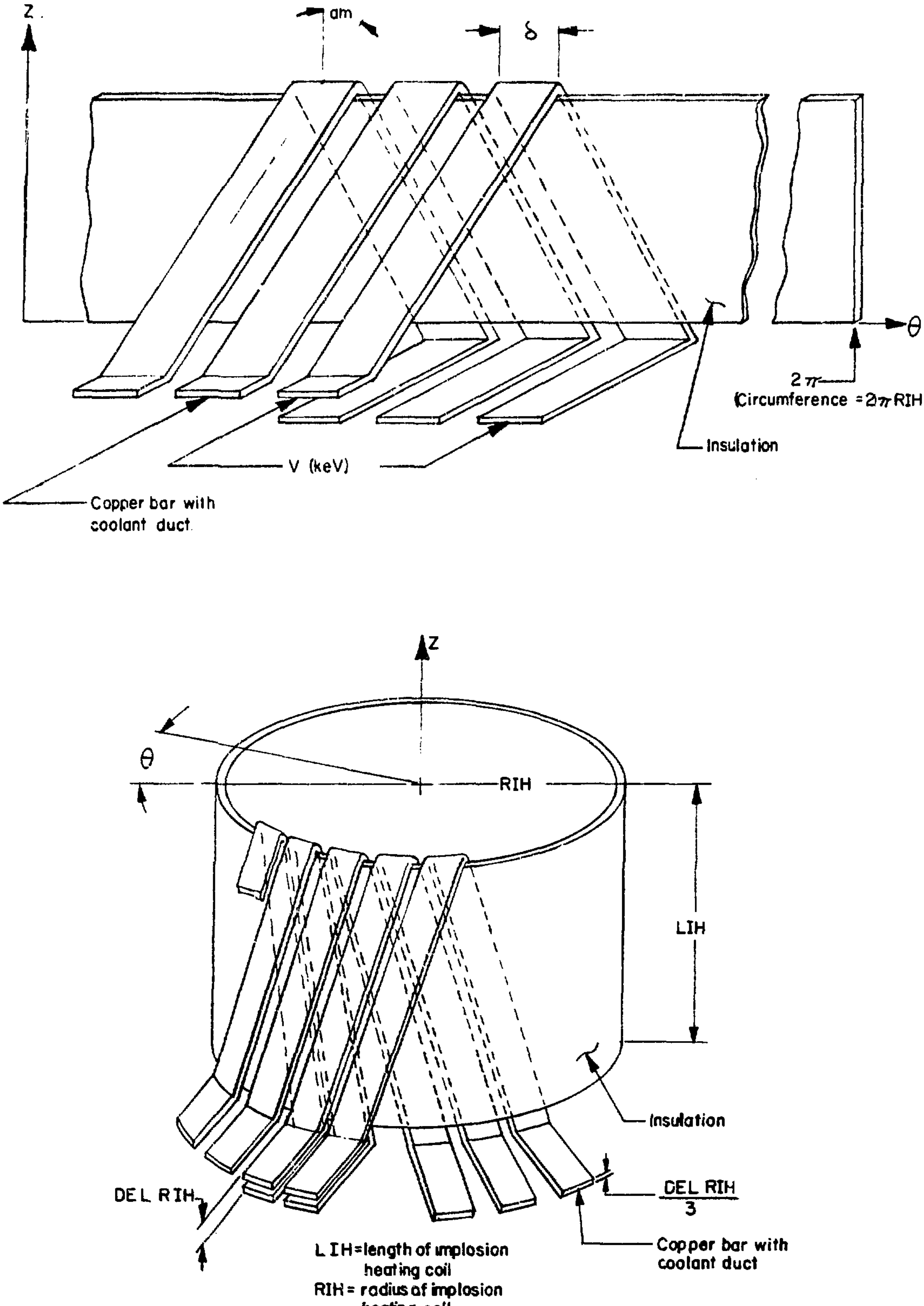


Fig. 2.1-1

Marshall coil

It was found that under conditions of large implosion heating fields and large plasma first wall radius, the angle of the Marshall coil winding became very small. As a result, huge currents were required to obtain sufficient current in the theta direction to supply the implosion heating field. This was caused by the arbitrary coil length of 100 cm. To avoid this ungainly loss, a provision to use coil lengths less than 100 cm was made. This provision worked as follows. It was first assumed that the Marshall coil was wound at a 30 degree angle. Using this assumed value of α_m, a first approximation of the length of the Marshall coil segment was found:

$$\text{LIHMC} = V \cdot 2\pi\text{RIH}/0.577 \cdot V_s \quad (0.575 = \tan 30°). \quad (2.1.4)$$

The number of Marshall coil segments per two meter module was then found by

$$\text{n\_MCS} = 200/\text{LIHMC 1A}. \quad (2.1.5)$$

This value is then rounded up to the next higher integer (in no case less than 2) and becomes the design number of Marshall coil segments per module. From this value, the design length of the Marshall coil segment is determined:

$$\text{LIHMC} = 200/\text{n\_MCS (integer)}, \quad (2.1.6)$$

and a final value of α_m calculated:

$$\alpha_m = \text{Tan}^{-1} [V \cdot 2\pi \text{ RIH}(\text{LIHMC} \cdot V_s)]. \quad (2.1.7)$$

The length of the coil leads, the distance from the capacitors to the coil, has been estimated from the RTPR plant design to be 1000 cm. For the Marshall coil, the insulator and the winding are assumed to have the same thickness, so the thickness of the conductor DELIHC is (see Fig. 2.1-1)

$$\text{DELIHC} = \text{DELRIH}/3. \quad (2.1.8)$$

For the segmented coil, where DELRIH is the thickness of the IH coil, the thickness of the conductor is the same as the coil, or

$$\text{DELIHC} = \text{DELRIH}. \qquad (2.1.9)$$

2.2 Skin Depth Calculation.

The magnetic field instantaneously penetrates the annulus and thermal and electrical insulators, but the magnetic field penetration of the electrical conducting portion of the blanket is a function of time. The depth to which a suddenly-applied (step wave) magnetic field penetrates in a given time, the skin depth, is calculated from the expression

$$\delta = 1.32 \sqrt{\frac{10^3}{0.4\pi}} \cdot \sqrt{\tau} . \qquad (2.2.1)$$

By substituting in the fast circuit time τ_f and the resistivity of niobium (e.g.), the skin depth at the end of the fast implosion heating time can be found from

$$\delta\text{SWF} = 1.32 \sqrt{\frac{10^8 \rho nb}{0.4\pi}} \cdot \sqrt{\tau_f} . \qquad (2.2.2)$$

The skin depth at slow circuit time τ_s, (δSWS) is found by the ratio

$$\delta\text{SWS} = \delta\text{SWF} \cdot \frac{\sqrt{\tau_s}}{\sqrt{\tau_f}} . \qquad (2.2.3)$$

Similarly the skin depth penetrated during the window on-time (δSWWH) is

$$\delta\text{SWWH} = 1.32 \sqrt{\frac{10^8 \rho nb}{0.4\pi}} \cdot \sqrt{\tau_{wh}} , \qquad (2.2.4)$$

where τ_{wh} is the hold time of the window.

The skin depth of the niobium during the ramped-window rise (δRWR) is

$$\delta\text{RWR} = \sqrt{\frac{10^8 \rho nb}{2}} \cdot \sqrt{\tau_{wr}} , \qquad (2.2.5)$$

where τ_{wr} is the window rise time and the difference between Eqs. (2.2.5) and (2.2.1) accounts for their different (ramp vs. step) wave forms. The blanket was assumed to be niobium here since the blanket segments are encased in niobium. Since the resistance of niobium is greater than other candidate blanket materials, a measure of conservatism is built into the calculations.

2.3 Area Calculations

The next step is to calculate the various cross-sectional areas from the known physical dimensions. The area between the compressed plasma and the first wall (APFWIH) is

$$APFWIH = \pi(1 - X\,IH^2)B^2, \qquad (2.3.1)$$

where XIH is the radius compression and B is the radius fo the first wall.

The area occupied by the compressed plasma (APIH) is

$$APIH = \pi B^2 X\,IH^2 \qquad (2.3.2)$$

The area of the blanket (ABLKT) is

$$ABLKT = \pi(R\,BO^2 - B^2), \qquad (2.3.3)$$

where RBO is the outer radius of the blanket. The current-carrying area of the IH coil leads (AIHL) (coaxial cables) is assumed to be the same as the IH coil before winding (Fig. 2.1-1); no provision is made for spacing between wraps of the Marshall coil:

$$AIHL = DELIHC \cdot LIHC. \qquad (2.3.4)$$

The area of the thermal insulator between the blanket and the IH coil is

$$AINS = \pi\left[(RIH - DELRIH)^2 - R\,BO^2\right], \qquad (2.3.5)$$

where the quantity (RIH - DELRIH) is the outer radius of the IH coil.

The cross sectional area of the segmented IH coil is

$$AIHC = 2\pi(RIH \cdot DELRIH) \cdot \qquad (2.3.6)$$

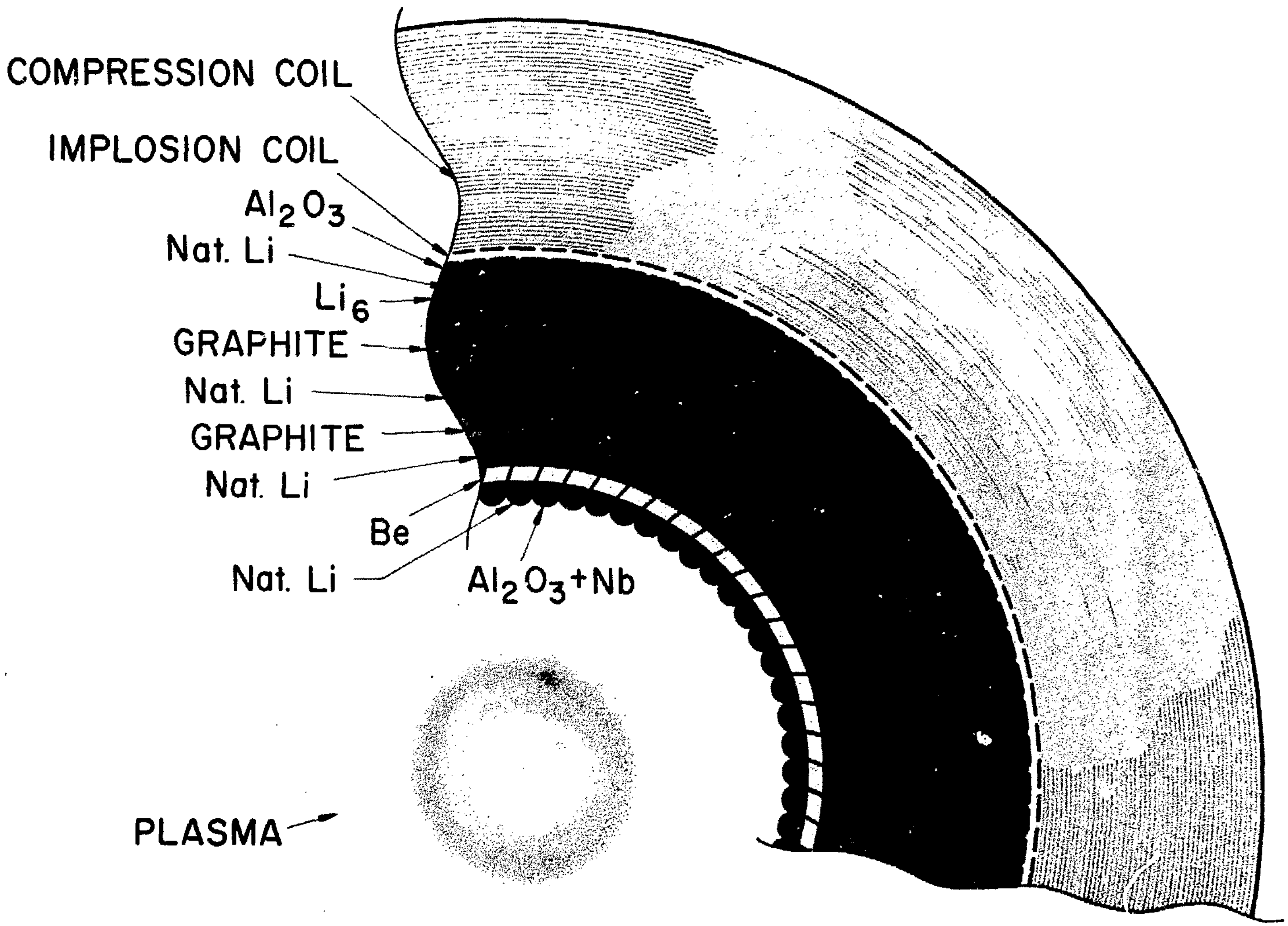


Fig. 2.3-1. Segmented blanket

The area of the annulus between the IH coil and the third coil (shield) used to prevent voltage multiplication within the compression coil is

$$AANN = \pi[RCI^2 - (RIH + DELRIH)^2], \qquad (2.3.7)$$

where RCI is the inner radius of the compression coil. This area was not considered in the original RTPR design shown in Fig. 2.3-1.

The blanket is segmented by electrical insulators in both the theta and r directions to control eddy current (Fig. 2.3-1). The blanket contains NRINGS insulators in the r direction and SEGS insulators in the theta direction. The mean radius of a radial insulator, assuming that they are equally spaced, is (RBO + B)/2, and the area occupied by the radial insulators is

$$2\pi\ \delta AISE\ \left(\frac{RBO + B}{2}\right)\ NRINGS, \qquad (2.3.8)$$

where δAISE is the thickness of each insulator ring.

The area occupied by the insulators in the r direction is

$$\delta AISE \cdot SEGS\ (RBO - B), \qquad (2.3.9)$$

assuming the insulator thickness between segments is equal to that between rings. Combining Eqs. (2.3.8) and (2.3.9), the total area occupied by the electrical insulators in the blanket is

$$AISE = \delta AISE\ [2\pi\left(\frac{RBO + B}{2}\right)\ NRINGS + SEGS\ (RBO - B)]. \qquad (2.3.10)$$

The additional length, due to the slight curvature of the insulator in the r direction, is neglected.

The area of the blanket penetrated by the magnetic field during the fast IH heating, ABLKTFF, is the area of the insulators in the blanket plus the area corresponding to the skin depth. The conducting blanket has contact length with the insulators of AISE/δAISE. Thus the area penetrated by the fast implosion field is

$$ABLKTFF = (AISE/\delta AISE)\,\delta SWF + AISE \qquad (2.3.11)$$

or

$$ABLKTFF = AISE\left(1 + \frac{\delta SWF}{\delta AISE}\right). \qquad (2.3.12)$$

Similarly, the area of the blanket penetrated by the slow IH spike is

$$ABLKTFS = AISE\,(1 + \delta SWS/\delta AISE). \qquad (2.3.13)$$

2.4 Resistance Calculation

Given the length, cross-section area, and resistivity of the conductor, and the resistance of the other circuit elements, the total resistance of the circuit can be determined. The resistance of the copper leads of the implosion coil is calculated by

$$REIHL = \rho_{cu}\,\frac{LIHL}{AIHC}, \qquad (2.4.1)$$

where ρ_{cu} is the resistivity of the copper and LIHL is the length of implosion coil leads to the power supply. The length of the conductor for the Marshall coil is

$$LIHCM = 2LIH/\cos(a_m). \qquad (2.4.2)$$

The cross-sectional area of the conductor through which the current flows in the Marshall coil can be expressed as

$$AMC = 2\pi RIH \cdot \frac{DELRIH}{3}. \qquad (2.4.3)$$

Combining Eqs. (2.4.2) and (2.4.3) and simplifying yields the expression for the resistance of the Marshall coil:

$$REIHC = \rho_{cu}\;LIHCM/\left(\frac{DELRIH}{3} \cdot 2\pi RIH\right). \qquad (2.4.4)$$

The resistances of the IH and staging circuits are the sum of the resistances of each element, since the elements all are in a series; that is,

Resistance total = resistance of coil + leads + switch.

Values used for switch resistances are shown in Table 2.4-1. These values are believed to be consistant with present day technology switch timing circuits and hold times as discussed in Sec. 2.6.

TABLE 2.4-1
SWITCH RESISTANCES

Code	Definition	Resistance (ohms)
REIHSF	Resistance of IH switch, fast	10^{-2}
REIHSS	Resistance of IH switch, slow	10^{-3}
RESGSF	Resistance of staging circuit switch, fast	10^{-4}
RESGSS	Resistance of staging circuit switch, slow	10^{-5}
REWS	Resistance of circuit window switch	10^{-3}

The resistance of the fast IH circuit is

$$REIHF = REIHC + REIHL + REIHSF, \qquad (2.4.6)$$

where REIHSF is the resistance of the fast switch. The resistance of the other circuits is calculated by correcting the switch term since the coil and leads are the same in each circuit. The resistance of the IH slow circuit is

$$REIHS = REIHF - REIHSF + REIHSS, \qquad (2.4.7)$$

where REIHSS is the resistance of the slow switch.

The resistance of the fast staging circuit is

$$RESGF = REIHF - REIHSF + RESGSF, \qquad (2.4.8)$$

where RESGSF is the resistance of the fast staging switch.

The resistance of the slow staging circuit is

$$RESGS = REIHF - REIHSF + RESGSS, \qquad (2.4.9)$$

where RESGSS is the resistance of the slow staging switch.

The resistance of the window circuit is

$$REW = REIHF - REIHSF + REWS, \qquad (2.4.10)$$

where REWS is the resistance of the circuit window switch.

The resistance to the fast IH eddy current circuit which heats the blanket is

$$REBF = \rho_{nb} \left(\frac{AISE}{\delta AISE} \right) / (\delta SWF \cdot 2 \cdot LIH), \qquad (2.4.11)$$

where the quantity (AISE/ AISE) is the length of the eddy current path, and the quantity (2 · SWF · LIH) is the area of the blanket through which the eddy current has penetrated. Simplification of the above expression yields

$$REBF = \frac{\rho_{nb}}{2} \frac{AISE}{\delta AISE \cdot \delta SWF \cdot LIH}. \qquad (2.4.12)$$

Similarly, for the eddy current heating during the slow implosion heating,

$$REBS = REBF \cdot \delta SWF/\delta SWS. \qquad (2.4.13)$$

It is assumed that the skin depth at the end of the pulse is achieved at the start of the pulse and does not change with time, building a degree of conservatism into calculations.

2.5 Magnetic Field Calculations

The magnetic field Bs required to obtain a compression fraction X can be found since the compression from XIH to XO is isentropic and the field strength is known at the end of the compression (BO and XO).

The expression relating pressure and volume for an isentropic compression of a monatomic gas is

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma} \qquad (2.5.1)$$

Since the magnetic pressure applied to the gas is equal to the internal pressure of the gas for $\beta = 1$, then

$$P = \frac{B_f^{\,2}}{8\pi}, \qquad (2.5.2)$$

where B_f is the applied magnetic field in gauss.

The volume of the plasma can be expressed as volume per meter length, or

$$V = B^2 X^2 L, \qquad (2.5.3)$$

where L is the length of the plasma chamber.

Substituting Eqs. (2.5.2) and (2.5.3) into Eq. (2.5.1), the relation becomes

$$\frac{B_1^2}{8\pi}\,[(BX_1)^2L]^\gamma = \frac{B_2^2}{8\pi}\,[(BX_2)^2L]^\gamma \qquad (2.5.4)$$

Then by solving and simplifying and substituting $\gamma = 5/3$ for a monatomic gas ,

$$B_1 = B_2\,(X_2/X_1)^{1.66}. \qquad (2.5.5)$$

Let $Bs = B_1$, $XIH = X_1$, $BO = B_2$, and $XO = X_2$. Then

$$Bs = BO(XO/XIH)^{1.66}\,. \qquad (2.5.6)$$

As the IH coil is energized, a magnetic field of opposite direction is created in the annulus by currents flowing in the conducting surface at the outer boundary of the annulus (near the inside surface of the compression coil). This causes the net flux linked by the compression coil to be zero and prevents the large implosion heating voltages from appearing in the compression coil. To recover some of the energy used in the creation of the magnetic field in the annulus, the conductor at the outer annulus surface is comprised of a helical (third) coil whose leads are connected to a storage capacitor. On the rapid field rise of the IH coil, the capacitor acts as a short circuit, but on a longer time scale the current charges the capacitors, transferring the magnetic energy in the annulus into them, except for ohmic heating losses. The voltage induced in the compression coil is reduced by the ratio of the rate of rise of the flux associated with implosion heating to the charging time of the capacitors.

As the IH field rises, the annulus (AANN), the thermal insulator between the IH coil and the blanket, the electrical insulator in the blanket (AISE), and the space between the plasma and the blanket are immediately permeated with the magnetic flux. The penetration of the blanket is assumed to be a function of time; the relation for this is in Sec. 2.2, Skin Depth Calculation. We assume that there is no penetration of the plasma by the magnetic fields during compression. Thus the total flux in the annulus is

$$B_2A_2. \tag{2.5.7}$$

Since the area penetrated by the magnetic field inside the coil does not change with time, and since flux is always conserved, the field strength inside the coil, B_1, can be found from the expression

$$B_1A_1 = B_2A_2, \tag{2.5.8}$$

if the field inside the annulus, the area of the annulus, and the area penetrated by the fields inside the coil are known. Schematic representation of the fields on the plasma and in the annulus is shown in Fig. 1.1-6 for the bucking option.

The magnetic field in the annulus during the fast IH pulse is

$$\mathrm{BRFF} = B_s(\mathrm{AINS} + \mathrm{APFWIH} + \mathrm{ABLKTFF} + \mathrm{AISE})/\mathrm{AANN}, \tag{2.5.9}$$

during the slow IH pulse

$$\mathrm{BRFS} = B_s(\mathrm{AINS} + \mathrm{APFWIH} + \mathrm{ABLKTFS} + \mathrm{AISE})/\mathrm{AANN}, \tag{2.5.10}$$

during the staging

$$\mathrm{BRFSG} = B_s(\mathrm{AINS} + \mathrm{APFWIH} + \mathrm{ABLKT})/\mathrm{AANN}, \tag{2.5.11}$$

during the window

$$\mathrm{BAW} = \mathrm{Bs}(\mathrm{AINS} + \mathrm{AANN} + \mathrm{APFWIH})/\mathrm{AANN}\ . \tag{2.5.12}$$

The compression field before the window was opened is

$$\mathrm{BPW} = \mathrm{BAW}(\mathrm{AANN})/\mathrm{RCI}^2 \cdot \pi\ . \tag{2.5.13}$$

The compression field immediately after it is closed is

$$\mathrm{BIHF} = \mathrm{BAW} \cdot \mathrm{AANN}/(\mathrm{AANN} + \mathrm{AINS} + \mathrm{AISE}). \tag{2.5.14}$$

The field in the annulus resulting from the negative IH spike is

$$\mathrm{BANIHW} = \mathrm{Bs}(\mathrm{AINS} + \mathrm{APFWIH} + \mathrm{ABLKTFF})/(\sqrt{2} \cdot \mathrm{AANN}). \tag{2.5.15}$$

2.6 Timing

The timing of the staging is critical to the system operation, and the times used in the staging calculations are very important, since the length and sequence of the capacitor discharges determine, to a degree, the efficiency of the system. The times used are the duration from the start of the IH sequence until the end of the pulse or cycle.

The duration of the entire staging sequence, τ_{hold}, is the length of time that is required for the compression field to rise from zero to the strength of the IH field (Bs). The rate of rise of the compression field is assumed to be linear and is expressed as

$$BO/\tau_r \, , \qquad (2.6.1)$$

where τ_r is the time for the compression field to rise to its maximum value, BO. Thus,

$$\tau_{hold} = \frac{B_s \quad \tau_r}{Bo} \qquad (2.6.2)$$

The staging time values chosen are listed in Table 2.6-1. These values are highly dependent upon switch and circuit technology and are believed to represent current state of the art of switching high voltage/current circuits with switches with resistances as shown in Table 2.4-1.

Table 2.6-1
STAGING TIME VALUES

Code	Definition	Time (sec)
τWR	Rise time of window	10×10^{-6}
τ_f	Hold time, fast IH circuit	5×10^{-6}
τ_s	Hold time, slow IH circuit	50×10^{-6}
τSGF	Hold time, fast staging circuit	0.5×10^{-3}
τSGS	Hold time, slow staging circuit	τ_{hold}
τWH	Hold time, window	20×10^{-6}

2.7 Currents

The currents are calculated from Ampere's law and the required magnetic fields. In convenient units $I/\ell(A/cm) = B/0.4\pi$, where B is the magnetic field in gauss. The eddy current in the blanket which is the opposite direction of the applied field is a function of the applied magnetic field:

$$IEB = Bs\ 100/0.4\pi \text{ amperes,} \tag{2.7.1}$$

where a length of 100 cm is employed. The current in the IH coil is also

$$IEB = Bs\ 100/0.4\pi \tag{2.7.2}$$

The current in the window is given by

$$IW = BAW \cdot 100/0.4\pi \tag{2.7.3}$$

For the Marshall coil, however, the current is not flowing in the theta direction. An additional, axial current component is needed to obtain the required implosion heating field:

$$IZ = I\alpha_m\ /\text{Sin}\ \alpha_m \tag{2.7.4}$$

$$I\theta = I\alpha_m\ /\text{Cos}\ \alpha_m \tag{2.7.5}$$

Since the desired current direction to obtain the implosion heating field is in the theta direction,

$$IIH = I\theta. \tag{2.7.6}$$

For the segmented coil, the current required to obtain the implosion field is

$$IIH = \frac{(BRFF+Bs)\times 100}{0.4\pi}. \tag{2.7.7}$$

For the Marshall coil, the current is cancelled in the Z direction at all locations except inside the coil layers. Here the current is additive, creating a very high flux tending to explode the coil. This equivalent magnetic pressure within the coil can be approximated by

$$PMARSHALL \cong (B\ COMP\ \alpha_m)^2 \times 2/8{,}000{,}000\pi \text{ atmospheres} \tag{2.7.8}$$

and

$$(B\ COMP\ \alpha_m) \cong \frac{B_s}{\cos\alpha_m}. \tag{2.7.9}$$

B COMP α_m is the field generated by the total current in the conductor, I α_m, and the factor of two is due to the current on both sides of the coil. Actually, the theta component of field is not additive, and the equation above for pressure within the Marshall coil is somewhat conservative.

2.8 Energy Calculation

The energy terms, for the purpose of analysis, can be separated into three different categories: 1) heating from the resistance in the wires, 2) energy dissipated in maintaining the magnetic fields, and 3) summation terms. The bucking field only rises to a value of Bs/ $\sqrt{2}$; then the energy required will be half that used by the brute force option. The terms which are different between the brute force and the window option are calculated in separate subprograms. Those terms which are the same are not calculated within the subprograms.

2.8.1 I^2R Terms

The heat generated by the resistance of conductors is given by

$$E = I^2 RE\ \tau, \qquad (2.8.1)$$

where I is the current flow through the circuit, RE is the electrical resistance of the circuit, and τ the time the current is closed. The heat generated by the entire staging system would be

$$QI2R = IIH^2 \cdot REIHF \cdot \tau_{hold} \cdot 10^{-6}, \qquad (2.8.2)$$

where IIH is the current in the IH coil, if only the fast circuit was used. The heat generated in the fast IH circuit is calculated using a time ratio of the total heat generated. The heat generated in the fast IH circuit is then

$$WI2RIHF = QI2R \cdot (\tau_f/\tau_{hold}). \qquad (2.8.3)$$

The heat generated during the three other periods is calculated by ratio to the heat during the IH fast period. The slow IH heat loss is

$$\text{WI2RIHS} = \text{WI2RIHF} \cdot (\tau_s - \tau_f)/\tau_f \cdot (\text{REIHS/REIHF}), \quad (2.8.4)$$

where $(\tau_s - \tau_f)$ is the duration of the slow IH circuit.

The heat generated in the fast staging circuit is

$$\text{WI2RSGF} = \text{WI2RIHF} \ (\tau_{SGF} - \tau_s)/\tau_f)(\text{RESGF/REIHF}), \quad (2.8.5)$$

where τ_{SGF} is the hold time of the fast staging circuit and $(\tau_{SGF} - \tau_s)$ is the duration of the circuit.

The heat generated in the slow staging circuit is

$$\text{WI2RSGS} = \text{WI2RIHF} \ (\tau_{SGS} - \tau_{SGF})/\tau_f(\text{RESGS/REIHF}), \quad (2.8.6)$$

where τ_{SGS} is the hold time in the slow staging circuit.

The heat generated by the eddy currents during the fast IH circuit is

$$\text{WI2RIHEBF} = \text{IEB}^2 \cdot \text{REBF} \cdot \tau_f \cdot 10^{-6}. \quad (2.8.7)$$

The heat generated by the circuit eddy current in the slow IH circuit is

$$\text{WI2RIHEBS} = \text{IEB}^2 \cdot \text{REBS} \cdot (\tau_s - \tau_f) \cdot 10^{-6}, \quad (2.8.8)$$

where REBS is the resistance of the slow eddy current.

For the window option, the implosion heating loss in the coil can be expressed as

$$\text{WI2RIHF} = \text{QI2R} \cdot \frac{\tau_f}{\tau_{hold}} \left[\frac{(\text{BAW} + \frac{\text{Bs}}{\sqrt{2}})}{\text{Bs}} \right]^2, \quad (2.8.9)$$

since the current must also sustain the field in the annulus during this period.

The heat generated by the window coil circuit is

$$\text{WI2RW} = \text{IW}^2 \cdot \text{REW} \ (\tau_{WH} + \tau_{WR}/3) \cdot 10^{-6}, \quad (2.8.10)$$

where the quantity $(\tau_{WH} + \tau_{WR}/3)$ is an approximation of the actual hold time since the current is increasing during the rise time. Other I^2R terms are discussed in Sec. 2.8.5.

2.8.2 Plasma Terms

The internal energy of the plasma is

$$E = 3/2\ nKT. \tag{2.8.11}$$

Then the increase in the energy is

$$\Delta E = \Delta[3/2\ nKT]. \tag{2.8.12}$$

The change in pressure of the plasma can be expressed as

$$\Delta P = \Delta[nKT]. \tag{2.8.13}$$

Since the change in the internal pressure is equal to the change in magnetic pressure on the plasma

$$\Delta P = \frac{\Delta(Bx^2)}{8\pi}, \tag{2.8.14}$$

then it follows that

$$\Delta(3/2nKT) = \Delta E = 3/2\ \frac{\Delta(Bx^2)}{8\pi}. \tag{2.8.15}$$

Since there is virtually no magnetic pressure on the plasma at the start of the IH cycle $\Delta Bx = Bs$, the total increase in the energy of the plasma WPIH is

$$WPIH = 3/2\ \frac{Bs^2}{8\pi} \cdot 10^{-11}. \tag{2.8.16}$$

2.8.3 Magnetic Energy

The energy in MJ/m required to maintain a magnetic field is $\frac{Bx^2}{8\pi} \cdot 10^{-11}A$, where the quantity $\frac{Bs^2}{8\pi}$ is the pressure produced by the field Bs, A is the area that the magnetic field occupies, and 10^{-11} is a unit conversion factor. The magnetic energy in the thermal insulator is

$$WBIHI = \frac{Bs^2}{8\pi} \cdot AINS. \tag{2.8.17}$$

The magnetic energy in the insulators in the blanket WBIHIS after the fast IH pulse, τ_f, is

$$WBIHIS = \frac{Bs^2}{8\pi} \cdot AISE \cdot 10^{-11}. \tag{2.8.18}$$

The field penetrates the conductor in the blanket as discussed in Sec. 2.2, Skin Depth Calculation. Assuming uniform field to that depth, the magnetic energy supplied to the conducting portion of the blanket is

$$\mathrm{WBIHF} = \frac{\mathrm{Bs}^2}{8\pi} \cdot 10^{-11}\ \mathrm{ABLKTFF}. \qquad (2.8.19)$$

The increase in magnetic energy in the blanket conductors from the slow IH field, WBIHS, is

$$\mathrm{WBIHS} = \frac{\mathrm{Bs}^2}{8\pi}\ (\mathrm{ABLKTFS} - \mathrm{ABLKTFF}) \cdot 10^{-11}, \qquad (2.8.20)$$

where the quantity (ABLKTFS - ABLKTFF) is the additional area penetrated by the slow IH field.

The magnetic energy in the blanket resulting from the staging circuits is

$$\mathrm{WBIHB} = \frac{\mathrm{Bs}^2}{8\pi}\ (\mathrm{ABLKT} - \mathrm{ABLKTFS}) \cdot 10^{-11}, \qquad (2.8.21)$$

where the quantity (ABLKT - ABLKTFS) is the area of the blanket not penetrated by the fast and slow IH pulses. The magnetic energy in the implosion heating coil is

$$\mathrm{WBIHM} = \frac{\mathrm{Bs}^2}{8\pi}\ \mathrm{AIHC} \cdot 10^{-11}. \qquad (2.8.22)$$

The magnetic energy in the space between the plasma and the first wall is

$$\mathrm{WBIHPW} = \frac{\mathrm{B}^2}{8\pi}\ \mathrm{APFWIH} \cdot 10^{-11} \qquad (2.8.23)$$

The magnetic energy required of the fast negative implosion heating coil circuit in the bucking option is similar to the brute force method, except WBIHI, WBIHIS, WBIHPW and WBIHM are all reduced by a factor of two because the implosion heating field in this option is less by a factor of $\sqrt{2}$.

The magnetic energy in the window bucking field, WBW, is

$$\mathrm{WBW} = \frac{10^{-11}}{8\pi}\ [(\mathrm{BAW}^2 \cdot \mathrm{AANN}) + (\mathrm{BPW})(\mathrm{ABLKT} - \mathrm{AINS} - \mathrm{AISE})] - \mathrm{BPW} \cdot \mathrm{RCI}^2, \qquad (2.8.24)$$

where the quantity $(BAW^2\ AANN) + (BPW)(ABLKT - AINS - AISE)$ is the positive magnetic flux and the quantity $(BPW^2\ RCI^2)$ is the area where there is negative magnetic flux. By simplification, Eq. 2.8.23 can be expressed as

$$WBW = \frac{10^{-11}}{8\pi} [BAW^2 \cdot AANN + (BPW^2)(ABLKT - AINS - AISE - RCI^2)]. \tag{2.8.25}$$

The magnetic energy in the return flux field during the fast IH pulse is

$$WBRFIHF = \frac{BRFF^2}{8\pi} \cdot AANN \cdot 10^{-11}. \tag{2.8.26}$$

for the brute force option and

$$WBRFIHF = \frac{[(BAW + BANIHW)^2 - BAW^2]}{8\pi} \cdot AANN \cdot 10^{-11} \tag{2.8.27}$$

for the bucking option.

The difference reflects change in the magnetic field from BAW before the IH pulse to BAW + BANIHW after the IH pulse.

The magnetic energy resulting from the return flux field during th slow IH pulse for the brute force option is

$$WBRFIHS = \frac{BRFS^2}{8\pi} \cdot AANN \cdot 10^{-11} - WBRFIHF. \tag{2.8.28}$$

The term WBRFIHF is subtracted because of the manner in which BRFS is calculated. In the bucking option WBRFIHS = 0 because the energy in the slow pulse was included in WBRFIHF.

The magnetic energy in the return flux from the staging pulses is

$$WBRFSGF = \frac{BRFSG^2}{8\pi} \cdot AANN \cdot 10^{-11} - WBRFIHF - WBRFIHS \tag{2.8.29}$$

Again the terms WBRFIHF and WBRFIHS are subtracted as a result of the manner in which BRFSG is calculated. This term does not exist for the bucking option since there is no staging circuit as such.

2.8.4 Magnetic Energy Conservation

The conversion process of the magnetic energy collected from the annulus by the third coil to electrical energy is assumed to have an efficiency ηRFR. A value of 70 per cent for ηRFR was assumed in our calculations. The energy that is returned to the system from the return flux is

$$\text{WDC2} = (\text{WBRFIHF} + \text{WBRFIHS} + \text{WBRFSGF})\ \eta\text{RFR} \qquad (2.8.30)$$

for the brute force option and

$$\text{WDC2} = (\text{WBRFIHF})\ \eta\text{RFR} \qquad (2.8.31)$$

for the bucking option, since the staging circuits are not used in the bucking option.

2.8.5 Heating Terms

The heat generated in the blanket by eddy current heating during the staging portion of the cycle is

$$\text{QIBES} = \frac{Bs^2}{8\pi}\ (\text{ABLKT} - \text{AINS}) \cdot 10^{-11} - \text{WI2RIHEBF} - \text{WI2RIHEBS} \qquad (2.8.32)$$

Since the rise time is so short, the total heat generated by the eddy currents in the blanket is equal to the magnetic energy in the volume. This is the basis for calculating the eddy current heating during the IH cycle. For the bucking option

$$\text{QIBE} = \text{QEHBW} + \text{WI2IHEBF} + \text{WI2RIHEBS}, \qquad (2.8.33)$$

where QEHBW is the eddy current losses in the blanket resulting from the window circuit which is

$$\text{QEHBW} = \text{WI2RIHEBF} \cdot [(\frac{\tau_{RW}}{\tau_f}) \cdot (\frac{\delta\text{SWF}}{\delta\text{RWR}}) + (\frac{\tau_{WH}}{\tau_f})\ (\frac{\delta\text{SWF}}{\delta\text{SWWH}})], \qquad (2.8.34)$$

the quantities ($\frac{\tau_{WR}}{\tau_f}$) and ($\frac{\tau_{WH}}{\tau_f}$) are used as time scaling factors to account for the different pulse times and ($\frac{\delta_{SWF}}{\delta_{RWR}}$)($\frac{\delta_{SWF}}{\delta_{SWWH}}$) are used for scaling factors of the resistance of the eddy currents to account for different skin depths.

The magnetic energy retained from the staging option in the conductor area of the blanket (ABLKT - AISE) is found from a ratio of areas and the energy in the blanket:

$$QIBE = WBIHB \cdot \frac{(ABLKT - AISE)}{ABLKT} \qquad (2.8.35)$$

for the brute force option.

The power leaked from the IH coil to the compression coil is

$$QIC = QICE + QICL, \qquad (2.8.36)$$

where QICE is power generated by electrical eddy currents caused by the IH coil, and QICL is heat conduction. These terms are both assumed to be zero in the current stage of our analysis.

The heat leaked from the blanket to the IH coil is calculated by ratio from the heat leak calculated in RTPR and the ratio of the radii and the thickness of the thermal insulation used in the calculations:

$$QBI = 0.43 \left(\frac{RIH}{RIH\_RTPR} \right) \left(\frac{RIH\_RTPR - RBO\_RTPR}{RIH - RBO} \right) \qquad (2.8.37)$$

where RIH_RTPR and RBO_RTPR are the dimensions in RTPR.

2.9 Energy Summations

For simplification the energy terms are summed into the needed energy flows for use in the TPFPP analysis as shown in Fig. 2.9-1.

The heat from IH system that is rejected to the cooling water is

$$QIW = WNI + QBI - QIC - QIBES + WSGS + WSGF + WIHS + WI2RIHF + WBRFIHF\,(1 - \eta RFR) + WBIHM + WI2RW - WDC2, \qquad (2.9.1)$$

where WNI is from nuclear radiation energy deposited in the IH coil, WSGS is the slow staging energy, WSGF is the fast staging energy, and WIHS is the slow IH energy.

The energy used by the window circuit WWIN is the sum of energy losses resulting:

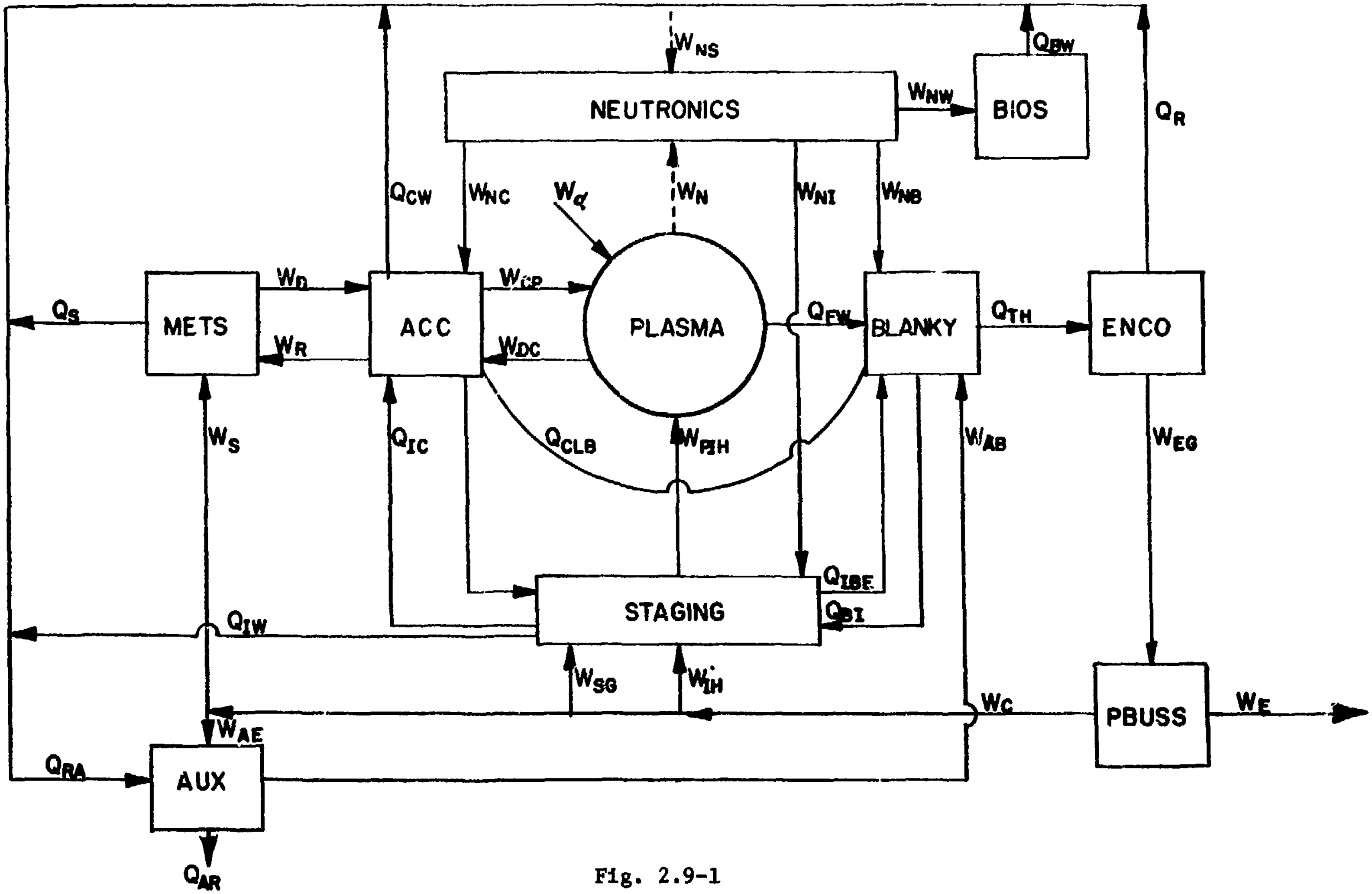


Fig. 2.9-1

Energy flows for TPFPP analysis

WWIN = WBW + WI2RW + QEHBW. (2.9.2)

The magnetic energy in the system at the end of the fast IH cycle WBIHF is

WBIHF = WBIHI + WBIHM + WBIHPW + WBIHIS +
WBRFIHF (2.9.3)

The energy used from storage during the fast IH pulse WIHF is

WIHF = WPIH + WBIHF + WI2RIHF + WI2RIHEBF.
(2.9.4)

The energy used during the slow IH pulse WIHS is

WIHS = WBIHS + WI2RIHS + WI2RIHEBS + WBIHS + (2.9.5)
WI2RIHS + WI2RIHEBS.

The total energy used by the IH fast and slow pulses, WIH, is

WIH = WIHF + WIHS (2.9.6)

where WIHF is the energy used during the fast IH pulse.

The energy used in the fast staging circuit WSGF is

WSGF = WI2RSGF + WBRFSGF + WBIHB + QIBES. (2.9.7)

The term WBIHB is added to the staging term to simplify the summations since the term does not exist for the bucking option. For the bucking option the term WWIN is set equal to WSGF to simplify the later calculations.

The energy in the slow staging circuit WSGS is

WSGS = WI2RSGS. (2.9.8)

In the bucking option WSGS = 0, since the staging circuit is not used. The total energy used in the staging system for the brute force option or window circuit for the bucking option is

WSG = WSGF + WSGS. (2.9.9)

The increase in the thermal energy of the plasma QPIH is

QPIH = WPIH, (2.9.10)

since there is no other work transported to the plasma.

3. RESULTS

Some of the first calculations are given in Table 3.1-1. The maximum allowed compression ratio shows a striking effect upon the staging and implosion energy requirements: as XO increases from 0.2 to 0.4, in the energy requirements increase nearly an order of magnitude. This is caused by the necessity of increasing the implosion heating magnetic field to obtain ignition at the required compression ratio with the maximum magnetic field (taken as 140 kG in these calculations). This increased magnetic field in turn leads to increased current and voltage requirements, resulting in larger energy requirements. In the particular case of XO of 0.4, the major energy requirement is for slow staging energy. This requirement is due to high joule losses in the coil, leads, and switch resulting from the need for both increased current and increased holding time, as shown in Eq. 2.6-2.

Another reason for the sharply increased energy is the return flux. This particular energy requirement can be reduced if larger annuli are used between the implosion heating coil and the compression coil shield. In practice, however, this approach is limited by the increased the compression coil diameter and a corresponding increase in the energy required for compression and for the compression coil energy store. Studies indicate that as the annulus increases above about 9 cm (for the conditions used here), further decreases in staging energy store are counterbalanced by compression requirements.

An option considered to reduce staging energy requirements was bucking field as described earlier. The results for calculations using this option are shown in Table 3.1-2 for the same conditions and parameters as used for the brute force method. It is seen that for high values of XO, the bucking field results in considerable energy savings compared to the brute force methods. At XO of 0.2, there is little difference between the methods, but as the staging time and magnetic field increase, the bucking method becomes more and more attractive.

TABLE 3.1-1

DEPENDENCE OF VARIOUS PARAMETERS ON XO USING BRUTE FORCE OPTION[a]

Parameter	XO 0.2	XO 0.227	XO 0.3	XO 0.4	Units
WIH	5.38	8.8	22.90	58.61	MJ/m
WSG	26.86	50.3	174.77	617.57	MJ/m
WIHF	3.1	5.01	13.00	33.34	MJ/m
WIHS	2.3	3.8	9.9	25.27	MJ/m
WSGF	16.6	27.09	70.44	180.38	MJ/m
WSGS	10.3	23.2	104.33	437.	MJ/m
WBIHB	1.63	2.48	6.27	16.29	MJ/m
WBRFIHF	0.77	1.17	2.96	7.696	MJ/m
WBRFSGF	7.04	10.7	27.07	70.35	MJ/m
WI2RSGF	6.87	12.24	32.96	82.98	MJ/m
WI2RIHF	1.15	2.04	5.51	13.86	MJ/m
BS	15.3	18.8	30	48	kG
BRFF	18.9	23.4	37	60	kG
BRFSG	60.3	74.4	118	191	kG
IIHF	4.8	6.4	10.5	16.6	MA
ISGF	10.6	14.1	23	36.7	MA
Pressure in Marshall Coil	27.45	39.05	96.23	253.0	atm

[a](ΔANN = 9 cm, BO = 140 kG, B = 50 cm)

TABLE 3.1-2

DEPENDENCE OF VARIOUS PARAMETERS ON XO USING BUCKING FIELD OPTION[a]

Parameter	XO			Units
	0.2	0.3	0.4	
WIH	7.19	31.495	80.26	MJ/m
WSG	4.49	17.27	44.88	MJ/m
WIHF	5.18	22.55	57.53	MJ/m
WIHS	2.01	8.9	22.7.	MJ/m
WSGF	4.49	17.27	44.88	MJ/m
WSGS	0.00	0.00	0.00	MJ/m
WBW	4.39	16.87	43.85	MJ/m
WBRFIHF	1.87	7.196	18.70	MJ/m
WBRFSGF	0.00	0.00	0.00	MJ/m
WI2RIHF	2.75	13.22	33.26	MJ/m
BS	15.3	29.9	48.2	kG
BAW	25.8	50.7	81.7	kG
BRFF	18.9	37.1	59.8	kG
BRFSG	60.3	118.2	190.5	kG
IIHF	4.8	10.5	16.6	MA
ISGF	10.6	23.2	36.7	MA
Pressure in Marshall Coil	27.45	96.23	253.0	atm

[a](ΔANN = 9 cm, BO = 140 kG, B = 50 cm)

The Marshall coil design was adopted in Ref. 1 for its ease of installation. It consumes extra energy, however, because the current is not purely tangential but has an added axial component. Although methods to install a coil with pure tangential current have not been pursued, the possible gains were investigated. Further, at high XO values, the magnetic pressures tending to crush the Marshall coil are very large. These results are shown in Table 3.1.3. The segmented coil makes some improvement but probably not enough to pursue the mechanical complexities of the segmented coil leads and accessibility

A second option to reduce the return flux losses was also examined. For illustrative purposes the insulator area within the compression coil was also utilized for return flux during implosion and staging. This possibility is discussed in Ref. 1, but induced voltage questions at the time dictated the use of the third coil to shield the compression coil from the implosion heating field. If the insulator within the compression coil could be utilized, the return flux losses would be decreased, as shown in Table 3.1.4.

Finally, an idea suggested by Dr. F. Ribe of LASL was modeled into the calculations. Ribe suggested that although the plasma is compressed to 76 per cént of the initial radius by the implosion, there is no reason why the staging portion of the cycle must hold the plasma at this compression. Instead, after implosion heating, the plasma may be allowed to expand to near the first wall, e.g., $x = 0.95$, and held until the compression field rises sufficiently to maintain plasma containment. This idea offers the possibility of not only reduced magnetic field requirements in staging, but also reduced holding time because the compression field does not need to be as high to provide containment at the larger radius. This option was also programmed, and some typical results are shown in Table 3.1.5.

TABLE 3.1-3

DEPENDENCE OF VARIOUS PARAMETERS ON STAGING OPTIONS[a] USING SEGMENTED COIL

Parameter	Option		Units
	Brute	Bucking	
WIH	14.68	17.23	MJ/m
WSG	86.65	17.27	MJ/m
WIHF	8.92	12.77	MJ/m
WIHS	5.8	4.5	MJ/m
WSGF	47.08	17.27	MJ/m
WSGS	39.6	0.0	MJ/m
WBIHB	6.27	---	MJ/m
WBRFIHF	2.96	7.196	MJ/m
WBRFSGF	27.07	0.00	MJ/m
WI2RSGF(WI2RW)	9.61	0.04	MJ/m
WI2RIHF	1.43	3.43	MJ/m
BS	29.9	29.9	kG
BRFF	37.1	37.1	kG
BRFSG	118.2	118.2	kG
IIHF	5.3	5.3	MA
ISGF	11.8	11.8	MA
E-Theta	5.06	5.06	kV/cm

[a](DANN = 9 cm, BO = 140 kG, XO = 0.3, B = 50 cm)

TABLE 3.1-4

DEPENDENCE OF VARIOUS PARAMETERS ON ALLOWING RETURN FLUX INTO COMPRESSION COIL USING MARSHALL COIL[a]

Parameter	Options Brute Force with Marshall Coil	Units
WIH	11.90	MJ/m
WSG	23.95	MJ/m
WIHF	6.21	MJ/m
WIHS	5.69	MJ/m
WSGF	15.70	MJ/m
WSGS	8.25	MJ/m
WBIHB	6.27	MJ/m
WBRFIHF	0.29	MJ/m
WBRFSGF	2.69	MJ/m
WI2RSGF	2.61	MJ/m
WI2RIHF	1.38	MJ/m
BS	29.9	kG
BRFF	3.68	kG
BRFSG	11.74	kG
IIHF	5.2	MA
ISGF	6.5	MA
Pressure in Marshall Coil	96.23	atm

[a](DANN = 9, BO = 140 kG, XO = 0.3, B = 50 cm)

TABLE 3.1-5

DEPENDENCE OF VARIOUS PARAMETERS ON XO ALLOWING PLASMA EXPANSION TO 0.95

Parameter	XO			Units
	0.2	0.3	0.4	
WIH	2.56	10.92	27.94	MJ/m
WSG	11.02	66.62	226.61	MJ/m
WIHF	1.48	6.198	15.89	MJ/m
WIHS	1.1	4.7	12.05	MJ/m
WSGF	7.93	33.58	85.99	MJ/m
WSGS	3.1	33.04	140.6	MJ/m
WBIHB	0.78	2.99	7.77	MJ/m
WBRFIHF	0.37	1.41	3.67	MJ/m
WBRFSGF	3.36	12.90	33.54	MJ/m
WI2RSGF	3.28	15.71	39.56	MJ/m
WI2RIHF	0.55	1.41	6.61	MJ/m
BS	10.5	20.6	33.3	kG
BRFF	13.1	25.6	41.3	kG
BRFSG	41.6	81.6	131	kG
IIHF	3.3	7.2	11.5	MA
ISGF	7.3	16.0	25.4	MA
Pressure in Marshall Coil	13.08	45.87	120.6	atm

[a](ΔANN = 9 cm, BO = 140 kG, B = 50 cm)

4. CONCLUSIONS

Results have indicated that the energy requirements for implosion and staging in a staged theta-pinch reactor can be very high with brute force techniques. More importantly, they show that potential reductions in those energy requirements are substantial. Further, specific, apparently feasible techniques offer very attractive alternatives to the brute force option. Both the bucking and the plasma relaxation schemes appear practical to greatly reduce the energy requirement.

A significant result of this study is verification of the extreme importance of the plasma compression ratio, XO. The need to determine the minimum value necessary for plasma stability is apparent. Methods to reduce the energy requirements for implosion and staging useful for one range of XO values may not be so useful for other ranges. Thus, until minimum permissible XO values necessary for toroidal plasma stability are determined, several staging options must be pursued.

The resulting code is now operational and incorporated into the larger parametric systems code, TPFPP. This latter code is being used to perform economic studies on the theta-pinch power plant. Some of these preliminary results will be given in Ref. 6.

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