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NONLINEAR INTERACTION OF AN ION FLUX WITH PLASMA

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SUMMARY

The present report discusses the interaction of an ion beam, formed during the charge exchange of injected neutral atoms, with a plasma. Methods of analytical study by means of quasi-linear equations as well as two-dimensional numerical modelling are used. It is shown that at a beam velocity $U_o/C_s \lesssim 1/2$, the relaxation process may be described by using the theory of quasi-linear relaxation of electron beams, at $U_o/C_s \gtrsim 10$; one can neglect the slowing down of the ion beam and consider only the angular spread. An analytical dependence of the spread angle on time was obtained. On the basis of the ion beam relaxation theory evolved, experiments on charge exchange of plasma fluxes on a gas target are analyzed. It is shown that the anomalous scattering of the plasma flux observed in a series of experiments may be explained by the interaction of ions of the flux with ion-acoustic oscillations of the target plasma. Consideration of damping of ion-acoustic noise by the plasma electrons and ions leads to a limitation of the relaxation of the angular distribution function. The relationships obtained are in good agreement with the experimental results.

At the present time, problems involving the injection of fast neutral beams into a tokamak plasma are attracting considerable interest.[1] As a result of the ionization, an ion beam that may interact intensively with the plasma is formed in the plasma of fast atoms. Moreover, as will be shown below, the nature of the interaction is substantially different for different beam energies: at $V \gg T_e$ (V being the energy of the beam particles and T_e, the plasma electron temperature), elastic scattering of the beam ions by the plasma oscillations predominates, occurring almost without energy loss; at $V \lesssim T_e$, the main process is retardation of the beam with effective energy transfer to the plasma. The study of the interaction of ion beams with plasma is also of great interest for the processes of formation of neutral atom fluxes during the charge exchange of ions on the gas target, since the ion flux may intensively interact with the plasma formed in the target during the charge exchange.[2] The paper deals with a theoretical study of the interaction of ion beams with plasma over the entire range. Methods of analytical study by means of quasi-linear equations were used along with numerical modelling, which has recently become widely employed in plasma physics.[3] On the basis of theoretical calculations, experiments on the charge exchange of plasma fluxes on a gas target were analyzed. It is known from linear theory[4] that when $T_e \gg T_i$ (T_e, T_i being the temperature of the plasma electrons and ions), an ion beam can start oscillations in plasma in the range of phase velocities $V_{T_i} < \omega/K < V_{T_e}$ ($V_{T_i} = \sqrt{2T_i/M}$, $V_{T_e} = \sqrt{2T_e/m}$, M and m being the masses of ions and electrons; ω and K being the frequency and wave vector of the oscillations), which are described by the following dispersion relation:

$$1+\frac{1}{K^2 d^2}-\frac{\omega_{pi}^2}{\omega^2}+\frac{\omega_{pi}^2}{K^2}\frac{n_e}{n_0}\int\frac{\vec{K}\frac{\partial f_e}{\partial v}}{\omega-Kv}\,dv=0$$

where $d=(T_e/4\pi n_0 e^2)^{1/2}$ is the Debye radius; $\omega_{pi}=\sqrt{4\pi n_0 e^2/M}$ is the ionic plasma frequency; n_o, n_b are the plasma and beam densities, and f_b is the velocity distribution function of the beam. In the absence of the beam, this equation describes oscillations with the dispersion law

$$\omega_k=\omega_{pi}/(1+1/k^2 d^2)^{1/2}$$

and the effect of the magnetic field on the plasma oscillations may be neglected.

Since the conditions $\omega_{He} \sim \omega_{pe}$ are characteristic of tokamak type devices, $\omega_{Hi} \sim \sqrt{\frac{m}{M}}\,\omega_{pi}$ holds for ionic frequencies, and the increment of oscillations started by the beam under typical experimental conditions is $\gamma \sim (0.1 \div 0.01)\,\omega_{pi} \gtrsim \omega_{Hi}$. The beam leads to an instability of these oscillations and to the appearance of intensive noise in the plasma. The reverse effect of the noise on the distribution function of the beam ions can be taken into account by use of the quasi-linear equations[5]

$$
\begin{aligned}
&\frac{\partial f_b}{\partial t} + v\frac{\partial f_b}{\partial r} = \frac{\partial}{\partial v_\alpha} D_{\alpha\beta}\frac{\partial f_b}{\partial v_\beta} \\
&D_{\alpha\beta} = \frac{8\pi^2 e^2}{M^2}\int \frac{k_\alpha k_\beta}{k^2}\,\frac{W_k}{\omega\frac{\partial \varepsilon}{\partial \omega}}\,\delta(\omega - kv)\,d^3k \\
&\frac{\partial W_k}{\partial t} + \frac{\partial \omega_k}{\partial k}\,\frac{\partial W_k}{\partial r} = 2\gamma_k W_k \\
&\gamma_k = \frac{\pi}{2}\,\frac{n_b}{n_0}\,\frac{\omega_k^3}{k^2}\int k\frac{\partial f}{\partial v}\,\delta(\omega_k - kv)\,d^3v
\end{aligned}
\qquad (1)
$$

where W_k is the spectral energy density of the oscillations. The basic features of the oscillation spectrum for a kinetic beam $\Delta V/U_0 > (n_b/n_0)^{1/3}$ (where ΔV is the velocity spread of the beam particles and U_0 is the beam velocity) can be explained by using the Maxwell distribution function as the model. One thus finds that for different ratios C_s/U_0, the angle of distribution of the fastest-growing oscillations changes. When $U_0 < \sqrt{\frac{2}{3}}\,C_s$, the oscillations develop primarily along the propagation of the beam ($\theta = 0$). At high beam energies ($U_0 > \sqrt{\frac{2}{3}}\,C_s$), the noise increases most intensively at an angle satisfying the inequality $\cos\theta \simeq \sqrt{\frac{2}{3}}\,\frac{C_s}{U_0}$. Thus, there are two regions where the interaction of the ion beam with the plasma is essentially different. When $V \lesssim T_e$, the beam starts ion-Langmuir oscillations mainly along the direction of its propagation, and is therefore effectively slowed. When $V \gg T_e$, the ion-acoustic oscillations started by the beam propagate almost at right angles to its velocity, causing angular scattering of the beam without energy loss. In the intermediate region $V \gg T_e$, the two processes are simultaneous.

Let us examine the interaction of an ion beam with a plasma at $U_o \lesssim C_s/2$. In this case, the formation of noise takes place in a narrow cone around the direction of propagation. The cone aperture may be estimated as follows:

$$\Delta\theta \sim 1/\sqrt{\Lambda}$$

where Λ is the Coulomb logarithm.

The narrowness of the noise spectrum makes it possible to use one-dimensional quasi-linear equations to within the replacement of m by M, equations which are the same as those describing the interaction of an electron beam with a plasma. The velocity V_o of the front of the distribution function is described by the following relation:[6]

$$\ln \frac{V_o}{U_o} + \frac{U_o}{V_o} = \pi \frac{\omega_{pi}}{\Lambda} \frac{n_e}{n_o} t + 1$$

During the characteristic relaxation time $\tau \sim 10 \frac{n_o}{n_o} \omega_{pi}^{-1}$, the beam gives up to 70% of its energy to the noise, and the noise spectrum has a sharp maximum near ω_{pi}/U_o. Consideration of the finiteness of the width of the noise spectrum with respect to $k_\perp$ does not change the one-dimensional characteristics of the process.[7]

If the beam velocity $U_o \gg C_s$, ion-acoustic noise with a vector almost non-perpendicular to the direction of propagation of the beam develops in the plasma, causing only a change in the direction of the velocity without any significant energy loss.[8] In this case, to study the process of spreading it is convenient to change to spherical coordinates. Using the axial symmetry of the problem and integrating over the polar angle, we write the quasi-linear equations in the form

$$\frac{\partial f_i}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left(A \frac{\partial f_i}{\partial v} + B \frac{\partial f_i}{\partial \theta} \right) + \frac{1}{v^3 \sin\theta} \frac{\partial}{\partial \theta} \times \qquad (2)$$

$$\times \left[\sin\theta \left(B \frac{\partial f_i}{\partial v} + C \frac{\partial f_i}{\partial \theta} \right) \right]$$

$$\left\| \begin{matrix} A \\ B \\ C \end{matrix} \right\| = \frac{8\pi^2 e^2}{M^2} \int \frac{W_k \sin\theta' d\theta' dk}{K\left(1+\frac{1}{k^2 d^2}\right)\sqrt{\sin^2\theta \sin^2\theta' - \left(\frac{\omega}{kv} - \cos\theta\cos\theta'\right)^2}} \times$$

$$\times \left\| \begin{matrix} \omega^2 \\ \frac{\omega^2}{\sin\theta}\left(\frac{\omega\cos\theta}{kv} - \cos\theta'\right) \\ \frac{\omega^2}{\sin^2\theta}\left(\frac{\omega\cos\theta}{kv} - \cos\theta'\right)^2 \end{matrix} \right\| \qquad (3)$$

$$\frac{\partial W_k}{\partial t} = 2\pi \frac{n_b}{n_0} \frac{\omega_k^3}{k^2} \int \frac{\sin\theta \frac{\omega}{k} \frac{\partial f_b}{\partial v} + \left(\frac{\omega}{kv}\cos\theta - \cos\theta'\right)\frac{\partial f}{\partial\theta}}{\sqrt{\sin^2\theta \sin^2\theta' - \left(\frac{\omega}{kv} - \cos\theta\cos\theta'\right)^2}} dv_d\, \theta \times W_k \qquad (4)$$

It is evident from formula (3) that the diffusion coefficients satisfy the proportion $A:B:C = 1 : \frac{U_o}{C_s} : \left(\frac{U_o}{C_s}\right)^2$ and when $U_o/C_s \gg 1$, the coefficient C, which allows for the angular spread of the beam, becomes predominant. In this case, in Eqs. (3, 4) one can consider only the angular spread of the beam, all the small terms being discarded.

We now turn to the angular distributions $F(\theta,t)$, $G/\theta',t)$

$$F(\theta,t) = \int v^2 f_b\, dv\, d\varphi$$

$$G(\theta',t) = \int k^2 W_k\, dk\, d\varphi'$$

and in integrating (3, 4), we will allow for the fact that f_b has a sharp maximum near U_o. W_k is maximum at the point where the increment C is maximum $K_0 = 1/\sqrt{2}\, d$. In actual experiments, one usually deals with narrow beams $\theta \ll 1$, and therefore, introducing the new variables we obtain the following equations describing the angular spread of the beam: $\cos\theta' - \omega_e/K_0 U_0 = \sqrt{2\Psi}$, $\sin\theta \simeq \theta = \sqrt{2\chi}$

$$\frac{\partial F}{\partial t} = \frac{\omega_{pi}^2}{3 n_0 M K_0 U_0^3} \frac{\partial}{\partial \chi}\left[\int_0^{\chi} \frac{G\sqrt{\Psi}}{\sqrt{\chi - \Psi}} d\Psi \frac{\partial F}{\partial \chi}\right] \qquad (5)$$

$$\frac{\partial G}{\partial t} = -\frac{n_e}{n_0} \frac{\omega_e^3}{K_0^2 U_0^3} \int_{\Psi}^{\infty} \frac{\partial F}{\partial \chi} \sqrt{\frac{\Psi}{\chi - \Psi}}\, d\chi \cdot G \qquad (6)$$

and the normalization condition $\int F d\chi = 1$. The solution of these equations can be found only numerically. The distribution function and spectrum of the noise formed during the spread of the beam are shown in Fig. 1. Equations (5, 6) have an integral, from which one can find the relationship between the total energy of ion-sound noise and the angular distribution function:

$$W = \int_0^{\infty} G\, d\Psi = \sqrt{\frac{2}{3}}\, n_e M U_0^2 \frac{C_s}{U_0} \int_0^{\infty} \chi F d\chi \simeq n_e M U_0^2 \frac{C_s}{U_0} \theta^{-2} \qquad (7)$$

where $\bar{\theta}$ is the angle of spread of the beam. Thus, the noise energy density turns out to be $\frac{C_s}{U_0} \bar{\theta}^2$ times smaller than the beam energy density. For long times, when the initial form of the distribution function becomes insignificant, the solution of Eqs. (5, 6) approaches the self-similar form. In self-similar variables $\xi = \Psi/t$, $\eta = \chi/t$, the equation for $\tilde{F}(\eta) = tF(\chi, t)$ and $\tilde{G}(\xi) = G(\Psi, t)$ has the form

$$\frac{\partial \tilde{G}}{\partial \xi} = \frac{n_e}{n_0} \frac{\omega_e^3}{K_0^2 U_0^2} \int_{\xi}^{\infty} \frac{\partial F}{\partial \eta} \frac{d\eta}{\sqrt{\eta - \xi}} \frac{\tilde{G}}{\sqrt{\xi}} \qquad (8)$$

$$\tilde{F} = -\frac{\omega_{pi}^2}{3 M n_0 U_0^3 K_0} \int_0^{\eta} \frac{G \cdot \sqrt{\xi}}{\sqrt{\eta - \xi}}\, d\xi \frac{1}{\eta} \frac{\partial \tilde{F}}{\partial \eta} \qquad (9)$$

The functions $\tilde{F}$, $\tilde{G}$ are the asymptotic form of the solution of Eqs. (5, 6), shown in Fig. 1 for large .* Equation (8) makes it possible to find

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*Symbol missing from the Russian original [translator].

the characteristic beam spread time. Dividing (8) by $\tilde{G}$ and integrating with respect to ξ from 0 to ∞, we obtain

$$-\ln \frac{\tilde{G}(0)}{\tilde{G}(\infty)} = \pi \frac{n_e}{n_0} \frac{\omega_0^3}{k_0^2 U_0^2} \left(\tilde{F}(\infty) - \tilde{F}(0)\right) \quad (10)$$

Replacing $\ln(\tilde{G}(0)/\tilde{G}(\infty))$ by the Coulomb logarithm Λ with logarithmic accuracy, and using $\tilde{F}(\infty)=0$, $\tilde{F}(0)=\frac{1}{l_0}$, where $l_0 = \bar{\theta}/2t$ is the characteristic width of the distribution function in self-similar variables, we obtain from formula (10)

$$\bar{\theta} = \frac{C_s}{U_0}\sqrt{\frac{6.5}{\Lambda}\frac{n_e}{n_0}\omega_{pi} t} \quad (11)$$

We have thus far been dealing with the spread of the beam, but even at large U_0/C_s, the spread is associated with slowdown. If we also consider the first three terms of Eq. (2), from a comparison of the values of the diffusion coefficients we can qualitatively estimate the ratio of the change in longitudinal velocity ΔV_z to the change in transverse velocity $\Delta V_z/\Delta V_\perp \sim \sim \frac{C_s}{U_0}\sqrt{\Lambda}$. A qualitative estimate of the broadening of the distribution function in transverse velocities is confirmed by the results of the numerical experiment.

The quasi-linear Eqs. (2) make it possible to study the interaction of kinetic beams in only two cases: $U_0 \leq C_s/2$; $U_0 \gg C_s$. An analytical study of the general case proves too difficult. Furthermore, quasi-linear equations are unsuitable for studying the initial stage of monoenergetic beams when $\Delta V/U_0 < (n_e/n_0)^{1/3}$. In this case, use may be made of the method of incomplete numerical modelling.[9] An important characteristic of the application of this method to this problem is the two-dimensional character of the problem. The basis of the method of incomplete numerical modelling as applied to beam problems is the representation of the entire plasma as a continuous medium and modelling by particles of the beam only. The electric fields are given in the form of a set of plane waves; only resonance waves generated in this process can be chosen. By using the results of the analytical discussion, one can specify waves with the value of the modulus of wave vectors $\sim \frac{1}{\sqrt{2}\,d}$ and on the basis of an analysis of the increment for the Maxwell distribution function in the direction of decreasing $|K|$. The distance between the waves in K space, not disturbing the continuity of the spectrum, is determined by the width of the wave interaction zone $\Delta v \sim \gamma/k$, which in the specific problem of spread of the beam means that $\Delta K_z \lesssim K_z\,\gamma/\omega$. The equations describing the interaction of the beam with the plasma include Poisson's equation, the equation of motion and continuity for plasma electrons and ions in the linear approximation, as well as the equations of motion of the particles modelling the beam. The equations were solved by the numerical Runge-Kutta method, and up to 200 waves and 1000 particles were considered.[3] The numerical calculation was carried out for $U_o/C_s = 9$ and $U_o/C_s = 3$ at a density ratio $n_e/n_o = 10^{-3}$ and for $U_o/C_s = 6$ and $U_o/C_s = 20$ when $n_e/n_o = 10^{-1}$. The results indicate that when $U_o/C_s = 9$ and $U_o/C_s = 20$, the slowdown may be neglected, and the beam spread is described by formula (11) to within the errors of the calculations. The initial "hydrodynamic" stage of relaxation of monoenergetic beams at such ratios U_o/C_s has the same qualitative course as the following "kinetic" stage, i. e., only an angular spread is observed. The characteristic time of the hydrodynamic stage is described by an inverse hydrodynamic increment. The relaxation from lower values of the ratio U_o/C_s has a substantially different course. When $U_o/C_s = 3$ ($n_e/n_o = 10^{-3}$), the spread is associated with an intensive slowing down of the beam, and the predominance of the spread over the slowdown is appreciable only in the hydrodynamic stage. Figure 2 shows the distribution function of the beam at three successive times: $\tau = 0, 150, 1000$ (the time is measured from the start of the nonlinear regime). It is obvious that for long times, the transverse and longitudinal velocity spreads become equal. Of considerable interest is the longitudinal velocity distribution function $f_z(v_z) = \int f_c v^2 dv$. As is evident from Figs. 3 and 4, when $U_o/C_s = 3$, the function f_z is nondiffusional in character, this being typical for $U_o/C_s \ll 1$, and when the ratio is slightly larger, $U_o/C_s = 6$ ($n_e/n_o = 0.1$) the slowdown is also significant, but the function f_z is diffusional in character (Fig. 4). The beam spread in the case of $U_o/C_s = 6$ as well as $U_o/C_s = 3$ is satisfactorily described by formula (11), and the shape of the angular distribution is similar to that shown in Fig. 1.

Thus, the slowing down of the beam may be neglected when $U_o/C_s \geq 10$. In this case the angle of spread is given by formula (11); it can be used up to $U_o/C_s \simeq 3$, but as soon as $U_o/C_s \leq 6$, one should not neglect the appearance of the energy spread. When $U_o/C_s < 3$, the transverse velocity spread becomes equal to the longitudinal velocity spread. When $\frac{U_o}{C_s} \ll 1$, the relaxation of the distribution function is almost one-dimensional, and the velocity of the front is determined by the one-dimensional theory of quasi-linear relaxation of electron beams.

The ion beam relaxation theory developed above may be applied to the analysis of experiments on the charge exchange of a plasma blob on a gaseous target. In experiments on charge exchange at high densities of the plasma blob of the flux ions, which cannot be explained by the theory of binary Coulombic collisions.[10*] Further studies showed that during the charge exchange of ions in the gaseous target, a plasma is formed in which the flux generates intense ion-acoustic oscillations propagating at an angle to the direction of motion of the flux.[2] Therefore, the anomalous scattering can be explained by the interaction of the flux ions with ion-acoustic oscillations started in a partly ionized gaseous target.

The oscillations in a plasma-blob system may be described by the following dispersion equation:

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*Sentence incomplete in the Russian original (translator).

$$1+\frac{1}{k^2 d^2} - \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pi}^2}{k^2}\frac{n_e}{n_0}\int \frac{k\frac{\partial f_{ei}}{\partial v}}{\omega - kv}\, d^3v = 0 \qquad (12)$$

where f_{ee}, f_{ei} is the distribution function of the flux electrons and ions.

In deriving Eq. (12), it was assumed that the phase velocities of the oscillations satisfy the condition $v_{Ti} < \frac{\omega}{k} < v_{Te}$. Since in these experiments the temperature of the flux electrons was on the order of the electron temperature of the target plasma, then, neglecting $\frac{n_e}{n_0}\frac{T_e}{T_{ee}} \ll 1$, we obtain a dispersion relation describing the interaction of the ion beam with the plasma, to which the quasi-linear theory of ion beam relaxation is applicable with certain qualifications. The point is that in analyzing the experiments, we encounter a somewhat different formulation of the problem: a beam of ions of radius R_0 is injected into a plasma of finite dimensions, and the pulse length is approximately 0.5 μsec. Moreover, the ion-acoustic oscillations undergo Landau damping by the plasma electrons and ions. Let us find out what results from the consideration of damping. As we know,[4] the Landau damping decrement of ion-acoustic noise is

$$\gamma_k^{(-)} = \sqrt{\frac{\pi}{8}}\,\frac{M}{m}\,\frac{\omega_k^4}{k^3 v_{Te}^3}\left\{1+\sqrt{\frac{M}{m}}\left(\frac{T_e}{T_i}\right)^{3/2} e^{-\frac{\omega_k^2}{2k^2 v_{Ti}^2}}\right\} \qquad (13)$$

and the condition for the development of ion-acoustic instability will be

$$\gamma_k - \gamma_k^{(-)} > 0$$

Using the estimate of the increment γ_k from Eq. (6) we obtain

$$\frac{n_e}{n_0}\,\frac{\omega_k^3}{k^2 u_0^2}\,\frac{1}{\bar{\theta}^2} > \gamma_k^{(-)} \qquad (14)$$

As is evident from (14), at large angles of spread $\bar{\theta}$, or at low densities of the beam, no instability develops. Above, we assumed that n_e/n_0 was small, i. e., the oscillations were due to the plasma particles. When

$n_e/n_o \gg 1$, one can change to the beam coordinate system and assume that the ionized target is incident on the plasma blob. One can then use the previous conclusion regarding the development of instability, and Eq. (14) now becomes

$$\frac{n_e}{n_e}\ \frac{\omega_E^3}{K^2 U_o^2}\ \frac{1}{\theta_M^2} > \gamma^{(-)} \tag{15}$$

where $\gamma^{(-)}$, ω_k, K are determined by the parameters of the flux; $\theta_M \sim \frac{v_{Mi}}{U_o}$; v_{Mi} is the thermal velocity of the target ions. Let us note that when $n_e/n_o \gg 1$), the ions of the beam do not undergo scattering. In the experiments under analysis, an intermediate case occurs in which $n_e \lesssim n_o$. In this connection, we will attempt to draw a qualitative comparison between the results of the experiment and those of the calculation for $n_e \ll n_o$.

The beam inhomogeneity along the radius R and axis Z makes it necessary to consider the spatial times in the quasi-linear equations (1). As was noted above, at velocities $U_o \gg C_s$, the ion-acoustic oscillations propagate almost at right angles to the Z axis. Under the experimental conditions, during the injection time τ_{in}, the oscillations are unable to leave the beam, i. e., $\frac{\partial \omega}{\partial K} \tau_{in} \approx C_s \tau_{in} \sim 2$ cm $< R_o \sim 6$ cm, and we can therefore neglect the term $\frac{\partial \omega}{\partial K}\ \frac{\partial W_k}{\partial r}$ in system (1). Integrating the first equation (1) with respect to Z from zero to Z_o (Z_o being the boundary of the gaseous target), we obtain

$$\frac{\partial}{\partial t}\int_0^{Z_o} f_e\, dz + U_o\left[f_e(Z_o) - f_e(0)\right] = \frac{\partial}{\partial v_\alpha}\int_0^{Z_o} D_{\alpha\beta}\,\frac{\partial f_e}{\partial v_\beta}\, dz$$

For a slight increase of the angle of spread $\bar{\theta}$, as is observed in the initial stage, we can set $\frac{1}{Z_o}\int_0^{Z_o} f_e dz \sim \bar{f}_e;\ \frac{1}{Z_o}\int_0^{Z_o} W dz \sim \bar{W}_k$, and for characteristic times of the process $\Delta t \ll \frac{Z_o}{U_o}$, we can neglect the term $U_o[f_e(Z_o) - f_e(0)]$. In the experiments, the characteristic time of the change $\bar{\theta}$ – $\Delta t \lesssim 10^{-7}$ sec, and $Z_o/U_o \approx 2 \times 10^{-7}$ sec. Thus, under the assumptions made above, in a qualitative description of the interaction of a blob with a plasma, one can neglect the spatial inhomogeneity of the system and use the results of quasi-linear theory, but taking into account the damping of oscillations, by introducing the damping into Eq. (6). The solution of Eqs. (5, 6) allowing for damping was obtained by the numerical method for an initial spread angle $\bar{\theta} = 0.1$ and 0.14 and different $\gamma^{(-)}$. The results of the computer calculation are presented in Figs. 5, 6, and 7. Figure 5 shows the ratio of the distribution function of the beam ions over angle θ at different instants of time to the initial distribution function for $\bar{\theta} = 0.14$. To compare the evolution of the distribution function in time, Fig. 6 shows this ratio, measured experimentally in Ref. 12. The following qualitative correspondence is observed: with increasing time after the start of injection, the magnitude of the distribution function decreases at small angles θ, and its maximum shifts toward larger angles. Figure 7 shows the time dependence of the width of the distribution function $\bar{\theta}$ at one half of its height for different values of $\gamma^{(-)}/\alpha = \alpha \frac{k_0^2 U_c^2}{\omega_c^3} \gamma^{(-)})$; the dashed curve shows the experimental time dependence of the mean ion scattering angle. It is evident that the experimental points are located in the region of the family of curves obtained for the initial distribution function width $\bar{\theta}_o = 0.14$, which is close to the experimental value. Figure 8 shows the increase of the energy density of the oscillations in time for $\bar{\theta}_o = 0.14$.

The above results indicate that the beginning of the interaction process involves a stage of linear increase of the noise, while $\bar{\theta}$ remains practically unchanged. Then the noise becomes much greater than the thermal noise, the stage of quasi-linear relaxation begins, and an increase of angle $\bar{\theta}$ is observed. In the presence of strong Landau damping ($\alpha \gtrsim 20$), the amplitude of the oscillations decreases, and the rate of spread of the beam declines. The final angle of spread may be estimated from expression (14).

Despite the fact that the experimental values of $\bar{\theta}$ (Fig. 6) were obtained in regimes with $n_e \lesssim n_o$, and the calculations are valid for $n_e \ll n_o$, qualitative agreement is observed between the calculated and the experimental time dependence of the mean ion scattering angle. However, one should not look for an exact quantitative agreement between the experimental and the calculated dependence for a fixed value of α, since in the case of a high density of the ion flux ($n_e \lesssim n_o$), the oscillations may be transported out of the plasma by the flux, and their level will be limited not only by the Landau damping by the particles, but also by the magnitude of the transport. The agreement between the observed and the calculated dependence is also indicated by the qualitative agreement between the experimental and the calculated energy density value of the oscillations in the plasma. The measured value of the oscillation amplitude $E \approx 15$ kV/cm corresponds to $W \approx 10^{-2}$ for $\tau \approx 0.5$ (see Fig. 9). The above analysis of the calculated and experimental results shows that for small beam scattering angles $\bar{\theta} < \frac{1}{2}\bar{\theta}_{KCH}$, one can obtain an estimate of the maximum rate of rotation by neglecting the Landau damping, since the increase of noise predominates at this stage. On the basis of system (5, 6), after some simple calculations we obtain an estimate of the rate of spread of the beam and the value of the noise energy

$$\frac{d\bar{\theta}}{dt} \simeq \frac{6.5}{\Lambda} \frac{n_e}{n_o} \frac{\omega_o^3}{K_o^2 U_o^2} \frac{1}{\bar{\theta}} \tag{16}$$

$$W \approx n_e M U_o^2 \frac{\omega_o}{K_o U_o} \frac{\bar{\theta}^2 - \bar{\theta}_o^2}{\bar{\theta}} \tag{17}$$

Here K_o, ω_o are the wave vector and frequency of oscillations started with the maximum increment. Moreover, while when the damping is neglected $K_o \approx 1/\sqrt{2}\ d$, when the damping is considered, K_o and ω_o will depend on the form of the distribution function, and hence, on time. Exact solutions, evidently at the initial stage, when the damping may be neglected, will not differ appreciably from estimates (16, 17). Moreover, the results of the comparison of the experimental data and calculations show that the quasi-linear approximation correctly describes the interaction of an ion beam with plasma up to $n^e/n_o \lesssim 1$ in view of the smallness of the parameters C_s/U_o and W/n_oT.

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FIGURES

Fig. 1. a - Angular distribution function of ion beam with

$$U_0 \gg C_s \quad \tau = \frac{2}{3\sqrt{3}} \frac{n_e}{n_0} \frac{C_s^2}{U_0^2} \omega_{pi} t$$

b - angular dependence of noise energy density with $U_0 \gg C_s$

$$\tau = \frac{2}{3\sqrt{3}} \frac{n_e}{n_0} \frac{C_s^2}{U_0^2} \omega_{pi} t; \quad G_H = \sqrt{\frac{2}{3}}\, n_e M U_0^2 \frac{C_s}{U_0}$$

Fig. 2. Distribution function of beam in velocity space with $U_0 = 3C_s$ (n_e/n_0 = 10^{-3} $\tau = \omega_{pi} t$)

Fig. 3. Distribution function of the beam over longitudinal velocity

$$U_0 = 3C_s \quad (n_e/n_0 = 10^{-3}, \quad \tau = \omega_{pi} t)$$

Fig. 4. Distribution function of the beam over longitudinal velocity with

$$U_0 = C \quad (n_e/n_0 = 0.1, \quad \tau = \omega_{pi} t)$$

Fig. 5. Calculated values of ratio of the distribution function of beam ions at an arbitrary instant to the initial distribution function with $\bar{\theta}_0 =$ 0.14 $\left(\tau = t \frac{n_e}{n_0} \frac{\omega_c^3}{k^2 U_0^2}\right)$ 1 - τ = 0.52; 2 - τ = 0.68.

Fig. 6. Experimental values of normalized distribution function of ion flux for different instants of time. The increase in the number of the curve corresponds to later instants of time.

Fig. 7. Time dependence of width of distribution function.

solid curves - $\bar{\theta}_0$ = 0.14, dashed curve - experimental.

Fig. 8. Change in noise energy density with time

$$\left(W = \frac{\bar{W} k_0}{n_e M U_0 \omega_c} \right)$$

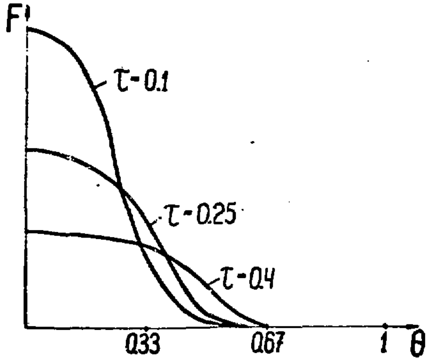


Fig. 1a

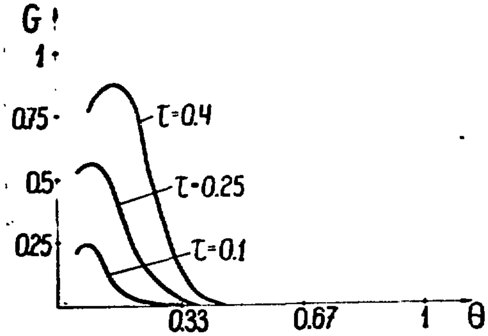


Fig. 1b

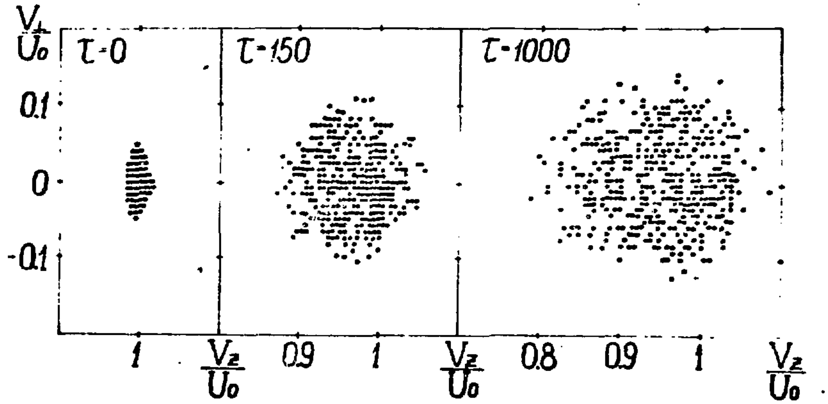


Fig. 2

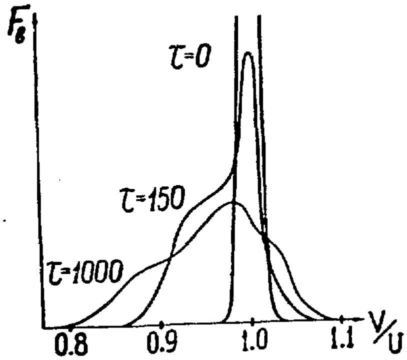


Fig. 3

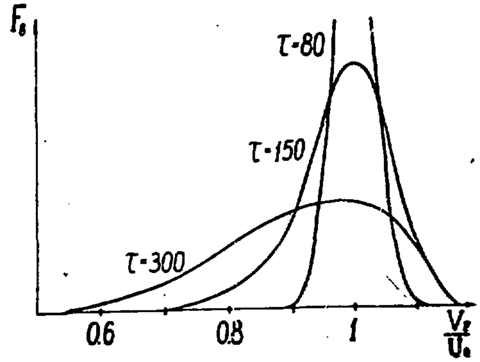
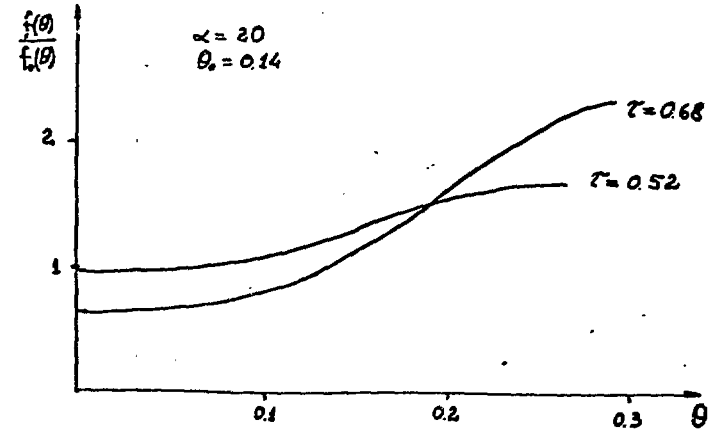
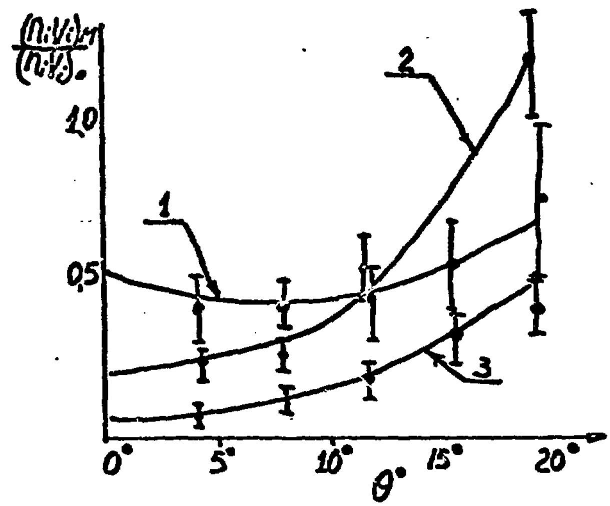


Fig. 4



f(θ)/f₀(θ)
α = 20
θ₀ = 0.14
τ = 0.68
τ = 0.52
2
1
0.1
0.2
0.3
θ

Fig. 5



(nᵢVᵢ)ₘ/(nᵢVᵢ)₀
1.0
0.5
1
2
3
0°
5°
10°
15°
20°
θ°

Fig. 6

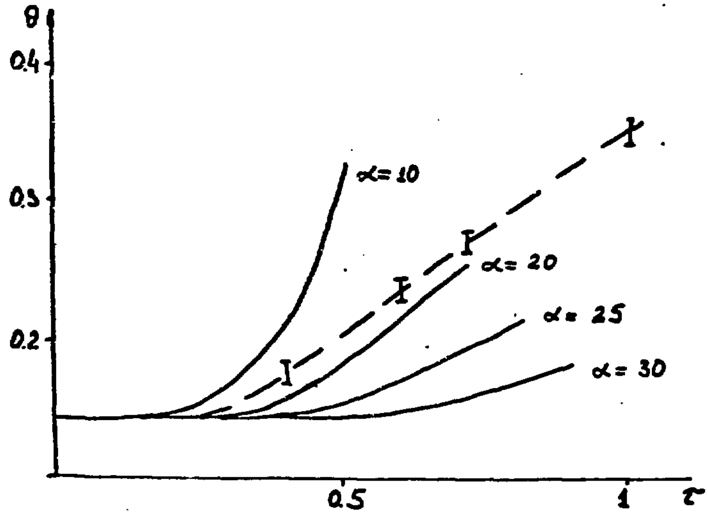


Fig. 7

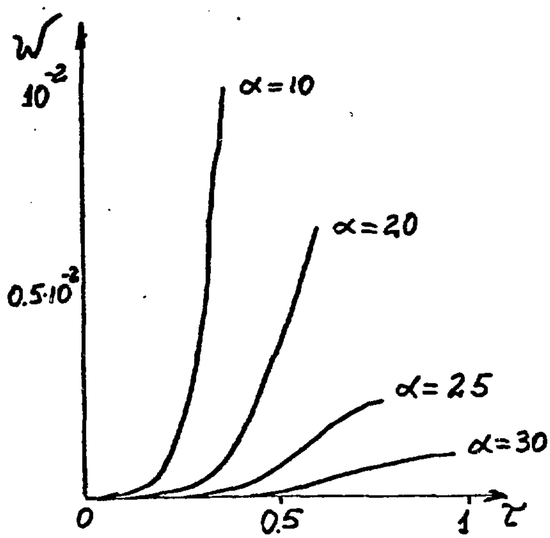


Fig. 8