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CAVITIES PRODUCED BY UNDERGROUND NUCLEAR EXPLOSIONS

T. R. Butkovich

MASTER

July 8, 1976

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LAWRENCE LIVERMORE LABORATORY
University of California, Livermore, California 94550

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CAVITIES PRODUCED BY UNDERGROUND NUCLEAR EXPLOSIONS

Abstract

This investigation studied the displacement of rock that formerly occupied cavities produced by underground nuclear explosions. There are three possible explanations for this displacement: the volume could be displaced to the free surface; it could occupy previously air-filled pores removed from the surrounding rock through compaction; or it could be accounted for by persisting compressive stresses induced by the outgoing shock wave.

The analysis shows it unlikely

that stored residual elastic stresses account for large fractions of cavity volumes. There is limited experimental evidence that free surface displacement accounts for a significant portion of this volume. Whenever the explosion mediums contain air-filled pores, the compaction of these pores most likely accounts for all the volume. Calculations show that 4% air-filled porosity can account for all the cavity volume within about 4 cavity radii and that even 1% can account for a significant fraction of the volume.

Introduction

Understanding cavity formation is important for studying the use of underground nuclear explosions to create porosity in a large mass of rock in which in situ processes are being considered. For example, in situ oil-shale retorting and copper-ore leaching require permeability significantly greater than what is present in deep ore bodies. Knowledge of cavity formation and its controlling factors is also important in containment studies.

When a nuclear device is

detonated deep underground, high-pressure gas is produced that pushes on the surrounding rock and forms a cavity. As the cavity grows, the internal pressure and temperature decay until the pressure comes into equilibrium with the stresses in the surrounding rock. Cavity volume can then be distributed as porosity. This can occur by the opening of cracks in the fractured rocks as the pressure decays in the fully formed cavity, or by collapse from gravitational force of the failed rock into the cavity void.

The primary aspect of this study of nuclear-explosion cavities was to determine how the volume of rock that formerly occupied the cavity void had been displaced. Another aspect of nuclear cavities considered here was the factors that control cavity growth. From underground nuclear explosions there are considerable data on cavity radius, chimney height, free field motion, free surface

motion, and collapse crater volume, as well as on the properties of the rock medium. Computer codes using measured rock properties have been developed that reproduce cavity radius, free field motion, and free surface motion with considerable accuracy. These same computer codes can also be used for parameter studies to expand on and supplement these measurements.

Explosion-Produced Cavities

The explosion of a nuclear device is essentially a point source of energy. The volume of the device can be considered independent of its energy content. When the explosive is detonated, all the energy is confined to a small volume with an enormous temperature and pressure. A strong shock wave generated by the explosion vaporizes the surrounding rock, which participates in the expansion and soon becomes the primary working gas. Calculations have shown that about 70 tonnes of rock are vaporized per kiloton (10^{12} cal) of energy released. The gases continue to expand until the pressure within the cavity comes into equilibrium with the counterbalancing stresses from overburden and from the strength of the rock. Butkovich calculated cavity pressures at full cavity growth to be 2 to 2.5

times the overburden stress (ρgh) for granite and salt of low water content, and about 1.4 ρgh for much weaker wet tuffs.¹ Higgins and Butkovich have developed a relationship from measurements of the cavity radii of 46 underground nuclear detonations in tuff, alluvium, salt, and granite:

$$R_c = \frac{100 W^{1/3}}{(\rho h)^\alpha} \quad (1)$$

where R_c is the cavity radius in metres, W is the energy released in kilotons, ρ is the average overburden density in grams per cubic centimetre, and h is the depth of burst in metres.² The constant 100 is derived for silicate rocks and in this sense is independent of medium, containing such things as the gravitational constant and dimensional conversions. The exponent $\alpha = 1/3\gamma$, where γ is the

adiabatic expansion coefficient that depends on the water content of the medium. The value α is derived from a simple silicate approximation that considers rock to be SiO_2 except for the water: It ranges from 0.3268 for zero weight-fraction water to 0.273 for 25% weight-fraction water.

The rock's water content strongly influences its shear strength.³

Michaud has developed a relationship of cavity radius similar to Eq. (1) that includes a rock-strength term, C_s , in the denominator:

$$R_c = \frac{52 \epsilon^{1/3} W^{1/3}}{(gh + C_s)^{1/3} f}, \quad (2)$$

where in this case ϵ refers to an emplacement geometry ($\epsilon = 1$ for tamped shots) and the units are in metres, kilotons, and bars.⁴

Cavity Displacement

An important aspect of understanding the cavity volume produced by underground explosions is to determine what happened to the rock that formerly occupied the volume of the cavity void. There are three possibilities: the volume could be displaced to the free surface; the volume could occupy previously air-filled pores removed from the surrounding rock through compaction;

Generally, the fractured rock above the cavity falls into the cavity void. The falling pieces rotate and bulk, and the cavity volume is distributed in the chimney as bulking porosity. In strong rocks such as granite, apical voids have been measured at the top of the chimney, indicating that all the cavity volume had not been distributed at the level the chimney height reached the maximum extent of fracture. It might be expected, then, that the bulking porosity would depend on the physical properties of the rock, the degree of pre-shot fracture, the fracturing due to the passage of the shock wave from the explosion, and the fracturing due to the collapse itself. Unfortunately it is not possible to separate these individual effects in the available data.

or the volume could be accounted for by persisting compressive stresses from the outgoing shock wave. Any one or combination of these is possible.

FREE SURFACE DISPLACEMENT

There are very little data regarding the free surface displacement of cavity volume. One rather obscure piece of information

regarding such measurement is from the Gnome Event in essentially pore-free bedded salt. The Gnome Event produced a standing cavity where the only collapse consisted of a roof fall to fill the lower hemisphere. The surveyed ground displacement around ground zero indicated a dome-shaped bulge.⁵ The approximate volume of the dome determined by surveys was 25 000 yd³ (19 200 m³). Measurements of cavity volume⁶ obtained by pressurization with air gave a volume of 29,000 ± 10% m³. This indicates that in this case most of the cavity volume was displaced to the free surface.

One can calculate the fraction of energy released by nuclear explosions necessary to lift the cavity volume to the surface. The energy required to lift a spherical cavity volume without friction is

$$E = (\rho gh) \left(\frac{4}{3} \pi R_c^3 \right), \quad (3)$$

where ρgh is the overburden stress and R_c is the cavity radius. Combining Eq. (3) with Eq. (1) and putting both in the same units, the fraction of energy required to lift this volume to the surface is

$$\frac{E}{W} = 0.0098 (\rho h)^{1-3\alpha}. \quad (4)$$

E/W was calculated for nuclear

events at the Nevada Test Site both in alluvium and in tuff. The assumptions were that cavities were spherical and that all the cavity volume was displaced to the free surface. Calculations of overburden stress used an average overburden density of 1.9 g/cm³. With these assumptions, and with the measured depth of burial, cavity radius, and energy yield, E/W was determined for each event. Table 1 summarizes the results.

The calculation for E/W merely demonstrates that only a small fraction of the energy released in nuclear explosions is needed to displace all the cavity volume to the surface. Of course, these are maximum values and will be smaller if some other process also accounts for part of the volume of the cavity void.

COMPACTION OF AIR-FILLED PORES

Almost all rocks contain some porosity, and small amounts of air-filled, nonconnected pores may be present even below the water table where rock is considered to be fully saturated. On passage of the shock wave, some or all of the air-filled pores are irreversibly removed from the rock, depending on intensity of the shock and duration of the pulse. Hydrostatic pressure-volume (P-V)

Table 1. Calculated fraction of energy required to displace entire cavity volume to surface for nuclear events at the Nevada Test Site (spherical cavities assumed).

| Shot-point material | No. of events | Av water content (wt fraction) | Av overburden density (g/cm^3) | Av E/W |
|---------------------|---------------|--------------------------------|---|---------------|
| Alluvium | 103 | 0.104-0.023 | 1.9 | 0.0164-0.0064 |
| Tuff | 90 | 0.139-0.044 | 1.9 | 0.0215-0.0021 |
| Salt ^d | 1 | 0.04 | 2.3 | 0.0208 |

^aGnome Event

measurements on porous rocks show that essentially all the air-filled pores are irreversibly removed when the rock is loaded above a certain pressure, P_m . Smaller fractions of the air-filled pores are removed at lower pressures. Below some threshold pressure, P_T , the material behaves more or less elastically. For a number of tuffs, P_m ranges between about 2.5 kbar to more than 40 kbar, depending primarily on the strength of the rock, which in turn is controlled primarily by its water content. The same data show that P_T is also affected by water content but controlled primarily by the initial amount of air-filled porosity present in the rock.⁷ Obviously P_T cannot be less than the lithostatic stress in the rock.

Calculations indicate what fraction of cavity volume could be accounted for by compaction of air-filled pores around nuclear explosions. The explosion environment chosen for the sample calculation was a high total porosity (40.93%) paintbrush tuff from the Nevada Test Site at four different saturations: dry, 50%, 90%, and 97%. Appendix A shows the technique and calculations, and Table 2 summarizes the results.

These results were produced assuming spherical cavities, which make them minimum distances or maximum percentages in which all the cavity volume can be accounted for by compaction. Since the water content of the rock strongly influences the rock strength, the results shown for higher saturations should be less reliable.

Table 2. Results from calculations of fraction of cavity volume from compaction of air-filled pores in rock surrounding nuclear explosions in paintbrush tuff of 40.93% total porosity with different saturations and energy yields at scaled depths of $120 W^{1/3}$ m.

| Material | 1 | 2 | 3 | 4 |
|---|--------|--------|-------------------|-------------------|
| Saturation (%) | 0 | 50 | 90 | 97 |
| Bulk density (mg/m ³) | 1.40 | 1.605 | 1.768 | 1.797 |
| Initial air-filled porosity | 0.4093 | 0.2047 | 0.0409 | 0.0123 |
| Wt-fraction water | 0 | 0.1275 | 0.2084 | 0.2209 |
| Cavity radius (m) | | | | |
| 1 kt | 16.9 | 20.5 | 22.0 | 22.2 |
| 10 kt | 28.3 | 35.2 | 38.2 | 38.6 |
| 100 kt | 47.4 | 60.7 | 66.5 | 67.3 |
| Cavity volume (m ³ × 10 ⁴) | | | | |
| 1 kt | 2.0 | 3.6 | 4.4 | 4.6 |
| 10 kt | 9.5 | 18.4 | 23.4 | 24.1 |
| 100 kt | 44.7 | 93.7 | 123.1 | 127.5 |
| D ^a | | | | |
| 1 kt | 2.3 | 2.3 | 0.60 ^b | 0.26 ^b |
| 10 kt | 1.9 | 2.1 | 4.4 | 0.49 ^b |
| 100 kt | 1.8 | 1.9 | 3.5 | 0.92 ^b |

^aMultiples of cavity radius where volume of compacted air-filled pores equals the cavity volume.

^bFraction of cavity volume accounted for by compaction of air-filled pores out to radius of $\bar{P} > P_t$.

EXPLOSION-INDUCED STRESSES

The introduction of explosion-induced stresses in the rock surrounding the explosion can also account

for cavity volume. There has been some speculation that permanent compressive stresses are induced in the rock by the explosion. That is, after passage of the shock wave, the

rock out to some distance is at a higher stress state than originally present. It might be expected that the amount of residual stresses would be some function of the distance from the explosion center, but no data exist. One can calculate the stored permanent residual stress above that present before the explosion by assuming the additional stress is at low levels; the compressibility of the surrounding rock is constant over the stress increment being considered. From the definition of bulk modulus,

$$K = \frac{\Delta P}{\frac{\Delta V}{V_0}}$$

where ΔP is the increase in stress above that preceding the explosion and $\frac{\Delta V}{V_0} = (V_0 - V)/V_0$ where $V_0 - V = \Delta V$, the corresponding change in specific volume. V_0 as defined here refers to the specific volume of the rock without additional stored stress, and V is the specific volume of the surrounding rock due to compression from the stored residual stress. If there are air-filled pores in the rock, then V_0 and K refer to compacted specific volume and bulk modulus of the surrounding rock:

$$\Delta V = \frac{\Delta P V_0}{K}, \quad v = \frac{V_0}{1+\mu}, \quad \text{and} \quad \Delta P = K\mu,$$

then

$$\Delta V = \frac{V_0 \Delta P}{1 + \mu + K}.$$

The spherical radius (R_a) for which all of the cavity volume can be accounted in uniformly stored compressive stress in the surrounding rock is

$$R_a = R_c \left(\frac{1 + \mu + K}{1 + \mu} \right)^{1/3}.$$

The fraction of the cavity volume (f) that can be accounted for at a given R is then

$$f = \left(\frac{R}{R_c} \right)^3 \frac{1 + \mu}{1 + \mu + K}. \quad (5)$$

Figure 1 is a plot of R/R_c , multiples of the cavity radius, where all of the spherical cavity volume can be accounted for in uniformly stored stress vs the elastic bulk modulus of the surrounding rock. An examination of this figure suggests that for the storage of residual stresses in the surrounding rock to account for all or even a large fraction of the cavity volume, either large volumes of rock or residual stresses in the hundreds of bars would be required.

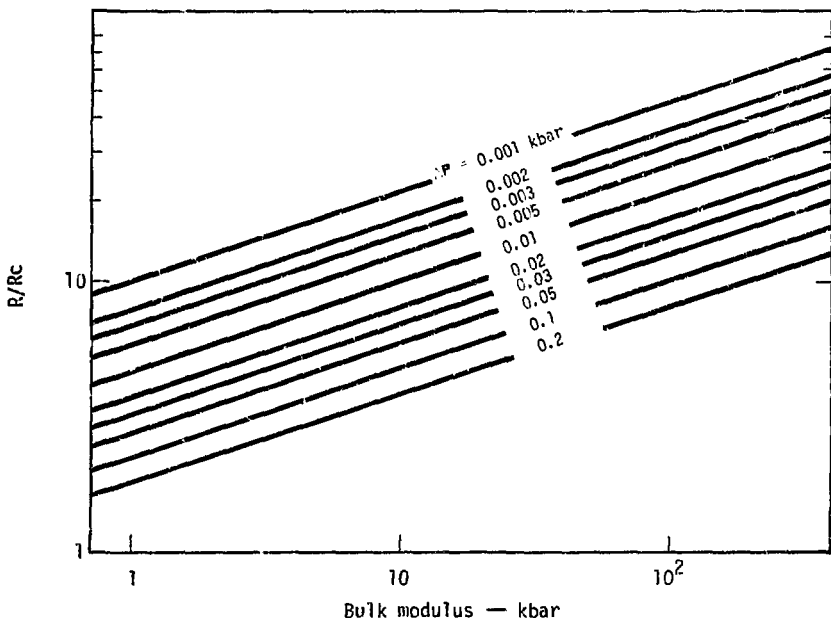


Fig. 1. Multiples of cavity radius where residual stresses can account for entire cavity volume vs bulk modulus of rock.

Aspherical Cavity Growth

It is generally assumed that the cavity formed by a nuclear explosion is spherical. However, calculations indicate that a spherical cavity from a point source is a limiting case.⁸ For instance, if the cavity is still growing when the rarefaction wave reflected from the free surface returns, an acceleration of cavity growth towards the free surface takes place. This is observed in shallow

buried detonations for crater formation. There is also evidence that cavity acceleration occurs for deeply buried detonations in high-velocity granites.⁹ The effect of the decreasing overburden stress in the direction of the free surface also causes greater cavity growth towards that surface. The size of the cavity and degree of asphericity decreases with greater depths of burial. The

strength of the rock is an important factor controlling cavity growth, and the amount of water present in the rock effectively determines the strength of the rock. Generally, wet rocks cannot support large deviatoric stresses and fail easily in shear.

In contrast, dry rocks, even those with significant initial porosities, will support large shear stresses once the material compacts. Calculations of cavity growth in high-porosity, dry, partially saturated, and fully saturated rock reported by Butkovich demonstrate this.⁸

Cavity radius is measured to the edge of the cavity below the shot point on reentry drilling and determined from radioactivity logs through the melt glass which is concentrated at the bottom of the cavity. On the basis of experimental measurements, it is generally assumed that all the refractory nuclides are associated with the melt. After post-shot collapse, no measurements on the upper part of the cavity are possible. The cavity radii calculated from Eqs. (1) or (2) are based on these measured values and are therefore minimum values.

Additional experimental evidence exists for aspherical cavities in the form of volume measurements of collapse craters. Sometime following cavity formation, the rock above the

cavity collapses. In many cases the collapse propagates to the free surface and forms a collapse crater. During the collapse it might be expected that the broken rock falling into the cavity would rotate and bulk. If the volume of the cavity were not completely occupied by bulking porosity in the chimney, a crater containing the residual volume would form on the surface. If there were no bulking during the collapse, then the crater volume (V_{cr}) would equal the cavity volume. Data show that $C = V_{cr}/(4/3 \pi R_c^3) > 1$ for about a third of the over 200 events with measured collapse craters (see Fig. 2). It might be suspected that the extra volume would come from compacting the rock around the cavity that eventually collapses to form the chimney. Assuming vertical chimney walls, one can calculate that portion from the ratio of the volume of the chimney-collapse material, V_v , to the cavity volume, V_c :

$$\frac{V_v \text{ (chimney)}}{V_c \text{ (cavity)}} = \frac{\pi R_c^2 \sum [\Delta h (\psi_i - \psi)]_{av}}{\frac{4}{3} \pi R_c^3} = \frac{3/4 \sum [\Delta h (\psi_i - \psi)]_{av}}{R_c}, \quad (6)$$

where Δh is the increment of height of chimney that had the initial air-filled porosity, ψ_i , and $(\psi_i - \psi)_{av}$ is

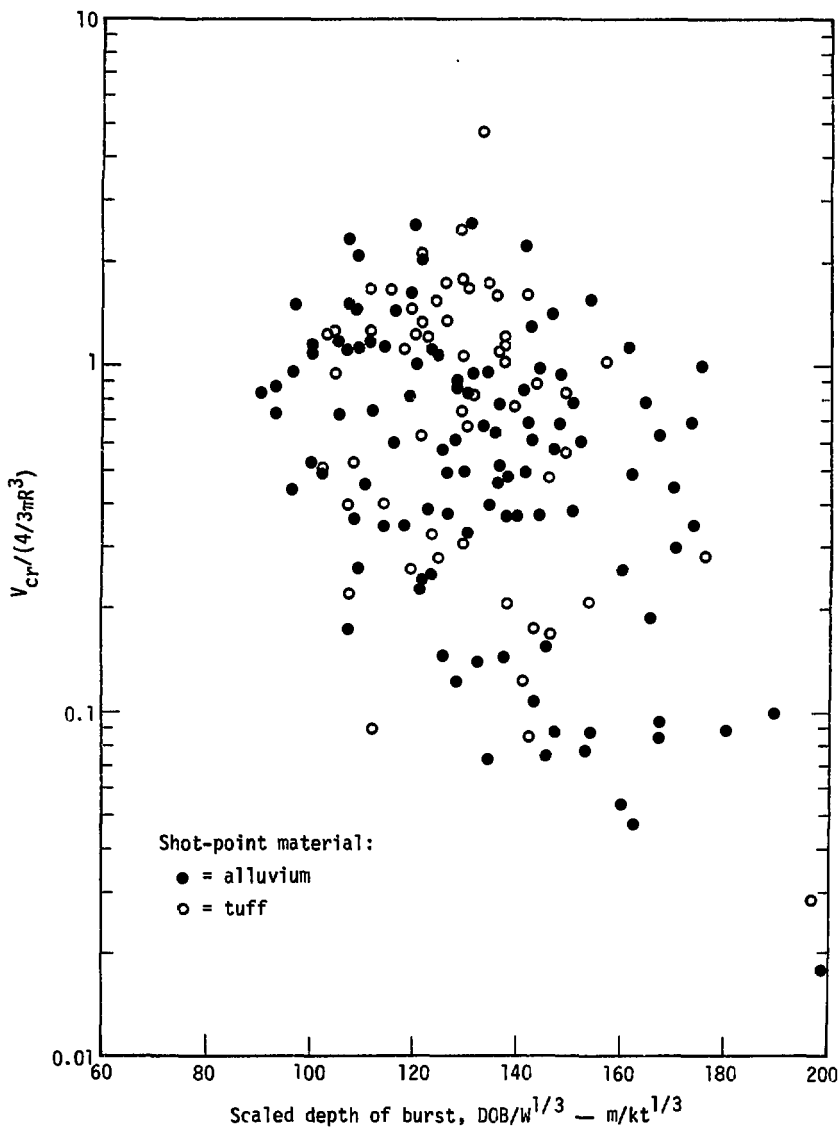


Fig. 2. Ratio of measured crater volume to calculated cavity volume from cavity radius measurements vs scaled depth of burst. Events at NTS prior to 1972.

the average porosity removed from the increment (see Appendix A).

Calculations were made using Eq. (6) for the materials discussed above, whose properties are shown in Table 2. Table 3 summarizes the results.

Shot-point material with air-filled porosity of 40% or even 20% at the Nevada Test Site is rare. Even so, the data show (Fig. 1) that about 15% of all shots with collapsed craters have ratios of crater volumes to assumed volumes of spherical cavity greater than 1.3. It is more likely that shot-point materials have air-filled porosities in the range of a few percent or less. The additional cavity volume would then be less than 10% for most shots.

This suggests that cavity volumes can be considerably larger

than those calculated by assuming spherical cavities and using measured R_c from the lower part of the cavity. As an approximation, if one considered the cavity to be made up of two hemispheres, the lower having a radius of R_l and the upper R_u , then

$$R_u = (2C-1)^{1/3} R_l,$$

where C , as defined earlier, is the ratio of the volume of a measured crater to the volume of a spherical cavity calculated from cavity radius measurements. If $C = 1.5$, then $R_u = 1.26 R_l$, and if $C = 2.0$, then $R_u = 1.44 R_l$. Taking the data at face value, one finds no obvious reason why $C > 1$ for a third of the events, and $C < 1$ for the other two-thirds. This phenomenon occurs equally as

Table 3. Fraction of cavity volume that can be accounted for by compaction of chimney material for four different air-filled porosity materials.

| Material | Air-filled porosity | Energy, W (kt) | | |
|----------|---------------------|----------------|------|------|
| | | 1 | 10 | 100 |
| 1 | 0.4093 | 0.30 | 0.34 | 0.36 |
| 2 | 0.2047 | 0.24 | 0.25 | 0.26 |
| 3 | 0.0409 | 0.06 | 0.07 | 0.08 |
| 4 | 0.0123 | 0.02 | 0.02 | 0.03 |

often in alluvium and tuff shot-point mediums and at scaled depths of burst between 100 and 160 $W^{1/3}$

$m/kt^{1/3}$. Lower values of C, however, occur at higher scaled depths of burst (see Fig. 2).

Summary and Discussion

The size of cavities formed by underground nuclear detonations depends on the energy yield of the explosive, the overburden stress, and the strength of the surrounding rock.

The displacement of rock formerly occupying the cavity volume was analyzed. The volume could be displaced to the free surface, it could occupy previously air-filled pores removed from the surrounding rock through compaction, or it could be accounted for by persisting compressive stresses induced by the outgoing shock wave. Any one or combination of these is possible.

There is limited experimental evidence that free surface displacement occurs that can account for all or at least a significant portion of the cavity volume. A calculation of the amount of energy required to move the cavity volume to the surface without friction shows the values to be about 2% of the energy released. This process is entirely feasible whenever there are insufficient air-filled pores in the surrounding rock to accommodate the cavity volume created.

When the explosion mediums contain air-filled pores, the compaction and removal of these pores most likely account for the cavity volume. Calculations show that the distance from the center of the cavity for which all the volume can be accounted is dependent on the amount of air-filled porosity present in the rock and on the yield of the explosive. The higher the air-filled porosity or the higher the energy yield, the lower the multiples of the cavity radius which account for all the cavity volume. In the cases presented, about 4% air-filled pores can account for all the cavity volume out to about $4 R_c$, and shot-point rock with as little as 1% air-filled porosity can account for a significant fraction of the cavity volume.

It is unlikely that stored residual elastic stresses account for large fractions of cavity volume. Calculations suggest (Fig. 1) that hundreds of bars of uniformly stored residual stress (which did not exist before the explosion) are required to account for all the cavity volume

within a reasonable multiple of the cavity radius.

Both calculation and experimental evidence indicate aspherical cavity growth. The size of the cavity and degree of asphericity decrease with greater depths of burial. From volume measurements of collapse craters, one knows that about a third of over 200 collapse craters are larger than would be assumed from spherical volume calculated from measured R_c of the lower hemisphere. By trying to account for the extra volume from the compaction of rock around the cavity that eventually collapses to form the chimney, one concludes that this would probably be less than 10% of the cavity volume for most events at the Nevada Test Site. An approximation of

asphericity using the same data shows a ratio of upper radius to lower radius as great as 1.5. This value was derived assuming no bulking and would be greater if there were some bulking. This study did not address the subject of the occurrence or nonoccurrence of bulking.

For aspherical cavities, the amount of energy necessary to lift the cavity volume to the surface is proportional to that volume. For aspherical cavities, the amount obtained for assumed spherical cavities should be multiplied by C. Likewise, the radii for which all the cavity volume can be accounted by compaction of air-filled pores are minimum values when obtained by assuming spherical cavities.

Appendix A: Results of Calculations of Contribution of Air-Filled Pores in Surrounding Rock on Cavity Volume

The relationship between peak pressures and $R/W^{1/3}$ (Fig. A1) was obtained from calculations shown in a report by Butkovich.⁸ The 97% saturation curve was estimated by interpolating between the 90° and 100% saturation curves. The relationship between air-filled porosity

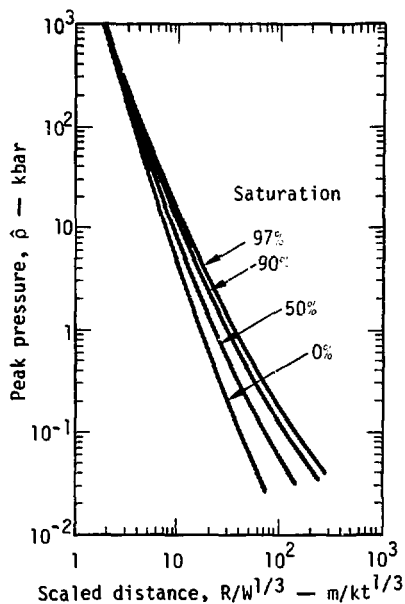


Fig. A1. Calculated peak pressure as a function of scaled distance for different saturations of porous dry tuff. See Table 2 for properties of materials.

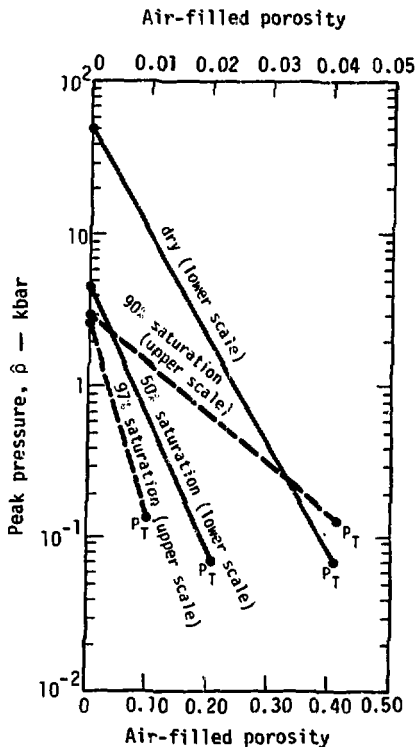
and peak pressure was developed from the model used in the PMUGEN code.⁷ P_T , the threshold pressure at which air-filled pores begin to fill, and P_m , the pressure at which all the air-filled pores are irreversibly removed, were calculated by the method shown in the same report (Fig. A2). Results shown in Tables A1-A4 were obtained from calculations in the following manner. The symbol ψ is air-filled porosity. When $\psi = 0$, all the air-filled pores have been removed. Fig. A2 gives the peak pressure, P_m , at which $\psi = 0$. Figure A1 gives the appropriate distance, $R/W^{1/3}$ for the P_m . V/W , the volume of rock per kiloton is calculated from

$$\frac{V}{W} = \frac{4}{3} \pi (R/W^{1/3})^3 = \frac{4}{3} \pi \frac{R^3}{W}$$

This is repeated for each increment of ψ . The volume of the voids (air-filled pores) per kiloton, V_v/W , is obtained by

$$(\psi_i - \psi)_{av} \cdot \frac{V}{W} = \frac{V_v}{W},$$

where ψ_i is the initial air-filled porosity and $(\psi_i - \psi)_{av}$ is the average porosity of the increment. Summing from $\psi=0$ to $\psi=\psi_i$ gives $\Sigma(V_v/W)$.



This was compared with the cavity volume obtained from

$$v_c = \frac{4}{3} \pi R_c^3 = \frac{4}{3} \pi \left[\frac{100w^{1/3}}{(\rho h)^\alpha} \right]^3.$$

This calculation assumed an average overburden density of 1.9 g/cm^3 and a scaled depth of burst of $120 W^{1/3} \text{ m/kt}^{1/3}$. The question of whether such a material can exist at a depth where $\rho gh > P_c$ was not considered.

Fig. A2. Relationship between air-filled porosity and peak pressure for four rocks of different saturations. Relationship used from PMUGEN.⁹

Table A1. Dry paintbrush tuff.

Wt-fraction water = 0

$\rho_0 = 1.40 \text{ g/cm}^3$

$\alpha = 0.3268$

$\psi_1 = 0.4093$

1 kt, $R_c = 16.87 \text{ m}$, $V_c = 2.01 \times 10^4 \text{ m}^3$

10 kt, $R_c = 28.28 \text{ m}$, $V_c = 9.47 \times 10^4 \text{ m}^3$

100 kt, $R_c = 47.41 \text{ m}$, $V_c = 4.46 \times 10^5 \text{ m}^3$

| ψ | $(\psi_1 - \psi)$ | \dot{P} (kbar) | $R/W^{1/3}$ (m/kt) ^{1/3} | V/W (m ³ /kt) $\times 10^4$ | V_v/W (m ³ /kt) $\times 10^3$ | $\Sigma(V_v/W)$ (m ³ /kt) $\times 10^3$ | V_v/V_c | | |
|--------|-------------------|---------------------|--------------------------------------|--|--|--|-----------|-------|--------|
| | | | | | | | 1 kt | 10 kt | 100 kt |
| 0 | 0.4093 | 50 | 4.7 | 0.043 | 0.018 | 0.018 | 0.009 | 0.02 | 0.04 |
| 0.02 | 0.38 | 37 | 5.1 | 0.012 | 0.0047 | 0.023 | 0.011 | 0.02 | 0.05 |
| 0.04 | 0.36 | 26 | 5.75 | 0.024 | 0.0089 | 0.031 | 0.015 | 0.03 | 0.07 |
| 0.06 | 0.34 | 19 | 6.3 | 0.025 | 0.0088 | 0.040 | 0.020 | 0.04 | 0.09 |
| 0.08 | 0.32 | 14 | 7.05 | 0.042 | 0.014 | 0.054 | 0.026 | 0.06 | 0.12 |
| 0.10 | 0.30 | 10 | 7.8 | 0.052 | 0.016 | 0.070 | 0.036 | 0.07 | 0.16 |
| 0.12 | 0.28 | 7.1 | 8.9 | 0.097 | 0.028 | 0.098 | 0.048 | 0.10 | 0.22 |
| 0.14 | 0.26 | 5.2 | 10.0 | 0.12 | 0.033 | 0.13 | 0.064 | 0.14 | 0.29 |
| 0.16 | 0.24 | 3.8 | 11.0 | 0.14 | 0.035 | 0.17 | 0.031 | 0.13 | 0.37 |
| 0.18 | 0.22 | 2.7 | 12.2 | 0.20 | 0.047 | 0.21 | 0.104 | 0.23 | 0.47 |
| 0.20 | 0.20 | 2.0 | 13.4 | 0.25 | 0.053 | 0.26 | 0.130 | 0.28 | 0.50 |
| 0.22 | 0.18 | 1.4 | 15.1 | 0.43 | 0.083 | 0.25 | 0.170 | 0.36 | 0.77 |
| 0.24 | 0.16 | 1.03 | 16.8 | 0.54 | 0.093 | 0.44 | 0.22 | 0.46 | 0.97 |
| 0.26 | 0.14 | 0.76 | 18.9 | 0.84 | 0.13 | 0.57 | 0.27 | 0.59 | 1+ |
| 0.28 | 0.12 | 0.54 | 21.5 | 1.3 | 0.17 | 0.74 | 0.35 | 0.74 | - |
| 0.30 | 0.10 | 0.4 | 24.0 | 1.6 | 0.18 | 0.92 | 0.46 | 0.97 | - |
| 0.32 | 0.08 | 0.29 | 27.5 | 2.9 | 0.26 | 1.2 | 0.79 | 1+ | - |
| 0.34 | 0.06 | 0.21 | 31.0 | 3.8 | 0.26 | 1.4 | 0.72 | - | - |
| 0.36 | 0.04 | 0.15 | 35.5 | 6.3 | 0.31 | 1.8 | 0.88 | - | - |
| 0.38 | 0.02 | 0.106 | 41.0 | 10.0 | 0.10 | 2.1 | 1+ | - | - |
| 0.4093 | 0. | 0.077 | 48.0 | 17.0 | 0.17 | 2.2 | - | - | - |

Radius at which $\Sigma V_v = V_c$ 2.3 R_c 1.9 R_c 1.8 R_c

Table A2. 50% saturated paintbrush tuff.

Wt-fraction water = .1275

$\rho_0 = 1.605 \text{ g/cm}^3$

$\alpha = 0.2914$

$\psi_1 = 0.2047$

1 kt, $R_c = 20.46 \text{ m}$, $V_c = 3.59 \times 10^4 \text{ m}^3$

10 kt, $R_c = 35.24 \text{ m}$, $V_c = 1.83 \times 10^5 \text{ m}^3$

100 kt, $R_c = 60.70 \text{ m}$, $V_c = 9.37 \times 10^5 \text{ m}^3$

| ψ | $(\psi_1 - \psi)$ | \hat{P} (kbar) | $R/W^{1/3}$ (m/kt) ^{1/3} | V/W (m ³ /kt) $\times 10^4$ | V_v/W (m ³ /kt) $\times 10^4$ | $\Sigma(V_v/W)$ (m ³ /kt) $\times 10^4$ | $\Sigma V_v/V_c$ | | |
|--------|-------------------|---------------------|--------------------------------------|--|--|--|------------------|-------|--------|
| | | | | | | | 1 kt | 10 kt | 100 kt |
| 0 | 0.2047 | 4.6 | 12.2 | 0.76 | 0.16 | 0.16 | 0.043 | 0.084 | 0.16 |
| 0.02 | 0.18 | 3.1 | 14.5 | 0.52 | 0.10 | 0.25 | 0.070 | 0.14 | 0.27 |
| 0.04 | | 2.1 | 17.0 | 0.78 | 0.13 | 0.39 | 0.11 | 0.21 | 0.41 |
| 0.06 | 0.14 | 1.4 | 20.0 | 1.3 | 0.19 | 0.58 | 0.16 | 0.31 | 0.61 |
| 0.08 | 0.12 | 0.91 | 24.2 | 2.6 | 0.34 | 0.92 | 0.25 | 0.48 | 0.96 |
| 0.10 | 0.10 | 0.61 | 29.0 | 4.3 | 0.47 | 1.39 | 0.38 | 0.74 | 1+ |
| 0.12 | 0.08 | 0.42 | 34.5 | 7.0 | 0.63 | 2.02 | 0.55 | 1+ | - |
| 0.14 | 0.06 | 0.27 | 43.5 | 17.0 | 1.2 | 3.22 | 0.88 | - | - |
| 0.16 | 0.04 | 0.18 | 52.5 | 26.0 | 12.0 | 46.0 | 1+ | - | - |
| 0.18 | 0.02 | 0.12 | 66.0 | 61.0 | 18.0 | 64.0 | - | - | - |
| 0.20 | 0 | 0.08 | 82.0 | 111.0 | 11.0 | 75.0 | - | - | - |

Radius at which $\Sigma V_v = V_c$ $\approx 2.3 R_c$ $\approx 2.1 R_c$ $\approx 1.9 R_c$

Table A3. 90% saturated paintbrush tuff.

Wt-fraction water = 0.2094

$$\rho_0 = 1.768 \text{ g/cm}^3$$

$$\alpha = 0.2784$$

$$\psi_1 = 0.0409$$

$$1 \text{ kt, } R_c = 21.96 \text{ m, } V_c = 4.43 \cdot 10^4 \text{ m}^3$$

$$10 \text{ kt, } R_c = 38.21 \text{ m, } V_c = 2.34 \cdot 10^5 \text{ m}^3$$

$$100 \text{ kt, } R_c = 66.47 \text{ m, } V_c = 1.23 \cdot 10^6 \text{ m}^3$$

| ψ | $(\psi_1 - \psi)$ | \hat{P} (kbar) | $R/W^{1/3}$ (m/kt ^{1/3}) | V/W (m ³ /kt) $\cdot 10^4$ | V_v/W (m ³ /kt) $\cdot 10^3$ | $\Sigma(V_v/W)$ (m ³ /kt) $\cdot 10^3$ | $\Sigma V_v/V_c$ | | |
|--------|-------------------|---------------------|---------------------------------------|---|---|---|------------------|-------|--------|
| | | | | | | | 1 kt | 10 kt | 100 kt |
| 0 | 0.0409 | 2.9 | 19.0 | 2.9 | 1.2 | 1.2 | 0.026 | 0.050 | 0.094 |
| 0.005 | 0.035 | 2.0 | 22.0 | 1.6 | 0.60 | 1.8 | 0.040 | 0.076 | 0.142 |
| 0.01 | 0.03 | 1.39 | 26.5 | 3.3 | 1.1 | 2.9 | 0.064 | 0.12 | 0.228 |
| 0.015 | 0.025 | 0.94 | 32.5 | 6.6 | 1.8 | 4.7 | 0.10 | 0.20 | 0.373 |
| 0.02 | 0.02 | 0.64 | 39.0 | 10.5 | 2.4 | 7.0 | 0.16 | 0.30 | 0.562 |
| 0.025 | 0.015 | 0.44 | 48.0 | 21.5 | 3.8 | 10.8 | 0.24 | 0.46 | 0.863 |
| 0.03 | 0.01 | 0.3 | 60.0 | 44.1 | 5.5 | 16.3 | 0.36 | 0.69 | 1.+ |
| 0.035 | 0.005 | 0.21 | 75.0 | 36.2 | 6.5 | 23.0 | 0.51 | 0.96 | - |
| 0.04 | 0 | 0.14 | 94.0 | 170.0 | 4.3 | 27.0 | 0.60 | 1+ | - |

Radius at which $\Sigma V_v = V_c$ 4.4 R_c - 1.5 R_c

Table A4. 97% saturated paintbrush tuff.

St-fraction water = 0.2209

$$\rho_0 = 1.797 \text{ g/cm}^3$$

$$c = 0.2767$$

$$v_i = 0.0123$$

$$1 \text{ kt, } R_c = 22.16 \text{ m } V_c = 4.55 \times 10^4 \text{ m}^3$$

$$10 \text{ kt, } R_c = 38.61 \text{ m } V_c = 2.41 \times 10^5 \text{ m}^3$$

$$100 \text{ kt, } R_c = 67.26 \text{ m } V_c = 1.27 \times 10^6 \text{ m}^3$$

| ψ | $(\psi_i - \psi)$ | \bar{p} (kbar) | $R/W^{1/3}$ (m/kt ^{1/3}) | V/W (m ³ /kt) $\times 10^4$ | V_v/W (m ³ /kt) $\times 10^3$ | $\Sigma V_v/W$ (m ³ /kt) $\times 10^3$ | $\Sigma V_v/V_c$ | | |
|--------|-------------------|---------------------|---------------------------------------|--|--|---|------------------|-------|--------|
| | | | | | | | 1 kt | 10 kt | 100 kt |
| 0 | 0.01 | 2.7 | 22.0 | 4.5 | 0.45 | 0.45 | 0.010 | 0.018 | 0.035 |
| 0.001 | 0.009 | 2.05 | 25.0 | 2.1 | 0.20 | 0.64 | 0.014 | 0.027 | 0.050 |
| 0.002 | 0.008 | 1.5 | 29.0 | 3.7 | 0.31 | 0.96 | 0.021 | 0.040 | 0.075 |
| 0.003 | 0.007 | 1.1 | 34.0 | 6.2 | 0.47 | 1.4 | 0.031 | 0.057 | 0.11 |
| 0.004 | 0.006 | 0.85 | 39.0 | 8.4 | 0.55 | 2.0 | 0.043 | 0.082 | 0.15 |
| 0.005 | 0.005 | 0.662 | 46.5 | 17.0 | 0.95 | 2.9 | 0.053 | 0.12 | 0.23 |
| 0.006 | 0.004 | 0.47 | 54.0 | 24.0 | 1.1 | 4.0 | 0.088 | 0.17 | 0.31 |
| 0.007 | 0.003 | 0.34 | 66.0 | 54.0 | 1.9 | 5.9 | 0.43 | 0.25 | 0.46 |
| 0.008 | 0.002 | 0.25 | 80.0 | 94.0 | 2.4 | 8.2 | 0.18 | 0.34 | 0.65 |
| 0.009 | 0.001 | 0.19 | 94.0 | 130.0 | 2.0 | 10.3 | 0.23 | 0.43 | 0.80 |
| 0.01 | 0 | 0.14 | 115.0 | 290.0 | 1.4 | 11.7 | 0.26 | 0.49 | 0.92 |

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