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H. CAPES - D. VOSLAMBER

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ABSTRACT.

The unified theory of line broadening by electron perturbers is generalized to include the case of a weakly turbulent plasma. The collision operator in the line shape expression is shown to be the sum of two terms, both containing effects arising from the non-equilibrium nature of the plasma. One of the two terms represents the influence of individual atom-particle interactions occuring via the non-equilibrium dielectric plasma medium. The other term is due to the interaction of the atom with the turbulent waves. Both terms contain damping and diffusion effects arising from the plasma turbulence.

I - INTRODUCTION

It was outlined by a number of authors [1,2] that oscillations of non-equilibrium plasmas may have significant effects on spectral line shapes. Theoretical treatments have generally been based on two different approaches. One of them, mostly employed for stable plasmas [3,4], consists essentially in modifying the usual equilibrium collision operator by introducing the corresponding non-equilibrium distribution function and dielectric constant. In the other one, employed for unstable plasmas [3], the collision operator is constructed on assuming that the radiating atoms interact only with the collective (Vlasov) field of the turbulent plasma.

It is our purpose to show that these two methods treat two different aspects of plasma turbulence which a consistent theory has to take into account simultaneously. Using a BBGKY formalism for line broadening, we will derive a line shape expression which constitutes an extension of the unified theory for electron broadening to include the case of non-equilibrium plasmas. It is characterized by a collision operator which is the sum of two terms, both of them containing collective effects arising from the non-equilibrium nature of the plasma. One of them describes principally the influence of individual particle fields, acting on the atom via a non-equilibrium dielectric medium ; the other one is due to the collective fluctuations of the turbulent plasma. Both contributions may be gathered in one compact expression involving the "total" (individual + collective) binary correlation function, and the dynamical dielectric constant of the non-equilibrium plasma.

II - MODEL.

The general theory of line shapes emitted by partially ionised plasmas involves the ensemble average of the time evolution operators of the radiating atoms. In order to evaluate this ensemble average, we adopt the following model. We assume that the entire system is composed of a collection of approximately independent subsystems, each of them containing just one radiating atom surrounded by a great number of charged particles. The ensemble average may then be carried out in two steps : a) an average performed with the distribution function of a given subsystem ; b) an average over the different subsystems, i.e. over different distribution functions. The latter average may be looked at as one over initial conditions for the distribution function. However, this is just a conceptional interpretation which we will not make use of explicitly. Rather we will relate averages of type b) to corresponding quantities known from the theory of plasma turbulence or from experiment. Note that this last average is not needed for a plasma in equilibrium.

In the present treatment we will focus our interest on high frequency turbulence which is produced by the electron component of the plasma. Low frequency (ion) turbulence will be accounted for only in a restricted manner, namely by means of a suitably chosen low frequency microfield distribution function [5, 6, 7]. This implies that ion dynamic effects, as well as possible coupling effects with the high frequency modes or with the motion of individual electron perturbers [8] are neglected. In such a model the line shape problem is essentially reduced to investigating the effects on the radiating atom arising independently from a turbulent electron gas in a positive neutralizing background and from a static stochastic electric microfield.

We will further assume that the turbulence is weak in a sense specified in Appendix A. The corresponding condition of validity involves the small parameters $\gamma_c = \rho_c / \nu m \overline{V^2}$ and $\lambda_c = \gamma_c\, b_w / b_L$. (Here ρ_c denotes the energy density of the turbulent waves ; ν , m , $\overline{V^2}$ are the density, mass and mean square velocity of the electrons ; b_w and b_L are the Weisskopf and Landau lengths respectively). These parameters are to be considered in addition to the smallness parameters $\gamma_i = b_w / \lambda_D$ and $\lambda_i = \gamma_i\, b_w / b_L$ (λ_D being the Debye length) which express essentially that the Debye sphere (which keeps its sense if the turbulence is weak) contains a large number of particles.

III - THEORY.

The beginning of our formalism follows closely the lines of Ref.[9] (hereafter referred to as I). As a first step we calculate the mean atomic time evolution operator in one of the above defined subsystems. Let us denote by $f_N(1\ldots N;t)$ the distribution function of the electron gas in the 6N-dimensional phase space of the subsystem. Further, let us define $\phi_N(1\ldots N;t) \equiv T(1\ldots N;t)\, f_N(1\ldots N;t)$, where T is the atomic time evolution operator in a given configuration $(\vec{r}_1,\vec{v}_1,\ldots,\vec{r}_N,\vec{v}_N)$ of the electron gas at time t. As in I, we define the reduced quantities

$$f_s(1\ldots s;t) = \int f_N(1\ldots N;t)\, d(s+1)\ldots d(N).$$

$$\phi_s(1\ldots s;t) = \int \phi_N(1\ldots N;t)\, d(s+1)\ldots d(N).$$

It was shown in I that ϕ_N obeys an equation of the Liouville type (see eq. I.8) from which the B B G K Y hierarchy of equations (I.10) may be derived (with initial conditions $\phi_s(1\ldots s;0) = f_s(1\ldots s;0)$).

The truncation procedure employed to close the hierarchy after the second $(s=1)$ equation may be carried out by means of the closure relation

$$\phi_2(1,2;t) = \phi_1(1;t)f_1(2;t) + \phi_1(2;t)f_1(1;t) + \phi_0(t)f_2(1,2;t) - 2\phi_0(t)f_1(1;t)f_1(2;t) \qquad (1)$$

which is formally identical to that used in I(see Eq.I.12). We have to bear in mind, however, that in our case $f_1(1;t)$ and $f_2(1,2;t)$ are now non-equilibrium, i.e time dependant and inhomogeneous distribution functions.

The validity of Eq.(1) is based on conditions similar to those in I, namely on the exclusion of simultaneous strong collisions and on the necessity that the cumulative effect of simultaneous weak collisions is sufficiently small as to cause only small changes of the atomic state during the correlation time of the electric microfield (see Appendix A).

With the help of the closure relation(1), and using the first B B G K Y equation for $f_1(1,t)$, we arrive at the following coupled system of equations for ϕ_0 and ϕ_1 :

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}-\frac{i}{\hbar}\vec{\bar{D}}.\vec{E}(t)\right]\phi_0(t) = -\frac{iN}{\hbar}\int \bar{V}(1)\,\Gamma(1;t)\,d(1) \ , \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}-\frac{i}{\hbar}\vec{\bar{D}}.\vec{E}(t)+\vec{v}_1.\frac{\partial}{\partial \vec{r}_1}-\frac{e}{m}\vec{E}_1(1;t).\frac{\partial}{\partial \vec{v}_1}+\frac{i}{\hbar}\bar{V}(1)\right]\Gamma(1;t)=$$
$$-\frac{i}{\hbar}\left[\bar{V}(1)f_1(1;t)+N\int \bar{V}(2)\,g(1,2;t)\,d(2)\right]\phi_0(t) + \frac{N}{m}\frac{\partial f_1(1;t)}{\partial \vec{v}_1}.\int\frac{\partial W(1,2)}{\partial \vec{r}_1}\Gamma(2;t)\,d(2) \tag{3}$$

where

$$\vec{E}(t) \equiv \int e\vec{r}_1/\,r_1^3 \ f_1(1;t)\ d(1) \ , \tag{4}$$

$$e\,\vec{E}_1(1;t) \equiv \int\frac{\partial W(1,2)}{\partial \vec{r}_1}f_1(2;t)\ d(2) \tag{5}$$

are the collective Vlasov-fields acting on the atom and particle 1, respectively ; and

$$\Gamma(1;t) \equiv \phi_1(1;t) - f_1(1;t)\,\phi_0(t) \ ,$$
$$g(1,2;t) \equiv f_2(1,2;t) - f_1(1;t)\,f_1(2;t)$$

Further $\bar{H}^{\circ}$ and $\vec{\bar{D}}$ denote the unperturbed atomic Hamiltonian and the atomic dipole operator. We also use the interaction potential of the atom with one electron

$$\bar{V}(1) = -e\ \vec{\vec{D}}\,\vec{r}_1 / r_1^3 \ ,$$

and the interaction energy between two electrons :

$$W(1,2) = e^2 / |\vec{r}_1 - \vec{r}_2| \ .$$

(-e denotes the electronic charge).

As in I, we will make use of the Green's function method to solve Eq.(3). Let us first define the time evolution operator for the atomic state in the presence of one electron perturber and the turbulent plasma field $\vec{E}(t)$. It satisfies the equation

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\bar{H}^\circ - \frac{i}{\hbar}\vec{\vec{D}}.\vec{E}(t) + Z_1^{-1}(t,t')\frac{i}{\hbar}\bar{V}(1)\,Z_1(t,t')\right]Q_1(t,t') = 0 \quad , \quad Q_1(t,t) = 1 \qquad (6)$$

where the propagator $Z_1(t,t')$ acting on the functions of $\vec{r}_1$ and $\vec{v}_1$, obeys the equation

$$\left[\frac{\partial}{\partial t} + \vec{v}_1.\frac{\partial}{\partial \vec{r}_1} - \frac{e}{m}\vec{E}(1;t).\frac{\partial}{\partial \vec{v}_1}\right] Z_1(t,t') = 0 \ , \qquad (7)$$

with $Z_1(t,t) = 1$.

We further introduce the operator

$$D_I(\vec{r}_1,\vec{r}_2;t,t') = i\,\delta(t-t')\,\delta(\vec{r}_1-\vec{r}_2) - \frac{N}{m}\int d^3\vec{v}_1\ \left[Z_1(t,t')\,Q_1(t,t')\right]_+ \frac{\partial}{\partial \vec{v}_1} f_1(1,t') \cdot \frac{\partial W(1,2)}{\partial \vec{r}_1} \ , \qquad (8)$$

(with $[ZQ]_+ = ZQ \cdot \Theta(t-t')$, $\Theta(t-t')$ being the Heaviside function, i.e. $\Theta(t-t') = 1$ for $t > t'$ and $= 0$ for $t < t'$))

and the Green's function $G_I(\vec{r}_1,\vec{r}_2;t,t')$ defined from

$$\int_0^\infty dt'' \int d^3r_3\, D_I(\vec{r}_1,\vec{r}_3;t,t'')\, G_I(\vec{r}_3,\vec{r}_2;t'',t') = 2\,\delta(t-t')\,\delta(\vec{r}_1-\vec{r}_2)\,, \tag{9}$$

for $t > t'$, and by

$G_I(\vec{r}_1,\vec{r}_2;t,t') \equiv 0$ for $t' > t$.

Equation (3) may then be formally solved and yields

$$\int d^3v_1\, \Gamma(1;t) = -\frac{i}{\hbar}\int_0^\infty dt' \int d(2)\, G_I(\vec{r}_1,\vec{r}_2;t,t') \int_0^\infty dt'' \left[Z_2(t',t'')\, Q_2(t',t'') \right]_+ \left[\bar{V}(2) f_1(2;t'') + N\int \bar{V}(3)\, g(2,3;t'')\, d(3) \right] \times \phi_0(t'') \tag{10}$$

Inserting this result into Eq.(2), we obtain the following integro-differential equation for $\phi_0(t)$:

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\bar{H}^{\circ} - \frac{i}{\hbar}\vec{D}.\vec{E}(t)\right]\phi_0(t) = \int_0^\infty dt'\, M(t,t')\,\phi_0(t') \tag{11}$$

with the initial condition $\phi_0(0)=1$.

Here the two time operator $M(t,t')$ is given by

$$M(t,t') = -\frac{N}{\hbar^2}\int d(1)\int d(2)\int_0^\infty dt''\, \bar{V}(1)\, G_I(\vec{r}_1,\vec{r}_2;t,t'')\left[Z_2(t'',t')\, Q_2(t'',t')\right]_+ \left[\bar{V}(2) f(2;t') + N\int \bar{V}(3)\, g(2,3;t')\, d(3)\right]. \tag{12}$$

Equation (11) describes the stochastic evolution of the atom in a given subsystem. Indeed, the statistics over individual particle collisions have now been carried out and are summarized in the operator $M(t,t')$, while the collective Vlasov field $\vec{E}(t)$ (which is also contained in $M(t,t')$) still presents its detailed behaviour. The statistics over collective fields $\vec{E}(t)$ will now be performed by the average over subsystems.

In order to carry out this second part of the ensemble average, it is convenient to introduce shorthand notations for the time integrals. Let $f(t,t')$, $g(t,t')$ and $h(t)$ be any functions such that $f = g = 0$ for $t < t'$. We then define

$$f * g = \int_0^\infty dt'' \, f(t,t'') g(t'',t') \,,$$

and

$$f * h = \int_0^\infty dt'' \, f(t,t'') h(t'') \,.$$

We will denote by $I = 2\,\delta(t-t')$ the identity with respect to this operation

$$I * g = g * I = g \,.$$

Further we will write Laplace transforms as

$$\int_0^\infty dt \, e^{-st} F(t) = \tilde{F}(s) \,.$$

Let us define the projection operators /10/ P and I-P such that $PX = \langle X \rangle$ and $(1-P)X = \delta X$, where $\langle \; \rangle$ and δ denote, respectively, an average over subsystems and the deviation from this average $(X = \langle X \rangle + \delta X)$.

For the further evaluation of $\langle \phi_0 \rangle$, we make the following simplifying assumptions. First, we will assume the turbulent plasma to be kept stationary (e.g. by some external source) and homogeneous (which implies $\langle \vec{E} \rangle = 0$). This allows us to write

$$\langle M(t,t') \rangle = \langle M(t-t',0) \rangle = \langle M(t-t') \rangle$$

Further, we will assume the plasma turbulence to be weak . This might suggest a series expansion of quantities like $\langle\phi_0\rangle$ in powers of the collective electric fields $\vec{E}(t)$ and $\vec{E}_1(1,t)$. However, such a series would involve secular coefficients arising, e.g., from the expansion of the true particle orbits around the unperturbed straight paths. As usual in quasi-linear theory of plasma turbulence /11,12,13/, we avoid this difficulty by expanding $\langle\phi_0\rangle$ only partly. As will be demonstrated in the next sections, this method amounts to keeping the effects of the mean turbulent field on the perturber trajectories and on the atomic state.

Applying P to Eq.(11), we obtain :

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}\right] P\phi_0 = -P\,\mathcal{H}*\phi_0 \tag{13}$$

where

$$\mathcal{H} \equiv -\frac{i}{\hbar}\,\vec{D}.\vec{E}\ I - M\ . \tag{14}$$

If we take the Laplace transform of Eq.(13), and write it in the form

$$\left[s+\frac{i}{\hbar}\bar{H}^{\circ} + \langle\widetilde{\mathcal{H}*\phi_0}\rangle\langle\tilde{\phi}_0\rangle^{-1}\right]\langle\tilde{\phi}_0\rangle = 1$$

we are led to introduce the formal collision operator

$$K(s) = \langle\widetilde{\mathcal{H}*\phi_0}\rangle\,\langle\tilde{\phi}_0\rangle^{-1} \tag{15}$$

This expression is formal because it contains the unknown $\langle\hat{\phi}_0\rangle$. However, we will now show that in the limit of weak turbulence we are considering here, $\langle\tilde{\phi}_0\rangle$ can be eliminated from expression (15).

As a first step, we express $\vec{E}(t)$ and $M(t,t')$ in Eq.(14) by using Eqs.(4) and (12) to obtain

$$\langle \mathcal{H}_* \phi_c \rangle = \frac{iN}{\hbar}\int d(1)\, \bar{V}(1) \Big[\langle \delta f_1(1;t)\, \phi_0(t) \rangle - \frac{i}{\hbar}\int d(2) \int_0^\infty dt' \int_0^\infty dt'' \langle G_I(\vec{r}_1, \vec{r}_2; t, t')$$

$$\times \left[Z_2(t',t'') Q_2(t',t'') \right]_+ \left[\bar{V}(2) f_1(2,t'') + N\int \bar{V}(3)\, g(2,3;t'')\, d(3) \right] \phi_0(t'') \rangle \Big] \tag{16}$$

Next, using Eq.(C1) for $\delta f_1(1;t)$ and Eq.(11) for $\phi_0(t)$, we derive an integro-differential equation for the quantity $\delta f_1 \phi_0$,

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\bar{H}^0 - \frac{i}{\hbar}\vec{D}.\vec{E}(t) + \vec{v}_1.\frac{\partial}{\partial \vec{r}_1} - \frac{e}{m}\vec{E}_1(1;t).\frac{\partial}{\partial \vec{v}_1} \right] \delta f_1 \phi_0 = \frac{e}{m}\vec{E}_1(1;t)\,\phi_0(t).\frac{\partial}{\partial \vec{v}_1}\langle f_1(1,t)\rangle$$

$$- \delta f_1(1,t) \int_0^\infty dt'\, M(t,t')\, \phi_0(t') .$$

Note that the unknown $\delta f_1 \phi_0$ also appears on the right side of this equation. In order to make this explicit, we express $\vec{E}_1(1;t)$ in the first term of the right side by means of Eq.(5), and $\delta f_1(1;t)$ in the second term by means of Eq(C2). We thus obtain :

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\bar{H}^0 - \frac{i}{\hbar}\vec{D}.\vec{E}(t) - M Z_1 * + \vec{v}_1.\frac{\partial}{\partial \vec{r}_1} - \frac{e}{m}\vec{E}_1(1,t).\frac{\partial}{\partial \vec{v}_1} \right] \delta f_1 \phi_0 = \frac{N}{m}\frac{\partial \langle f_1 \rangle}{\partial \vec{v}_1}.\int \frac{\partial W(1,2)}{\partial \vec{r}_1}\delta f_1(2,t)\phi_0(t)\, d(2)$$

$$+ S_1(t) \tag{17}$$

where

$$S_1(t) = -\frac{N}{m}\int_0^\infty dt'\, M(t,t') \int_{t'}^{t} dt''\, Z_1(t,t') \frac{\partial}{\partial \vec{v}_1}\langle f(1,t'')\rangle . \int d(3)\, \frac{\partial W(1,3)}{\partial \vec{r}_1}\, \delta f(3,t'')\, \phi_0(t')$$

In order to solve formally Eq.(17), we introduce the operator U(t,t') (acting in the atomic Hilbert space) which satisfies the same equation as ϕ_0, namely

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\bar{H}^0 - \frac{i}{\hbar}\vec{D}.\vec{E}(t) \right] U(t,t') = \int_{t'}^{\infty} M(t,t'')\, U(t'',t')\, dt'' \tag{18}$$

but with initial condition U(t,t) = 1.

We further define the Green's function $G_{II}(\vec{r}_1,\vec{r}_2;t,t')$ from

$$\int_0^\infty dt'' \int d^3r_3\, D_{II}(\vec{r}_1,\vec{r}_3;t,t'')\, G_{II}(\vec{r}_3,\vec{r}_2;t'',t') = 2\,\delta(t-t')\,\delta(\vec{r}_1-\vec{r}_2)\,, \text{ for } t>t' \tag{19}$$

and by

$$G_{II}(\vec{r}_1,\vec{r}_2;t,t') \equiv 0 \qquad \text{for } t'>t$$

Here the kernel D_{II} is given by

$$D_{II}(\vec{r}_1,\vec{r}_2;t,t') = 2\,\delta(t-t')\,\delta(\vec{r}_1-\vec{r}_2) - \frac{N}{m}\int d^3v\,\left[Z_1(t,t')\,U(t,t')\right]_+ \frac{\partial\langle f(1,t')\rangle}{\partial\vec{v}_1}\cdot\frac{\partial W_{12}}{\partial\vec{r}_1} \tag{20}$$

with $[Z_1 U]_+ = Z_1 U\,\theta(t-t')$.

Eq.(17) may then be formally solved and yields

$$\int d^3v_1\, \delta f_1\, \phi_0 = \int_0^\infty dt' \int d(2)\, G_{II}(\vec{r}_1,\vec{r}_2;t,t')\Big[\left[Z_2(t',0)\,U(t',0)\right]_+ \delta f_1(2;0)$$
$$+ \int_0^\infty dt''\,\left[Z_2(t',t'')\,U(t',t'')\right]_+ S_2(t'')\Big]$$

Inserting this result into Eq.(16) leads to the following expression for $\langle \mathcal{H}_* \phi_0\rangle$:

$$\langle \mathcal{H}_*\phi_0\rangle = \frac{iN}{\hbar}\int d(1)\,\bar{V}(1)\int d(2)\int_0^\infty dt'\Big[\Big\langle G_{II}(\vec{r}_1,\vec{r}_2;t,t')\Big[\left[Z_2(t',0)\,U(t',0)\right]_+ \delta f_1(2,0)$$
$$+\int dt''\left[Z_2(t',t'')\,U(t',t'')\right]_+ S_2(t'')\Big]\Big\rangle - \Big\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\left[Z_2(t',t'')\,Q_2(t',t'')\right]_+ \tag{21}$$
$$\times\Big[\bar{V}(2)\,f_1(2,t'') + N\int \bar{V}(3)\,g(2,3;t'')\,d(3)\Big]\,\phi_0(t'')\Big\rangle\Big].$$

This expression may be simplified by neglecting $S_1(t)$ and by replacing g with $\langle g\rangle$. Indeed, both $S_1(t)$ and δg yield relative contributions of order λ_c with respect to the term containing G_I and of order λ_i with respect to the term containing G_{II}.

A more convenient form of $\langle \mathcal{H}* \phi_0 \rangle$ may then be obtained by using the relation

$$f_1(1;t) = Z_1(t,0) f_1(1;0) \tag{22}$$

which follows from Eqs. (C3) and (7) for $f_1(1;t)$ and $Z_1(t,t')$, respectively. Making also use of the relation

$$\phi_0(t) = U(t,0)$$

in the second term of Eq. (21), we finally obtain

$$\begin{aligned}\langle \mathcal{H}*\phi_0\rangle = \frac{iN}{\hbar}\int d(1)\,\bar{V}(1)\int_0^\infty dt' \int d(2) \Big[&\langle G_{II}(\vec{r}_1,\vec{r}_2;t,t')\,[Z_2(t',0)U(t',0)]_+\, df_1(2;0)\rangle \\ -\frac{i}{\hbar}\int_0^\infty dt'' &\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\,[Z_2(t',t'')\,Q_2(t',t'')]_+\,\bar{V}(2)\,[Z_2(t'',0)\,U(t'',0)]_+\, f_1(2;0)\rangle \\ -\frac{i}{\hbar}\int_0^\infty dt'' &\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\,[Z_2(t',t'')\,Q_2(t',t'')]\,N\int \bar{V}(3)\,\langle g(2,3;t'')\rangle\, d(3)\, U_+(t'',0)\rangle . \end{aligned} \tag{23}$$

with $U_+ = U\,\Theta(t-t')$.

A more explicit expression for $\langle \mathcal{H}* \phi_0 \rangle$ can be derived by expressing the average quantities in the integrands of Eq. (23) in terms of factors like $\langle G_I \rangle$, $\langle G_{II} \rangle$, $\langle [Z\,U]_+ \rangle$ and $\langle [ZQ]_+ \rangle$. As will be demonstrated in Appendix F, this procedure leads to a rather complicated series expansion in power of λ_c. This expansion, however, does not contain singular terms since all its terms depend on the average propagator $\langle [ZU]_+ \rangle$ or $\langle [ZQ]_+ \rangle$ and thus include the mean effect of the turbulent plasma field on the perturber trajectories and the atomic state. Therefore, in our case of weak turbulence, we may keep only the terms to lowest order in λ_c, to obtain

$$\langle \mathcal{H} * \phi_0 \rangle = \frac{iN}{\hbar}\int d(1)\,\bar{V}(1)\int_0^{\infty} dt' \int_{t'}^{\infty} dt'' \int d(2)\,\langle G_{II}(\vec{r}_1,\vec{r}_2;t,t')\rangle \langle [Z_2(t',t'')\bar{Q}_2(t',t'')]_+\rangle \left\langle \frac{i}{\hbar}\vec{D}.\vec{E}(t'')\,\delta f_1(2,t'')\right\rangle \langle U_+(t'',0)\rangle$$

$$-\frac{iN}{\hbar}\int d(1)\,\bar{V}(1)\int_0^{\infty} dt' \int_0^{\infty} dt'' \int d(2)\,\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\rangle \langle [Z_2(t',t'')Q_2(t',t'')]_+\rangle \Big[\bar{V}(2)\langle f_1(2,t'')\rangle$$

$$+ N\int \bar{V}(3)\,\langle g(2,3;t'')\rangle\, d(3)\Big]\langle U_+(t'',0)\rangle \qquad (24)$$

where the propagator $\bar{Q}_1$ is defined from the Dyson equation

$$\bar{Q}_1 = U - \frac{i}{\hbar}\bar{Q}_1\bar{V} * U \qquad (25)$$

By virtue of the stationarity of the turbulent plasma state, the time dependance of the quantities $\langle U(t,t')\rangle$, $\langle Z_1(t,t')\,U(t,t')\rangle$, $\langle Z_1(t,t')\,Q_1(t,t')\rangle$, $\langle Z_1(t,t')\,\bar{Q}_1(t,t')\rangle$ and consequently $\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\rangle$ and $\langle G_{II}(\vec{r}_1,\vec{r}_2;t,t')\rangle$ only occurs via the difference t-t'. For instance we may write

$$\langle U_+(t,t')\rangle = \langle U_+(t-t',0)\rangle = \langle \phi_0(t-t')\rangle \qquad (26)$$

This allows us to perform the Laplace transform of Eq.(24). Inserting the result into Eq.(15) leads to the following result for the collision operator $K(s)$

$$K(s) = K_I(s) + K_{II}(s)$$

where

$$K_I(s) = \int_0^{\infty} d(t-t'')\, e^{-s(t-t'')}\frac{N}{\hbar^2}\int d(1)\,\bar{V}(1)\int_0^{\infty} dt' \int d(2)\,\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\rangle \langle [Z_2(t',t'')Q_2(t',t'')]_+\rangle$$

$$\times\Big[\bar{V}(2)\,\langle f_1(2,t'')\rangle + N\int \bar{V}(3)\langle g(2,3;t'')\rangle\, d(3)\Big]. \qquad (27)$$

$$K_{II}(s) = \int_0^{\infty} d(t-t'')\, e^{-s(t-t'')}\frac{N}{\hbar^2}\int d(1)\,\bar{V}(1)\int_0^{\infty} dt' \int d(2)\,\langle G_{II}(\vec{r}_1,\vec{r}_2;t,t')\rangle \langle [Z_2(t',t'')\bar{Q}_2(t',t'')]_+\rangle$$

$$\times \langle \vec{D}.\vec{E}(t'')\,\delta f_1(2,t'')\rangle . \qquad (28)$$

The first part, K_I, characterizes the influence of individual particle collisions on the radiating atom. However, it also includes collectives effects contained mainly in $\langle G_I \rangle$ which describes the response of the non-equilibrium medium to the individual particle fields. The second part, K_{II} , just represents the influence of the turbulent plasma modes on the radiating atom.

Let us now turn to a more convenient form of $K_I(s)$ by introducing the Fourier transforms of the quantities $\bar{V}(1)$, $\langle G_I \rangle$ and $\langle [Z_1 Q_1]_+ \rangle$ in Eq.(27), and carrying out the Laplace transform :

$$K_I(s) = \frac{N}{\hbar^2 (2\pi)^9} \int d^3k \int d^3k' \int d^3k'' \tilde{V}(\vec{k}) \langle \tilde{G}_I(\vec{k},\vec{k}';s) \rangle \int d^3v \, T(-\vec{k}';-\vec{k}'',\vec{v};s) \, \tilde{V}(\vec{k}'') [f(\vec{v}) + N g_I(\vec{k}'',\vec{v})] . \quad (29)$$

Here the following notations have been used :

$$\tilde{V}(\vec{k}) = \int d^3r \, e^{-i\vec{k}.\vec{r}} \quad \bar{V}(\vec{r})$$

$$g_I(\vec{k},\vec{v}) = \int d^3r_{12} \int d^3v' \langle g(\vec{r}_{12}, \vec{v}, \vec{v}') \rangle \quad e^{-i\vec{k}.\vec{r}_{12}}$$

$$f(\vec{v}) \quad = \langle f_1(1;t) \rangle$$

Further, $\langle \tilde{G}_I \rangle$ and T_I stand, respectively, for the Fourier and the Laplace transforms of the Green's function $\langle G_I \rangle$ and of the propagator $\langle (Z_1 Q_1)_+ \rangle$:

$$\langle \tilde{G}_I(\vec{k};\vec{k}';s) \rangle = \int d^3r_1 \int d^3r_2 \int_0^\infty d(t-t') \, e^{-s(t-t')} \, e^{-i(\vec{k}.\vec{r} + \vec{k}'.\vec{r}')} \langle G_I(\vec{r}_1,\vec{r}_2;t,t') \rangle ,$$

$$T_I(-\vec{k};-\vec{k}',\vec{v};s) = \int d^3r_1 \int_0^\infty d(t-t') \, e^{-s(t-t')} \quad e^{+i\vec{k}.\vec{r}} \langle [Z_1(t,t') Q_1(t,t')]_+ \rangle \, e^{+i\vec{k}'.\vec{r}'} . \quad (30)$$

Note that the propagator T_I acts on all functions of $\vec{v}$ standing on its right.

For a further evaluation of expression (29) we need to specify the quantities $\langle \tilde{G}_I(\vec{k},\vec{k}',s)\rangle$ and $T_I(\vec{k},\vec{k}',\vec{v};s)$. In order to determine the propagator T_I, we use the following equation, derived in Appendix E :

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{0}+\frac{i}{\hbar}\bar{V}(1)+\frac{i}{\hbar}\,A_1 * +\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}+R_1 * +\frac{\partial}{\partial\vec{v}_1}\cdot\overleftrightarrow{C}_1\cdot\frac{\partial}{\partial\vec{v}_1} *\right]\langle(Z_1Q_1)_+\rangle=\delta(t-t'). \tag{31}$$

This equation describes the time evolution of the atom and of one perturbing electron under the influence of the turbulent collective field. It contains three terms of a diffusion type. Two of them, namely

$$R_1(t-t')=-\frac{i}{\hbar}\left\langle \vec{\vec{D}}.\vec{E}(t)\,\left[Z_1(t,t')Q_1(t,t')\right]_P\frac{i}{\hbar}\vec{\vec{D}}.\vec{E}(t')\right\rangle \tag{32}$$

and

$$\overleftrightarrow{C}_1(t-t')=-\frac{e^2}{m^2}\left\langle \vec{E}_1(1;t)\left[Z_1(t,t')Q_1(t,t')\right]_P\vec{E}_1(1,t')\right\rangle \tag{33}$$

account respectively for a damping of the atomic state and for a diffusion of the electron perturber in the turbulent collective field. Here $[Z_1Q_1]_P$ satisfies the equation

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{0}+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}+(1-P)\left(-\frac{i}{\hbar}\vec{\vec{D}}.\vec{E}\;-\frac{e}{m}\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}\right)\right]\left[Z_1Q_1\right]_P=\delta(t-t') \tag{34}$$

The third diffusion type term

$$A_1(t-t')=-\frac{ei}{m\hbar}\left\langle\vec{\vec{D}}.\vec{E}(t)\,\left[Z_1(t,t')Q_1(t,t')\right]_P\,\vec{E}_1(1;t')\right\rangle\cdot\frac{\partial}{\partial\vec{v}_1} \tag{35}$$

$$-\frac{ei}{m\hbar}\frac{\partial}{\partial\vec{v}_1}\cdot\left\langle\vec{E}_1(1;t)\,\left[Z_1(t,t')Q_1(t,t')\right]_P\vec{\vec{D}}.\,\vec{E}(t')\right\rangle$$

cannot be given an easy interpretation. It contains diffusion effects on the atom and on the perturbing electrons.

We will show that in most cases the above diffusion type terms will have only small effects on the line profile. Therefore it is sufficient to calculate them approximately, especially with regard to the determination of $[Z_1 Q_1]_p$ from Eq.(34). Since to lowest order, this quantity may be approximated as

$$\left[Z_1(t,t')Q_1(t,t')\right]_p \simeq \exp\left[-\frac{i}{\hbar}\bar{H}^\circ(t-t') - (t-t')\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}\right] \tag{36}$$

we get for R_1

$$R_1(t-t') = M_{II}(t-t')\exp\left[-(t-t')\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}\right]$$

where

$$M_{II}(t-t') = \left\langle \frac{i}{\hbar}\vec{\vec{D}}.\vec{E}(t)\exp\left[-\frac{i}{\hbar}\bar{H}^\circ(t-t')\right]\frac{i}{\hbar}\vec{\vec{D}}.\vec{E}(t')\right\rangle$$

Note that the Laplace transform $K^\circ_{II}(s)$ of $M_{II}(t-t')$ equals to lowest order the expression for $K_{II}(s)$, we will obtain below [see Eq.(49)].

The same approximation for $[Z_1 Q_1]_p$ leads to

$$\overleftrightarrow{C}_{11}(t-t') = \overleftrightarrow{C}^\circ(\vec{v}_1, t-t')\exp\left[-\frac{i}{\hbar}\bar{H}^\circ(t-t') - (t-t')\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}\right]$$

where the Laplace transform of $\overleftrightarrow{C}^\circ(\vec{v}, t-t')$ is given by

$$\overleftrightarrow{C}^\circ(\vec{v};s) = \frac{N^2}{m^2(2\pi)^3}\int d^3k'\, i\vec{k}'\,\widetilde{W}(\vec{k}')\,\epsilon^{-1}(\vec{k}';s)\int d^3v'\left[s+i\vec{k}'.\vec{v}'\right]^{-1}\tilde{g}_{II}(\vec{k}';\vec{v}')\, i\vec{k}'\,\widetilde{W}(\vec{k}')$$

Here

$$\widetilde{W}(\vec{k}) = \int d^3r\, e^{-i\vec{k}.\vec{r}}\, W(r)$$

is the Fourier transform of the interaction potential $W(r)$

and

$$\epsilon(\vec{k};s) = 1 - \frac{N}{m}\, i\, \vec{k}\, \widetilde{W}(k) . \int \frac{\partial f(\vec{v})}{\partial \vec{v}} \left[s + i\vec{k}.\vec{v}\right]^{-1} d^3 v$$

is the dielectric constant ; further

$$\widetilde{g}_{II}(\vec{k},\vec{v}) = \int d^3 v_2 \; e^{-i\vec{k}.\vec{r}_{12}} \; g_{II}(1,2) \;\; d^3 r_{12} \;, \qquad (37)$$

is the Fourier transform of the collective binary correlation function,

$$g_{II}(1,2) = \langle \delta f_1(1;t)\, \delta f_1(2;t) \rangle \;,$$

which we will discuss in more detail in Appendix B .

Finally, using again approximation (36), we obtain for A_1

$$A_1(t-t') = A^\circ(1;t-t') \;\; \exp\left[-\frac{i}{\hbar}\overline{H}^\circ(t-t') - (t-t')\vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1}\right]$$

where the Fourier and Laplace transforms of A° are given by

$$\widetilde{A}^\circ(\vec{k},\vec{v};s) = \frac{N^2}{m}\left[\widetilde{V}(\vec{k})\epsilon^{-1}(\vec{k};s)\int \widetilde{g}_{II}(\vec{k},\vec{v}')\left[s+i\vec{k}.\vec{v}'\right]^{-1} d^3v' \;\; i\vec{k}.\widetilde{W}(\vec{k})\frac{\partial}{\partial \vec{v}}\right.$$
$$\left. + \;\frac{\partial}{\partial \vec{v}} . \, i\vec{k}\,\widetilde{W}(\vec{k})\epsilon^{-1}(\vec{k};s)\int \widetilde{g}_{II}(\vec{k},\vec{v}')\left[s+i\vec{k}.\vec{v}'\right]^{-1} d^3v' \;\; \widetilde{V}(\vec{k})\right]$$

For the determination of $\langle G_I(\vec{r}_1,\vec{r}_2;t,t')\rangle$ we take the Fourier and Laplace transforms of Eq.(D3) to arrive at

$$\int \langle \widetilde{D}_I(\vec{k},\vec{k}'';s)\rangle \langle \widetilde{G}_I(\vec{k}'',\vec{k}';s)\rangle\, d^3k'' = \delta(\vec{k}+\vec{k}') \qquad (39)$$

An explicit expression for $\langle \widetilde{G}_I \rangle$ may be obtained along the same lines as in paper I. Expanding T_I in the expression for $\langle \widetilde{D}_I \rangle$ to zero order in $\overline{V}(1) + A_1$ (which assumes $\lambda_i \ll 1$

and $\lambda_c \ll 1$), Eq.(39) may be solved to yield

$$\langle \tilde{G}_I(\vec{k},\vec{k}';s)\rangle = \epsilon_I^{-1}(\vec{k};s)\ \delta(\vec{k}+\vec{k}') \tag{40}$$

where

$$\epsilon_I(\vec{k};s) = 1 - \frac{iN}{m}\ \vec{k}\,\tilde{W}(\vec{k}) \int d^3v \left[s + \frac{i}{\hbar}\bar{H}^o + i\vec{k}.\vec{v} + K_I^o(s+i\vec{k}.\vec{v}) + \frac{\partial}{\partial\vec{v}}\cdot\tilde{\overleftrightarrow{C}}{}^{10}(\vec{v};s+\frac{i}{\hbar}\bar{H}^o+i\vec{k}.\vec{v}).\frac{\partial}{\partial\vec{v}}\right]^{-1}\cdot\frac{\partial f(\vec{v})}{\partial\vec{v}} \tag{41}$$

The meaning of $\epsilon_I(\vec{k};s)$ is also essentially that of a dielectric constant, the only difference being its dependance on atomic variables via $\bar{H}_o$ and $K_{II}^o(s+i\vec{k}.\vec{v})$ Inserting expression (40) into Eq.(29), the collision operator $K_I(s)$ takes the following form :

$$K_I(s) = \frac{-N^2}{(2\pi)^3\hbar^2}\int d^3k \int d^3k'\ \tilde{V}(\vec{k})\,\epsilon_I^{-1}(\vec{k};s)\int d^3v\ T_I(\vec{k},-\vec{k}',\vec{v};s)\left[f(\vec{v})+N g_I(\vec{k},\vec{v})\right]\ \tilde{V}(k') \tag{42}$$

Let us now turn to a more explicit evaluation of K_{II}. Carrying out Fourier and Laplace transforms we are led to the following expression :

$$K_{II}(s) = \frac{N^2}{(2\pi)^6\hbar^2}\int d^3k\int d^3k'\int d^3k''\ \tilde{V}(k)\langle\tilde{G}_{II}(\vec{k},\vec{k}';s)\rangle\int d^3v\ T_{II}(-\vec{k}',-\vec{k}'',\vec{v};s)\ g_{II}(\vec{k}'',\vec{v})\ \tilde{V}(k'') \tag{43}$$

Here, two quantities, the Green's function $\langle\tilde{G}_{II}\rangle$ and the propagator T_{II} remain to be determined. Let us first consider $T_{II}(-\vec{k},-\vec{k}',\vec{v};s)$ which is the Fourier and Laplace transform of the propagator $\langle[Z_1\bar{Q}_1]_+\rangle$.

$$T_{II}(-\vec{k},-\vec{k}';\vec{v};s) = \int d^3r_1\int_0^\infty d(t-t')\,e^{-s(t-t')}\,e^{+i\vec{k}.\vec{r}_1}\left\langle\left[Z_1(t,t')\bar{Q}_1(t,t')\right]_+\right\rangle e^{+i\vec{k}'.\vec{r}_1} \tag{44}$$

For the determination of $\langle [Z_1\bar{Q}_1]_+ \rangle$, an integro-differential equation is derived in Appendix G :

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^\circ+\frac{i}{\hbar}\bar{V}(1)+\frac{i}{\hbar}A_1'* +\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}+R_1'* - \langle MZ_1\rangle * +\frac{\partial}{\partial\vec{v}_1}\cdot\overleftrightarrow{C}'\cdot\frac{\partial}{\partial\vec{v}_1}*\right]\langle [Z_1\bar{Q}_1]_+\rangle \tag{45}$$
$$= \delta(t-t') .$$

This equation describes the time evolution of the atom and of one perturbing electron, including diffusion effects represented by $\langle MZ\rangle$ and the three terms

$$R_1'(t-t') = -\left\langle\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-(1-P)MZ_1*\right][Z_1\bar{Q}_1]_P\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-*(1-P)MZ_1\right]\right\rangle$$

$$\overleftrightarrow{C}_1'(t-t') = -\frac{e^2}{m^2}\left\langle\vec{E}_1(1,t)[Z_1(t,t')\bar{Q}_1(t,t')]_P\vec{E}_1(1,t')\right\rangle$$

$$A_1'(t-t') = \frac{e}{m}\left\langle\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-(1-P)MZ_1*\right][Z_1\bar{Q}_1]_P\,\vec{E}_1\right\rangle\frac{\partial}{\partial\vec{v}_1}$$
$$+\left\langle\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}[Z_1\bar{Q}_1]_P\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-*(1-P)MZ_1\right]\right\rangle$$

Here $[Z_1\bar{Q}_1]_P$ satisfies the following equation :

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^\circ+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}+(1-P)\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-MZ_1*\right]-(1-P)\vec{E}_1\frac{\partial}{\partial\vec{v}_1}\right][Z_1\bar{Q}_1]_P=\delta(t-t') \tag{46}$$

Note that Eq.(45) differs from Eq.(31) for $\langle [Z_1Q_1]_+\rangle$ essentially by the presence of $\langle MZ\rangle$ which may be approximated by $M_I(t-t')\exp\left[-(t-t')\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}\right]$ where the Laplace transform $K_I^\circ(s)$ of $M_I(t-t')$ equals to lowest order in λ_c, the expression for $K_I(s)$ given by Eq. (42) . For a further simplification of Eq.(45) we shown in Appendix G that R_1', $\overleftrightarrow{C}_1'$ and A_1', may be approximated by R_1, $\overleftrightarrow{C}_1$ and A_1.

Let us now consider the Green's function $\langle \tilde{G}_{II}'\rangle$ which satisfies the equation

$$\int d^3k'' \langle \tilde{D}'_{II}(\vec{k},\vec{k}'';s)\rangle \langle \tilde{G}_{I}(\vec{k}'',\vec{k}';s)\rangle = \delta(\vec{k}+\vec{k}')$$

The kernel $\langle \tilde{D}_{II} \rangle$ is given by

$$\langle \tilde{D}_{II}(\vec{k},\vec{k}';s)\rangle = \delta(\vec{k}+\vec{k}') - \frac{iN}{\hbar(2\pi)^3}\int d^3v\, T^{\circ}_{II}(-\vec{k},-\vec{k}';\vec{v};s)\, \vec{k}'\, \tilde{W}(k')\cdot\frac{\partial f(v)}{\partial \vec{v}}$$

with

$$T^{\circ}_{II}(-\vec{k},-\vec{k}';\vec{v};s) = \int d^3r_1 \int_0^{\infty} d(t-t')\, e^{-s(t-t')}\, e^{i\vec{k}.\vec{r}_1} \langle [z_1(t,t')\,V(t,t')]_+ \rangle\, e^{+i\vec{k}'.\vec{r}_1}$$

As for $\langle \tilde{G}_I \rangle$, an explicit expression for $\langle \tilde{G}_{II} \rangle$ may be given by expanding $\langle \tilde{D}_{II} \rangle$ to zero order in the atom-turbulent field interaction. The result thus obtained is

$$\langle \tilde{G}_{II}(\vec{k},\vec{k}';s)\rangle = \epsilon^{-1}_{II}(\vec{k};s)\ \delta(\vec{k}+\vec{k}') \tag{47}$$

where

$$\epsilon_{II}(\vec{k};s) = 1 - \frac{iN}{m}\vec{k}.\tilde{W}(\vec{k})\int d\vec{v}\left[s+\frac{i}{\hbar}\tilde{H}^{\circ}+i\vec{k}.\vec{v}+K^{\circ}_{I}(s+i\vec{k}.\vec{v})+K^{\circ}_{II}(s+i\vec{k}.\vec{v})+\frac{\partial}{\partial\vec{v}}\cdot\tilde{\overleftrightarrow{C}}^{\circ}(\vec{v};s+\frac{i}{\hbar}\tilde{H}_{\circ}+i\vec{k}.\vec{v})\cdot\frac{\partial}{\partial\vec{v}}\right]^{-1}\frac{\partial f(\vec{v})}{\partial\vec{v}} \tag{48}$$

Inserting expression (47) into Eq.(42), K_{II} takes the following form:

$$K_{II}(s) = \frac{-N^2}{\hbar^2(2\pi)}\int d^3k \int d^3k'\ \tilde{V}(\vec{k})\,\epsilon^{-1}_{II}(\vec{k};s)\int d\vec{v}\ T_{II}(-\vec{k},\vec{k}',\vec{v};s)\ \tilde{g}_{II}(\vec{k}';\vec{v})\ \tilde{V}(\vec{k}')\,. \tag{49}$$

Similarly to the expression (42) for $K_I(s)$, this expression also contains the diffusion type terms C°, K°_{II} and A_1 due to the collective fields, and in addition K°_{I}. Note that K_I and K_{II} do not depend on the same binary correlation function. While K_I involves g_I, K_{II} depends on g_{II} and therefore involves the turbulent spectral energy density, $\mathcal{E}$, which is related to g_{II} by [18]

$$\mathcal{E}_{\vec{k}} = \frac{(4\pi\nu e)^2}{8\pi k^2}\int d^3v\ \tilde{g}_{II}(\vec{k},\vec{v}) \tag{50}$$

The expressions (42) and (49) for K_I and K_{II} yield a rather general formulation of the problem of line broadening in a turbulent plasma, including all aspects of the unified theory. The expression for K_{II} involves the spectral energy density of the turbulent waves. Both operators, K_I and K_{II}, contain the dielectric properties of the non-equilibrium plasma, including turbulent diffusion effects on the atomic state and on the perturber orbits.

The fact that K_I does not occur as a diffusion type term in Eq.(42); /via Eqs.(31) and (30) for T / is a consequence of the closure relation (1). A treatment involving a more complete closure relation /14/ would also lead to diffusion terms due to individual atom-perturber and perturber-perturber collisions, in addition to the collective diffusion terms mentionned above. In particular, K_I would then also include the combined diffusion type term $K_I^0(s+i\vec{k}.\vec{v}) + K_{II}^0(s+i\vec{k}.\vec{v})$ Indeed, the effect of diffusion terms like $K_I^0(s+i\vec{k}.\vec{v})$ and $K_{II}^0(s+i\vec{k}.\vec{v})$ in Eqs.(42) and (49) is essentially to damp the influence of the resonant modes in the dielectric constant. This is effective only when another "damping" effect, namely the smoothing of the resonance peak by the k-integration in Eqs.(42) and (49) is less significant. This may occur in situations where the turbulent modes are relatively strongly excited. However in these cases, $K_{II}^0(s+i\vec{k}.\vec{v})$ is generally dominant and, compared to this quantity, $K_I^0(s+i\vec{k}.\vec{v})$ might be neglected if it were present in Eq.(42) (via Eqs.(31) and (30) for T_I).

Including $K_I^0(s+i\vec{k}.\vec{v})$ into Eq.(42) might be of some importance when condition (A1) of Appendix A is only marginally fulfilled (plasmas with only a few particles in the Debye sphere, or spectral lines with high principal quantum number). Indeed, $K_I^0(s+i\vec{k}.\vec{v})$ would then account for the cumulative influence of simultaneous weak collisions, its effect being essentially a long range cut-off (similar to the screening cut-off). Note that the more general form for K_I, mentioned above, would have the advantage of presenting the same formal features as K_{II}. K_I might then be combined with K_{II} to yield the following compact formula :

$$K(s)=\frac{-N}{\hbar^4(2\pi)^6}\int d^3k\int d^3k'\ \tilde{V}(\vec{k})\,\epsilon_{II}^{-1}(\vec{k};s)\int d^3v\ T_{II}(\vec{k},\vec{k}',\vec{v};s)\left[f(\vec{v})+N\tilde{g}_T(\vec{k}',\vec{v})\right]\tilde{V}(\vec{k}') \tag{51}$$

In this expression $\tilde{g}_T(\vec{k},\vec{v})$ is the total two-electron correlation function

$$\tilde{g}_T(\vec{k},\vec{v})=\tilde{g}_I(\vec{k},\vec{v})+\tilde{g}_{II}(\vec{k},\vec{v}),$$

combining both, the individual part $\tilde{g}_I$ and the collective part $\tilde{g}_{II}$. Note that the static limit of the inverse of the dielectric constant $\epsilon_{II}^{-1}(\vec{k};s)$ (or $\epsilon_I^{-1}(\vec{k};s)$) does not generally coincide, as in the equilibrium case, with

$$1+N\int d^3v\ \tilde{g}_T(\vec{k},\vec{v})$$

even if the diffusion terms $K_I^0(s+i\vec{k}.\vec{v})$, $K_{II}^0(s+i\vec{k}.\vec{v})$ and $\tilde{\vec{C}}^0(\vec{v};s)$ are neglected. Moreover $\tilde{g}_T(\vec{k},\vec{v})$ cannot be generally written as being equal to $f(\vec{v})\int dv\ \tilde{g}_T(\vec{k},\vec{v})$. It thus appears that the expression (51) for the total collision operator cannot be obtained by a simple generalization of the previous unified theory derived for the equilibrium case. Indeed, to obtain the non equilibrium case (stationary and homogenous) it is not sufficient to just replace the equilibrium dielectric constant and the velocity distribution by the corresponding non equilibrium quantities.

IV - CONCLUSION.

We have used the general formalism of the unified theory of electron broadening to derive an expression for the line shape in a weakly turbulent and quasi-stationary plasma.

Our result for the collision operator /Eqs.(37) and (43)_7 reveals two complementary aspects of the influence of plasma turbulence on line profiles. On the one hand, collective fields due to the turbulent plasma fluctuations affect the atomic state in addition to the individual particle collisions. On the other hand, the individual fields, in turn, are modified due to the response of the non equilibrium plasma via the corresponding dielectric constant.

In the vicinity of the turbulent resonance frequencies the line profile may show a structure (satellites). The shape of these satellites may depend on several factors : the general form of the spectral line, the energy spectrum of the turbulent waves and the diffusion type terms $K_{I}^{\circ}(s+i\vec{k}.\vec{v})$, $K_{II}^{\circ}(s+i\vec{k}.\vec{v})$ and $\widetilde{\overleftrightarrow{C}}^{\circ}(\vec{v},s+\frac{i}{\hbar}\bar{H}^{\circ}+i\vec{k}.\vec{v})$.

Far from the resonant frequencies, the effect of $K_{I}(s)$ and $K_{II}(s)$ is essentielly to broaden and to shift the line. Due to the presence of $K_{II}(s)$, the line width is generally larger in the presence of turbulence than in the case of an equilibrium plasma.

APPENDIX.

A- Validity conditions.

Apart from the conditions

$$b_W / \nu^{-1/3} \ll 1 \quad , \quad b_L / \nu^{-1/3} \ll 1 ,$$

which express the fact that strong collisions are well separated in time, we also have to assure that the mean cumulative effect on the atom of weak collisions remains weak during the correlation time t_c of the total electr microfield. We therefore require that the matrix elements of the mean time evolution operator in the interaction representation, $\varphi_0 = e^{+i/\hbar H^0 t} \langle \phi_0 \rangle$, undergo little relative changes during t_c :

$$\left| t_c \; \varphi_{0\,\alpha\alpha'} \frac{d}{dt} \varphi_{0\,\alpha\alpha'} \right| \ll 1$$

This is essentially equivalent to

$$\| K_I + K_{II} \| \cdot t_c \ll 1 .$$

t_c should be taken to be the largest of $t_{cI} = \omega_p^{-1}$ (corresponding to K_I) and $t_{cII} = (\Delta\omega)^{-1}$ (corresponding to K_{II}), where $\Delta\omega$ is the width of the turbulent spectrum. Since the latter is of the order of ω_p in many cases of interest (e.g in the case of beam-plasma instabilities /15-16/) we evaluate our condition for $t_c = \omega_p^{-1}$.

An estimation of $K_I + K_{II}$ may be performed with the help of the auto-correlation functions of the total microfield. By putting roughly $\int_0^\infty dt \langle \vec{E}(0)\vec{E}(t)\rangle \gtrsim E_0^2 t_c$ ($E_0^2 = (\gamma_i + \gamma_c)\nu kT$ being the energy density of the field), we arrive at the following condition :

$$(b_W/b_\perp)^2 (\gamma_i + \gamma_c) \ll 1$$

In this estimate, the logarithmic factor contained in K_I has been omitted. Apart from this factor, the condition (c) of paper I is recovered for the case $\gamma_c \ll \gamma_i$.

B- Evolution equations for g_I and g_{II} .

We start from the first two equations of the B B G K Y hierarchy for the distribution function $f_1(1;t)$ and the correlation function $g(1,2;t) = f_2(1,2;t) - f_1(1;t) f_1(2;t)$.

$$\left[\frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} - \frac{e}{m} \vec{E}_1(1;t) \cdot \frac{\partial}{\partial \vec{v}_1}\right] f_1(1;t) = \frac{N}{m} \int \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \frac{\partial}{\partial \vec{v}_1} g(1,2;t)\, d(2) \tag{B1}$$

$$\left[\frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} + \vec{v}_2 \cdot \frac{\partial}{\partial \vec{r}_2} - \frac{e}{m} \vec{E}_1(1;t) \cdot \frac{\partial}{\partial \vec{v}_1} - \frac{e}{m} \vec{E}_1(2;t) \cdot \frac{\partial}{\partial \vec{v}_2} - \frac{1}{m} \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \left(\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right)\right] g(1,2;t) =$$

$$\frac{1}{m} \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \left(\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right) f_1(1;t) f_1(2;t) + \left[\frac{N}{m} \frac{\partial f_1(1;t)}{\partial \vec{v}_1} \cdot \int \frac{\partial W(1,3)}{\partial \vec{r}_1} g(3,2;t)\, d(3)\right.$$

$$\left. + \frac{N}{m} \int \frac{\partial W(1,3)}{\partial \vec{r}_1} \cdot \frac{\partial}{\partial \vec{v}_1} h(1,2,3;t)\, d(3) \quad + \quad 1 \longleftrightarrow 2 \right] \tag{B2}$$

These equations involve the three particle correlation function

$$h(1,2,3;t) \equiv f_3(1,2,3;t) - f_2(1,2;t) f_1(3;t) - f_2(1,3;t) f_1(2;t) - f_2(2,3;t) f_1(1,t)$$

Taking the average over subsystems leads to

$$\left[\frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1}\right] \langle f_1(1;t) \rangle = \frac{N}{m} \int \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \frac{\partial}{\partial \vec{v}_1} \langle g(1,2;t) \rangle \, d(2) + \frac{N}{m} \int \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \frac{\partial}{\partial \vec{v}_1} \langle \delta f_1(2;t) \delta f_1(1,t) \rangle \, d(2) \tag{B3}$$

$$\left[\frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} + \vec{v}_2 \cdot \frac{\partial}{\partial \vec{r}_2} - \frac{1}{m} \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \left[\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right]\right] \langle \delta f_1(1;t) \delta f_1(2;t) \rangle =$$
$$\frac{N}{m} \int \frac{\partial W(1,3)}{\partial \vec{r}_1} \cdot \frac{\partial}{\partial \vec{v}_1} \left[\langle \delta f_1(3;t) \, \delta g(1,2;t) \rangle + \langle \delta f_1(1;t) \delta f_1(2;t) \delta f_1(3;t) \rangle \right] d(3) + 1 \leftrightarrow 2 \tag{B4}$$

$$\left[\frac{\partial}{\partial t} + \vec{v}_1 \cdot \frac{\partial}{\partial \vec{r}_1} + \vec{v}_2 \cdot \frac{\partial}{\partial \vec{r}_2} - \frac{1}{m} \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \left[\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right]\right] \langle g(1,2;t) \rangle = \frac{1}{m} \frac{\partial W(1,2)}{\partial \vec{r}_1} \cdot \left[\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right] \left[\langle f_1(1;t) \rangle \langle f_1(2;t) \rangle + \langle \delta f_1(1;t) \delta f_1(2;t) \rangle \right]$$
$$+ \left\{ \frac{N}{m} \frac{\partial \langle f_1(1,t) \rangle}{\partial \vec{v}_1} \cdot \int \frac{\partial W(1,3)}{\partial \vec{r}_1} \langle g(3,2;t) \rangle \, d(3) + \frac{N}{m} \int \frac{\partial W(1,3)}{\partial \vec{r}_1} \cdot \frac{\partial}{\partial \vec{v}_1} \left[\langle \delta f_1(1;t) \, \delta g(2,3;t) \rangle + \langle h(1,2,3;t) \rangle \right] d(3) + 1 \leftrightarrow 2 \right\} \tag{B5}$$

The equations for $\langle g \rangle$ and $\langle \delta f_1(1;t) \delta f_1(2;t) \rangle$ are coupled via the terms containing $\langle \delta f \, \delta g \rangle$ and the term $\frac{1}{m} \frac{\partial W(1,2)}{\partial \vec{r}_1} \left[\frac{\partial}{\partial \vec{v}_1} - \frac{\partial}{\partial \vec{v}_2}\right] \langle \delta f_1(1;t) \delta f_1(2;t) \rangle$ which are of order $\gamma_i \gamma$. We neglect these terms, which amounts to retaining contributions to first order in γ_i and to second order in γ_c /17/. Note, however, that the terms of order γ_c^2 lead to significant contributions only in cases of not too weak turbulence.

The equation for g_{II} is identical to that occuring in the hierarchy for a Vlasov plasma. We refer to ref./18/ for the further evolution of this function.

C - Integral equation for δf.

Starting from Eq.(B1), we derive the following coupled system of equations for $\langle f \rangle$ and δf:

$$\left[\frac{\partial}{\partial t}+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}\right]\langle f_1(1;t)\rangle=\frac{e}{m}\left\langle\vec{E}_1(1;t)\cdot\frac{\partial}{\partial\vec{v}_1}\,\delta f_1(1;t)\right\rangle+\frac{N}{m}\int\frac{\partial W(1,2)}{\partial\vec{r}_1}\cdot\frac{\partial}{\partial\vec{v}_1}\,\langle g(1,2;t)\rangle\,d(2)$$

$$\left[\frac{\partial}{\partial t}+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}-\frac{e}{m}\vec{E}_1(1;t)\cdot\frac{\partial}{\partial\vec{v}_1}\right]\delta f_1(1;t)=\frac{e}{m}\vec{E}_1(1,t)\cdot\frac{\partial}{\partial\vec{v}_1}\langle f_1(1;t)\rangle-\frac{e}{m}\left\langle\vec{E}_1(1,t)\cdot\frac{\partial}{\partial\vec{v}_1}\,\delta f_1(1;t)\right\rangle$$
$$+\frac{N}{m}\int\frac{\partial W(1,2)}{\partial\vec{r}_1}\cdot\frac{\partial}{\partial\vec{v}_1}\,\delta g(1,2;t)\,d(2).$$

We neglect the term containing δg, which gives rise to ternary correlations $\langle\delta f\,\delta g\rangle$ already suppressed in Appendix B. The plasma state being homogenous and stationary, we assume that g_I depends on $\vec{r}_1$ and $\vec{r}_2$ only via $|\vec{r}_1-\vec{r}_2|$.
The equation for $\delta f_1(1;t)$ then reads

$$\left[\frac{\partial}{\partial t}+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}-\frac{e}{m}\vec{E}_1(1;t)\cdot\frac{\partial}{\partial\vec{v}_1}\right]\delta f_1(1;t)=\frac{e}{m}\vec{E}_1(1;t)\cdot\frac{\partial}{\partial\vec{v}_1}\langle f_1(1;t)\rangle \tag{C1}$$

The solution may be formally written using the propagator $Z_1(t,t')$ defined by Eq.(7) :

$$\delta f_1(1,t)=Z_1(t,t')\,\delta f_1(1,t')+\frac{e}{m}\int_{t'}^{t}Z_1(t,t'')\,\vec{E}_1(1;t'')\cdot\frac{\partial}{\partial\vec{v}_1}\langle f_1(1;t'')\rangle\,dt'' \tag{C2}$$

Expressing $E_1(1;t'')$ in terms of the fluctuating part $\delta f_1(1;t'')$ (See Eq.(5)), leads to the following integral equation for $\delta f_1(1;t)$

$$\delta f_1(1;t)=Z_1(t,t')\,\delta f_1(1;t')+\frac{N}{m}\int_{t'}^{t}dt''\,Z_1(t,t'')\int d(2)\,\frac{\partial W}{\partial\vec{r}_1}\,\delta f_1(2;t'')\frac{\partial}{\partial\vec{v}_1}\langle f_1(1;t'')\rangle \tag{C3}$$

Note that with the above approximations for g, we are led to the following equation for $f_1(1;t)$:

$$\left[\frac{\partial}{\partial t}+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}-\frac{e}{m}\vec{E}_1(1;t)\cdot\frac{\partial}{\partial\vec{v}_1}\right]f_1(1;t)=0 \tag{C4}$$

D - Series expansions for $\langle G_I \rangle$, $\langle G_{II} \rangle$, δG_I and δG_{II}

Using the shorthand notation

$$\int d(3) \int_0^{\infty} dt' \, f(1,3;t,t') \, g(3,2;t',t'') = f \circ g$$

equations (9) and (41) may be written in the form

$$D \circ G = J$$

where J is the identity with respect to the operation $\circ$. From this equation two coupled equations for $\langle G \rangle$ and δG may be derived :

$$\langle D \rangle \circ \langle G \rangle + \langle \delta D \circ \delta G \rangle = J$$

$$\langle D \rangle \circ \delta G + \delta D \circ \delta G + \delta D \circ \delta G - \langle \delta D \circ \delta G \rangle = 0$$

Solving the second equation by iteration we obtain

$$\delta G = \delta N \circ \langle G \rangle \qquad \text{(D1)}$$

with $\delta N = \sum_{n=0}^{\infty} [G^{\circ} \circ [1-P] \delta D]^n \circ G^{\circ} \circ \delta D$, (D2)

where G° is defined from

$$\langle D \rangle \circ G^{\circ} = J \qquad \text{(D3)}$$

Inserting the expression for G into the first equation of the coupled system for $\langle G \rangle$ and δG , we arrive at

$$\langle G \rangle = G^{\circ} + G^{\circ} \circ M \circ \langle G \rangle \qquad \text{(D4)}$$

with $M = \langle \delta D \circ \delta N \rangle$ (D5)

Eq.(D4) yields the following series expansion

$$\langle G \rangle = \sum_{n=0}^{\infty} (G^{\circ} \circ M)^n \circ G^{\circ} \qquad \text{(D6)}$$

The expansions (D2), (D5) and (D6) require the knowledge of $\langle D \rangle$ and δD. In order to obtain these quantities, we take the average of Eqs.(8) and (20). We have successively :

$$\langle D_I \rangle = 2\,\delta(\vec{r}_1-\vec{r}_2)\,\delta(t-t') - \frac{N}{m}\int d^3v_1 \left\langle [Z_1 Q_1]_+ \frac{\partial}{\partial \vec{v}_1} f_1(1,t') \right\rangle \cdot \frac{\partial W(1,2)}{\partial \vec{r}_1} \tag{D7}$$

$$\delta D_I = -\frac{N}{m}\int d^3v_1 \;\delta\left([Z_1 Q_1]_+ \frac{\partial}{\partial \vec{v}_1} f_1(1;t')\right) \cdot \frac{\partial W(1,2)}{\partial \vec{r}_1} \tag{D8}$$

$$\langle D_{II} \rangle = 2\,\delta(\vec{r}_1-\vec{r}_2)\,\delta(t-t') - \frac{N}{m}\int d^3v_1 \left\langle [Z_1 U]_+ \right\rangle \frac{\partial \langle f_1(1,t')\rangle}{\partial \vec{v}_1} \cdot \frac{\partial W(1,2)}{\partial \vec{r}_1} \tag{D9}$$

$$\delta D_{II} = -\frac{N}{m}\int d^3v_1 \;\delta\left([Z_1 U]_+\right) \frac{\partial \langle f_1(1;t')\rangle}{\partial \vec{v}_1} \cdot \frac{\partial W(1,2)}{\partial \vec{r}_1} \tag{D10}$$

E - Calculation of $\langle [Z_1 Q_1]_+ \delta f \rangle$, $\langle [Z_1 \bar{Q}_1]_+ \delta f \rangle$ etc...

For the final evaluation of K_I and K_{II} , we need quantities of the kind $\langle [Z_1(t,t')\, Y_1(t,t')]_+ X(t') \rangle$, where $X(t')$ represents an arbitrary operator, independant of t , and $Y_1(t,t')$ an operator which stands successively for $\bar{Q}_1$, Q_1 and U_+. We use for $(Z_1 Y_1)_+$ the general equation

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\left(\bar{H}^\circ - \vec{D}.\vec{E} + \bar{V}(1)\right) + \vec{v}_1 . \frac{\partial}{\partial \vec{r}_1} - \frac{e}{m}\vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} - M Z_1 * \right] [Z_1 Y_1]_+ = \delta(t-t')$$

A differential equation determining $[Z_1 Y_1]_+ X$ can easily be derived from the above equation. We obtain

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\left(\bar{H}^\circ - \vec{D}.\vec{E} + \bar{V}(1)\right) + \vec{v}_1 . \frac{\partial}{\partial \vec{r}_1} - \frac{e}{m}\vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} - M Z_1 * \right] [Z_1 Y_1]_+ X = \delta(t-t')\, X(t')$$

This equation is equivalent to the following two coupled equations for $\langle [Z_1 Y_1]_+ \rangle$ and $\delta\left([Z_1 Y_1]_+ X\right)$:

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}-\langle M Z_1\rangle *\right]\langle [Z_1Y_1]_+X\rangle = \delta(t-t')\,\langle X\rangle$$
$$-\left\langle (1-P)\left[-\frac{i}{\hbar}D.\vec{E}-\frac{e}{m}\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}-MZ_1*\right]\delta([Z_1Y_1]_+X)\right\rangle \quad , \tag{E1}$$

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}+(1-P)\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-\frac{e}{m}\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}-M.Z_1*\right]\right]\delta([Z_1Y_1]_+X)$$
$$-(1-P)\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-\frac{e}{m}\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}-MZ_1*\right]\langle [Z_1Y_1]_+X\rangle + \delta(t-t')\;\delta X \; .$$

Using the propagator $[Z_1Y_1]_p$ which satisfies the equation

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}+(1-P)\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-\frac{e}{m}E_1\cdot\frac{\partial}{\partial v_1}-MZ_1*\right]\right][Z_1Y_1]_p = \delta(t-t') \tag{E2}$$

we express $\delta([Z_1Y_1]_+X)$ in terms of $\langle [Z_1Y_1]_+X\rangle$ and δX :

$$\delta([Z_1Y_1]_+X) = [Z_1Y_1]_p\,\delta X - [Z_1Y_1]_p * (1-P)\left[-\frac{i}{\hbar}\vec{D}.\vec{E}-\frac{e}{m}\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}-MZ_1*\right]\langle [Z_1Y_1]_+X\rangle \tag{E3}$$

Inserting this expression into Eq.(E1), leads to a differential equation for $\langle [Z_1Y_1]_+X\rangle$:

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\frac{\partial}{\partial\vec{r}_1}-\langle MZ_1\rangle * - \langle (1-P)H[Z_1Y_1]_p * (1-P)H\rangle\right]\langle [Z_1Y_1]_+X\rangle =$$
$$\delta(t-t')\langle X\rangle - \langle (1-P)H\,[Z_1Y_1]_p\,\delta X\rangle \tag{E4}$$

with

$$H \equiv -\frac{i}{\hbar}\vec{D}.\vec{E}-\frac{e}{m}\vec{E}_1\cdot\frac{\partial}{\partial\vec{v}_1}-MZ_1*$$

In order to solve formally Eq.(E4), we use the operator $\langle [Z_1Y_1]_+\rangle$, which is deduced from Eq.(E4) for the special case $X\equiv 1$; we have

$$\left[\frac{\partial}{\partial t}+\frac{i}{\hbar}\bar{H}^{\circ}+\frac{i}{\hbar}\bar{V}(1)+\vec{v}_1\cdot\frac{\partial}{\partial\vec{r}_1}-\langle MZ_1\rangle * - \langle (1-P)H\;[Z_1Y_1]_p * (1-P)H\rangle\right]\langle (Z_1Y_1)_+\rangle$$
$$= \delta(t-t') \tag{E5}$$

The solution of Eq.(E4) thus reads

$$\left\langle [Z_1 Y_1]_+ X \right\rangle = \left\langle [Z_1 Y_1]_+ \right\rangle \langle X \rangle - \left\langle [Z_1 Y_1]_+ \right\rangle * \left\langle (1-P) H \, [Z_1 Y_1]_P \, \delta X \right\rangle \qquad \text{(E6)}$$

Eqs.(G3-6) will be used below by putting Y_1 successively equal to $\bar{Q}_1$, Q_1, U.

F - Derivation of Eq.(24)

Temporarilly we will write expression (16) in the following abbreviated form:

$$\langle \mathcal{H} * \phi_0 \rangle = \frac{iN}{\hbar} \int \bar{V}(1) \, L(1;t) \, d(1)$$

where

$$L = \left\langle G_{II} \circ [Z_1 U]_+ \delta f_1 \right\rangle - \frac{i}{\hbar} \left\langle G_I \circ [Z_1 Q_1]_+ * \bar{V} \quad [Z_1 U]_+ f_1 \right\rangle - \frac{i}{\hbar} \left\langle G_I \circ [Z_1 Q_1]_+ \mathcal{V} * U_+ \right\rangle$$

whith

$$\mathcal{V}(1) = N \int \bar{V}(3) \, \langle g(2,3) \rangle \, d(3)$$

Using the relations $f_1 = \langle f_1 \rangle + \delta f_1$, $G_{I,II} = \langle G_{I,II} \rangle + \delta G_{I,II}$, L becomes $L = L_I + L_{II}$

with

$$L_I = -\frac{i}{\hbar} \Big[\langle G_I \rangle \circ \langle [Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+ \rangle \langle f_1 \rangle + \langle \delta G_I \circ \delta([Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+ \rangle \langle f_1 \rangle + \langle G_I \rangle \circ \langle [Z_1 Q_1]_+ \mathcal{V} * U_+ \rangle + \langle \delta G_I \circ \delta([Z_1 Q_1]_+ \mathcal{V} * U_+) \rangle \Big] ,$$

$$L_{II} = \langle G_{II} \rangle \circ \langle [Z_1 U]_+ \delta f_1 \rangle - \frac{i}{\hbar} \langle G_I \rangle \circ \langle [Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+ \delta f_1 \rangle + \langle \delta G_{II} \circ \delta([Z_1 U]_+ \delta f_1) \rangle - \frac{i}{\hbar} \langle \delta G_I \circ \delta\left([Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+ \delta f \right) \rangle .$$

With the help of Eqs.(E6) and (E3) used successively for $\langle [Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+ \rangle$, $\langle [Z_1 Q_1]_+ \mathcal{V} * U_+ \rangle$ and $\delta([Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+)$, $\delta([Z_1 Q_1]_+ \mathcal{V} * U_+)$, and by expressing $\langle [Z_1 U]_+ \rangle$ in terms of $\langle Z_1 \rangle$ and $\langle U_+ \rangle$ by means of the relation

$$\langle [Z_1 U]_+ \rangle = \langle Z_1 \rangle \langle U_+ \rangle + \langle Z_1 \rangle * \left\langle \frac{e}{m} \vec{E}_1 \cdot \frac{\partial}{\partial \vec{v}_1} Z_{1p} (1-P) U_p (1-P) \left[-\frac{i}{\hbar} \vec{D} . \vec{E} - M* \right] \right\rangle * \langle U_+ \rangle ,$$

we obtain the following result for L_I :

$$L_I = -\frac{i}{\hbar} \bar{G}_I \circ \langle [Z_1 Q_1]_+ \rangle \; \bar{\bar{V}} \langle f_1 \rangle * \langle U_+ \rangle ,$$

where

$$\bar{G}_I = \langle G_I \rangle - \left\langle \delta G_I * [Z_1 Q_1]_p (1-P) \left[-\frac{i}{\hbar} \vec{D} . \vec{E} - \frac{e}{m} \vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} \right] \right\rangle ,$$

$$\bar{\bar{V}} = \left\{ \bar{V} + \left\langle \left[-\frac{i}{\hbar} \vec{D} . \vec{E} - \frac{e}{m} \vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} \right] [Z_1 Q_1]_p * \bar{V} [Z_1 U]_p (1-P) \left[-\frac{i}{\hbar} \vec{D} . \vec{E} - \frac{e}{m} \vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} - M Z_1 * \right] \right\rangle \right\}$$

$$\times \left\{ I + \langle Z_1 \rangle * \left\langle \frac{e}{m} \vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} Z_{1p} (1-P) U_p (1-P) \left(-\frac{i}{\hbar} \vec{D} . \vec{E} - M* \right] \right\rangle \right\}$$

$$+ \mathcal{V} \langle f_1 \rangle^{-1} + \left\langle \left[-\frac{i}{\hbar} \vec{D} . \vec{E} - \frac{e}{m} \vec{E}_1 . \frac{\partial}{\partial \vec{v}_1} \right] [Z_1 Q_1]_p \mathcal{V} U_p (1-P) \left[-\frac{i}{\hbar} \vec{D} . \vec{E} - M* \right] \right\rangle \langle f_1 \rangle^{-1}$$

For a further simplification of $\bar{\bar{V}}$ and $\bar{G}_I$, we derive a Dyson equation for the operator $[Z_1 Y_1]_p$ (which will stand successively for $[Z_1 Q_1]_p$, $[Z_1 U]_p$, U_p and Z_{1p}) by starting from Eqs.(E2) and (E5). We obtain

$$[Z_1 Y_1]_p = \langle [Z_1 Y_1]_+ \rangle + [Z_1 Y_1]_p * \left[(1-P) H + \langle M Z_1 \rangle + \left\langle (1-P) H [Z_1 Y_1]_p * (1-P) H \right\rangle \right] \langle [Z_1 Y_1]_+ \rangle .$$

This equation gives rises to series expansions of the quantities $[Z_1 Y_1]_p$, which we insert into the expression for $\bar{\bar{V}}$ and $\bar{G}_I$. Using the results for $\langle G_I \rangle$ and δG_I derived in App. D and keeping $\bar{\bar{V}}$ and $\bar{G}_I$ only to lowest order, we have

$$L_I = -\frac{i}{\hbar} G_I^0 \circ \langle [Z_1 Q_1]_+ \rangle \left[\bar{V} \langle f_1 \rangle + \mathcal{V} \right] * \langle U_+ \rangle ,$$

where G_I^0 is defined by Eq.(D3) written for $G = G_I$.

Let us now turn to the evaluation of L_{II}. After some manipulations similar to those employed in the derivation of L_I, we obtain

$$L_{II} = \bar{G}_{II} \circ \langle [\,[Z_1 U]_+ - \frac{i}{\hbar}[Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+\,] \delta f_1 \rangle + (\bar{G}_{II} - \bar{G}_I) \circ \langle [Z_1 Q_1]_+ \bar{V} [Z_1 U]_+ \delta f_1 \rangle$$
$$+ \langle \delta G_{II} \circ [Z_1 U]_P \, \delta f_1 \rangle - \langle \delta G_I \circ \frac{i}{\hbar} [Z_1 Q_1]_P \bar{V} * [Z_1 U]_P \, \delta f_1 \rangle$$

where

$$\bar{G}_{II} = \langle G_I \rangle - \langle \delta G_I * [Z_1 U]_P (1-P) [-\frac{i}{\hbar} \vec{D}\cdot\vec{E} - \frac{e}{m} \vec{E}_1 \cdot \frac{\partial}{\partial \vec{v}_1} - * M Z_1] \rangle$$
$$+ \frac{i}{\hbar} \langle \delta G_I * [Z_1 Q_1]_P \bar{V} * [Z_1 U]_P (1-P) [-\frac{i}{\hbar} \vec{D}\cdot\vec{E} - \frac{e}{m} \vec{E}_1 \cdot \frac{\partial}{\partial \vec{v}_1} - * M Z_1] \rangle$$

Using the propagator $(Z\bar{Q})_+$ which satisfies the Dyson equations

$$[Z_1 \bar{Q}_1]_+ = [Z_1 U]_+ - \frac{i}{\hbar} [Z_1 \bar{Q}_1] \bar{V} * [Z_1 U]_+$$

$$[Z_1 \bar{Q}_1]_+ = [Z_1 Q_1]_+ - [Z_1 Q_1]_+ * M Z_1 * [Z_1 \bar{Q}_1]_+$$

we have

$$L_{II} = \bar{G}_{II} \circ \langle [Z_1 \bar{Q}_1]_+ \delta f_1 \rangle + \frac{i}{\hbar} \bar{G}_{II} \circ \langle [Z_1 Q_1]_+ * M Z_1 * [Z_1 Q_1]_+ \bar{V} * [Z_1 U]_+ \delta f_1 \rangle$$
$$- [\bar{G}_I - \bar{G}_{II}] \circ \frac{i}{\hbar} \langle [Z_1 Q_1]_+ \bar{V} [Z_1 U]_+ \delta f_1 \rangle + \langle \delta G_{II} \circ [Z_1 U]_P \delta f_1 \rangle$$
$$- \langle \delta G_I \circ \left[[Z_1 Q_1]_P - [Z_1 U]_P + \frac{i}{\hbar} [Z_1 Q_1] \underset{P}{*} (1-P) M Z_1 * [Z_1 Q_1]_P \bar{V} * [Z_1 U]_P \right] \delta f_1 \rangle$$

In this equation, we express the quantities $\langle [Z_1 \bar{Q}_1]_+ \delta f_1 \rangle$ and $\langle [Z_1 Q_1]_+ \bar{V} [Z_1 U]_+ \delta f_1 \rangle$ by means of Eq.(E6`. The resulting expression involves terms containing $M(t,t')$ which lead to corrections of order λ_i^2 in the collision operator $K(s)$. Neglecting these terms we arrive at

$$I = \bar{G}_{II} \circ \langle [Z_1 \bar{Q}_1]_+ \rangle * \langle \delta H_2 [Z_1 \bar{Q}_1]_P \delta f_1 \rangle - \frac{i}{\hbar} [\bar{G}_I - \bar{G}_{II}] \circ \langle [Z_1 Q_1]_+ \rangle * \bar{V} \Big\{ I -$$
$$\langle \delta H_1 [Z_1 Q_1]_P (1-P) * [Z_1 U]_P \delta H_2 \rangle + \langle Z_1 U \rangle * \langle \delta H_2 [Z_1 Q_1]_P * (1-P) [Z_1 U]_P \delta f_1 \rangle$$
$$+ \langle \delta H_1 [Z_1 Q_1]_P (1-P) * [Z_1 U]_P \delta f_1 \rangle \Big\} + \langle \delta G_{II} \circ [Z_1 U]_P \delta f_1 \rangle$$
$$+ \langle \delta G_I \circ \left[[Z_1 Q_1]_P - [Z_1 U]_P \right] \delta f_1 \rangle$$

with

$$H_1 = -\frac{i}{\hbar}\,\vec{D}.\vec{E} - \frac{e}{m}E_1.\frac{\partial}{\partial \vec{v}_1},$$

$$H_2 = H_1 - MZ_1 * .$$

For the further evaluation of L_{II} we follow similar lines as in the case of L_I, except that we express $[Z,\bar{Q}_1]_\rho$ in terms of $[Z_1 U]_\rho$ by means of the relation

$$[Z_1\bar{Q}_1]_\rho = [Z_1 U]_\rho - \frac{i}{\hbar}[Z_1\bar{Q}_1]_\rho \bar{V} * [Z_1 U]_\rho .$$

The lowest order result for L_{II} thus obtained is

$$L_{II} = \frac{i}{\hbar} G_{II}^{\circ} \circ \langle [Z_1\bar{Q}_1]_+ \rangle \langle \vec{D}.\vec{E}\, \delta f_1 \rangle * \langle U_+ \rangle$$

with

$$\langle D_{II} \rangle \circ G_{II}^{\circ} = J$$

<u>G - Expressions for the diffusion terms R_1, R_1', $\langle MZ_1 \rangle$ and C_1, C_1'.</u>

In this section we first derive expressions for the diffusion type terms R_1, R_1' and $\langle MZ_1 \rangle$ showing their close relation to the collision operators K_{II} and K_I. Indeed, R and R' taken to lowest order in λ_c turn out to be related to the lowest order of K_{II}. Similarly, $\langle MZ_1 \rangle$ and K_I are related to lowest order.

Starting from Eq.(15) for K and inserting successively the relations

$$U_+ = U_\rho - U_\rho * P\mathcal{H} * U_+$$

$$U_\rho = U_+^\circ - U_\rho * (1-P)\mathcal{H} * U_+^\circ$$

where U_+° satisfies the equation

$$\left[\frac{\partial}{\partial t} + \frac{i}{\hbar}\bar{H}^\circ\right] U_+^\circ(t,t') = \delta(t-t'),$$

the following expression for $K(s)$ may be derived,

$$K(s) = -\langle\tilde{M}\rangle - \langle \delta\mathcal{H} * \widetilde{U_p * \delta\mathcal{H}} \rangle$$

where the two terms of this expression are identical to K_I and K_{II} respectively. In order to establish the relation between K_{II} on the one hand, and R_1 and R_1' on the other hand, we put

$$U_p \propto e^{-\frac{i}{\hbar}\bar{H}^{\circ}(t-t')} \quad , \quad \mathcal{H} \propto -\frac{i}{\hbar}\vec{D}.\vec{E}$$

in the expression for K_{II}, and

$$[Z_1\bar{Q}_1]_p \simeq [Z_1 Q_1]_p \simeq e^{-\frac{i}{\hbar}\bar{H}^{\circ}(t-t')} \; e^{-(t-t')\, \vec{v}_1 . \frac{\partial}{\partial \vec{r}_1}}$$

in the expression for R and R'. We thus obtain

$$K_{II} \simeq K_{II}^{\circ} = -\langle \frac{i}{\hbar}\vec{D}.\vec{E}(t) \, \widetilde{\exp[-\tfrac{i}{\hbar}\bar{H}^{\circ}(t-t')]} \frac{i}{\hbar}\vec{D}.\vec{E}(t') \rangle$$

and

$$R_1 = R_1' = M_{II}(t-t') \exp - (t-t')\, \vec{v}_1 . \frac{\partial}{\partial \vec{r}_1}$$

Along similar lines we arrive at

$$\langle M.Z_1\rangle \simeq \langle M\rangle\langle Z_1\rangle \simeq M_I(t-t') \exp - (t-t')\, \vec{v}_1 . \frac{\partial}{\partial \vec{r}_1}$$

where the Laplace transform, $K_I^{\circ}(s)$ of $M_I(t-t')$ equals to lowest order in λ_c the expression for $K_I(s)$.

The same kind of approximations as used above also leads to the identities

$$\overleftrightarrow{C}_1' = \overleftrightarrow{C}_1 \quad , \quad A_1' = A_1 .$$

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