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V.Stančić

NEW STATISTICAL MODEL OF INELASTIC FAST NEUTRON SCATTERING

"BORIS KIDRIČ" INSTITUTE
BEOGRAD-VINČA
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Abstract

A new statistical model for treating the fast neutron inelastic scattering has been proposed by using the general expressions of the double differential cross section in impuls approximation. The use of the Fermi-Dirac distribution of nucleons makes it possible to derive an analytical expression of the fast neutron inelastic scattering kernel including the angular momenta coupling. The obtained values of the inelastic fast neutron cross section calculated from the derived expression of the scattering kernel are in a good agreement with the experiments. A main advantage of the derived expressions is in their simplicity for the practical calculations.

INTRODUCTION

In a precedent paper we used the Van Hove general expressions of the double differential scattering cross section to derive a formal statistical model for calculating the fast neutron inelastic scattering cross sections, where the nucleus has been described by using the Fermi-Dirac statistics[1]. The aim of this paper is to include the angular momenta distribution of nucleons too. Firstly, we start from the following general expression of the double differential cross section[2]

$$\frac{d^2\sigma}{dEd\Omega} = \sum_f |\langle kf|T|k_o i\rangle|^2 \frac{k}{(2\pi)^3 k_o} \delta(E+E_f-E_o-E_i) \qquad (1.1)$$

in the system of units h=1, m=1, where

E_o, E - the incoming and outgoing neutron energy,
k_o, k - the incoming and outgoing neutron momenta,
T - reaction matrix.

The expression (1.1) may be further transformed in a more conveninent form

$$\frac{d^2\sigma}{dEd\Omega} = \frac{k}{(2\pi)^3 k_o} \langle i|T^+ \exp(i\vec{K}\vec{r})\delta(\omega+H_N-E_i)\exp(-i\vec{K}\vec{r})|T|I\rangle \qquad (1.2)$$

$$T = \langle \vec{k}, s_n', \vec{P}-\vec{K}|T(S_t)|\vec{k}_o, s_n, \vec{P}\rangle ,$$

where

$\vec{K}=\vec{k}-\vec{k}_o$ - transfered momentum,
$\omega=E-E_o$ - transfered energy,
E_i - ground state energy of nucleons,
r - coordinate of the nucleus,
S_t - its spin,
s_n, s_n' - initial and final state neutron spin.

Furthermore, if we use the identity

$$\exp(i\vec{K}\vec{r})H_n(\vec{p},\vec{r})\exp(-i\vec{K}\vec{r}) = H_n(\vec{p}+\vec{K},\vec{r}),$$

we have instead of (1.2)

$$\frac{d^2\sigma}{dEd\Omega} = \frac{d\sigma^{(0)}_{if}}{d\Omega} S_{if}(\vec{K},\omega) \tag{1.3}$$

which is similar to the Van Hove expression of the spin independent double differential cross section[2,4]. The main difference appearing here lies in the fact that we projected the transition operator T into some final state f≠i. As usually we used the impuls approximation[2] of the T operator

$$T = \sum_{a=1}^{A} T_a \tag{1.4}$$

which allows to express $S_{if}(K,\omega)$ in the following form

$$S_{if}(\vec{K},\omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} dt e^{-i\omega t} X_{if}(\vec{K},t) \tag{1.5}$$

$$X_{if}(\vec{K},t) = \sum_{a,b} X^{ab}_{if}(\vec{K},t) \tag{1,6}$$

$$X^{ab}_{if}(\vec{K},t) = \langle f|e^{iHt}e^{-iH'_a t}\exp(iK(r_a-r_b))|i\rangle \tag{1.7}$$

Here, H'_a represents the total nuclear Hamiltonian perturbed in its a-th particle Hamiltonian by $\vec{K}$ momentum, i.e. $\vec{p}_a$ $\vec{p}_a+\vec{K}$. The term $d\sigma^{(0)}_{if}/d\Omega$ in (1.3) becomes a form factor of scattering by a model particle of the target.

2. Definition of the statistical model

As earliar[1] let us consider only the diagonal elements a=b in (1.7). We have simply

$$X_{if}^{aa}(\vec{K},t) = \langle f|\exp(iHt)\exp(-iH_a't)|i\rangle, \qquad (2.1)$$

where

$$H_a' = \sum_{b\neq a} H)\vec{p}_b,\vec{r}_b) + (\vec{p}_a-\vec{K})^2/2 + (\vec{j}_a-\Delta\vec{j}_n)^2/2J_a, \qquad (2.2)$$

j_a - the total angular momentum of a-th particle of the target and J_a- its moment of inertia. If we introduce new variavles $q_a=p_a-K$ and $\vec{j}_a'=\vec{j}_a-\Delta\vec{I}$,($\Delta\vec{I}=\vec{I}_i-\vec{I}_f$ - change of the nuclear spin) and by using

$$\exp(iHt)\exp(-iH_a't) \cong \exp(it(H-H_a'))$$

we have

$$X_s(\vec{K},t)=\exp(-itk^2/2A)\sum_a\langle f|\exp(it(Kq_a+\frac{q_a^2}{2}-\frac{p_a}{2}+\frac{\vec{j}_a'\Delta\vec{I}_n}{J_a}+\frac{j_a'^2}{2J_a}-\frac{j_a^2}{2J_a}))|i\rangle \qquad (2.3)$$

Roughly speaking the terms in exponent of the expression (2.3) represent a difference of the ground and excited state Hamiltonians of the a-th parrticle of the target plus interference terms. Since we are dealing with the energy interval 0 - 10 MeV which is far from the Fermi energy E_F=37 MeV, we can introduce the following approximations:

- the states of the target nucleons will be described with the use of the Fermi-Dirac distribution including angular momenta[3]

$$f(a) = (1+\exp(\beta_o\varepsilon(a)+\gamma_o m(a)))^{-1} \qquad (2.4)$$

$$\beta_o^2 = \frac{\pi^2 g_o}{6(E_i - \frac{M^2 h^2}{2J_{c\ell}})}, \quad \frac{\gamma_o}{\beta_o} = -\frac{Mh^2}{J_{c\ell}}, \quad \langle m^2 \rangle = \frac{J_{c\ell}}{g_o h^2}, \quad g_o \simeq A/E_f$$

And the second propostition:

- only small number of nucleons are excited when the ecitation energy being less then 10 MeV.

Therefore (2.3) reads

$$X_{if}(\vec{K},t) = \exp(-it(E_f + K^2/2A)) \sum_a \langle f|\exp(it(\vec{K}\vec{q}_a + \frac{\Delta \vec{i}_n \vec{J}_a}{J_a}|i\rangle_{av} \qquad (2.5)$$

The averaging in (2.5) will be performed by using (2.4) distribution. Hence

$$X_{if}(\vec{K},t) \simeq \frac{\beta_o}{\gamma_o \langle m^2 \rangle 2\pi^2 Kt} \exp(-it(E_f + K^2/2A)) X$$

$$X \int_{-\infty}^{\infty} \frac{q \sin((qK + m/J_c)t)\,dq\,dm}{1 + \exp(\beta_o(q^2 - q_F^2)/2 + \gamma_o m)} \qquad (2.6)$$

where

q=4 - statistical degeneration of nucleons,
A - mass of nucleus in amu,
J_c - classical moment of inertia of the target.

The calculation of $d\sigma_{if}^{(o)}/d\Omega$ is straightforward. In first Born approximation we have

$$\frac{d\sigma_{if}^{(o)}}{d\Omega} = \frac{k}{(2\pi)^3 k_o} |\langle f|V|i\rangle|^2 \qquad (2.7)$$

The transition element in (2.7), after time consuming calculations, being

$$|\langle f|V|i\rangle|^2 = |\sum_{\substack{ll' \\ ss'}} V_{ll'}(k,k_o) \sum_{\substack{LS \\ m\nu}} Y_l^m(\Omega) F(LS,ll';ss')|^2 \qquad (2.8)$$

Here

- l,l' - incoming and outgoing neutron angular momenta
- s,s' - their spins,
- L,S - total neutron-nucleon angular momentum and spin respectively,
- M,ν - their spin z-componenta.

The form factor $F(LS,ll';ss')$ is given by

$$F(LS,ll';ss') = |\langle LS;M|L+S,M+\nu\rangle|^2 X$$

$$X\langle (l,L-l)L,(ss)S;L+S,m+\nu|(ls)j,(L-l,s)L-l+s,m+\nu\rangle X$$

$$X\langle (l',L-l')L,(s's')S';L+S,m+\nu|(ls')j',(L-l',s')L-l-\Delta I+s,$$

$$+ L+S,m+\nu\rangle \qquad (2.9)$$

where by $\langle (ab)c, (de)f:, i|\ (ad)q, (be)h; i\rangle$ we denoted the 9j - coupling constants which may be further expressed versus Rach coefficients[9]. We note that the averaging over the directions of the incoming and outgoing neutron and nuclear spins in (2.8) have to be included too.

3. Fast neuton inelastic scattering kernel

Now we are prepared to calculate the energy distribution cross section, usually called as a scattering kernel, defined by

$$\frac{d\sigma}{dE} = \int \frac{d^2\sigma}{dEd\Omega}\, d\Omega \tag{3.1}$$

In order to simplify the calculations we shall use the so called short time approximation[2]. Namely, we can expand the second exponential factor in (2.6) in a power series giving

$$X^s_{if}(\vec{K},t) = \exp(-it(E_f+K^2/2A)) \; \frac{g_o V\gamma_o}{2\pi^2} \sum_{n=o} (-1)^n \frac{(Kt)^{2n}}{(2n+1)!} C_n \tag{3.2}$$

$$C_n = \int_0^\infty \frac{(q+m/J_c)^{2n+2}}{1+\exp(\beta_o(q^2-q_F^2)/2+\gamma_o m)}\, dqdm$$

Introducing (3.2) into (1.5) we have

$$S(\vec{K},\omega) = \sum_{n=o} S_n(\vec{K},\omega), \tag{3.3}$$

where

$$S_n(\vec{K},\omega) = \frac{gV\gamma_o K^{2n}}{2\pi^2(2n+1)!}\, \delta^{(2n)}\, (\omega+K^2/2A+E_f) \tag{3.4}$$

Here $\delta^{(n)}$ denotes a derivation of the Dirac function.
The fast neutron scattering kernel has a very simple form if we restrict to zero term in (3.3), where the appearence of the Dirac function represents the energy conservation. Hence, by using (2.7-9) and (3.3) we have the zero order expression of the

scattering kernel of fast neutrons from (3.1):

$$\sigma_k^{(o)}(E_o \to E) = \frac{gV\gamma_o A}{16\pi^4 E_o}\left| \sum_{ll'} V_{ll'}(k,k_o) \sum_{\substack{LS \\ m}} P_L^m(\xi x - \frac{\eta}{x}) F(LS, ll'; ss') \right|^2 \tag{3.5}$$

$$0 \leq E \leq E_o - E_k,$$

where

$$x = (E/E_o)^{1/2}, \xi = \frac{1}{4}(1+\frac{1}{A}), \eta = \frac{1}{4}(1-\frac{1}{A}) - \frac{E_k}{4E_o},$$

E_k - nuclear energy level.

By integrating the scattering kernel (3.5) over all possible final neutron energies we can find the fast neutron inelastic cross section $\sigma_k(E_o)$ and the inelastic scattering function defined by $f(E_o,E) = \sigma_k^{(o)}(E_o \to E)/\sigma_k^{(o)}(E_o)$.

As an illustration of the proposed model we give few values of the inelastic fast neutron scattering cross section for $(0^+ \to 4^+)Fe^{56}$ transition compared with the experimental data which are available[8]:

E_o (MeV)	$\sigma^I_{o,teor}$(b)	$\sigma^{II}_{o,teor}$(b)	σ_{exp}(b)
3.	0.17	0.162	0.165
4.	0.164	0.159	0.15
5.	0.157	0.13	0.1

E_k = 2.084 MeV

$\sigma^I_{o.teor}$ - calculated without angular momentum coupling[1],
$\sigma^{II}_{o,teor}$ - calculated by (3.5),
σ_{exp} - ref.8.

4. Conclusion

The fast neutron inelastic scattering may be successfully described by using the Fermi-Dirac statistics for nucleons for the incoming neutron energy between 0 - 10 MeV. A better agreement of the theory with the experiments may be obtained including some other degrees of freedom of nucleons which are not included by Fermi gas model. Nevertheless, the proposed model here may be a good approximation for some practical calculations.

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