PLASMA WAVES IN AN INHOMOGENEOUS CYLINDRICAL PLASMA

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ABSTRACT: The complete dispersion equation governing small amplitude plasma waves propagating in an inhomogeneous cylindrical plasma confined by a helical magnetic field is solved numerically. The efficiency of the wave energy thermalization in the lower hybrid frequency range is studied.

1. INTRODUCTION

The lower hybrid (LH) plasma heating provides an efficient means of thermalization of the energy added to the plasma. This relatively rapid heating method is characterized by an important efficiency. It offers the possibility to communicate the RF energy preferentially to the ion perpendicular motion via collisionless damping mechanisms. The availability of gigahertz high power RF sources, the rather attractive possibility of wave launching by an array of phased waveguides and the high heating efficiency indicate that the LH heating method has a favourable thermonuclear prospect.

The LH heating scheme is based on the field amplification and concomitant linear wave conversion. These phenomena are peculiar to an actual inhomogeneous plasma where waves propagate through regions of separate identification and may reach a wave resonance in whose vicinity they merge smoothly and convert one to the other /1/. The electromagnetic energy which reaches the wave conversion region is in general, partially carried off by plasma waves. Of particular interest is the resonant interaction of charged particles with the excited LH plasma waves. The propagation of plasma (electrostatic) waves with finite $k_{\parallel}$ value in hot homogeneous magnetized plasma has been the subject of extensive theoretical investigations /2-6/. The straightline orbit approximation or rather simplified dispersion equations have been used in most of these investigations /2-4/. The complete dispersion equation describing small-amplitude electrostatic waves with finite $k_{//}$ in hot homogeneous magnetized plasma has been solved numerically in Refs /5,6/. The LH dispersion curves, spatial damping rates of externally driven plasma waves as well as temporal damping rates of stable eigen oscillations have been reported. Recently these investigations have been completed by allowing the presence of moderate magnetic field gradients in hot quasi-homogeneous plasma /7,8/.

In order to gain an insight into the LH wave energy absorption in toroidal discharges in the present paper we deal with plasma waves propagating in an inhomogeneous cylindrical plasma confined by a helical magnetic field. The dispersion equation governing small-amplitude electrostatic waves is derived in the framework of the small Larmor radii approximation. The obtained dispersion equation is solved numerically for plasma parameters corresponding to typical present-day toroidal devices.

2. DISPERSION EQUATION

We consider the wave propagation in two-component non-uniform cylindrical plasma confined by a helical magnetic field $\vec{B}_o = (0, B_{o\theta}(r), B_{oz}(r))$. Here r, θ and z are the coordinates of a circular cylindrical system which is used in the paper. The particle motions and the electromagnetic field will be treated in a self-consistent way by solving the closed system of equations formed by the Vlasov equation and Maxwell equations. In a static circular cylindrical plasma the electric and magnetic field acting on the charged particles are independent of t, θ and z. The equilibrium distribution function can be constructed from the following constants of the motion: the energy $\epsilon_\alpha = m_\alpha v_\alpha^2/2$ and the components of the generalized angular momentum $P_{\theta\alpha} = m_\alpha r\, v_{\theta\alpha} + q_\alpha \int^r B_{oz}\, r' dr'$ and $P_{z\alpha} = m_\alpha v_{z\alpha} - q_\alpha \int^r B_{o\theta} dr$. In what follows we shall assume that the particle Larmor radius ρ_α is small compared to r and a, where a is the characteristic length of the plasma parameters variation. Expansion of the equilibrium distribution function in powers of ρ_α/a then yields,

$$f_{o\alpha}(\epsilon_\alpha, P_{\theta\alpha}, P_{z\alpha}) = \left(1 - \frac{v_{c\alpha}}{\Omega_\alpha}\frac{\partial}{\partial r}\right) F_{o\alpha}\ (\epsilon_\alpha,\ r), \tag{1}$$

where $v_{c\alpha}$ is the particle velocity component in the direction determined by the unit vector $\vec{e}_c = \vec{e}_r \times \vec{e}_b$, $\vec{e}_b = \vec{B}_o/B_o$, $\Omega_\alpha = q_\alpha B_o/m_\alpha$ is the particle cyclotron frequency and $F_{o\alpha}$ is an arbitrary function of ε_α and r. We choose a Maxwellian distribution function with non-uniform density and particle temperature,

$$F_{\alpha o}(\varepsilon_\alpha, r) = n(r) \left(\frac{m_\alpha}{2\pi T_\alpha(r)}\right)^{3/2} \exp\{-\varepsilon_\alpha/T_\alpha(r)\}. \qquad (2)$$

In deriving (1) we have only retained the term involving the first order derivative in space. It can be shown that the inclusion of higher order derivatives leads to negligible corrections in the dielectric tensor of non-uniform plasma /9/.

The response of charged particles for electromagnetic fields can be found by applying the linear perturbation analyses of non-uniform plasmas /9/. Throughout this paper we consider small amplitude waves and consequently, media which depart only slightly from thermal equilibrium. By virtue of the cylindrical symmetry of the initial state we assume that the perturbed quantities have the following time-space dependence,

$$\vec{E}(\vec{r}, t) = \int \vec{E}(k_r) \exp(i\, k_r r)\, dk_r \exp\{i[m\theta + k_z z - \omega t]\}, \qquad (3)$$

and analogously for $\vec{B}(\vec{r},t)$ and the first-order perturbated distribution function $f_{1\alpha}(\vec{v}, \vec{r}, t)$. The dielectric tensor for non-uniform cylindrical plasma can be expressed then as,

$$\varepsilon_{ij}(\omega,\vec{k},r) = \delta_{ij} + \sum_\alpha \frac{1}{T_\alpha}\left\{1 - \frac{k_c v_{t\alpha}^2}{2\omega\Omega_\alpha}\left(\frac{\partial n}{\partial r}\frac{\partial}{\partial n} + \frac{\partial T_\alpha}{\partial r}\frac{\partial}{\partial T_\alpha}\right)\right\} \times$$
$$\times\, T_\alpha\left[\varepsilon_{ij}^{(o)}(\omega,\vec{k},r) - \delta_{ij}\right], \qquad (4)$$

where $k_c = (mB_{oz}/r - k_z B_{o\theta})/B_o$, $v_{t\alpha} = (2T_\alpha/m_\alpha)^{1/2}$ and $\varepsilon_{ij}^{(o)}(\omega,\vec{k},r)$ is a dielectric tensor which is identical in the form to the dielectric tensor for uniform plasma. In obtaining the dielectric tensor (4) it is assumed that the plasma density is so low that the particles are unable to perturb the magnetic field. In these low β conditions the characteristic length of the static magnetic field inhomogeneity is large compared to the characteristic length of variation of the non-uniform equilibrium distribution function and in a first approximation, the magnetic field can be taken to be uniform.

The dispersion equation governing small-amplitude electrostatic waves in non-uniform cylindrical plasma reads,

$$\varepsilon_{\ell}(\omega,\vec{k},r) = 1 + \sum_{\alpha} A_{\alpha} \equiv 1 + \sum_{\alpha} \frac{2\omega_{p\alpha}^{2}}{k^{2}v_{t\alpha}^{2}} \left\{ 1 + z_{o\alpha} \sum_{n} \left[1 - \right.\right.$$

$$\left.\left. - \frac{c v_{t\alpha}^{2}}{2\omega\Omega_{\alpha}} \left(\frac{\partial \ln n}{\partial r} + \frac{\partial T_{\alpha}}{\partial r} \frac{\partial}{\partial T_{\alpha}} \right) \right] e^{-\lambda_{\alpha}} I_{n}(\lambda_{\alpha})\, Z(z_{n\alpha}) \right\} = 0 . \qquad (5)$$

Here $\omega_{p\alpha}$ is the particle plasma frequency, $\lambda_{\alpha} = k_{\perp}^{2} v_{t\alpha}^{2}/2\Omega_{\alpha}^{2}$, $k_{\perp} = (k_{r}^{2} + k_{c}^{2})^{1/2}$, $z_{n\alpha} = (\omega - n\Omega_{\alpha})/|k_{/\!/}|v_{t\alpha}$, $k_{/\!/} = (k_{z}B_{oz} + mB_{o\theta}/ /r)/B_{o}$, $I_{n}(z)$ is the modified Bessel function of the first kind and $Z(z)$ is the plasma dispersion function. Since the transverse electric field is non-vanishing in magnetized plasma the waves described by (5) are in fact nearly longitudinal (electrostatic). The coupling of longitudinal to transverse (electromagnetic) waves can be shown to be small for $N^{2} \gg \omega_{p\alpha}^{2}/\omega^{2}$ where N is the wave refractive index.

3. RESULTS

The dispersion equation $\varepsilon_{\ell}(\omega,\vec{k},r) = 0$ relates the angular frequency ω and the wave vector $\vec{k}$ of electrostatic waves in non-uniform cylindrical magnetized plasma. In the present paper an initially quiescent system externally excited at some (real) frequency is considered. Therefore the equation (5) is regarded as giving the local complex wave vector $\vec{k}$ in non-uniform plasma in terms of the independent real variable ω.

In certain limiting cases the equation (5) can be simplified so that it may be investigated analytically. In the LH frequency range the phase variation along the field lines is too slow to be important that is $z_{oe} \gg 1$. Since $|z_{ne}| \gg z_{oe}$ the term n=0 dominates in A_{e}. Using the asymptotic expansion of the plasma dispersion function and the power series representation of $I_o(\lambda_e)$ ($|\lambda_e| \ll 1$ even for large λ_i values) one obtains a fairly good approximation of A_e. The situation is however more complicated in the simplification of the ion term A_i. Anticipating the establishment of ion Cerenkov interaction for $\lambda_i > 1$ the ion trajectory over the wavelength is frequently approximated by a straight line. The analysis of the conditions under which the recent particle memory dominates its motion has shown /6,10/ that the collisionless long time ion behaviour can be neglected only when the ion Larmor radius is of the order or larger than the parallel wavelength. Generally the resonant contribution of the plasma dispersion function does not assure the convergence of the series in A_i. A large number of terms are significant in the summation of $Re(A_i)$ while the resonant contribution of the term $n=h=\omega/\Omega_i$ is dominant in $Im(A_i)$ only for $Re\lambda_i \gg Im\lambda_i$ and $Re\lambda_i < Im\lambda_i$. Finally we remark that in replacing the equation (5) by a simplified algebraic equation certain roots are a priori lost while the identification of the remainder ones is ambiguous.

The complete dispersion equation (5) is solved numerically for a wide range of plasma parameters in the LH frequency range. The root tracing algorithm is analogous to the one used in our previous investigations /6,10/. We have been interested only in complex $\vec{k}$ solutions with $Im\,\vec{k} > 0$. The parameters chosen correspond to typical toroidal discharges.

The wave vector component k_z which is fixed by the wave launching structure or the periodic boundary conditions (in toroidal geometry $k_z = l/R$ where l is the toroidal mode number and R is the major radius) is taken to be real. We confine ourselves to the consideration of the azimuthal mode m=1. In adapting solutions for cylindrical geometry to toroidal one, the z component of the magnetic field is allowed to vary as $B_z = B_{zo}/(1 \pm x/A)$, where A is the aspect ratio, while the plasma density is assumed to have the following profile $n = n_o(1-x^\gamma)$ where $\gamma = 2,3$. We remark that the small Larmor radii approximation $\rho_\alpha \ll a$ is justified in toroidal discharges with the exception of the narrow region near the plasma edge of inconsequential importance for the subject under consideration. In connexion with this it should be pointed out that the wave propagation up to the wave coupling region is not materially affected by the particle thermal motion /11/. Hence the problem of wave penetration and the propagation up to this region as well as the problem of wave density coupling can be handled appropriately within the framework of the cold plasma theory of non-uniform bounded plasmas.

We shall briefly analyse now the plasma wave trajectory in non-uniform magnetized plasma. In general, by approaching the coupling region as a travelling backward wave the resonant electromagnetic mode converts into a backward fast plasma wave. In the flat-topped toroidal plasma the wave conversion occurs relatively far from the cold wave resonance position. The coupling region moves toward lower densities as the ion temperature or $N_{\parallel}^2$ increases /6,12/. This suggests that the existence of the cold LH resonance in the plasma is not a necessary condition for the wave energy thermalization /8/. If the waves are launched in the direction of increasing density and magnetic field, the cold wave resonance position displaces toward higher densities, passes on the other side of the plasma column and finally disappears as the maximal plasma density decreases. However, the conversion between electromagnetic and plasma waves takes place even in the absence of the cold wave resonance.

Moving radially inwards the excited fast plasma wave encounters a branch point. We want to underline that the density at which this first branch point takes place n_c is the parameter which is relevant in the LH heating scheme. In order to communicate the wave energy to the bulk of charged particles the maximal plasma density should be slightly larger than n_c. Using a simplefied dispersion equation which is regarded as a function

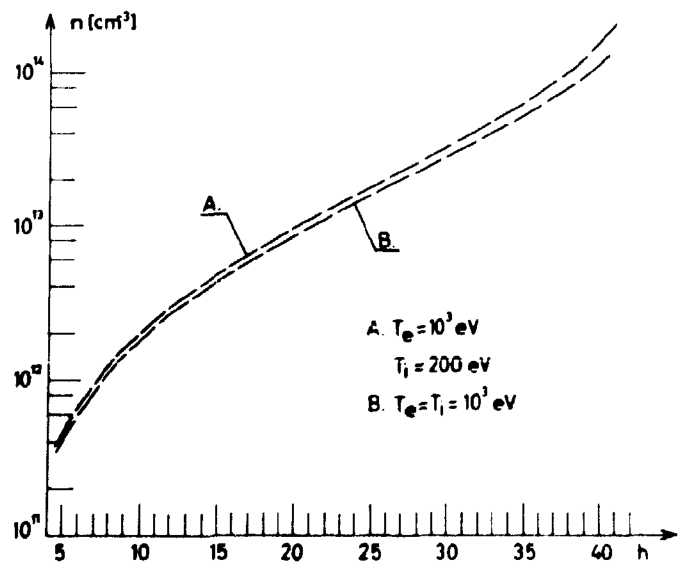

Fig.1.The characteristic density n_c vs h.

$\omega_p^2=\omega_p^2(\lambda_i)$ /10/ we have evaluated the characteristic density n_c. The value of n_c obtained in that way is in very good agreement with the numerical results. The variation of the characteristic density n_c with $h=\omega/\Omega_i$ in hydrogen plasma for a fixed value of the magnetic induction B_{oz}=2T is represented on Fig.1. One concludes that at relatively low magnetic field intensities high h values should be choosen in order to heat a dense plasma.

There are several linearly independent solutions which are generated at the branch point/6,10/. The solution trajectory which can be correlated to the least damped temporal plasma mode describes an outward-moving forward wave. Propagating in the direction of decreasing plasma density this wave either encounters a cyclotron harmonic at which it can be efficiently damped or another branch point at which it converts into an inward-moving slow plasma mode/5-8/. There is another possible wave trajectory on which we precisely focus our attention in the present paper. Namely,at the branch point the excited plasma wave can be converted into an inward-moving mode. This mode is slightly dependent on the plasma density. Propagating in the direction of increasing density it is highly damped. We have illustrate the previous discussion by Figs.2. and 3. on which the variation of the perpendicular refractive index of plasma waves with x is represented. The parameters are those foreseen in JET: B_{oz}=3T,A=2.34,n_o=4-5x$10^{13}$$cm^{-3}$,$T_{eo}=T_{io}$= 5keV. Deuterium plasma with parabolic density and temperature profile is considered. It is assumed that the slow waves are launched with $N_{\parallel}^2$=1.2 from the inner circumference of the torus. We remark in passing that a relatively small wave retardation verifies the accessi-

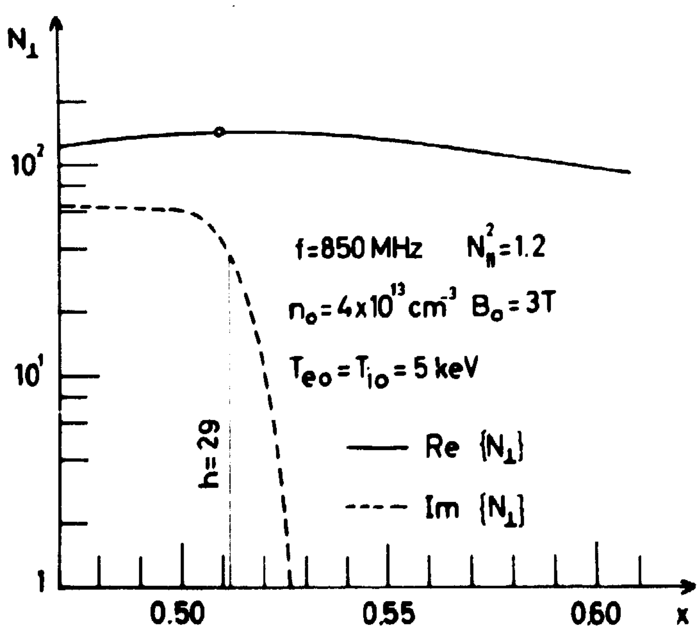

Fig.2. Re$\{N_\perp\}$and Im$\{N_\perp\}$ vs $x=r/R_o$.

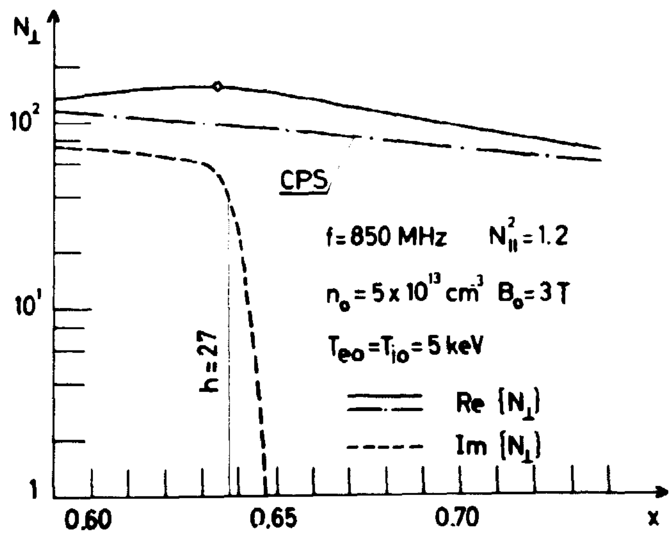

Fig.3.Re$\{N_\perp\}$ and Im$\{N_\perp\}$ vs $x=r/R_o$.

bility criterion in this case.As one can note the injected RF energy is efficiently absorbed in the plasma core. The curves reoresented demonstrate that the problem of frequency tracking of the maximal density is overestimated.The results obtained offer the possibility to evaluate the power absorbed in the plasma $P_a=\vec{S}_p\cdot Im\vec{k}$ ($\vec{S}_p$ is the energy flux flowing coherently with the wave) as well as the heating rate. The influence of distant collisions on the ion resonant interaction in the LH frequency range has been previously discussed /6,10/. From the standpoint of the wave absorption mechanism and that of excitation of parametric effects there is a substantial difference between the current LH experiments the ignition ones/10/.Namely,the LH wave field amplification decreases for increasing the ion temperature and the linear collisionless interaction is the dominant process of the wave energy thermalization in high-temperature plasmas.

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