ATOMIC PHYSICS TESTS OF QUANTUM ELECTRODYNAMICS*

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I. INTRODUCTION

Experiments with simple atomic systems, for which the effects of quantum electrodynamics can be accurately predicted, have steadily become more precise and have been extended to systems with a wide variety of constituent particles. The corresponding increased demands on quantum electrodynamics theory have so far been met successfully. At this time, there does not appear to be any serious disagreement between theory and experiment.[1]

In this paper, the tests of quantum electrodynamics derived from bound systems and the free electron and muon magnetic moments will be reviewed. The emphasis will be on the areas in which recent developments in theory or experiment have taken place. In addition, determinations of the fine structure constant from the Josephson effect and the fine structure of helium will be discussed.

II. JOSEPHSON EFFECT DETERMINATION OF THE FINE STRUCTURE CONSTANT

The most accurate value for the fine structure constant α is presently derived from the Josephson effect determination of e/h. The value of α is related to the value of e/h through a series of precisely known constants:[2]

$$\alpha^{-2} = \left[\left(\frac{1}{4R_\infty}\right)\left(\frac{c\Omega}{\Omega_{NBS}}\right)\left(\frac{\mu'_p}{\mu_B}\right)\left(\frac{2e}{h}\right)\right]\frac{1}{\gamma'_p}$$

The constants in the square brackets have been measured with uncertainties of 5 parts in 10^8 or less. The gyromagnetic ratio of the proton in water γ'_p has been the least well known, and has recently been measured with improved precision by Olsen and Williams[2] at the National Bureau of Standards. Their result

$$\gamma'_p = 2.675\ 131\ 4(11) \times 10^8\ \text{Rad}\ \text{s}^{-1}\ \text{T}^{-1}_{\text{NBS}} \qquad 0.42\ \text{ppm}$$

combined with measured values for the remaining constants yields[2]

$$\alpha^{-1} = 137.035\ 987(29) \qquad 0.21\ \text{ppm}$$

The value of α obtained this way does not depend directly on quantum electrodynamics theory.

III. FINE STRUCTURE IN HELIUM

The calculation and measurement of the fine structure of the 2^3P levels of helium to a precision of 1 ppm provides a test of the theory of the two-electron system; or assuming the validity of the theory, the theoretical and experimental results can be used to determine an accurate value for the fine structure constant.[3] The theoretical program for such a calculation was outlined by Schwartz in 1964.[4] Precision experiments were undertaken by Hughes and collaborators at Yale. The experimental values they obtained are[5]

$$\nu_{01} = [E(2^3P_0) - E(2^3P_1)]/h = 29\ 616.864(36)\ \text{MHz} \qquad 1.2\ \text{ppm}$$

$$\nu_{12} = [E(2^3P_1) - E(2^3P_2)]/h = 2\ 291.196(5)\ \text{MHz} \qquad 2.2\ \text{ppm}$$

The theory is based on a reduced Hamiltonian H_R obtained from the Bethe-Salpeter equation:[6]

$$H_R = H_0 + H_2 + H_a + H_4 + \ldots$$

The reduced Hamiltonian consists of the nonrelativistic two-electron Pauli-Schrödinger Hamiltonian H_0, the relativistic corrections H_2 of order α^2Ry associated with the Breit equation, a Hamiltonian associated with the anomalous magnetic moment of the electron H_a, and the remaining corrections H_4 of order α^4Ry. The corrections are evaluated by perturbation theory starting with numerical eigenfunctions of H_0. The leading term is the first order correction due to the spin-dependent part of H_2, which must be evaluated accurately.[4,7] This correction is of order α^2Ry. The first order correction due to H_a contributes in order α^3Ry and α^4Ry.[4] The first order correction due to H_4 is of order α^4Ry.[8] In addition, H_2 contributes through second order perturbation theory terms of

order $\alpha^5 Ry$, and through cross terms between the mass polarization operator and H_2 terms of order $(m/M)\alpha^2 Ry$.[9,10,11] There are additional corrections of order $(m/M)\alpha^2 Ry$.[12,8] Earlier results are summarized in Ref. 13.

The largest theoretical uncertainty has been in the terms evaluated in second order perturbation theory. Recent work of Lewis, Serafino, and Hughes has reduced the uncertainty of these terms.[11] Their method of evaluation is based on Dalgarno-Lewis variational perturbation theory with up to 455 term Hylleraas trial functions.

Assuming the 1973 recommended value for α,[14] $\alpha^{-1} = 137.036\ 04(11)$, the theoretical fine structure splittings are[11]

$\nu_{01} = 29\ 616.883(43)$ MHz — 1.4 ppm

$\nu_{12} = 2\ 291.282(81)$ MHz — 35 ppm

in good agreement with the experimental values. Using the calculated and measured values of ν_{01} to obtain a value for α, one finds

$\alpha^{-1} = 137.036\ 08(13)$ — 0.94 ppm

This result is consistent with other determinations of α (see Section IX).

IV. ELECTRON G-FACTOR ANOMALY

The electron g-factor anomaly $a_e = (g_e - 2)/2$, which is a purely quantum electrodynamical quantity, is the deviation of the g-factor of the electron from its value predicted by the Dirac equation $g_e = 2$.

The experimental value of Wesley and Rich is[15,16]

$a_e = 1\ 159\ 656.7(3.5) \times 10^{-9}$ — 3 ppm

Work on a new experiment, which is expected to improve the accuracy by up to an order of magnitude, is in progress by Rich and co-workers at Michigan.[17] A recent preliminary value measured in a resonance experiment by Van Dyck, Ekstrom, and Dehmelt at the University of Washington is[18]

$a_e = 1\ 159\ 655(5) \times 10^{-9}$ — 4 ppm

This value was obtained with a single electron trapped in a magnetic bottle.[19] The precision is comparable to the precision of the

Wesley-Rich measurement.

In view of the prospects for improved accuracy in the experiments, it is appropriate to review the present status of the theory. The theoretical value for the g-factor anomaly is expressed as a series in powers of α/π

$$a_e = a_e^{(2)}\frac{\alpha}{\pi} + a_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + a_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots$$

where each coefficient $a_e^{(n)}$ is determined by evaluation of the n^{th} order radiative corrections. The first two coefficients in this series are known exactly:[20,21]

$$a_e^{(2)} = \frac{1}{2}$$

$$a_e^{(4)} = \frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta(3) = -0.328\ 48$$

The term $a_e^{(6)}$ requires the evaluation of 72 Feynman diagrams.[22] Recent work has led to improved numerical accuracy in the evaluation of the diagrams and analytic evaluation of many diagrams. The contribution of graphs with vacuum polarization insertions, such as the one shown in Fig. 1(a), is known analytically.[23] The contribution of graphs with light-by-light insertions, such as in Fig. 1(b), is known accurately numerically.[24] Some combinations of the remaining graphs which contain no electron loops, such as in Fig. 1(c), are now known analytically.[25,26] Combining the exact value of the analytically known graphs with numerical values for the remaining graphs, one obtains the following theoretical values for the sixth order coefficient (the three values correspond to three independent numerical evaluations):[26]

$a_e^{(6)} = 1.206(49)$ Levine and Wright[27]

$a_e^{(6)} = 1.188(17)$ Cvitanović and Kinoshita[28]

$a_e^{(6)} = 1.070(39)$ Carroll and Yao[29]

Higher order corrections, hadronic effects, muon loops, and weak interaction effects are expected to have the effect of a change of less than 0.01 in $a_e^{(6)}$.[26]

The sixth order coefficient can be compared with the derived experimental value, obtained by subtracting the lower order theoretical values (assuming $\alpha^{-1} = 137.035\ 987(29)$) from the experimental

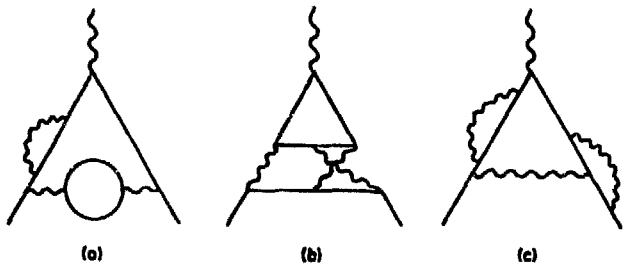


Fig. 1. Feynman diagrams which contribute to $a_e^{(6)}$.

value of Kesley and Rich:

$$a_e^{(6)} = 1.55(33)$$

The theoretical uncertainty in $a_e^{(6)}$ is approximately an order of magnitude smaller than in the experimentally determined value.

V. MUON G-FACTOR ANOMALY

A recent measurement of the muon g-factor anomaly by Bailey et al. at CERN has yielded the value[30]

$$a_\mu = 1\ 165\ 895(27) \times 10^{-9} \qquad 23\ \text{ppm}$$

In this experiment, the g-factor anomaly is determined by observation of the precession frequency of muons in a storage ring.[31]

The quantum electrodynamical contributions to the muon anomaly are expressed in a power series in α/π in analogy with the electron anomaly:

$$a_\mu(\text{QED}) = a_\mu^{(2)}\frac{\alpha}{\pi} + a_\mu^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + a_\mu^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots$$

There are two classes of Feynman graphs which contribute to $a_\mu^{(n)}$. One class consists of the graphs obtained by replacing the electrons by muons in the graphs associated with $a_e^{(n)}$. The contribution of these graphs to $a_\mu^{(n)}$ is just $a_e^{(n)}$. The other class consists of mass-dependent graphs with electron loop insertions such as the one shown in Fig. 2:

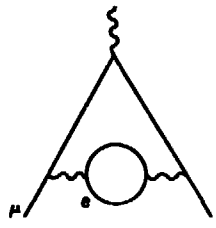


Fig. 2. Fourth order mass-dependent contribution to $a_\mu^{(4)}$.

$$a_\mu^{(n)} = a_e^{(n)} + \text{graphs with electron loops}$$

In second order, there are no such mass-dependent graphs:

$$a_\mu^{(2)} - a_e^{(2)} = 0$$

In fourth order, there is one mass-dependent graph, which is shown in Fig. 2:[32]

$$a_\mu^{(4)} - a_e^{(4)} = \frac{1}{3} \ln \frac{m_\mu}{m_e} - \frac{25}{36} + \frac{\pi^2}{4} \frac{m_e}{m_\mu} + \ldots = 1.094\ 26$$

In sixth order, all the mass-dependent graphs except those with light-by-light insertions are known analytically.[33] Recent work by Samuel and Chlouber has improved the numerical evaluation of the graphs with light-by-light insertions.[34] The combined contribution of analytically known graphs plus light-by-light graphs is

$$a_\mu^{(6)} - a_e^{(6)} = 1.944 + 21.32(5) = 23.26(5)$$

An estimate for the eighth order term, obtained by evaluating the eighth order graphs which are expected to give the dominant contribution, is given by[35]

$$a_\mu^{(8)} = 150(70)$$

In addition to the quantum electrodynamical contributions to a_μ, there is a hadronic contribution which arises from the graph in Fig. 2 with the electron loop replaced by virtual hadronic intermediate states. The value for this contribution is obtained from measured values for the cross section $\sigma(e^+e^- \to \text{hadrons})$ by means of a dispersion relation.[22] The result of a recent calculation is[36]

$$a_\mu(\text{hadronic}) = 66(10) \times 10^{-9}$$

Weak interaction contributions are expected to be small ($a_\mu(\text{weak}) \sim 2 \times 10^{-9}$).

The total theoretical value for a_μ is

$$a_\mu = 1\ 165\ 918(10) \times 10^{-9}$$

in good agreement with the experimental value. The hadronic contribution is necessary in order to have this agreement.

VI. GROUND STATE FINE STRUCTURE OF POSITRONIUM

Two recent measurements of the ground state fine structure of positronium have been made. Mills and Bearman obtained[37]

$$\Delta\nu(\text{exp}) = 203.3870(16)\ \text{GHz}$$

Egan, Frieze, Hughes, and Yam have determined[38]

$$\Delta\nu(\text{exp}) = 203.3849(12)\ \text{GHz}$$

The current theoretical value is less accurate. There remain terms of order α^2 relative to the main term which have not yet been evaluated. The theoretical expression up to this order is[39]

$$\Delta\nu(\text{th}) = \alpha^2 \text{Ry}\left[\frac{7}{6} - \frac{\alpha}{\pi}\left(\frac{16}{9} + \ln 2\right) + \frac{1}{2}\alpha^2 \ln \alpha^{-1} + O(\alpha^2)\right]$$

corresponding to a value of

$$\Delta\nu(\text{th}) = 203.404\ \text{GHz} + O(\alpha^4 \text{Ry})$$

Some of the corrections of order α^4Ry have been evaluated. The quantum electrodynamical correction without recoil yields[40]

$$-1.937\alpha^4 \text{Ry} = -0.018\ \text{GHz}$$

Vacuum polarization corrections to the one-photon annihilation diagram contribute[41]

$$-0.298\,\alpha^4 \text{Ry} = -0.003\ \text{GHz}$$

These contributions, combined with the lower order result, give a tentative theoretical value of

$$\Delta\nu(\text{th}) = 203.383\ \text{GHz}$$

However, a definitive test of the theory must await evaluation of the remaining terms of relative order α^2.

VII. $2^3S_1 - 2^3P_2$ ENERGY SPLITTING IN POSITRONIUM

The first measurement of an excited state energy splitting in positronium has been reported by Mills, Berko, and Canter.[42] For the $2^3S_1 - 2^3P_2$ splitting, they obtained the value

$$\Delta E(2^3S_1 - 2^3P_2) = 8628.4(2.8)\ \text{MHz}$$

The theoretical value has been calculated to lowest order in α by Ferrell:[43]

$$\Delta E(\alpha^2 Ry) = 8394\ \text{MHz}$$

The corrections of relative order α, which include the leading order radiative corrections, have been calculated by Fulton and Martin:[44]

$$\Delta E(\alpha^3 Ry) = 231\ \text{MHz}$$

The total theoretical result

$$\Delta E(2^3S_1 - 2^3P_2) = 8625\ \text{MHz} + O(\alpha^4 Ry)$$

is in good agreement with the experimental value.

The experiments on positronium mentioned in this section and the preceding section, and the recent measurement of the lifetime of orthopositronium in a powder by Gidley, Marko, and Rich[45] are reviewed by Mills, Berko, and Canter in these proceedings.[46]

VIII. MUONIUM HYPERFINE INTERVAL

A precision measurement of the muonium hyperfine interval by Casperson et al. at LAMPF has yielded[47]

$$\Delta\nu = 4463.3022(14)\ \text{MHz} \qquad 0.3\ \text{ppm}$$

This result is consistent with the "world average" of previous measurements quoted by Kobrak et al:[48]

$$\Delta\nu = 4463.3040(18)\ \text{MHz} \qquad 0.4\ \text{ppm}$$

The theoretical value is given by[22]

$$\Delta\nu = \frac{16}{3}\alpha^2 Ry \frac{\mu_\mu}{\mu_B}\left(1 + \frac{m_e}{m_\mu}\right)^{-3}\left(1 + b + q + \delta_\mu\right)$$

where b is the Breit correction, q is the quantum electrodynamic correction, which includes the anomalous magnetic moment of the electron, and δ_μ is the recoil correction of order m_e/m_μ. The theoretical expression for $\Delta\nu$ can be written in terms of the experimentally determined parameters α and μ_μ/μ_p:[47]

$$\Delta\nu = \alpha^2 \frac{\mu_\mu}{\mu_p}\left[2.632\ 957\ 87 \pm 0.6\ \text{ppm}\right] \times 10^7\ \text{MHz}$$

The error quoted in the square brackets arises from the estimated uncertainty in the quantum electrodynamical contributions of relative order α^3.[49] Equating the theoretical and experimental values, assuming the value for α given in Section II, one obtains[47]

$$\frac{\mu_\mu}{\mu_p} = 3.183\ 329\ 9(25) \qquad 0.8\ \text{ppm}$$

and

$$\frac{m_\mu}{m_e} = 206.769\ 27(17) \qquad 0.8\ \text{ppm}$$

The value for μ_μ/μ_p obtained this way differs by two standard deviations from the value[50]

$$\frac{\mu_\mu}{\mu_p} = 3.183\ 346\ 7(82) \qquad 2.6\ \text{ppm}$$

determined by observation of the muon precession frequency in liquids.

IX. DERIVED VALUES OF α

It is of interest to compare the values of α derived from comparison of theory and experiment for various precision experiments. The values from the hydrogen fine structure and hydrogen hyperfine structure are discussed in Ref. 14. The results from muonium hyperfine structure, helium fine structure, electron anomalous g-value, and a c Josephson effect are discussed in the preceding sections. The derived values are shown in Fig. 3.

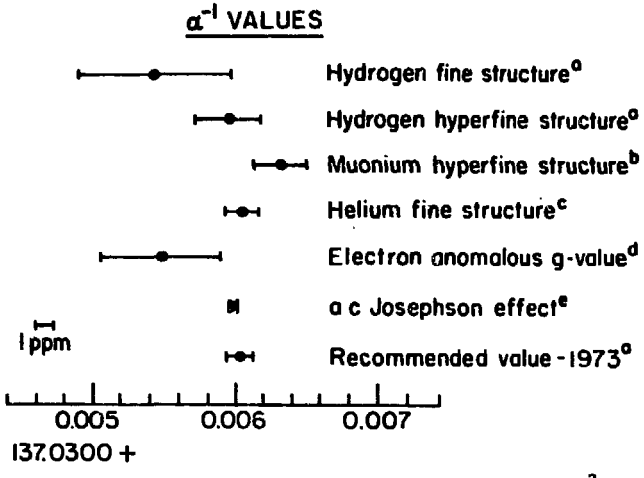


Fig. 3. Values of α derived from various measurements: [a]Ref. 14; [b]Sec. VIII; [c]Sec. III; [d]Sec. IV; [e]Sec. II.

X. HYPERFINE SPLITTINGS IN $^3He^+$

The theoretical expression for the hyperfine splitting in the $nS_{\frac{1}{2}}$ state of a hydrogen-like system is given by

$$\Delta\nu_n = \Delta\nu_n^F\left(1 + \frac{m}{M}\right)^{-3}\left(1 + b_n + q_n + \delta_n\right)$$

where $\Delta\nu_n^F$ is the Fermi splitting, M is the mass of the nucleus, b_n is the Breit correction, q_n is the quantum electrodynamic correction for a fixed point nucleus, and δ_n represents the remaining corrections including the effect of nuclear size and recoil. In $^3He^+$, δ_n is of the order of 10^{-4} and can be calculated only with limited precision. Uncertainty arising from this term limits the accuracy of a direct comparison of theory and experiment as a test of quantum electrodynamics. The main contribution to δ_n is proportional to $|\psi_n(0)|^2 \propto n^{-3}$ as is the Fermi splitting $\Delta\nu_n^F$, so the dominant part of δ_n is independent of n. Therefore, by comparing theory and experiment for the difference $8\Delta\nu_2 - \Delta\nu_1$, the main contribution of the nuclear structure effects, which is proportional to $\delta_2 - \delta_1$ in the difference, is eliminated. This fact was recognized some time ago and calculations of the necessary differences have been done. The results for the case of $^3He^+$ are given by

$$\Delta\nu_1^F\left(1 + \frac{m}{M}\right)^{-3}(b_2 - b_1) = 1.152\ 98\ \text{MHz}$$

$$\Delta\nu_1^F\left(1 + \frac{m}{M}\right)^{-3}(q_2 - q_1) = 0.036\ 03\ \text{MHz}$$

$$\Delta\nu_1^F\left(1 + \frac{m}{M}\right)^{-3}(\delta_2 - \delta_1) = 0.000\ 80\ \text{MHz}$$

The difference $b_2 - b_1$ includes Breit corrections through order $(Z\alpha)^4$.[51] The difference $q_2 - q_1$ has been calculated by Zwanziger through order $\alpha(Z\alpha)^2$.[52] The difference $\delta_2 - \delta_1$ was first calculated by Sternheim[53] through order $(m/M)(Z\alpha)^2$ and includes corrections calculated by Schwartz.[54]

An improved comparison for $^3He^+$ has been made possible by a recent measurement of $\Delta\nu_2$ by Prior and Wang,[55] which is described by Prior in these proceedings.[56] A precise measurement of $\Delta\nu_1$ has been made by Schuessler, Fortson, and Dehmelt.[57] Both of these measurements were made by an ion storage method. The experimental values are

$$\Delta\nu_2 = 1083.354\ 982\ 5(76)\ \text{MHz} \qquad 7\ \text{ppb}$$

$$\Delta\nu_1 = 8665.649\ 867(10)\ \text{MHz} \qquad 1\ \text{ppb}$$

The differences are

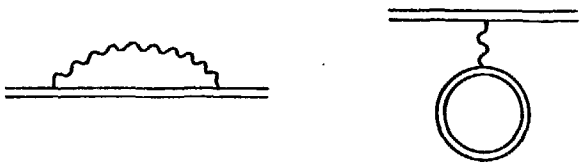
Exp: $8\Delta\nu_2 - \Delta\nu_1 = 1.189\ 993(62)$ MHz

Th: $8\Delta\nu_2 - \Delta\nu_1 = 1.189\ 80$ MHz

Comparison of the experimental and theoretical values confirms the quantum electrodynamical correction to approximately 0.5%. The residual difference between theory and experiment is presumably due to uncalculated contributions to $\delta_2 - \delta_1$, including terms of order $(Z\alpha)^2\delta_1$, and uncalculated contributions to $q_2 - q_1$ of order $\alpha(Z\alpha)^3$.

XI. LAMB SHIFT

The main theoretical contribution to the Lamb shift $S = \Delta E(2S_{\frac{1}{2}}) - \Delta E(2P_{\frac{1}{2}})$ in a hydrogen-like system arises from the lowest order self energy and vacuum polarization corrections corresponding to the Feynman diagrams shown in Fig. 4. The contribution of the self energy to the Lamb shift S_{SE} is given by



(a) (b)

Fig. 4. Feynman diagrams for the lowest order self energy (a) and vacuum polarization (b) corrections.

$$S_{SE} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{6} mc^2 \left[\ln(Z\alpha)^{-2} - \ln\frac{K_0(2,0)}{K_0(2,1)} + \frac{11}{24} + \frac{1}{2} + 3\pi\left(1 + \frac{11}{128} - \frac{1}{2}\ln 2\right)(Z\alpha) - \frac{3}{4}(Z\alpha)^2 \ln^2(Z\alpha)^{-2} + \left(\frac{299}{240} + 4\ln 2\right)(Z\alpha)^2 \ln(Z\alpha)^{-2} + (Z\alpha)^2 G_{SE}(Z\alpha)\right]$$

and the contribution of the vacuum polarization to the Lamb shift S_{VP} is

$$S_{VP} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{6} mc^2 \left[-\frac{1}{5} + \frac{5}{64}\pi(Z\alpha) - \frac{1}{10}(Z\alpha)^2 \ln(Z\alpha)^{-2} + (Z\alpha)^2 G_{VP}(Z\alpha)\right]$$

In each expression, the energy shift is divided into known parts of lowest order in $Z\alpha$ and a function which contains the exact higher order remainder. Calculations of the lower order terms are summarized in Ref. 22. The remainder, contained in $G_{SE}(Z\alpha)$ and $G_{VP}(Z\alpha)$, gives a significant contribution (-0.17 MHz) to the Lamb shift in hydrogen. Erickson and Yennie calculated the main contribution $G_{SE}(0) = -19.08 \pm 5$.[58] Erickson has estimated the value $G_{SE}(\alpha) = -17.1 \pm 0.6$ and has obtained an approximation for $G_{SE}(Z\alpha)$ for a wide range of Z.[59] Recently, the self energy has been evaluated numerically to all orders in $Z\alpha$ for the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ states for Z in the range 10-110.[60] The results, in terms of the function $G_{SE}(Z\alpha)$, are shown in Fig. 5. The value at Z = 1, $G_{SE}(\alpha) = -23.4 \pm 1.2$, is the extrapolated value obtained by fitting the function

$$a + b(Z\alpha)\ln(Z\alpha)^{-2} + c(Z\alpha)$$

to the calculated values at Z = 10, 20, and 30.[60]

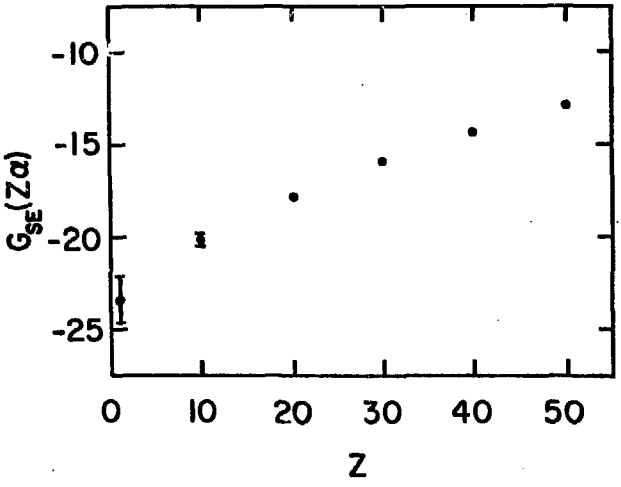


Fig. 5. Calculated values of $G_{SE}(Z\alpha)$ and the extrapolated value at Z = 1, from Ref. 60.

The numerical evaluation is based on the expansion of the coordinate space Green's function in the expression for the self-energy shift as a sum over eigenfunctions of angular momentum.[61] Each term in the sum factorizes into a radial Green's function and a part which contains the functional dependence on the coordinate angles. The radial Green's functions for a Coulomb potential are known in terms of Whittaker functions which can be evaluated to high accuracy. The numerical work consists of evaluating the radial Green's functions, summing the series associated with the angular momentum expansion, and evaluating a three-dimensional integral over the sum. It is necessary to isolate and remove the mass renormalization term. Technical difficulties, such as numerical cancellation for small Z and slow convergence of the angular momentum sum, are dealt with by algebraic rearrangement of the expression, and by the introduction of suitable subtraction terms.

Evaluations of the self-energy contribution to the $1S_{\frac{1}{2}}$-state energy at high Z which take into account electron screening effects and finite nuclear size effects have been done by an independent method based on summation over angular momentum eigenfunctions by Desiderio and Johnson[62] and recently by Cheng and Johnson.[63] The results of Cheng and Johnson for the $1S_{\frac{1}{2}}$-state in a Coulomb poten-

tial and the results of the method described in the preceding paragraph for the $1S_{\frac{1}{2}}$-state[61] are in close agreement for Z in the range 50-130.

Evaluation of the vacuum polarization contribution to the Lamb shift is facilitated by expanding the vacuum polarization potential in powers of the external potential. The first term, which is linear in the external potential, is the Uehling potential[64] which gives rise to the lowest order terms in S_{VP} and the dominant part $G_U(Z\alpha)$ of $G_{VP}(Z\alpha)$:[65,60]

$$G_{VP}(Z\alpha) = G_U(Z\alpha) = -\frac{1199}{2100} + \frac{5}{128}\pi(Z\alpha)\ln(Z\alpha)^{-2} + 0.5(Z\alpha) + \ldots$$

There are additional well-known corrections to the Lamb shift which are reviewed in Ref. 22. They arise from fourth order radiative corrections, reduced mass effects, relativistic recoil corrections, and the effect of nuclear size. The nonrelativistic size correction

$$S_{NS} = \frac{(Z\alpha)^4}{12}(R/\lambda\!\!\!^{-})^2 mc^2$$

depends on the r.m.s. nuclear charge radius $R = \langle r^2 \rangle^{\frac{1}{2}}$ which is determined by experiments on muonic atoms and by electron scattering experiments. In the case of hydrogen, only the latter information is presently available.

The result of a recent analysis of electron-proton elastic scattering data by Borkowski et al. is R = 0.87(2) fm.[66] This value is larger than the values R = 0.805(11) fm[67] and R = 0.80(2) fm[68] which have been generally used in evaluating the Lamb shift in hydrogen.[58,59,22,60] The difference, ΔR = 0.07 fm, produces a change in the theoretical value for the Lamb shift of 0.02 MHz which is at the level of precision of the most recent measurement.[69] In order to choose a "best" value for R to evaluate the Lamb shift, we consider the relevant electron scattering data. Fig. 6 shows a sample of measured cross sections over a wide range of the four-momentum transfer squared q^2. Fig. 7 shows data at low q^2 which is included in the analysis of Ref. 66. A normalization error of 3-4% in the data in the figures is not included in the error bars. The data are normalized to the dipole fit which is the cross section based on the Rosenbluth formula with

$$G_E^D(q^2) = (1 + q^2/18.23\ \text{fm}^{-2})^{-2}$$

$$G_M^D(q^2) = \mu G_E^D(q^2)$$

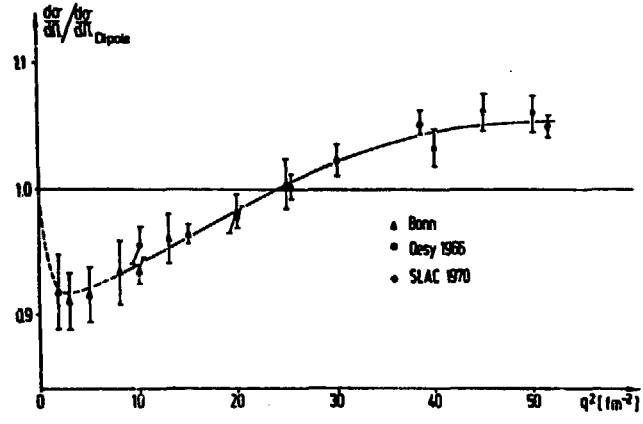


Fig. 6. Elastic electron - proton cross sections normalized to the dipole fit, from Berger et al., Ref. 70. The DESY data have been multiplied by 0.989 and the SLAC data by 0.984.

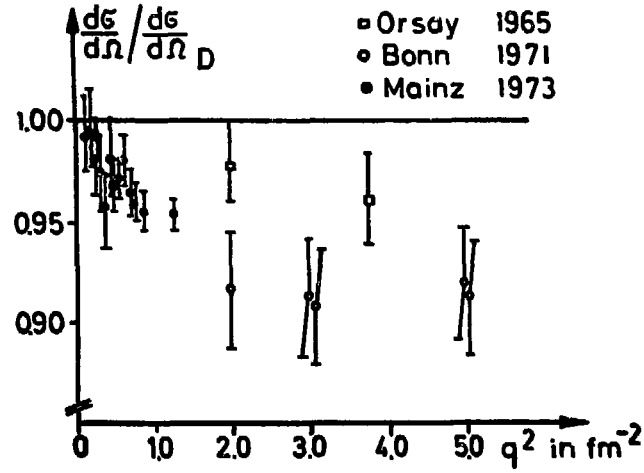


Fig. 7. Low q^2 elastic electron-proton cross sections normalized to the dipole fit, from Borkowski et al., Ref. 71.

substituted for the electric and magnetic form factors $G_E(q^2)$ and $G_M(q^2)$ of the proton; μ is the magnetic moment of the proton. The proton r.m.s. charge radius is related to the electric form factor by

$$R^2 = -6 \left. \frac{dG_E(q^2)}{dq^2} \right|_{q^2 = 0}$$

The dipole form factor $G_E^D(q^2)$ gives R = 0.81 fm. If the approximation of one photon exchange together with the scaling relation $G_M(q^2) = \mu G_E(q^2)$ describes the scattering, then the cross section is proportional to $G_E^2(q^2)$, and deviation of the slope of the data from zero at $q^2 = 0$ signifies deviation of the value of R from the dipole value. The data in Figs. 6 and 7 indicate a systematic deviation of the cross section from the dipole fit and appear to approach $q^2 = 0$ with a negative slope. The apparent slope of the data is consistent with the value R = 0.87 fm, but not with R = 0.80 fm which would require a small positive slope. Hence, we shall tentatively assume R = 0.87(2) fm in evaluating the Lamb shift in hydrogen. Additional experimental information on the proton charge radius would be useful.

The contributions to the Lamb shift are listed in Table I. The theoretical value, with an estimated uncertainty for uncalculated terms, is[22,60]

$$S_{TH} = 1057.888(13) \text{ MHz}$$

where R = 0.87(2) fm[66] has been assumed.

TABLE I. Contributions to the Lamb shift in hydrogen

CORRECTION	ORDER (mc^2)	VALUE (MHz)
Self energy	$\alpha(Z\alpha)^4[\ln(Z\alpha)^{-2}, 1, Z\alpha, \ldots]$	1085.812
Vacuum polarization	$\alpha(Z\alpha)^4[1, Z\alpha, \ldots]$	-26.897
Fourth order	$\alpha^2(Z\alpha)^4$	0.101
Reduced mass	$\alpha(Z\alpha)^4 m/M[\ln(Z\alpha)^{-2}, 1]$	-1.636
Relativistic recoil	$(Z\alpha)^5 m/M[\ln(Z\alpha)^{-2}, 1]$	0.359
Nuclear size	$(Z\alpha)^4(R/\lambda\!\!\!^{-})^2$	0.148
TOTAL		1057.888

TABLE 11. Comparison of theory and experiment for the Lamb shift ($Z \geq 3$)

ION	THEORY	EXPERIMENT	REF.
Li^{2+}	62 737.5(6.6) MHz	62 765(21) MHz	76
		62 790(70) MHz	77
		63 031(327) MHz	78
C^{5+}	781.99(21) GHz	780.1(8.0) GHz	79
O^{7+}	2196.21(92) GHz	2215.6(7.5) GHz	80
		2202.7(11.0) GHz	81
F^{8+}	3343.1(1.6) GHz	3339(35) GHz	74
Ar^{17+}	38.250(25) THz	38.3(1.2) THz	73

There has been a substantial improvement in the measurement of the Lamb shift in hydrogen. Lundeen and Pipkin, using the separated oscillatory field method to produce a resonance narrower than the natural linewidth, have obtained[69]

$$S_{EXP} = 1057.893(20) \text{ MHz}$$

which agrees with the theoretical value. For discussion of the experiment, see Ref. 72.

Measurements of the Lamb shift in high Z hydrogen-like ions have been extended to Ar^{17+} by Gould and Marrus who have reported a preliminary value at this conference.[73] A result for F^{8+} has been reported by Kugel et al.[74] The theoretical and experimental values are listed in Table 11; the theoretical values are from Ref. 75. There is generally good agreement between theory and experiment. Measurements at high Z provide a test of strong field binding effects in the Lamb shift.

XII. FINE STRUCTURE IN MUONIC HELIUM

The first measurement of the fine structure $E(2P_{3/2}) - E(2S_{1/2})$ in $(\mu^- He)^+$ has been reported by Bertin et al.[82] The experiment was described in the previous conference of this series.[83] The measured value is

$$\Delta E(2P_{3/2} - 2S_{1/2}) = 1527.4(0.9) \text{ meV}$$

TABLE III. Theoretical contributions to the fine structure $E(2P_{3/2}) - E(2S_{1/2})$ in muonic helium (in meV)

Fine structure	145.7
Vacuum polarization, order α	1666.1
Vacuum polarization, order α^2	11.6
Self energy, muon vac. pol.	-10.7 ± 1
Finite nuclear size	$-103.1 \langle r^2 \rangle$ fm^{-2}
Nuclear polarization	3.1 ± 0.6
TOTAL	$1815.8 \pm 1.2 - 103.1 \langle r^2 \rangle$ fm^{-2}

Early theoretical studies of low-Z muonic atoms were done by Di Giacomo,[84] and by Campani.[85] Two conflicting estimates have been made for the nuclear polarization correction. Bernabéu and Jarlskog [86] estimated 3.1 ± 0.6 meV with a calculation based primarily on measured photoabsorption cross sections as input data. Henley, Krejs, and Wilets[87] estimated a correction of 7.0 ± 1.5 meV, based on a harmonic oscillator model for the helium nucleus, in agreement with an earlier calculation of Joachain.[88] Bernabéu and Jarlskog,[89] in an analysis of the two results, point out that the harmonic oscillator model predicts a nuclear electric dipole polarizability which disagrees with the value deduced from measured photoabsorption cross sections; hence, the smaller value appears more likely to be correct. An independent calculation by Rinker confirms the value 3.1 meV.[90]

Theoretical contributions to the energy separation are listed in Table III. The values in that table are taken from the compilation of Borie,[91] with the exception of the vacuum polarization correction of order α which is taken from Rinker,[90] and the nuclear polarization correction which is taken from Bernabéu and Jarlskog.[86] It is interesting to note that the electron vacuum polarization contribution to the fine structure is an order of magnitude larger than the Dirac fine structure splitting. The splitting is sensitive to the nuclear size, so that it is convenient to parameterize the corresponding energy shift in terms of the r.m.s. nuclear charge radius. The self energy and muon vacuum polarization terms are point nucleus values with an estimated uncertainty due to finite nuclear size corrections. Relativistic recoil corrections, which are estimated to be less than 0.5 meV,[90,91] are not included.

Assuming the value $\langle r^2 \rangle^{1/2} = 1.650(25)$ fm for the nuclear radius, based on a weighted average of electron scattering results,[82] the theoretical value for the fine structure is

$$\Delta E(2P_{3/2} - 2S_{1/2}) = 1535(9) \text{ meV}$$

which is in good agreement with the experimental value. The error is mainly due to uncertainty in the nuclear radius. If the theory is assumed to be correct, one can use the measured energy splitting to deduce the value

$$\langle r^2 \rangle^{\frac{1}{2}} = 1.673(4) \text{ fm}$$

for the nuclear charge radius.

XIII. HIGH Z MUONIC ATOMS

Accurate measurements of the x rays emitted in transitions between large n states in high Z muonic atoms provide a test of quantum electrodynamic corrections to the energy levels. In particular, the muon levels are sensitive to the effect of vacuum polarization, which is tested to better than 1% with present-day experimental precision. Recent experiments by Tauscher et al.[92] and by Dixit et al.[93] have yielded results for muon transition energies which are in good agreement with theory, in contrast to the results of earlier experiments which disagreed with theory.[94,95]

A recent improvement in the theory has been made by the evaluation of the effect of nuclear size on the higher order vacuum polarization correction. This effect has been calculated by Arafune,[96] by Brown, Cahn, and McLerran,[97] and by Gyulassy.[98] For the $5g_{9/2} - 4f_{7/2}$ transition in muonic lead, the finite nuclear size correction decreases the magnitude of the higher order vacuum polarization correction by 6 eV from the Coulomb value of about -50 eV to -44 eV. A calculation of the higher order vacuum polarization correction which takes finite nuclear size into account has been done by Rinker and Wilets.[99]

There has been interest in a correction of order $\alpha^2(Z\alpha)^7$ corresponding to the Delbrück-like diagrams shown in Fig. 8. It was suggested that the correction could be large (~ -35 eV) for the $5g_{9/2} - 4f_{7/2}$ transition in muonic lead.[100] However, Wilets and Rinker estimated a range of 1 to 3 eV for the correction.[101] The

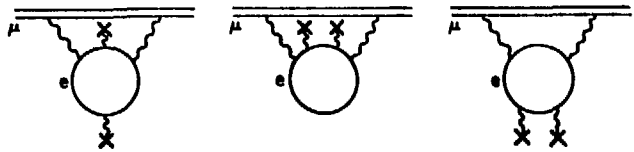


Fig. 8. Feynman diagrams for order $\alpha^2(Z\alpha)^2$ Delbrück-like corrections to muon energy levels.

TABLE IV. Muonic Atom Energy Level Differences (eV)

Transition	Point Nucleus	Finite Size	Vacuum Polarization I	II	III	IV	Self En.	Rel. Rec.	Nuc. Pol.	Elec. Scr.	Total
$_{56}^{Ba}$											
$4f_{5/2}$-$3d_{3/2}$	439 069±1	-146±8	2436	-21±2	17	1	9±3	3	7	-18±1	441 357±9
$4f_{7/2}$-$3d_{5/2}$	431 654±1	-55±5	2328	-20±2	16	1	-8±2	3	7	-18±1	433 908±6
$5g_{7/2}$-$4f_{5/2}$	200 544±1	0	761	-9±1	5	0	2±1	1	0	-31±2	201 273±3
$5g_{9/2}$-$4f_{7/2}$	199 194±1	0	747	-9±1	5	0	-2±1	1	0	-31±2	199 905±3
$_{80}^{Hg}$											
$5g_{7/2}$-$4f_{5/2}$	414 182±1	-8±1	2047	-42±2	14	1	7±2	2	3	-78±4	416 128±5
$5g_{9/2}$-$4f_{7/2}$	408 465±1	-2	1972	-40±2	14	1	-6±2	2	3	-79±4	410 330±5
$^{203}_{81}Tl$											
$5g_{7/2}$-$4f_{5/2}$	424 850±1	-9±1	2117	-44±2	15	1	7±2	2	4	-79±4	426 864±5
$5g_{9/2}$-$4f_{7/2}$	418 837±1	-3	2039	-43±2	14	1	-7±2	2	4	-81±4	420 763±5
$_{82}^{Pb}$											
$5g_{7/2}$-$4f_{5/2}$	435 666±1	-10±1	2189	-46±2	15	1	7±2	2	4	-81±4	437 747±5
$5g_{9/2}$-$4f_{7/2}$	429 344±1	-4	2106	-45±2	15	1	-7±2	2	4	-83±4	431 333±5

I. $\alpha(Z\alpha)$ Uehling term. II. $\alpha(Z\alpha)^{3+}$ + finite size. III. $\alpha^2(Z\alpha)$. IV. $\alpha^2(Z\alpha)^2$ Delbrück term.

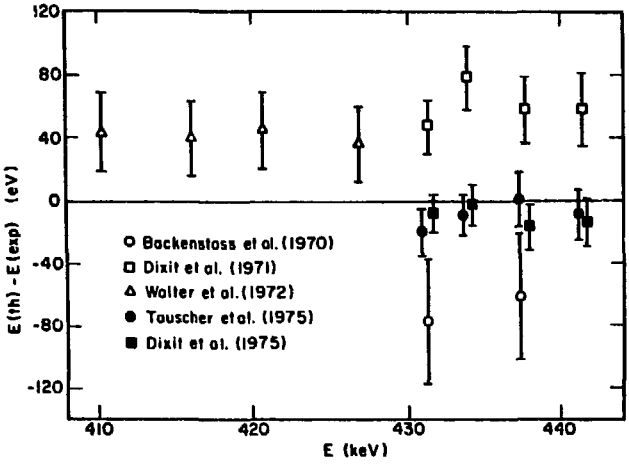


Fig. 9. Comparison of theory and experiment for muonic atom transition energies. The experimental values are from Backenstoss et al.,[105] Dixit et al.,[94] Walter et al.,[95] Tauscher et al.,[92] and Dixit et al.[93]

small value has been confirmed by Fujimoto;[102] Borie has recently reported a value of 1 eV for the correction.[103]

The contributions to the muon energy level differences for a selection of transitions are listed in Table IV. The point nucleus contribution is the reduced-mass Dirac energy separation. The value employed here for the mass of the muon is based on the muonium hyperfine structure determination described in Section VIII. The remaining numerical values are from Table 2 of the review by Watson and Sundaresan,[104] with the following exceptions. The vacuum polarization correction in column II includes a finite size correction based on the formula of Arafune[96] (which predicts a 5 eV correction in the $5g_{9/2}$ - $4f_{7/2}$ transition in lead). The $\alpha^2(Z\alpha)^2$ term is based on results in Refs. 101-103. Theory and experiment for muonic atom transitions with energy > 410 keV are compared in Fig. 9. We note that the most recent experimental results would disagree with theory without the higher order vacuum polarization correction.

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