To be presented at 1976 Proton *Linear Accelerator Conference*

LBL-5536

PARTICLE ACCELERATION BY COLLECTIVE EFFECTS*

D. Keefe
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

Successful acceleration of protons and other ions has been achieved experimentally in this decade by a number of different collective methods. The attainment of very high accelerating fields has been established although so far the acceleration distance has been confined to only a few centimeters. Efforts are in progress to understand the accelerating mechanisms in detail and, as a result, to devise ways of extending considerably the acceleration distance. This paper is intended to review the current progress, expectations, and limitations of the different approaches.

1) *Introduction*

The accelerating electric field in conventional accelerators is typically limited to values in the 1 - 10 MV/m range because of sparking and other practical reasons. The promise of collective effect methods of acceleration is attainment of much higher fields; this promise has been fulfilled and ion-acceleration by collectively-produced fields of 100 MV/m and more has been clearly demonstrated in a wide variety of experiments. So far, however, the distance over which acceleration has been achieved has been limited to some tens of centimeters; the major challenge now is to extend and control the acceleration process over much longer lengths. Although light ions of various charge-states have been accelerated most of the experiments have studied proton acceleration and to avoid confusion in what follows I shall mainly refer to the results for protons.

My intention here is to survey the recent progress, to report on the most striking new results, and to describe new theoretical proposals in the field of acceleration of ions by collective effects. The electron-ring method continues to look promising especially for medium-energy ions but there are no new ion-acceleration results to report; the E-beam in gas-cells method has had the benefit of a great deal of new experimental results and the various mechanisms at play are beginning to be elucidated; the E-beam in vacuum-diodes method still holds the record for peak ion-energy produced (45 MeV protons by J. Luce and collaborators) but also lacks a convincing theoretical description; experimental work is beginning on the auto-resonant acceleration concept (ARA) by Drummond and co-workers; finally there have been several suggestions for new methods of ion-acceleration by means of slow, controlled, space-charge waves on electron-beams in vacuum pipes.

2) Electron-Ring Accelerators

Research at the Lawrence Berkeley Laboratory on this approach ended a few years ago but succeeded in defining experimentally and theoretic ly the limits to the promise of this technique s·· by instabilities of the ring. The hurtful i stabilities that limit performance are the negative-ma s instability, the ion electron instability a the resistive-wall instability. A major conc ion of that work was that realistically one cou d not expect to achieve peak accelerating fields greater than about 80 MV/m [1] but that creation of rings with fields of about this value were close to experimental accomplishment [2].

The Maryland group are actively pursuing their novel technique of forming rings by injection of a hollow E-beam along an axial magnetic field and then through a cusp field to convert much of the axial velocity to transverse motion [3]. The rings formed so far have axial dimensions that are still rather large (some few centimeters) so that the peak electric fields are small (perhaps ≈ 1MV/m). The group is at present working on techniques for slowing down the axial motion of the ring to allow ion loading and subsequent controlled launching into the appropriately-tailored accelerating magnetic field.

A year and a half ago the group at the Max-Planck Institute for Plasma Physics, Garching, gave convincing evidence of acceleration of helium ions to a low energy (≳ 200 keV) in a compressor experiment called "Schuko" [4]. In the meantime they have continued ring-forming experiments, but much of their effort is now concentrated on fabricating a larger apparatus called "Pustarex" [5]. The Garching group's approach has been characterized by an extremely short compression cycle to form the rings, viz., ≲ 10μsec, which is some two orders of magnitude less than that used at Berkeley or Dubna. This has the advantage of rapid crossing of resonances during compression. After compression, however, times of the order of hundreds of microseconds are needed to load the ring with ions. In their new experiment (Pustarex) they will continue to use fast pulsed compression followed by use of static magnetic fields, first to hold the ring in a "waiting-room" for ion loading, and thereafter for axial magnetic acceleration. This experiment will have the advantage that the accelerator can be extended in length by addition

*This work was done with support from the U.S. Energy Research and Development Administration.

of further magnetic-field sections at a later time to allow higher energy ions to be obtained. The goal of the new experiment is to accelerate ions in the mass range from A = 1 to A = 40 to an energy of 5 MeV/amu in a total acceleration length of only 1.25 meters.

Work by Sarantsev and his colleagues at the Joint Institute for Nuclear Research at Dubna, has concentrated on construction of a large accelerator with provision for electric acceleration of the rings by superconducting rf cavities. They have recently formed rings in their new compressor and claim to have achieved accelerating fields of 50 MV/m; they have not yet, however, successfully extracted the rings in an axially compact form.

Under the direction of Kapchinsky a group at the Institute for Theoretical and Experimental Physics, Moscow, is also working on electron-ring devices. The main thrust of their efforts is towards understanding the formation and compression of the ring. Progress has been inhibited by difficulties with the electron-beam injector.

3) Electron Beams in Gas-Cells

Ion acceleration is found to occur when an electron-beam with a current above the space-charge limited value (I_{SCL}) is injected into a gas of pressure about one-tenth Torr. Typical parameters for these experiments are $E_e \geq 1$ MeV, $I_e \geq 10{,}000$ amps, and $\tau \simeq 0.1$ usec. Figure 1 is a schematic of the experimental arrangement, including representative diagnostics, which has been widely used.

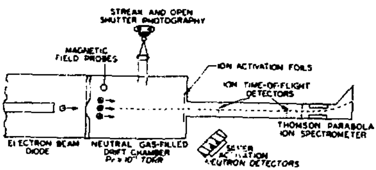

Fig. 1: Typical experimental arrangement for ion acceleration in gas cell [8]

The space-charge-limited current is often defined in the following terms: If an intense cylindrical beam of radius b is propagating inside a grounded pipe of radius a there is a potential depression at the axis given by: $\phi = I(1+2\ln a/b)/\beta c$. If partial ionization of the gas occurs then the potential is modified to $\phi' = \phi[1-f_e(t)]$ where $f_e(t)$ is the fractional neutralization and depends on time because of continuing ionization by the beam. A necessary condition that an electron should have found its way from the grounded anode to the beam center is that its kinetic energy exceed $e\phi$, i.e.

$$mc^2(\gamma-1) > e\phi\;[1-f_e(t)]$$

which leads to the condition on the current, I:

$$I < I_{SCL} = \frac{mc^3}{e}\,\beta(\gamma-1)(1+2\ln\frac{a}{b})^{-1}(1-f_e(t))^{-1} \quad (1)$$

that is necessary for propagation. If the injected current exceeds I_{SCL}, the beam does not propagate; instead a virtual cathode is formed. This problem was addressed by Smith and Hartman [6] for the somewhat unreal situation of constant current density. More careful examination leads to the expression:

$$I_{SCL} = \frac{mc^3}{e}\,(\gamma^{2/3}-1)^{3/2}\,(1+2\ln\frac{a}{b})^{-1}\,[1-f_e(t)]^{-1} \quad (2)$$

It is interesting to refer here to another well-known limiting current-the Lawson-Alfvèn limit. This limit is appropriate to the case of a fully-neutralized beam ($f_e=1$) - wherein there is no electric field-at an intensity such that the self-magnetic field can bend beam particles around with a Larmor radius on the order of the beam dimension, b. This leads to

$$I_{L-AL} = \frac{mc^3}{e}\,\beta\gamma \quad [\simeq 17{,}000\ \beta\gamma \text{ amps, for electrons}] \quad (3)$$

This limit corresponds to the Budker parameter, ν, which is the number of electrons in a classical electron radius (e^2/mc^2) measured along the beam, being equal to γ, that is: $\nu/\gamma=1$. Note that Eqn (3) applies for the case of a space-charge neutralized beam; if current-neutralization occurs, larger currents can be propagated. (In fact, at Physics International beams with $\nu/\gamma = 10$ have been generated).

If we use the approximate expression for I_{SCL} from Eqn. (1) above, we note:

$$\frac{I_{SCL}}{I_{L-AL}} \simeq \frac{\gamma-1}{\gamma}\,(1+2\ln a/b)^{-1}\,[1-f_e(t)]^{-1} = [\frac{\nu}{\gamma}]_{SCL} \quad (4)$$

In practice, therefore, most ion acceleration experiments use beams with ν/γ values between 0.1 and 1.

Early data were scrappy and inconsistent from group to group and several models were proposed that fitted some data but not others. In the last couple of years, however, the experimental results have become sufficient to identify two distinct processes, although at least two more may also be playing a role or at least are expected to become operative under certain conditions. We can summarize these processes as follows:

(i) Virtual Cathode Formation and Relaxation (Quasi-Stationary Well):

This process explains most of the experiments performed by many groups. The bulk of the ions produced in all experiments (1-5X10^{14}, typically) have an energy spectrum distributed from the electron-beam energy E_e to about 3 E_e with an average value close to 1.5 E_e.

For ion acceleration to occur the injected current I_e must exceed I_{SCL} so that virtual cathode formation occurs. Note, however, that I_{SCL} is a function of time and as the fractional neutralization $f_e(t)$ builds up because of ionization, I_{SCL} increases. For ion-acceleration one must arrange that I_{SCL} crosses over I_e at some time during the pulse duration. This sets a condition on the ionization rate and hence on the gas pressure. If the pressure is too high I_{SCL} increases so rapidly that it exceeds I_e before the beam current has risen to its full value. If the pressure is too low a virtual cathode can presist throughout the pulse; any ions formed may oscillate in the potential at the virtual cathode but receive no net energy gain. In the appropriate pressure window (0.01 - 1 torr) I_{SCL} is less than I_e at the beginning of the pulse and a deep potential well formed - later I_{SCL} exceeds I_e, the well relaxes, the beam front propagates, and ions can emerge with high energy generated in the transient well. The mechanism of well-formation is complicated and was first described by Poukey and Rostoker [7] who showed that the potential (times charge) could attain a depth of two to three times the beam-electron kinetic energy i.e. 2-3 E_e. The exact dynamical effects during formation and relaxation are not completely understood.

Recently Straw and Miller set out to verify in a systematic way the role of I_{SCL} in determining the onset of acceleration [8]. In an elegant series of experiments they varied many of the parameters in Eqn (2) over a wide range, e.g. the ratio b/a, current, energy, pressure (which changes $f_e(t)$) and convincingly demonstrated agreement with the model described.

(ii) Moving Potential well at the Front of the Propagating Beam (Travelling Well):

When during the beam pulse I_{SCL} comes to exceed I_e the virtual cathode relaxes and the beam front begins to propagate with a speed on the order of 0.1 c. The head of the beam is being continually ablated because of beam-electron loss to the wall. The beam front carries with a co-moving potential well because ahead of it there is essentially no beam (ϕ=0), behind it there is full space-charge neutralization (ϕ=0), and in between there is only partial neutralization ($\phi > 0$). Early results at Physics International had shown the presence of a group of fast ions $E_p > 3E_e$ whose speed was equal to that of the beam-front [9]. These data provided strong evidence for ions being trapped and transported in the travelling well at the beam front. More recent work at P.I. confirms this conclusion. How the ions become trapped is not understood - they may be born in the well when the fields are sufficiently gentle to allow trapping or they may be injected with suitable velocity by pre-acceleration in the quasi-stationary well (see (i)).

It seems safe at this point to assume the existence of the travelling well and its ability to accelerate ions to high energy (> $3E_e$), but there remains some disagreement over what phenomena determine the speed of propagation of the beam-front and concomitant well. It is extremely important to elucidate this question since a proper understanding may lead to a technique of controlling this method of collective acceleration so as to allow substantially higher ion energies to be achived.

On the one hand is a model developed as a result of computations by Olson [10] and by Poukey and Olson [11]. This model included refinements to the Poukey-Rostoker model, delineated the conditions for cessation of acceleration in the high-pressure condition (beam--propagation), was a two-dimensional model, and it seemed to explain the anomalously high energy observations by the P.I. group at that time. Basically the description of the beam-front potential-well propagation was in terms of a beam-front axial extent of some 2a, which leads to a front speed of $2a/\tau_i$, where τ_i is a characteristic ionization time proportional to gas pressure although it should also depend on current. The data at that time seemed to support this model - a low pressure region where E_i was constant and on average ~ 1.5 - 2 E_e, an intermediate pressure region where the average values of the PI energies did show a rise that was proportional to pressure and a high-pressure region where beam propagation occurred and no ion-acceleration was observed.

Recent work by Ecker and Putnam, however, has shown that at constant pressure (therefore constant energy in the Olson interpretation) they can obtain ions of a very wide variety of energies (2 - 14 MeV) by varying other conditions [12]. It is important to note that the P_I work, in contrast to that at other laboratories, has been with high ν/γ beams (0.5 - 1). Recently, in exploring an ever-wider range of parameters, Straw and Miller observed that at high ν/γ they also obtained a high-energy ion component ($E_p > 5\ E_e$) [13]. For ν/γ approaching unity Ecker and Putnam pointed out that the beam-front speed can be limited simply by a power balance condition. As the front propagates, energy in the self-magnetic-field must be supplied, work also is being done by the electrostatic well at the beam-front in accelerating secondary electrons to the walls; these processes clearly cannot proceed at a rate greater than the input supply of power I_eE_e. On this rather fundamental basis they derive the following expression for the beam-front (or ion) velocity, β_f:

$$\frac{1}{\beta_f} = \frac{geI_e}{E_e} + \frac{\kappa}{\beta_{11}} \qquad (5)$$

where $g = \frac{\mu_0 c}{16\pi}\left(1 + 4\ \ln\frac{a}{b}\right)$, $\kappa\ E_e$ is the mean kinetic energy supplied to each secondary electron (adjustable), and $\beta_{11}c$ is the injection velocity of the beam. In a variety of experiments, E_e was varied from 0.45 to 1.35 MeV, I_e from 32 to 150 kA, a from 3.8 cm to 6.9 cm. Their data are shown in Fig. 2 where the proton energy is plotted against $Z = E_e/I_e$, together with the predictions of Eq. 5 and of the Olson model (β_f = constant at constant pressure). The power-balance model appears to be convincingly borne out as an explanation for the P.I. data.

Where does this leave us? Needing more experiment, of course! At this time one can only state an opinion about what may be going on. At high ν/γ (≈ 1) the existence of a travelling well, in which ions can be trapped and accelerated, seems well-established and its velocity is properly described on the basis of power-balance. But what about Olson's prediction of a beam-front potential well propagating at a rate determined by the ionization rate? In my opinion this mechanism ought to be effective at lower ν/γ ($\lesssim 0.5$) and the fact that we have no satisfactory experimental verification may have to do with the subtleties of launching ions (perhaps from the precursor quasi-stationary well) so that they will be appropriately trapped. Olson has proposed to study ion acceleration by controlling the speed of the beam-front by external control of the ionization rate [14]. In such an Ionization-Front Accelerator (IFA), the ionization time τ_i at successive space locations would be determined by pulsed light sources so that the speed of propagation ($\simeq 2a/\tau_i$) could be adjusted to keep the desired ion-species in synchronism with the travelling space-charge potential well.

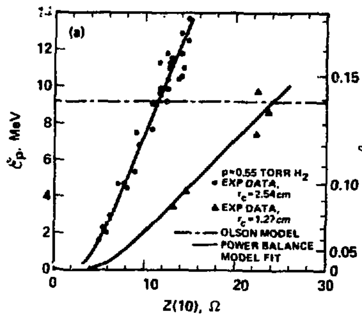

Fig. 2 The Recent data of Ref. 12.

4) Acceleration in Vacuum Diodes:

Luce's studies of the characteristics of ions accelerated in vacuum diodes constitute a tour de force in illustrating what one can achieve by experimental persistence [15]. Relying mainly on radiochemical diagnostic techniques he and his co-workers have painstakingly studied a wide variety of ion-species in diodes of many different geometric configurations. The basic experimental feature is the use of a pierced anode of insulating material such as polyethylene. Several cathode materials have been tried and considerable success achieved with pointed insulators. The energy source for the diode is a standard E-beam generator producing a few megavolts, and a few tens of kiloamperes. Most of the beam passes through the hole in the center of the anode but some electrons land on the insulated anode which charges up and, during the beam pulse, flashes over to ground. This discharge creates an anode plasma which is the source of the ions that are later accelerated by collective effects. That the anode material is the source of the accelerated ions has been clearly demonstrated by Luce and co-workers at Livermore - this is an important distinction between these experiments and those by Plyutto in which the ions were generated from the cathode. It was discovered empirically that placing a sequence of insulated co-axial rings (so-called "plasma lenses") downstream of the anode led both to an increase in ion-energy and a focussing effect upon the ion beam (to a focal spot of one-centimeter diameter and less). Unusually high energies and fluxes have been produced by this method, for example more than 10^{14} protons with individual energies of 45 MeV. Many of Luce's results have been confirmed by Zorn and co-workers at the University of Maryland [16].

No convincing explanation of the physics of the acceleration process has been given. Attempts have been made to formulate exotic plasma-vortex discriptions. It is possible that transient virtual-cathode formation occurs followed by propagation of the potential well as successive discharges occur first at the anode and later at the succeeding "plasma lenses", which events (if the spacing of the elements is correctly chosen) can lead to a trapping condition that would be essential for acceleration to the energies that have been observed.

Three points worth emphasing about these results are:

a) Energies have been achieved (45 MeV protons) in distances measured in centimeters compared with some tens of meters that would be needed in proton rf linacs - a gain by two orders of magnitude or more in rate of energy gain.

b) High-intensity per pulse (10^{14}) is observed - again a gain of some orders of magnitude.

c) A beginning to the ordered control of both optics and energy enhancement by placing of a succession of elements has been achieved. The electron-ring method has by its nature a built-in orderliness for extension once the basic principles are proven. Apart from this method, the vacuum-diode work is unique in having demonstrated some degree of iterative control of the acceleration process.

5) Auto-Resonant Acceleration:

This method which utilizes a travelling wave on an electron-beam was first proposed by Sloan and Drummond in 1973 [17]. In the meantime these authors and co-workers have made extensive theoretical studies of how a practical accelerator based on this method could be made to work. Recently experimental work has begun in connection with developing a suitable E-beam source.

Just as one can analyze the travelling-wave modes in a vacuum waveguide, so also one can analyze such modes in a pipe structure in which an electron beam is present - and hence $j \neq 0$, $\rho \neq 0$, away from the walls. Sloan and Drummond have analyzed such modes in the presence of an axial magnetic field and have identified one mode as having especial properties that make it suitable for ion-acceleration - the so-called lower Doppler-shifted cyclotron mode. This wave, corresponding to propagation of radius and charge - density modulations, and hence of a potential well that can serve as a vehicle for ion-acceleration, has a low phase-velocity given by:

$$v_{ph} = \frac{\gamma_p \omega_p}{\omega_0 + \Omega} \qquad (6)$$

where v_e = electron velocity, $\Omega = \frac{eB}{mc}$ is the cyclotron frequency, and ω_o is the initial frequency externally excited. Thus by tailoring the magnitude of the axial field as a function of distance, i.e. $B = B(z)$, one can arrange for the phase-velocity to increase in a prescribed way and accelerate bunches of ions while keeping them trapped in the potential well. An important feature of this particular mode is that it is a "negative energy" mode so that the transfer of energy to the ions leads to further growth in the wave amplitude; thus energy is pumped from the E-beam acting as a power source, into the ion beam.

There are practical difficulties to be overcome; one must have an E-beam (~ 3 MeV, 30 kA) of very low momentum spread, the right mode has to be excited by an external source and allowed to grow in a section of dissipative liner material, and unwanted modes must be suppressed. Most of these problems and others have been closely examined and several possible solutions identified. In an initial " proof-of-principle" experiment to take place over the next few years Drummond and his group hope to demonstrate acceleration of some tens of amperes of protons to 30 MeV in a flared magnetic field two or three meters in length.

6) Recent Proposals of New Methods for Ion Acceleration

There have been several recent suggestions of possible ways of exciting slow space-charge waves in electron beams and using such travelling waves for ion-acceleration. In a brief highly-idealized and non-relativistic treatment Yadavalli has enumerated four possible schemes [18]. These are based upon the dispersion relation for slow space-charge waves in an electron beam propagating in a pipe (see, for example, [19] which yields for the velocity, v_s, of the slow space-charge wave

$$v_s = \frac{v_e \omega_o}{\omega_o + F\omega_p}$$

where ω_p = plasma frequency and F is the so-called plasma-frequency reduction factor for a cylindrical beam in a pipe. Thus if one can excite a modulation, by means of an rf cavity, at an applied frequency ω_o and arrange for F to be a suitable function of distance then the phase-velocity of the potential well can be increased and ions accelerated. The space-charge reduction factor depends on both the radius of the beam and of the pipe and can be made smaller as the beam proceeds forward by converging the walls or by allowing the beam to expand. The other two suggestions by Yadavalli involve creating an extra degree of control of the phase velocity, v_s, by also introducing an axial dependence of the longitudinal speed of the electron beam, v_e. He points out that under conditions of Brillouin and Harris flow in which the beam particles have significant rotational velocity it may be possible to change the ratio of transverse velocity to longitudinal velocity by changing the magnetic field along the pipe, and so arrive at a programmed z-dependence of v_e.

Yadavalli is careful to point out that his first suggestion (convergent beam-pipe) was in fact the basis of a scheme previously published by Sprangle, Drobot and Manheimer [20]. In contrast to Yadavalli's rather simple treatment Sprangle et al had presented a more sophisitcated discussion and had given quantitative estimates of the performance of a "convergent wave-guide" accelerator, including such details as the expected electrical conversion efficiency to the ion-beam. Depending on how the non-linear saturation phenomena are found to behave it may be possible to achieve 0.5 amps of protons at 300 MeV in a length of 15 meters.

Finally, Miller has drawn attention to the possibility of using controlled motion of the potential well associated with a virtual cathode to effect a ram type of acceleration. Similarities to some of the schemes referred to above will be preceived because of the basic electrostatics underlying many of the considerations but there are some quantitative differences in Miller's approach. He points out that several of the quantities in the condition for virtual cathode formation, $I > I_{SCL}$, can be controlled as a function of axial position, z, and time, t. Rewriting Eqn. (2) to indicate explicit dependences, we have the condition for virtual cathode formation:

$$I(z,t) > I_{SCL}(z,t) = \frac{mc^3[\gamma^{2/3}(z,t) - 1]^{3/2}}{e[1+2\ln a(z,t)/b(z,t)][1-f_e(z,t)]}$$

One can identify five parameters that can be varied either singly or in combination to change the strength of the inequality which in turn will change the position of the virtual cathode in time. Some examples are the following: the beam pipe radius, viz. the cylindrical surface at ground potential, can be effectively altered in time by propagating along it a suitably-designed voltage pulse; a propagating constriction in the beam can effectively alter b(x,t); finally, external ionizers can change $f_e(z,t)$ in space and time (this can be recognized as the case of Olson's I.F.A.). As an example of changing combinations of parameters, Miller discusses the case of a ramped current I(z,t) injected into a drift tube with flared radius a(z,t).

This proposed ramming action of the virtual cathode should work to some degree in enhancing the energy of collectively-accelerated ions but probably not by a very large factor. It must be noted, however, that this approach is based upon creating a deep potential-well in an established fashion and then moving it in a controlled way; in one sense, it is a slow space-charge wave. In another it is in sharp contrast with the usual approach of starting with an externally imposed perturbation to generate a small space charge wave which, it is expected, will grow to a large value at the same time avoiding growth of unwanted modes.

Conclusion

Understanding of collective effect acceleration has come a long way since the last Proton Linac Conference was held. There seems still a long way to go before one can think of supplanting high energy synchrotrons as machines at the frontier of high-energy physics. Nonetheless, there is an intermediate energy region of a few hundred

MeV per nucleon which could benefit greatly from use of collective-effect acceleration - particularly for heavy ions - if one can learn how to extend and control acceleration over a longer length. It should be pointed out, however, that at low energies, i.e. tens of MeV which have already been achieved, a collective accelerator may offer a suitable solution to special injection problems. In high current conventional machines there is trouble handling the self space-charge field of the beam after the column, viz., at the 1 MeV point. To have a source of protons or ions that is already at tens of MeV before the self-field problems have to be contended with, could be a great benefit.

References

1) L. J. Laslett: IEEE Trans. Nucl. Sci. NS-20, 271 (1973).

2) D. Keefe: Proc. IVth All-Union Conf. on Particle Accelerators (Nov. 1974) Nauka, Moscow 1975, Vol. I, p. 109.

3) M. Reiser: IEEE Trans. Nucl. Sci. NS-20, 310 (1973).

4) U. Schumacher, C. Andelfinger, M. Ulrich: IEEE Trans, Nucl. Sci., NS-22, 989 (1975).

5) C. Andelfinger, E. Buchelt, W. Dommaschk, J. Fink, W. Hermann, I. Hofmann, D. Jacobi, P. Merkel, H. B. Schilling, A. Schluter, U. Schumacher, M. Ulrich: Max-Planck Institute for Plasma Physics Internal Report, IPP, 0/30 (January 1976).

6) L. P. Smith and P. L. Hartman: Jour. App. Phys. 11, 220 (1940).

7) J. W. Poukey and N. Rostoker: Plasma Physics 13, 897 (1971).

8) D. C. Straw and R. B. Miller: App. Phys. Lett. 25, 379 (1974); IEEE Trans. Nucl. Sci. NS-22, 1022 (1975).

9) J. Rander, B. Ecker, G. Yonas, D. J. Drickey: Phys. Rev. Lett. 24 283 (1970); J. Rander: Phys. Rev. Lett. 25, 893 (1970).

10) C. L. Olson: Phys. Rev. A 11, 288 (1975); Phys. Fluids 18, 585 (1975).

11) J. W. Poukey and C. L. Olson: Phys. Rev. A 11, 691 (1975).

12) B. Ecker and S. Putnam: To be presented at the Symposium on Collective Methods of Acceleration, Dubna (USSR), Oct. 1976.

13) D. C. Straw and R. B. Miller: Jour. Appl. Phys. (to be published)

14) C. L. Olson: Proc. Int. Top. Conf. on Electron Beam Research and Technology, Albuquerque, N. M. (Nov. 1975) Vol. II, 312.

15) J. S. Luce: Lawrence Livermore Laboratory Report UCRL 75933 (1974).

16) G. T. Zorn, H. Kim, C. N. Boyer: IEEE Trans. Nucl. Sci. NS-22, 1006 (1975).

17) M. L. Sloan and W. E. Drummond: Phys. Rev. Lett. 31, 1234 (1974).

18) S. V. Yadavalli: Appl. Phys. Lett. 29, 272 (1976).

19) A. H. W. Beck: Space-Charge Waves (Pergamon Press, New York, 1958) Chap. 4.

20) P. Sprangle, A. T. Drobot, W. M. Manheimer: Phys. Rev. Lett. 36, 1180 (1976).