université paris sud
INSTITUT DE PHYSIQUE NUCLEAIRE
BP N° 1 - 91406 ORSAY TEL. 941.51.10
laboratoire associé à l'IN2P3

MASS EXCHANGE AND ANGULAR DISTRIBUTION IN A DYNAMICAL TREATMENT OF HEAVY ION COLLISIONS

Christian NGÔ

and

Helmut HOFMANN

IPNO-RC-77-01

Mass exchange and angular distribution in a dynamical treatment of heavy ion collisions

Christian NGÔ

Institut de Physique Nucléaire, Laboratoire de Chimie Nucléaire,
BP n°1, 91406 - Orsay, France.

and

Helmut HOFMANN

Physik Department der technischen Universität München,
teilinstitut theorie, D 8046 Garching, B.R.D.

Abstract.

We present a first numerical computation of the absolute value of the double differential cross section as a function of mass asymmetry and detection angle including a dynamical coupling between relative motion and mass asymmetry. We apply it to the $^{63}Cu+^{197}Au$ experiment at two different energies. The equation of motion used is a Fokker Planck equation for distribution function in classical phase space. The coefficients needed are those known from classical model calculations, besides a friction coefficient introduced for the mass asymmetry degree. We find encouraging agreement between the calculated and experimental curves.

1. Introduction.

Deep inelastic heavy ion collisions[1] are commonly described in terms of a few collective degrees of freedom. Their final values which determine the cross section are obtained by solving some kind of equation of motion. All the other degrees of freedom (referred to as intrinsic degrees) are only assumed to be involved in the process in an indirect way. Different from the conventional collective model[2], the intrinsic degrees are supposed to be statistically excited and hence absorb energy irreversibly. For the collective degrees, usually quantum effects, can be and are neglected. For a justification of this picture, we refer to the literature[3-18]. It is almost obvious that the coupling among the relevant collective degrees of freedom should be very important. Thus the relative motion, for instance, will influence the mass exchange as well as shape vibrations and vice versa. It is the main purpose of this peper to present a first numerical treatment of the dynamics of the relative motion and the mass exchange, including their statistical fluctuations. For this we apply the theory of ref.15). Its basic assumptions are mentioned and briefly discussed in section 2. The Fokker Planck equation of ref.15) is correct only for a cartesian coordinats system. In a curvilinear system there arise additional terms as will be shortly explained in section 3. These terms have already been used in ref.17) in which the application was restricted to the relative motion. The specific model within which the numerical application is performed shall be discussed in section 4. The results will be given and commented on in section 5.

2. Basic assumptions.

For the theory of ref.15) to apply, the following assumptions must be fulfilled :

a) A clear separation between slow degrees and fast one must be possible.

b) The latter ones (intrinsic degrees) are then treated as if they were in a statistical equilibrium at all times.

c) The concept of classical trajectories must be meaningful.

d) The magnitude of the locally defined collective frequency should not be too big compared with the temperature ($\omega_{coll} \lesssim 3.5\ T$).

Let us briefly comment on these assumptions. Condition a) is necessary not only for b) to hold, but also that the definition of a local friction force is possible. This has been already strengthened in ref.12). As a measure for the intrinsic time scale, we may use the relaxation time τ as defined in ref.12,15). It is the time after which the response of the intrinsic system to an external perturbation has diminished. In ref.16), τ was computed for a typical case. A value of 2.10^{-22}s was found, which is smaller than the typical total reaction time by about one fifth or less. The model used assumes that the intrinsic structure can mainly be described by a shell model hamiltonian. In the work of ref.16) (which is based on the random matrix model), residual interactions are taken into account implicitly. If, for the matrix elements which are relevant to define the internal response, we assume the form and the values of the parameters as given in ref.14,16), τ appears to be smaller by at least a factor of two.

Condition b) being probably the strongest of all is also basic for the work of Agassi, Ko and Weidenmüller[16]. Different to their using a microcanonical ensemble, in ref.15), the formal derivation is performed for a canonical ensemble. This implies the use of a time dependant temperature. It is defined by the intrinsic system and is to be computed from the mean value of the intrinsic excitation energy as given by the irreversible loss of collective energy. The use of the concept of a temperature is not necessarily in contradiction with the fact that the reacting nuclei conform an isolated body. The reason for this is that we are interested in the statistical distribution of the collective degrees only which can be thought as a subsystem of the total system. For a discussion of this point we refer to text books on statistical mechanics[19] where the case of complete statistical equilibrium is discussed. For deep inelastic reactions, the situation differs only in that the distribution of the collective degrees we are going to calculate will not be given by a thermal equilibrium.

Condition c) involves that the fluctuations in those coordinates for which the conservative force is not linear, should remain small (cf. the text between eqs(92) and (98) of ref.15); see also below). This is certainly the case for the relative distance r. Indeed the computation presented in ref.17) has shown that for physically reasonable trajectories the fluctuations remain fairly small (see also below).

The last condition is necessary for the Fokker Planck equation to have a simple form[15]. A release of this assumption would imply the appearance of coefficients not present in the newtonian equations. In this paper, we want to avoid this complication. Moreover, in the opposite case $\omega_{coll} >> T$, quantal fluctuations might be important.

The conditions of slow collective motion and low temperature (i.e. relatively small intrinsic excitation energy) are most likely to be invalid[18] at the beginning of the reaction. This stage should not be treated by statistical methods. For the comparison of our computation with the experimental results, we hope the influence of this initial stage on the final results to be not very important. For the mass exchange process at least this should be a fair assumption.

3. Formal preludes.

a. Equations of motion.

The basic quantity from which the cross sections will be deduced is the distribution function $d(Q^\mu, P_\mu, t)$ for the collective variables Q^μ and their conjugate momentum P_μ. It is the solution of a Fokker Planck equation (FPE) like the one derived in ref.15) (see eq.(96)). Under the assumptions mentioned in section 2, this FPE is almost linear in P_μ and Q^μ. This implies that d is a gaussian entirely determined by its first and second moments :

$$Q^\mu_c(t) = \int d\Gamma\, Q^\mu\, d(t) \qquad (1)$$

$$P^c_\mu(t) = \int d\Gamma\, P_\mu\, d(t) \qquad (2)$$

$$\omega_{\mu\nu}(t) = \frac{1}{2} \int d\Gamma (P_\nu - P^c_\nu(t))(P_\mu - P^c_\mu(t))\, d(t) \qquad (3)$$

$$\chi^{\mu\nu}(t) = \frac{1}{2} \int d\Gamma (Q^\mu - Q^\mu_c(t))(Q^\nu - Q^\nu_c(t))\, d(t) \qquad (4)$$

$$\phi_\mu{}^\nu(t) = \frac{1}{2} \int d\Gamma (P_\mu - P^c_\mu(t))(Q^\nu - Q^\nu_c(t))\, d(t) \qquad (5)$$

with $d\Gamma = \prod_{\mu\nu} dP_\mu\, dQ^\nu$ being the volume element in classical phase space.

The first moments, $Q^\mu_c(t)$ and $P^c_\mu(t)$, are solutions of classical equations of motion for the trajectories like the ones used in phenomenological descriptions of deep inelastic reactions[4-8]. The second moments $\omega_{\mu\nu}$, $\chi^{\mu\nu}$

and $\psi_\mu^{\ \nu}$ can be found as solutions of first order differential equations. They have been derived in ref.15) for the case where the inertial tensor is given by a constant times the unity matrix. For practical applications it is, however, convenient to use curvilinear coordinates. In the following, we shall, for instance, describe the relative motion in terms of the polar coordinates r and θ. We shall however not include those effects of the nucleus nucleus interaction which will make the reduced mass μ r-dependent. That is to say, we neglect the last term of the FPE (9S) of ref.15) (in the notation of ref.15), they are proportional to $\frac{m_{\mu\nu}}{\mu}$).

In the following, all the equations will be written in covariant form and we shall make use of Einstein's summation convention. The covariant (contravariant) form of the inertial tensor for instance will be denoted by $m_{k\ell}$ ($m^{k\ell}$) . Then the equations of motion for the moments are given by :

$$\frac{dQ_c^\mu}{dt}(t) = m_c^{\mu\nu} P_\nu^c(t) \tag{6}$$

$$\frac{dP_\mu^c}{dt}(t) = -\frac{\partial m_c^{\nu\eta}}{\partial Q_c^\mu(t)} P_\nu^c(t)\, P_\eta^c(t) - \frac{\partial U(Q_c^\nu)}{\partial Q_c^\mu} - \gamma_{\mu\nu}^c \frac{dQ_c^\nu(t)}{dt} \tag{7}$$

$$\begin{aligned}\frac{d\omega_{\alpha\beta}}{dt} = &- P_\mu^c \left(\frac{\partial m_c^{\eta\mu}}{\partial Q_c^\alpha}\right)\omega_{\beta\eta} - P_\mu^c \left(\frac{\partial m_c^{\eta\mu}}{\partial Q_c^\beta}\right)\omega_{\alpha\eta} - P_\mu^c P_\nu^c \left(\frac{\partial^2 m_c^{\mu\nu}}{\partial Q_c^\eta \partial Q_c^\xi}\right)\left(\delta_\alpha^\xi\, \psi_\beta^\eta + \delta_\beta^\xi\, \psi_\alpha^\eta\right) \\ &- C_{\alpha\eta}^c\, \psi_\beta^\eta - C_{\beta\eta}^c\, \psi_\alpha^\eta - \gamma_{\alpha\nu}^c\, m_c^{\eta\nu}\, \omega_{\eta\beta} - \gamma_{\beta\nu}^c\, m_c^{\eta\nu}\, \omega_{\eta\alpha} - \gamma_{\alpha\nu}^c P_\eta^c \left(\frac{\partial m_c^{\eta\nu}}{\partial Q_c^\xi}\right)\psi_\beta^\xi \\ &- \gamma_{\beta\nu}^c P_\eta^c \left(\frac{\partial m_c^{\eta\nu}}{\partial Q_c^\xi}\right)\psi_\alpha^\xi + \gamma_{\alpha\beta}^c\, T(t)\end{aligned} \tag{8}$$

$$\frac{d\chi^{\alpha\beta}}{dt} = m_c^{\mu\alpha}\, \psi_\mu^\beta + m_c^{\mu\beta}\, \psi_\mu^\alpha + P_\mu^c \left(\frac{\partial m_c^{\alpha\mu}}{\partial Q_c^\xi}\right)\chi^{\xi\beta} + P_\mu^c \left(\frac{\partial m_c^{\beta\mu}}{\partial Q_c^\xi}\right)\chi^{\xi\alpha} \tag{9}$$

$$\begin{aligned}\frac{d\psi_c^\beta}{dt} = &\; m^{\mu\beta}\, \omega_{\mu\alpha} + P_\mu^c \left(\frac{\partial m^{\beta\mu}}{\partial Q_c^\xi}\right)\psi_\alpha^\xi - P_\mu^c \left(\frac{\partial_{\eta_1}\eta\mu}{\partial Q_c^\alpha}\right)\psi_\eta^\beta - P_\mu^c P_\nu^c \left(\frac{\partial^2 m^{\mu\nu}}{\partial Q_c^\eta \partial Q_c^\alpha}\right)\chi^{\eta\beta} \\ &- C_{\alpha\eta}\, \chi^{\eta\beta} - \gamma_{\alpha\nu}\, m^{\eta\nu}\, \psi_\eta^\beta - \gamma_{\alpha\nu} P_\eta^c \left(\frac{\partial m^{\eta\nu}}{\partial Q_c^\xi}\right)\chi_\beta^\xi\end{aligned} \tag{10}$$

Here, $U(Q_c^\nu)$ is the interaction potential between the two heavy ions. $\gamma^c_{\mu\nu} = \gamma_{\mu\nu}(Q_c^\eta)$ and $C^c_{\mu\nu} = C_{\mu\nu}(Q_c^\eta)$ represent friction and stiffness tensor, respectively, calculated along the classical path and T(t) is the temperature at time t.

In writing eqs.(6) to (10), we anticipated for $d(Q^\mu, P_\mu, t)$ to have a gaussian form also for the curvilinear coordinate system. To achieve this, a manipulation for the FPE is necessary, which is not included in the derivation of ref.15). If the FPE (86) of ref.15) is transformed to a curvilinear system, the factors $\frac{P_\mu}{\mu}$ have to be replaced by $P_\eta\, m^{\eta\mu}(Q^\nu)$. The problem arises from the fact that the inertial coefficients will depend on the coordinates Q^μ. It can be overcome in the same way as the potential energy $U(Q^\nu)$ has been treated in ref.15) (see the discussion between eq(92) and (96)). To make the analogy complete, we relate $P_\eta\, m^{\eta\mu}(Q^\nu)$ to the kinetic energy using the definition of the Poisson brackets :

$$P_\eta\, m^{\eta\mu} = \left\{ \frac{1}{2} P_\eta\, P_\xi\, m^{\eta\xi}, Q^\mu \right\} = \left\{ E_{kin}, Q^\mu \right\} \qquad (11)$$

Within this Poisson bracket we only have to expand E_{kin} around the classical path up to second order in $P_\mu - P^c_\mu$ and $Q^\mu - Q^\mu_c$:

$$\begin{aligned} 2\, E_{kin} &= P^c_\eta\, P^c_\xi\, m^{\eta\xi}_c + 2\, P^c_\eta (P_\xi - P^c_\xi)\, m^{\eta\xi}_c + P^c_\eta\, P^c_\xi\, (Q^\mu - Q^\mu_c)\, \frac{\partial m^{\eta\xi}_c}{\partial Q^\mu_c} \\ &+ \frac{1}{2} (P_\eta - P^c_\eta)(P_\xi - P^c_\xi)\, m^{\eta\xi}_c + \frac{P^c_\eta P^c_\xi}{2} (Q^\mu - Q^\mu_c)(Q^\nu - Q^\nu_c)\, \frac{\partial^2 m^{\eta\xi}_c}{\partial Q^\mu_c \partial Q^\nu_c} \\ &+ (P_\eta - P^c_\eta)(Q^\mu - Q^\mu_c)\, P^c_\xi\, \frac{\partial m^{\eta\xi}_c}{\partial Q^\mu_c} \end{aligned} \qquad (12)$$

b. Analytic form of the distribution function in phase space.

Within the approximations described in the last section, the form of d(t) is the same as the one given in ref.15) (eq(107) and (108))

$$d(t) = (4\pi)^{-n}\, \Delta^{-1/2} \exp\left\{ -\frac{1}{4} \sum_{i,k} (\chi_i - \chi^c_i(t))(\chi_k - \chi^c_k(t))\, A_{ik}(t) \right\} \qquad (13)$$

with $$X_i = \begin{pmatrix} P_\mu \\ Q^\mu \end{pmatrix}, \quad [A^{-1}]_{ik} = \begin{pmatrix} \omega_{\mu\nu} & \psi_\mu^\nu \\ \psi_\mu^\nu & \chi^{\mu\nu} \end{pmatrix} \tag{14}$$

$i, k = 1, 2 \ldots, 2n$; $\mu, \nu = 1, 2 \ldots, n$ and $\Delta^{-1} = \det (A_{ik})$

This solution is normalized in the following sense :

$$\int_{-\infty}^{\infty} \prod_\mu dQ^\mu \int_{-\infty}^{+\infty} \prod_\nu dP_\mu \, d(Q^\mu, P_\mu, t) = 1 \tag{15}$$

In order to avoid unnecessary confusion, two remarks should be made here. The first one concerns the limits of integration in eq(15). They are choosen to be $-\infty$ and $+\infty$. Usually curvilinear coordinates extend over finite intervals only, so the integration used in (15) makes sense only if the distance from the center of the distribution to the ends of the interval is smaller than the width of the distribution which in our case is always fulfilled.

Secondly, we want to note that a transformation to a new coordinate system with $Q^\mu = Q^\mu(\chi^k)$ should only be performed to lowest order

$$Q^\mu - Q_c^\mu(t) \simeq \left(\chi^k - \chi_c^k(t)\right) \frac{\partial Q_c^\mu}{\partial \chi_c^k} (\chi_c^k) \tag{16}$$

Then the solution will be gaussian in the new system too. It is only in this sense that the second moments (see the definitions (3),(4) and (5)) behave like tensors (see above). The fact that the transformation matrix $\frac{\partial Q_c^\mu}{\partial \chi_c^k}$ is to be taken at its classical value implies the same for the functional determinant defining the volume element.

c. Cross sections.

From the solution (13), we can compute cross sections for any physically observable quantity which can be expressed as functions of the collective degrees taken into account. For this we need the limiting value of the distribution function (13) for $t \to \infty$ and we simply have to integrate over the remaining degrees and the impact parameter. Let us briefly describe

this for two cases. In the following, we shall mainly be interested in the cross section $\frac{d^2\sigma}{d\theta dx}$, where θ is the scattering angle and x is the mass asymmetry defined as $x = \frac{A_2 - A_1}{A_1 + A_2}$, with $A_{1,2}$ being the mass of nucleus 1,2. We shall furthermore restrict ourselves to a model in which r , θ and x are the only coordinates taken into account. Then the double differential cross section will be given by :

$$\frac{d^2\sigma}{d\theta dx} = \int 2\pi b db \int dr \int dP_\theta\, dP_r\, dP_x\; \omega(r,\theta,x,P_r,P_\theta,P_x,t\to\infty) \qquad (17)$$

This formula can easily be derived from the definition of the cross section, as given in any textbook on classical mechanics. It should be noted that the volume element taken for polar coordinates is drdθ instead of rdrdθ (see discussion in paragraph b). The reason is that for d we take the gaussian form in curvilinear coordinates (this should not be confused with formula (1) of ref.17)). There d is meant to be a function of cartesian coordinates ; compare however formula (2) of ref.17) with eq(19) below .

In formula (17) the last four integrations can easily be performed. According to our discussion in paragraph b , the remaining function of θ and x will be a gaussian centered at $x_c(t)$ and $\theta_c(t)$ with the widths $\chi^{\theta\theta}$, $\chi^{\theta x}$ and χ^{xx} . Thus $\frac{d^2\sigma}{d\theta dx}$ will be given by :

$$\frac{d^2\sigma}{d\theta dx} = \int 2\pi b db \lim_{t\to\infty} \frac{1}{4\pi\sqrt{\Delta(t)}} \cdot \exp\left\{-\frac{1}{4\Delta(t)}\left[(\theta-\theta_c(t))^2\,\chi^{xx}(t) - 2(\theta-\theta_c(t))\,(x-x_c(t))\,\chi^{\theta x}(t) + (x-x_c(t))^2\,\chi^{\theta\theta}(t)\right]\right\} \qquad (18)$$

with $$\Delta(t) = \chi^{xx}(t)\,\chi^{\theta\theta}(t) - (\chi^{x\theta}(t))^2$$

The integration over the impact parameter has to be understood in the following sense : as the solutions of eqs(6) to (10), the first and second moments depend on the initial condition and thus on b.

The cross section for the angular distribution, $\frac{d\sigma}{d\theta}$ is easily obtained from eq(18) by integrating over x. This leads to the form :

$$\frac{d\sigma}{d\theta} = \int 2\pi b db \lim_{t\to\infty} \frac{\exp\left\{-\frac{(\theta-\theta^c(t))^2}{4\chi^{\theta\theta}(t)}\right\}}{\sqrt{4\pi\chi^{\theta\theta}(t)}} \qquad (19)$$

Formula (19) has already been used in ref.17). There, it has been shown also that in the limit of a negligibly small width, (19) reduces to the limit :

$$\frac{d\sigma}{d\theta} = \sum_n 2\pi b_n \frac{1}{\left|\frac{d\theta^c}{db_n}\right|} \qquad (20)$$

known from classical mechanics.

4. Practical application of the model.

We performed numerical calculations for the system Cu+Au at two different bombarding energies 365 MeV and 443 MeV in the laboratory system. Experimental studies of this system were performed and discussed extensively in ref.13,20).

a. Model with no mass transfer.

Although, in principle, the friction tensor $\gamma^c_{\mu\nu}$, the mass tensor $m^{\mu\nu}_c$ and the stiffness tensor $C^c_{\mu\nu}$ can be calculated microscopically using formulas of ref.12,15), we have not performed such a calculation but proceeded as follows :

- For sake of simplicity, we restricted ourselves to only two degrees of freedom for the relative motion : r and θ. The first moments (Q^μ_c and P^c_μ) are calculated from a phenomenological model as those proposed in ref.4-8].We have choosen the model developped by Gross et al.[4] with a frictional tensor diagonal in the system of polar coordinates ($\gamma^c_{rr} = C_r f(r)$, $\gamma^c_{\theta\theta} = C_t r^2 f(r)$]. Instead of taking the Krappe and Nix potential[21] as in ref.4] for the nuclear interaction potential $U_N(r)$, we used the energy density potential of ref.22) which can be written in the simple analytical form :

$$U_N(r) = \bar{V}\ \frac{A_1^{1/3} A_2^{1/3}}{A_1^{1/3}+A_2^{1/3}}\ \exp(-.27s^2) \quad \text{for } s > 0$$

and

$$U_N(r) = (-\bar{V} + 6.3\ s^2)\ \frac{A_1^{1/3} A_2^{1/3}}{A_1^{1/3}+A_2^{1/3}} \quad \text{for } s \leq 0 \qquad (21)$$

with

$$s = r - \bar{r}(A_1^{1/3}+A_2^{1/3})$$

As proposed in ref.22), $\tilde{r} = 1$ fm and $\tilde{V} = -30$ MeV. For the friction force, the constant c_r and c_t as well as the form, factor f(r) have been taken to be those of ref.4) : $f(r) = (\frac{\partial LN}{\partial r})^4$. Compared to the model using the Krappe and Nix potential, the use of the energy density potential gives similar results. The energy loss is however a bit larger and the deflection function for the low energy system is closer to the one expected experimentally. However this model gives a too large fusion cross section compared to experiment. This is also observed for the same model using the folded potential. Despite this fact, it will give us at least some qualitative estimate. It should however be kept in mind that this system is one of the few cases where the model of Gross et al[4] does not reproduce well the fusion cross section and that for other systems one should expect a better agreement.

b. Inclusion of mass transfer.

To keep this first application simple enough we make the (possibly drastic) assumption that the relative motion and mass exchange are statistically uncorrelated. This means, we suppose the total FPE to be separable, i.e. the total solution to be given by :

$$d(r,\theta,x,P_r,P_\theta,P_x,t) = d_r(r,\theta,P_r,P_\theta,t) \cdot d_x(x,P_x,t) \qquad (22)$$

(In this case $\chi^{x\theta} = \chi^{\theta x}$ appearing in eq(18) is equal to zero). This is equivalent to neglecting in the FPE direct coupling terms of the kind $\sim (x - x_c(t)) \frac{\partial}{\partial P_i}$ ($i \equiv r$ or θ) etc... However we do not neglect the coupling between mass exchange and relative motion completely. The equations for the first moments, i.e. for the classical values $Q_c^\mu(t)$, are coupled to each other. We hope that in this way the main effect is taken into account to a fair approximation, namely the influence of the mass exchange on the relative motion which so far, has not been possible to be studied in other models. At least for the Cu+Au system, studied here, the decoupling (22) could be reasonnable if we notice that the experiment indicates that the collective relaxation time connected with the mass asymmetry degree of freedom is much larger than the one connected with the kinetic energy in the relative motion[13,20].

The next model assumption will be to neglect the acceleration term for the x-degree, i.e. the mean value of x shall fulfill the equation :

$$\gamma^{c}_{xx}\,\frac{dx^{c}}{dt} = -\frac{\partial U^{c}}{\partial x} \qquad (23)$$

In other words we assume the x-motion to be overdamped. As an indication for this to be fulfilled one might take an observed experiment fact: there are no oscillations of the position of the maximum of the mass distribution around the equilibrium value as a function of the detection angle (which can be taken as a measure of the collision time). One should however notice that this could also be explained in the opposite case of strongly underdamped motion: if the period for an x-oscillation is much bigger than the total reaction time. A final answer can only be given after computing the forces microscopically, especially the inertia and friction coefficient for the x-degree.

In the case where (23) is a good approximation to the Newton equations, the distribution function in phase space should be rather independent to the momentum P_x. It has been shown by Kremers[23] how the FPE for $d_x(x,P_x,t)$ can be reduced to a simpler one of Smoluchowski type for a distribution function $\sigma(x,t)$. For the form of this equation, we refer to eq(73) of ref.15) and we only write explicitly the equation for the moment $\chi^{xx}(t)$:

$$\gamma^{c}_{xx}\,\frac{d\chi^{xx}(t)}{dt} = -2\,\frac{\partial^{2}U^{c}}{\partial x^{2}}\,\chi^{xx}(t) + T(t) \qquad (24)$$

(It is easily deduced from eq(73) of ref.15)). We should note that the Smoluchowski equation for $\sigma(x,t)$ has already been used in ref.9,13).

We shall now briefly explain the choice of the quantities which appears in eq(23) and (24). The potential $U^{c} = U(r^{c},P^{c}_{\theta},x^{c})$ represents the sum of the nuclear (interaction potential plus liquid drop energy), the coulomb and the centrifugal potential. The friction coefficient γ^{c}_{xx} is choosen to be a step function: we took, $\gamma^{c}_{xx} = 40000$ MeV 10^{-23}s (value which gives a reasonable order of magnitude for the mass transfer) as soon as the energy loss is equal to 2 MeV. Of course, it is a drastic simplification because γ^{c}_{xx} certainly depends on the size of the neck of the composite system. We hope, however, that this simple ansatz will give us a qualitative insight into the problem.

The temperature of the intrinsic degrees T(t) depends on the time. It has been calculated from the excitation energy E_{int} using a Fermi gas model.

It is given by the relation :

$$a\,T^2(t) = E_{int}(t) \qquad (25)$$

with the level density parameter $a = \frac{A_1+A_2}{10}$. It should be mentioned that this parameter has only a small effect on the results which are mainly dominated by the collision time.

We finally need a connection between the mass and the atomic number of the fragments. We use the fact that in deep inelastic reactions the charge equilibration is very fast[24]. Consequently, we assumed that the charge equilibrium is reached at any time after which the mass transfer starts (that is to say this degree of freedom is treated as an internal one). The ratio between the number of neutrons and the number of protons $\frac{N_1}{Z_1}$, $\frac{N_2}{Z_2}$ in each nucleus has been taken to be the value which minimizes the potential energy of two tangents spheres with a given number of nucleons[25]. For the light fragment for instance, the value obtained using this method differs by about 1 unit of charge with the value calculated assuming that the equilibrium charge corresponds to that of the compound nucleus $(\frac{N_1+N_2}{Z_1+Z_2})$.

5. Results.

a. Double differential cross section $\frac{d^2\sigma}{d\theta dx}$.

We shall mainly discuss the results for the low energy system because they are similar for the high energy system.

On fig.1 is displayed the computed contour plot of $\frac{d^2\sigma}{d\theta dM}$ in mb/a.m.u/rd as a function of the mass and of the center of mass angle for the quasi fission products. We can observe the following points :

- As the detection angle decreases from the grazing value which is around 110°, the position of maximum of the mass distribution is shifted towards larger masses and, at the same time, its width increases. This can be seen more easily on fig.2 showing different cuts corresponding to different angles θ_{cm}.

- Beckwerd the grazing angle, quasi fission products are also observed but the differential cross section decreases more rapidly as a function of θ_{cm} than for forwards angles. Note than in a pure classical picture, these

products should not be observed. In our theory, they appear in a natural way as a result of the fluctuations.

Fig.3 corresponds to cuts in fig.1 corresponding to given masses. The main point is that the angular distributions of the products broaden as the mass transfer increases and becomes flatter and flatter. At the same time the position of the maximum is shifted towards smaller angles.

Let us now compare these results with the experimental data of ref.13,20). In ref.20), the angular distributions of the quasi fission products have been measured for different slice of mass. We refer the reader to ref.20) for this figure. We can observe that all the trends we mention above are reproduced. In addition, the order of magnitude of the cross section is reproduced.

The mass distributions displayed in ref 13,20) are those measured in the laboratory system. They have to be transformed to the center of mass system before comparison with the calculated one. For a given laboratory angle, the different masses do not correspond to the same center of mass angle. Therefore we proceeded as follows : for each angle and each mass we have calculated the corresponding center of mass angle as well as the center of mass double differential cross section. This leads to fig.4 which has to be compared with fig.1. We see that in both cases the pattern looks very similar.

Cuts in fig.4 give the mass distributions displayed on fig.5 which should be compared to fig.2. Due to the low statistic, errors on the experimental mass distributions are of the order of 30-40%. Also here we observe a good agreement between the calculated curves and the experimental ones. One should particularly notice that the asymmetry of the mass distributions on the right, towards large masses, is reproduced. This asymmetry is due to the dynamics which is now conjointly introduced in the Smoluchowski equation. We want to draw attention also to the fair agreement of the absolute values of the cross sections for a large part of the curves. Note that no renormalization has been performed.

On fig.6 is displayed a similar curve as fig.1 for the high energy system. In that case one gets a non negligible contribution from negative angles. We also observe a small amount of cross section for masses greater

than 130 a.m.u which corresponds to symmetry. This means that for masses between 120 and 130 a.m.u, one gets contributions from distributions centered at masses between 130 and 140 a.m.u. This contribution is simply the one plotted here above 130 a.m.u. It should be added at the supplementary angle.

b. Mass integrated angular distribution $\frac{d\sigma}{d\theta}$

On fig.7-8 are displayed the angular distribution of the quasi fission and fission following complete fusion products for both energies. Since the cross section above can only account for quasi fission, we added the calculated angular distribution for fission following complete fusion. For the latter we have assumed the usual $\frac{1}{\sin\theta}$ form for $(\frac{d\sigma}{d\Omega})_{fission}$. The dashed curve has been calculated without mass transfer. The full curve corresponds to a calculation including mass transfer. In both cases, the fusion cross section is increased when mass transfer is included. This is essentially due to the charge equilibration we introduce before the mass transfer occurs. As a matter of fact, the product Z_1Z_2 decreases when the charge equilibration is performed, the mass of the ions remaining fixed. Consequently, as the nuclear force remains almost the same, the coulomb force which is decreased, makes the fusion cross section to be increased. The change in the width is essentially due to the dynamics.

In both cases the experiment is qualitatively reproduced. We should particularly note that in the contrary to a pure classical model, $\frac{d\sigma}{d\theta}$ is finite at the rainbow angle[17]. The calculated FWHM is however too small. The discrepancy can come from the following points :

α - The calculation has been performed for the primary products. These products, due to their excitation energy, will sequentially decay to final products which one really measures. Consequently, all the final distributions are broadened compared to the initial ones.

β - Further reasons may origin from deficiencies of the special model used in our computation (see below).

c. Discussion.

We have seen above that most of the characteristic features found experimentally can be reproduced in our model. In particular, this holds for

phenomena which cannot be accounted for in a purely classical picture. Many of the discrepancies left may be due to deficiencies of the special model which could be improved without major difficulties. Indeed, we did not try to readjust the parameters for the forces (especially the friction forces) after having included the x degree of freedom. For the $\vec{r}$ motion, for instance, we took the values given in ref.4) which were adjusted for a two-dimensionnal model only. Furthermore, in our model, not all the relevent degrees are taken into account, as for instance neck formation (cf.ref.18)) and deformations of the fragments. It has been already stressed in ref.8) that the inclusion of deformations implies an increase in the total reaction time. On the other hand, the computations showed that the widths of the distribution function depend strongly on time.

From a comparison of fig.1, we see that for angles around the grazing angle the maximum of the theoretical mass distribution is shifted too much whereas for small angles the shift is not big enough. This should mainly be an effect of a poor parametrization of the forces which define the trajectories. Furthermore, it was not possible to obtain the correlation between the position of the maximum and the square of the width of the mass distribution observed in ref.13).

This width, of course, is directly connected to the width of our distribution d(t) for each trajectory. As we have seen above, the latter depends on the reaction time which will be changed after including deformations. In addition, the choice of a step function for Y_{xx}^{c} and also, possibly the neglect of the correlation $\chi^{x\theta}$ may be too drastic.

We are grateful to Pr. Marc LEFORT for discussions and careful reading of the manuscript.

References

1) We refer the reader to the following review papers where references to the experimental works can be found :
J. Galin, European Conference on Nuclear Physics with Heavy Ions, Caen (1976) J. Phys. C5 37 (1976) 83.
M. Lefort, Rep. Prog. Phys. (1976) 39 129-173.
L.G. Moretto and R. Schmitt, European Conference on Nuclear Physics with Heavy Ions, Caen (1976) J. Phys. C5 37 (1976) 109.

2) A. Bohr and B.R. Mottelson, Nuclear Structure vol.II W.A. Benjamin, INC (1975) .

3) W.J. Swiatecki, J. de Phys. 33 (1972) 45.
W.J. Swiatecki and S. Bjornholm, Phys. Lett. Rep. 4C (1972) 326.
P.J. Siemens, Triestre publications STI/PUB/386 paper IAEA-SMR-14/8 vol.I.

4) D.H.E. Gross and H. Kalinowski, Phys. Lett. 48B (1974) 302.
D.H.E. Gross, H. Kalinowski and J.N. De, Proc. Heidelberg Symposium on Heavy Ion Reactions (1974), lecture notes in physics 33 (1975) 194.
H. Kalinowski, Thesis (1975).
J.N. De, D.H.E. Gross and H. Kalinowski, preprint (1976).

5) J.P. Bondorf, M.I. Sobel and D. Sperber, Phys. Lett. Rep. 15C (1975) 83.
J.P. Bondorf, J.R. Huizenga, M.I. Sobel and D. Sperber, Phys. Rev. C11 (1975) 1265.

6) A.J. Sierk and J.R. Nix, Proc. of the 3rd IAEA Symposium on the Physics and Chemistry of Fission, Rochester vol.II (1973) 273 (IAEA Vienna).
J.R. Nix and A.J. Sierk, Phys. Script., 10A (1974) 401.

7) C.F. Tsang, Phys. Script., 10A (1974) 90.
F. Beck, Phys. Lett. 62B (1976) 385.
K. Siwek-Wilczynska and J. Wilczynski, Nucl. Phys. A264 (1976) 115.

8) H.H. Deubler and K. Dietrich, Phys. Lett. 56B (1975) 241.
H.H. Deubler and K. Dietrich, Nucl. Phys., in press.

9) W. Nörenberg, Phys. Lett. 53B (1974) 289.
W. Nörenberg, Z. Phys. A274 (1975) 241 and A276 (1976) 84.

S. Ayik, B. Schürmann and W. Nörenberg, Z. Phys. A277 (1976) 299 and Z. Phys. A279 (1976) 145.
W. Nörenberg, European Conference on Nuclear Physics with Heavy Ions, Caen (1976).

10) L.G. Moretto and J.S. Sventek, Phys. Lett. 58B (1975) 26.
J.S. Sventek and L.G. Moretto, Lawrence Berkeley Report LBL-5012 (1976)

11) R.A. Broglia, C.M. Dasso and Aa. Winther, Phys. Lett. 53B (1974) 301 and 61B (1976) 113.

12) H. Hofmann and P.J. Siemens, Nucl. Phys. A257 (1976) 165.
see also : H. Hofmann, Phys. Lett. 61B (1976) 243.
H. Hofmann and P.J. Siemens, Phys. Lett. 58B (1975) 417.
H. Hofmann, XIIIth Int. Winter Meeting on Nucl. Phys., Bormio (Italy) 1975.

13) C. Ngô, J. Péter, B. Tamain, M. Berlanger and F. Hanappe, Nucl. Phys. A267 (1976) and Proceeding of the International Workshop Hirschegg AED-Conf-76-015-000 p.122.

14) C.M. Ko, J.J. Pirner and H.A. Weidenmüller, Phys. Lett. 62B (1976) 248.

15) H. Hofmann and P.J. Siemens, Nucl. Phys. A275 (1977) 464.

16) D. Agassi, C.M. Ko and H.A. Weidenmüller, preprint MPI-H (1976) - 25

17) H. Hofmann and C. Ngô, Phys. Lett. 65B (1976) 97.

18) P.J. Johansen, P.J. Siemens, A.S. Jensen and H. Hofmann, (preprint 1976).

19) L.D. Landau and E.M. Lifschitz, Statistical Mechanics, Pergamon press, London, 1958.
R. Becker, Theorie der Wärme, Springer Verleg, Heidelberg (1964).
F. Reif, Fundamental of statistical and thermal physics, Mc Graw Hill Company (1965).

20) J. Péter, C. Ngô and B. Tamain, J. Phys. Lett. 36 (1975) L23.
C. Ngô, Thesis (1975).
J. Péter, C. Ngô, F. Plasil, B. Tamain, M. Berlanger and F. Hanappe, Nucl. Phys. in press.

21) H.J. Krappe and J.R. Nix, Proc. 3rd IAEA Symp. on Physics and Chemistry of Fission, Rochester 1973, vol.I p.159, Vienna, 1974.

22) C. Ngô, B. Tamain, M. Beiner, R.J. Lombard, D. Mas and H.H. Daubler, Nucl. Phys. A252 (1975) 237, see also :
C. Ngô, B. Tamain, J. Galin, M. Beiner and R.J. Lombard, Nucl. Phys. A240 (1975) 353.

23) H.A. Kramers, Physica VII (1940) 284.

24) B. Gatty, D. Guerreau, M. Lefort, J. Pouthas, X. Tarrago, J. Galin, B. Cauvin, J. Girard and H. Nifenecker, Z. Phys. A273 (1975) 65 and Nucl. Phys. A253 (1975) 511.
J. Galin, B. Gatty, D. Guerreau, M. Lefort, X. Tarrago, R. Babinet, B. Cauvin, J. Girard and H. Nifenecker, Z. Phys. A278 (1976) 347.

25) J. Galin, private communication.

26) J. Wilczynski, Phys. Lett. 47B (1973) 484.

Figure captions

Figure 1. Two dimensional plot where $\frac{d^2\sigma}{d\theta dM}$ in mb/rd/a.m.u of the quasi fission products has been calculated as a function of the mass and of the center of mass angle.

Figure 2. Calculated quasi fission mass distributions at different center of mass angles.

Figure 3. Calculated angular distributions corresponding to quasi fission products of various masses.

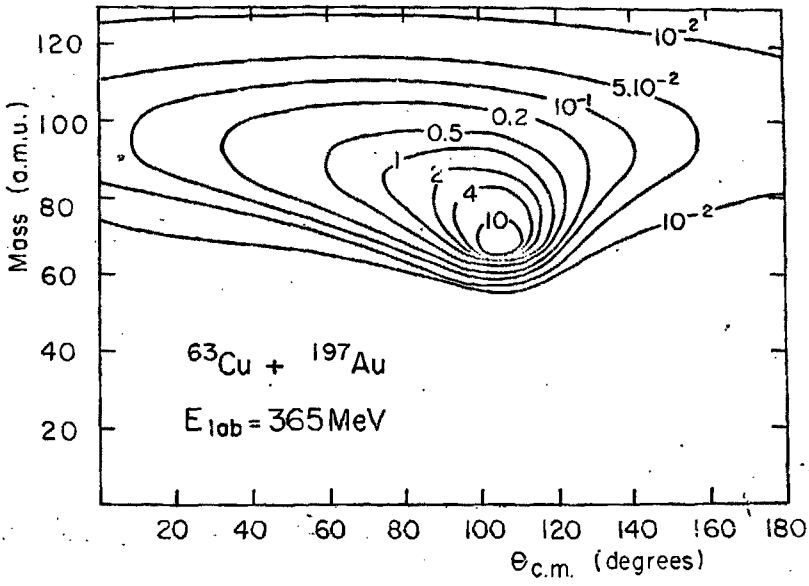
Figure 4. Experimental curve corresponding to fig.1 (extracted from ref.13,20]).

Figure 5. Experimental quasi fission mass distributions in the center of mass (extracted from ref.13,20]).

Figure 6. Same as fig.1.

Figure 7. The calculated angular distributions (the dashed curve corresponds to no mass transfer and the full curve corresponds to the case where mass transfer is included) are compared with the experimental points (dots).

Figure 8. Same as fig.7.


Mass (a.m.u.)
120
100
80
60
40
20
10^{-2}
5.10^{-2}
10^{-1}
0.2
0.5
1
2
4
10
10^{-2}
$^{63}Cu + ^{197}Au$
$E_{lab} = 365 MeV$
20 40 60 80 100 120 140 160 180
$\theta_{c.m.}$ (degrees)

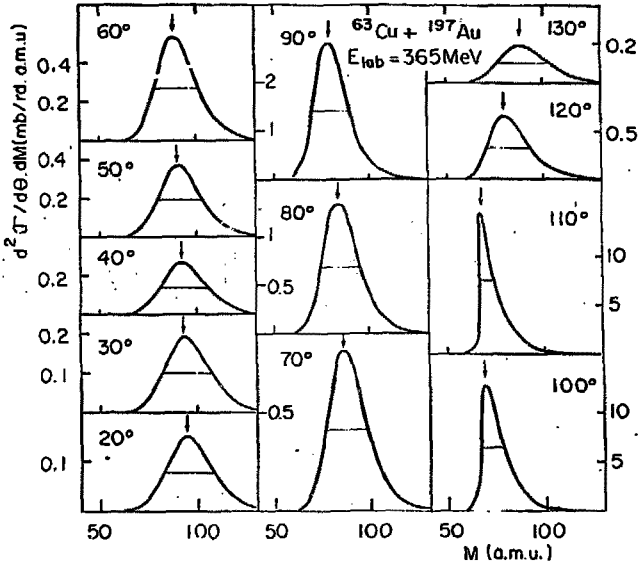
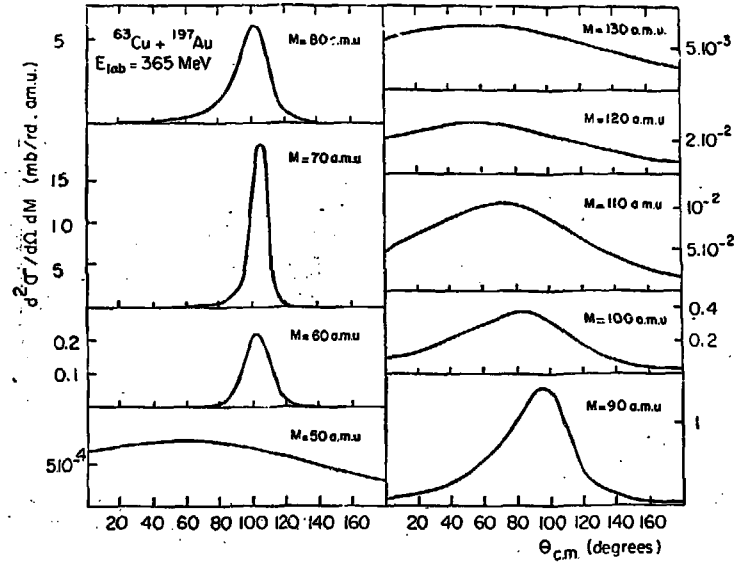

$^{63}Cu + ^{197}Au$
$E_{lab} = 365 MeV$
60°
50°
40°
30°
20°
90°
80°
70°
130°
120°
110°
100°
$d^2\sigma/d\theta.dM$(mb/rd.a.m.u)
M (a.m.u.)

Fig. 2


$^{63}Cu + ^{197}Au$
$E_{lab} = 365$ MeV
M = 80 a.m.u
M = 70 a.m.u
M = 60 a.m.u
M = 50 a.m.u
M = 130 a.m.u.
M = 120 a.m.u
M = 110 a.m.u
M = 100 a.m.u
M = 90 a.m.u
$d^2\sigma/d\Omega\, dM$ (mb/rd. a.m.u.)
$\theta_{c.m.}$ (degrees)

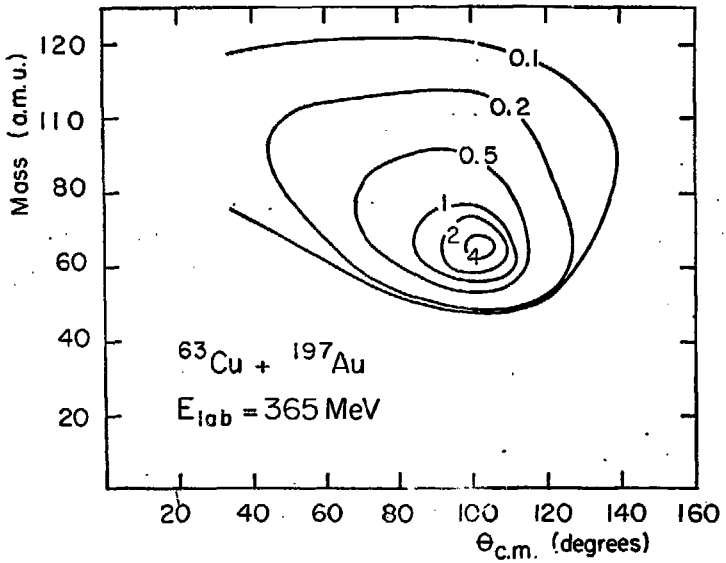

Mass (a.m.u.)
120
110
80
60
40
20
0.1
0.2
0.5
1
2
4
$^{63}Cu + {}^{197}Au$
$E_{lab} = 365$ MeV
20
40
60
80
100
120
140
160
$\theta_{c.m.}$ (degrees)

Fig. 4

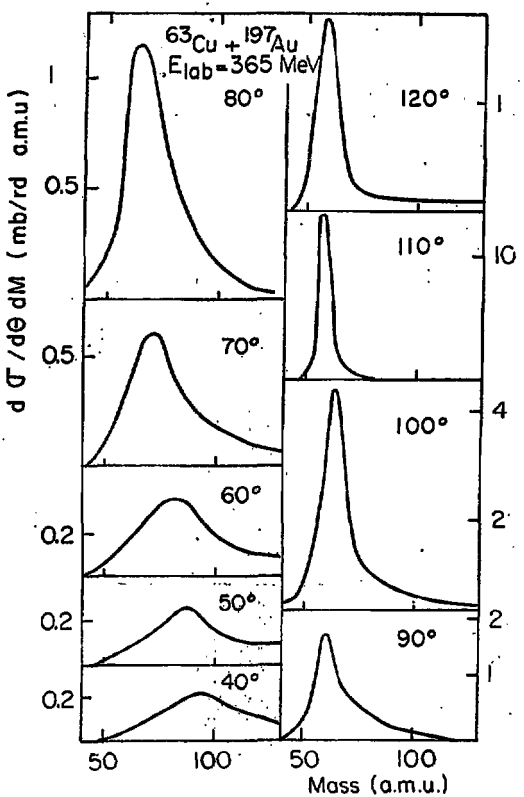

63Cu + 197Au
Elab=365 MeV
80°
120°
110°
70°
100°
60°
50°
90°
40°
d σ / dθ dM (mb/rd a.m.u)
Mass (a.m.u.)
50
100
0.2
0.5
1
2
4
10

Fig. 5

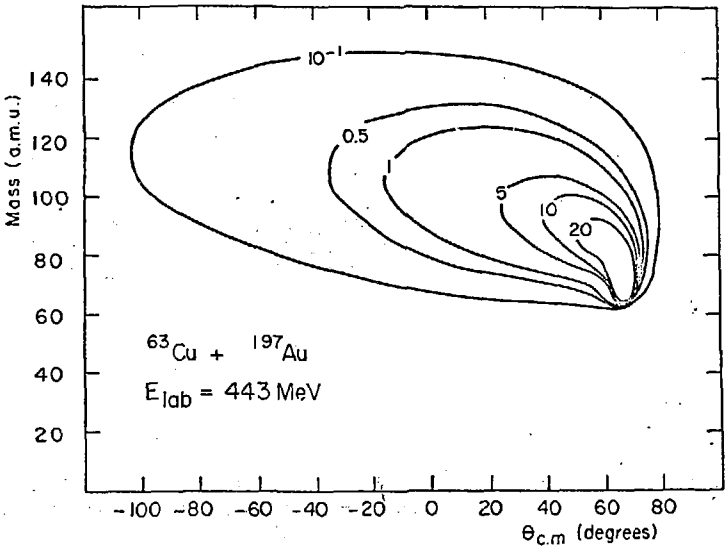

Mass (a.m.u.)
140
120
100
80
60
40
20
10⁻¹
0.5
1
5
10
20
⁶³Cu + ¹⁹⁷Au
E_lab = 443 MeV
-100 -80 -60 -40 -20 0 20 40 60 80
θc.m (degrees)

Fig. 6

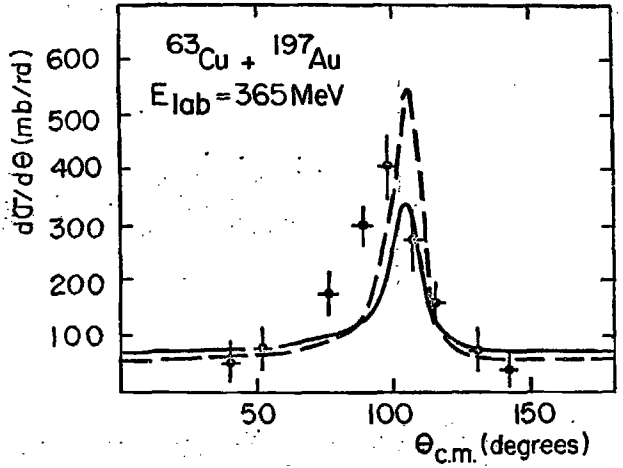
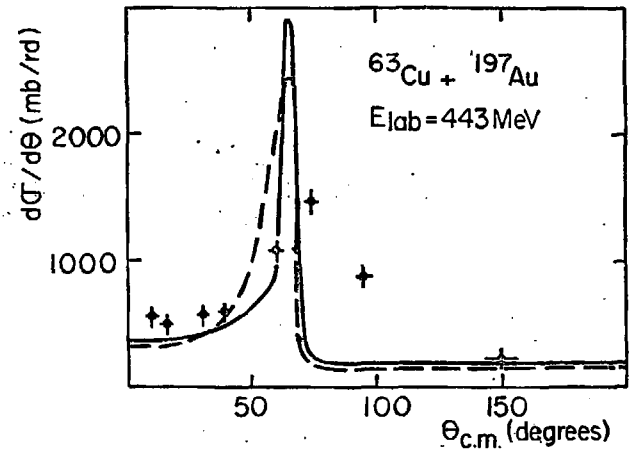

$^{63}Cu + ^{197}Au$
$E_{lab} = 365 MeV$
$d\sigma/d\theta$ (mb/rd)
600
500
400
300
200
100
50
100
150
$\theta_{c.m.}$ (degrees)

Fig. 7


$^{63}Cu + {}^{197}Au$
Elab = 443 MeV
dσ/dθ (mb/rd)
2000
1000
50
100
150
θc.m. (degrees)