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**AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS**

**AUS DIFFUSION MODULE POW CHECKOUT – 1- AND 2- DIMENSIONAL
KINETICS CALCULATIONS**

by

J. P. POLLARD

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ABSTRACT

POW is the diffusion module 'workhorse' of the AUS reactor neutronics modular code system; its steady state calculations have been checked out against other diffusion codes (particularly CRAM and GOG). Checkout of kinetic aspects, however, is difficult as kinetic codes are not freely available. In this report POW has been checked against three benchmark calculations as well as a calculation on the 100 kW Argonaut reactor Moata.

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1. INTRODUCTION

For several years the AUS reactor neutronics modular code system [Robinson 1975] has been used on the AAEC's IBM360/50 computer at the Research Establishment, Lucas Heights. Presently, some of the peripheral modules are being written; however, the main 'workhorse' modules are available and have been tested. Even so, extensive crucial testing of the scheme is yet to be undertaken, particularly of the system as a whole.

This report describes the checkout of the diffusion module 'workhorse' POW [Pollard 1974] for multidimensional kinetics calculations. Extensive earlier checkout of POW has been carried out for steady state comparisons [J. Pollard, unpublished data], particularly against the codes CRAM [Hassitt 1962] and GOG [Hopkins & Oakes 1968]. However, checkout of kinetic aspects is difficult as kinetic codes are not freely available. A further difficulty is encountered in trying to repeat published multidimensional kinetic calculations since the reported calculations frequently do not list all of the required data. The main reason for the oversights is probably that the authors present comparisons of different methods and are not particularly concerned with presenting 'benchmark' calculations. Another difficulty is that sometimes authors are using undefined, although apparently universal, notation. As an example, even with commonly used notation, the 2-group fast removal cross section is sometimes the downscattering cross section, sometimes the absorption cross section and sometimes the sum of the two (as it is with POW).

The kinetic checkout adopted here uses three well-documented benchmark type calculations and reports a calculation on the AAEC research reactor Moata. The Moata calculation is well-documented but, on account of the low computing power at the time of writing, extensive verification of dependence of the results on choice of groups, mesh spacing, time step *etc.* is not pursued. Even so, the results would be of use to others pursuing 'benchmark' type kinetic calculations.

2. THE MULTIGROUP NEUTRON DIFFUSION EQUATION

To present some of the quantities required by POW, the reader needs to be acquainted with the types of equations that POW solves. The time, t , dependent equations to be solved at a general point \underline{r} of the reactor under study [Stacey 1969; Pollard 1975] are as follows:

- (i) The neutron flux diffusion equation which gives the connection: the rate of decrease of neutron flux (velocity times neutron density) for energy group g , $\phi_g(\underline{x}, t)$ (the sought after quantity) = that due to leakage and removal via absorption reactions and out scattering reactions, less production from in scattering reactions and prompt fission from all groups, less production from decay of precursors. Mathematically these are

$$\begin{aligned}
 -\frac{1}{v} \frac{\partial}{\partial t} \phi_g(\underline{x}, t) &= \{-\nabla \cdot D_g(\underline{x}, t) \nabla + \sigma_{\text{rem}g}(\underline{x}, t)\} \phi_g(\underline{x}, t) - \\
 &-\sum_{g'} [\sigma_{sg' \rightarrow g}(\underline{x}, t) + \chi_{pg}(1-\beta) \frac{v}{k} \sigma_{fg'}(\underline{x}, t)] \phi_{g'}(\underline{x}, t) - \\
 &-\sum_d \chi_{dg} \lambda_d C_d(\underline{x}, t), \quad g = 1, 2, \dots, G. \quad (1)
 \end{aligned}$$

- (ii) The precursor concentration equation, which gives the relation: the rate of increase of precursor concentration for the delayed group d , $C_d(\underline{x}, t)$ (an intermediate quantity only) = that due to production from fission less decay of the precursor. Mathematically, this is

$$\begin{aligned}
 \frac{\partial}{\partial t} C_d(\underline{x}, t) &= \beta_d \sum_{g'} \frac{v}{k} \sigma_{fg'}(\underline{x}, t) \phi_{g'}(\underline{x}, t) - \lambda_d C_d(\underline{x}, t), \\
 d &= 1, 2, \dots, \text{ usually } 6, \quad (2)
 \end{aligned}$$

where the following are supplied data:

- χ_{pg} is the prompt fission spectrum (normalised to unit group sum);
- χ_{dg} is the delayed fission spectrum (normalised to unit sum) for delayed group d with decay constant λ_d and which yields fraction β_d of total fission emissions, of which $\beta (= \sum_d \beta_d)$ are delayed (β is usually about 0.7 per cent of total fissions);
- $\sigma_g(\underline{x}, t)$ denotes various macroscopic cross sections (spatially constant for one material and possibly subject to step or ramp time variation);
- $\sigma_{sg' \rightarrow g}$ = scattering matrix from group g' to g (with g' to g term set to zero);

$$\begin{aligned} \sigma_{\text{rem}g} &= \text{removal} \\ &= \sigma_{\text{ag}} + \sum_{g'} \sigma_{\text{sg} \rightarrow g}, \quad (\text{absorption} + \text{outscatters}); \\ v\sigma_{\text{fg}} &= \text{fission emission}; \end{aligned}$$

$D_g(\underline{x}, t)$ denotes diffusion coefficient which is usually given as $D_g(\underline{x}, t) = 1/\{3\sigma_{\text{tr}g}(\underline{x}, t)\}$, $\sigma_{\text{tr}g}$ being the transport cross section;

v_g denotes group g average velocity; and

k is the effective steady state multiplication eigenvalue supplied from a preceding calculation (equation (5b)), and sums are taken over all groups (1,2,...,G for energy groups and 1,2,..., usually 6 for delayed groups).

Solution of the equations is subject to the following

boundary conditions:

(iii) the outer boundary condition

$$D_g(\underline{x}, t) \frac{\partial \phi_g(\underline{x}, t)}{\partial n_g} + \frac{\phi_g(\underline{x}, t)}{3c} = 0 \quad \text{for each group } g, \quad (3)$$

where n = outward normal,

c = transport (mean free path) extrapolation distance,

$$= \begin{cases} 0.71 & \text{for an actual outer boundary of the reactor,} \\ \infty & \text{for a symmetric boundary (with POW, input of } c = 0 \text{ sets this condition).} \end{cases}$$

(iv) The internal boundary conditions for boundaries separating different materials are all assumed to lie parallel to axes, say $L|R$:

(a) continuity of flux

$$\phi_g(\underline{x}, t) \Big|_L - \phi_g(\underline{x}, t) \Big|_R = 0 \quad \text{for each group } g; \quad (4a)$$

(b) continuity of current

$$n \cdot \{D_g(\underline{x}, t) \nabla \phi_g(\underline{x}, t) \Big|_L - D_g(\underline{x}, t) \nabla \phi_g(\underline{x}, t) \Big|_R\} = 0 \quad (4b)$$

for each group g .

(v) In addition the initial conditions are

$$\frac{\partial \phi_g(\underline{x}, t)}{\partial t} = 0, \quad \frac{\partial C_d(\underline{x}, t)}{\partial t} = 0, \quad t \leq 0 \quad \text{for all groups} \quad (5a)$$

which give the steady state equation (dropping time dependence)

$$\begin{aligned} & \{-\nabla \cdot D_g(\underline{x}) \nabla + \sigma_{\text{remg}}(\underline{x})\} \phi_g(\underline{x}) - \sum_{g'} [\sigma_{sg' \rightarrow g}(\underline{x}) + \\ & + \chi_g \frac{\nu}{k} \sigma_{fg'}(\underline{x})] \phi_{g'}(\underline{x}) = 0 \quad , \end{aligned} \quad (5b)$$

where $\chi_g = \chi_{pg}(1-\beta) + \sum_d \chi_{dg} \beta_d$ is the equilibrium fission spectrum, k is the effective steady state multiplication (eigenvalue), $\phi_g(\underline{x})$ is the steady state (eigenfunction) flux and $\phi_g(\underline{x}, 0) = \phi_g(\underline{x})$, except for an arbitrary normalising factor.

3. POW CALCULATION PROCEDURE IN BRIEF

POW uses a finite difference approximation for the spatial derivative involved in equation (1) and requires calculation of group fluxes at the intersection of chosen grid lines $i = 1, 2, \dots, I$ and $j = 1, 2, \dots, J$. Temporal integration from time t_{p-1} to t_p , a step of δt_p , is carried out using the following assumptions:

- (i) for the temporal variation of the cross sections a constant value is taken

$$\sigma_g(\underline{x}, t) = \int_{t_{p-1}}^{t_p} \sigma_g(\underline{x}, t') dt' / \delta t_p \quad , \quad (6)$$

- (ii) and for the temporal variation of the flux, a linear form is assumed

$$\phi_g(\underline{x}, t) = \{(t_p - t) \phi_g(\underline{x}, t_{p-1}) + (t - t_{p-1}) \phi_g(\underline{x}, t_p)\} / \delta t_p \quad . \quad (7)$$

When we do this, the precursor concentration equation (2) may be (time) integrated analytically and the result substituted into the diffusion equation (1).

Leaving aside essentially all the details [see Stacey 1969; Pollard 1973, 1974] we arrive at the matrix equation

$$M\phi(p) = -M'\phi(p-1) + LC(p-1) \quad , \quad (8)$$

where M and M' are matrices of order IJG , L is a matrix of order IJ times the number of delayed groups (usually 6), and $\phi(p-1)$ and $C(p-1)$ are flux and precursor concentration vectors. The precursor concentration is updated using the solution of equation (8) as

$$C(p) = HC(p-1) + F\phi(p) , \quad (9)$$

where H and F are matrices containing precursor decay and fission emission elements respectively.

TABLE 1

POW FLOW PROCEDURE FOR SOLUTION OF THE DIFFUSION EQUATION

DO 1 p=0,1,...,P	<u>temporal loop</u> no iteration required, calculate t_p , cross sections and coefficient ^p matrix
IF(p=0) CONTINUE	<u>steady state trial solution</u> generate for bare homogeneous system
IF(p=0) DO 5 $l_o=1,2,\dots,*$	<u>outer (fission) loop</u> calculate total fission source from last flux ϕ and last eigenvalue k and combine with earlier sources (Chebyshev extrapolation)
IF(p \neq 0) $l_o=1$	<u>right hand calculation</u> calculate right hand of equation (8)
$g_1 = 1$	
DO 7 $l_u = 1,2,\dots,*$	<u>upscatter (and fission p > 0) loop</u> update groups using Gauss-Seidel method
DO 6 $g = g_1, g_1+1, \dots, G$	
IF($g=g_u$, 1st upscatter group)	<u>group rebalance</u> rebalance groups on a whole plane basis when 1st upscatter group (or 1st group p > 0) encountered
	<u>region rebalance</u> rebalance regions of the whole plane for a group
DO 4 $l_I = 1,2,\dots,*$	<u>inner plane solution</u> for a group
DO 4 j = 1,2,...,J	<u>line solution, SLOR</u> solve directly for a line (i=1,2,...,I) as matrix is tridiagonal (i and j could be swapped here)
4 CONTINUE	combine lines using SLOR method
6 CONTINUE	
7 $g_1 = g_u$	
5 CONTINUE	
IF(p=0) CONTINUE	<u>precursor concentration initial condition</u> calculate precursor concentration
IF(p \neq 0) CONTINUE	<u>precursor concentration update</u> update precursor concentration following equation (9)
1 CONTINUE	output results (disk, printer, plotter, etc.)
STOP	

* these loops terminate on convergence tests

The solution procedure adopted by POW employs successive line over-relaxation (SLOR) for the inner plane solution for each group and relies on a region and group rebalance technique [Pollard 1975] to hasten convergence for the whole plane (Table 1).

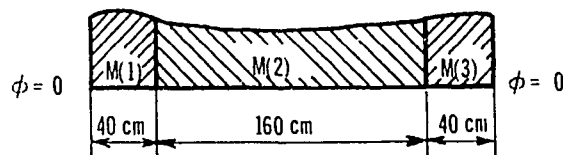
4. 1D BENCHMARK-SPATIALLY ASYMMETRIC NEGATIVE RAMP OF $1/10 \text{ } \$/ \text{ s}^{-1}$

The need for well documented reactor calculations able to serve as 'benchmark' standards for the development and testing of codes has been recognised for many years. Mostly, the calculations are taken to higher accuracy than normally required for everyday work to show up possible small discrepancies between the different codes. The Argonne Code Center has produced two books on benchmark problems [Argonne National Laboratory 1968, 1972] and from the latest book we draw our first example (also reported by Fuller 1972). Results are compared from the four codes:

RAUMZEIT [Adams & Stacey 1967] - best benchmark solutions;
 WIGLE [Cadwell, Henry & Vigilotti 1964];
 QX1 [Meneley, Ott & Wiener 1971]; and
 POW [Pollard 1974].

4.1 Problem Description

Following the data of W.M. Stacey [Argonne National Laboratory 1972] a very loosely coupled 1D slab reactor is subject to a negative reactivity ramp in one part only of an otherwise symmetric 3-region system.



M(3) = material 3; M(2) = material 2; M(1) = M(3) initially.

4.2 Initial Data Required

4.2.1 Mesh spacing

A mesh spacing of $\delta x = 2 \text{ cm}$ is used throughout the system. This results in 121 mesh points for POW (an edge of mesh code). The boundary condition $\phi = 0$ is taken to be essentially at the reactor extremes with a transport extrapolation distance of $c = 10^{-6}$ rather than the usual value of $c = 0.71$.

4.2.2 Initial nuclear data

Two (energy) group data are used as indicated (according to POW requirements).

	M(1)	M(2)	M(3)	
$\sigma_{tr} \text{ cm}^{-1}, g=1$	0.222222	0.333333	same	D=1.5 1 for M(1), M(2)
$=2$	0.666667	0.666667	as	0.5 0.5 for M(1), M(2)
$\sigma_{rem} \text{ cm}^{-1}, g=1$	2.6-2	2.-2	M(1)	rem = absorption
$=2$	1.8-1	8.-2	↓	plus outscatters
$\nu\sigma_f \text{ cm}^{-1}, g=1$	1.-2	5.-3		2.6-2 = 2.6 x 10 ⁻²
$=2$	2.-1	9.9-2		
$\sigma_{sg' \rightarrow g} \text{ cm}^{-1}, g'=1$	0,1.5-2	0,1.-2		downscatters from
$=2$	0,0	0,0		group 1=1.5-2 for M(1)
$\chi_p, g=1$	1	same as		
$=2$	0	M(1)		
$\nu \text{ cm s}^{-1}, g=1$	1.+7	↓		
$=2$	3.+5			

4.2.3 Delayed group data

Six delayed group data are used:

Family d	Effective Delay Fraction (β_d)	Decay Constant ($\lambda_d \text{ s}^{-1}$)
1	2.50-4	1.24-2
2	1.64-3	3.05-2
3	1.47-3	1.11-1
4	2.96-3	3.01-1
5	8.60-4	1.14
6	3.20-4	3.01
$\beta=7.5-3$		

The delayed group fission spectra are taken to be the same as the prompt spectrum, χ_p .

4.3 Steady State Calculation, $t \ll 0$

The steady state calculation, solution of equation (5b), is compared for different codes following the Argonne National Laboratory

[1972] proposals. In the comparison, power, $p(t)$, is taken to be

$$p(t) = f \sum_g \int_{\text{reactor}} v \sigma_{fg}(\underline{x}, t) \phi_g(\underline{x}, t) d\underline{x} \quad , \quad (10)$$

where f is an arbitrary normalising factor, and group g variation of v_g (the number of neutrons released per fission) is ignored. (This power expression is apparently that adopted by the benchmark proposers.) From this calculation we have the following results:

	<u>RAUMZEIT</u>	<u>WIGLE</u>	<u>QX1</u>	<u>POW</u>
multiplication, k	0.90156	0.90156	0.90156	0.90156
fractional power, $M(1)$	0.2790	0.2790	0.2790	0.2790
$M(2)$	0.4421	0.4421	0.4421	0.4421
$M(3)$	0.2790	0.2790	0.2790	0.2790

Starting from a symmetric cosine distribution trial flux, POW took 31 outer iterations to drop the error indicator down to 10^{-5} . (The error indicator used in POW is

$$S_{\text{err}}^{(n)} = \max_{i,j} S_{ij}^{(n)} / S_{ij}^{(n-1)} - \min_{i,j} S_{ij}^{(n)} / S_{ij}^{(n-1)} \quad , \quad (11)$$

$$\text{where } S_{ij}^{(n)}(t) = \sum_g \int_{i,j} v \sigma_{fg}(\underline{x}, t) \phi_g(\underline{x}, t) d\underline{x} \quad , \quad (12)$$

the integration being taken over a box about the point i, j and which has edges midway between mesh lines.) The number of outer iterations is not surprising for the very loosely coupled core considering the effective dominance ratio (ratio of successive k -eigenvalues corresponding to symmetric eigenfunctions) is 0.937. (Starting with an asymmetric trial flux, the convergence is far worse, since the dominance ratio $d = 0.9984$.)

4.3.1 Steady state normalisation, $t=0$

POW has several different normalisation procedures by which it can generate

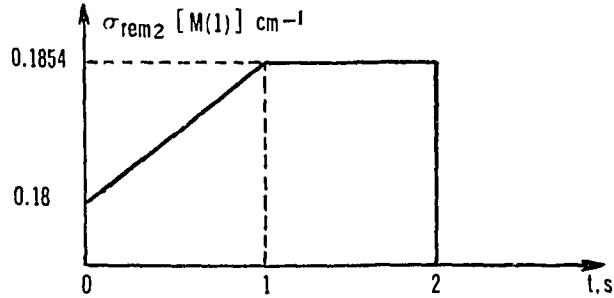
$$\phi_g(\underline{x}, 0) = a \phi_g(\underline{x}) \quad , \quad (13)$$

where a is a constant. The normalisation adopted in the present calculation is

$$a \sum_g \int_{\text{reactor}} \phi_g(\underline{x}) d\underline{x} = \int_{\text{reactor}} d\underline{x} \quad . \quad (14)$$

4.4 Time Dependent Data

The only quantity varied with time is the group 2 removal cross section (equivalent to the absorption cross section) for M(1). The variation is a ramp of 3 per cent as illustrated.



The asymptotic reactivity corresponding to the full ramp

$$\rho = \frac{\delta k}{(k + \delta k) \beta} \quad (15)$$

is -0.094 which is obtained from a second steady state calculation with $\sigma_{rem2} [M(1)] = 0.1854$.

4.4.1 Temporal step length, δt

The temporal step length, δt , adopted is:

	<u>RAUMZEIT</u>	<u>WIGLE</u>	<u>QX1</u>	<u>POW</u>
$\delta t, s$	10^{-3}	10^{-3}	ave. 5×10^{-2}	10^{-2}

POW convergence for the kinetics part corresponds to the condition:

$$S_{err}^{(n)}(t) \leq 10^{-4} \quad (\text{see equation (11)}).$$

4.5 Kinetics Calculation, $t \geq 0$

Results of the calculation are plotted in Figure 1 (thermal flux) and summarised in Tables 2 and 3.

4.6 Observation on Results

From the results, it is clear that POW is able to calculate the 1D benchmark job to acceptable accuracy (normally a factor of 10 lower than the accuracy implied by the tables).

5. 2D TWIGL TEST-POSITIVE RAMP OF $2\frac{1}{2} \text{ s}^{-1}$

Documentation of 2D kinetics calculations in the literature unfortunately tends to be incomplete. Sometimes vital basic data are not given (or worse, wrongly given) and, at other times, the calculated quantities

TABLE 2

TOTAL POWER

Time, seconds	RAUMZEIT	WIGLE	QX1	POW
0.	1.0000	1.0000	1.0000	1.0000
0.1	0.9299	0.9298	0.9298	0.9298
0.2	0.8733	0.8732	0.8733	0.8732
0.5	0.7597	0.7596	0.7597	0.7596
1.0	0.6588	0.6588	0.6588	0.6587
1.5	0.6432	0.6432	0.6433	0.6432
2.0	0.6307	0.6306	0.6307	0.6306

*normalised here

TABLE 3

RELATIVE REGIONAL POWER

	Time, seconds						
	0.	0.1	0.2	0.5	1.0	1.5	2.0
<u>Region 1</u>							
RAUMZEIT	1.0000	0.8621	0.7521	0.5337	0.3453	0.3236	0.3066
WIGLE	1.0000	0.8621	0.7520	0.5336	0.3452	0.3235	0.3066
QX1	1.0000	0.8621	0.7521	0.5336	0.3452	0.3235	0.3066
POW	1.0000	0.8622	0.7522	0.5337	0.3453	0.3236	0.3066
<u>Region 2</u>							
RAUMZEIT	1.0000	0.9340	0.8805	0.7724	0.6753	0.6588	0.6455
WIGLE	1.0000	0.9339	0.8804	0.7724	0.6753	0.6587	0.6455
QX1	1.0000	0.9340	0.8805	0.7724	0.6753	0.6588	0.6455
POW	1.0000	0.9339	0.8805	0.7723	0.6752	0.6588	0.6455
<u>Region 3</u>							
RAUMZEIT	1.0000	0.9910	0.9831	0.9655	0.9463	0.9382	0.9312
WIGLE	1.0000	0.9910	0.9830	0.9655	0.9462	0.9381	0.9311
QX1	1.0000	0.9910	0.9831	0.9656	0.9463	0.9383	0.9312
POW	1.0000	0.9908	0.9829	0.9653	0.9460	0.9380	0.9310

Note the tilt in power

↑
normalised here

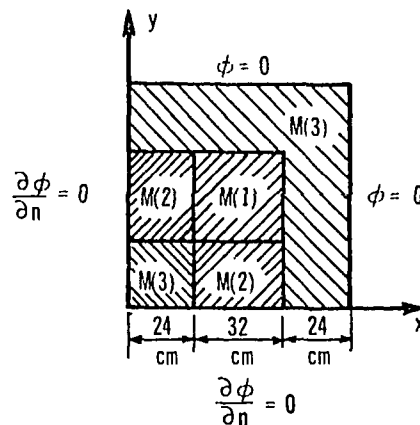
being compared are imprecise. Test cases are offered in ENEA/EURATOM [1972], but the present author has had little success with them. There is thus a real need for 2D benchmark kinetics problems to be defined.

Some problems originally set up to test the 2D, 2-group TWIGL code [Yasinsky, Natelson & Hageman 1968] and reported by Hageman & Yasinsky [1969] and Wight, Hansen & Ferguson [1971] are discussed in Sections 5.1 and 5.2. Results are compared from the five codes:

TWIGL [Yasinsky, et al. 1968] - best comparison solutions;
 LUMAC [McCormick 1969];
 MITKIN [Reed & Hansen 1970];
 SADI [Wight, et al. 1971]; and
 POW .

5.1 Problem Description

A 2D symmetric square section reactor is subject to a positive reactivity ramp in part of the system. The quarter reactor layout is as follows:



M(3) = blanket material; M(2) = core material;
 M(1) = driver material
 = M(2) initially.

5.2 Initial Data Required

5.2.1 Mesh spacing

A mesh spacing of $\delta x = \delta y = 8$ cm is taken throughout the system. The result is an 11×11 spatial mesh arrangement for POW. The boundary condition $\phi = 0$ is taken close to the actual boundary, using a transport extrapolation distance of $c = 10^{-3}$.

5.2.2 Initial nuclear data

POW 2-group data for the problem are given below:

	M(1)	M(2)	M(3)		
$\sigma_{tr} \text{ cm}^{-1}, g=1$	0.238095	same	0.256410	D=1.4	1.3 for M(1),M(3)
=2	0.833333	as	0.666667	0.4	0.5 for M(1),M(3)
$\sigma_{rem} \text{ cm}^{-1}, g=1$	2.-2	M(1)	1.8-2		
=2	1.5-1	↓	5.-2		
$\nu\sigma_f \text{ cm}^{-1}, g=1$	7.-3		3.-3		
=2	2.-1		6.-2		
$\sigma_{sg' \rightarrow g} \text{ cm}^{-1}, g'=1$	0,1.-2		0,1.-2		
=2	0,0		0,0		
$\chi_p, g=1$	1		same		
=2	0		as		
$\nu \text{ cm s}^{-1}, g=1$	1.+7		M(1)		
=2	2.+5		↓		

5.2.3 Delayed group data

A one delayed group data set is used:

$$\beta = 7.5-3, \lambda = 8.-2 \text{ s}^{-1} .$$

The delayed group fission spectrum is the same as the prompt spectrum, χ_p .

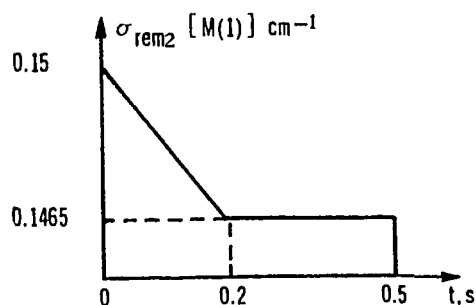
5.3 Steady State Calculation, $t \leq 0$

The results obtained for the steady state calculation are:

	<u>TWIGL</u>	<u>POW</u>
multiplication, k	0.914193	0.914194

5.4 Time Dependent Data

The thermal removal cross section of the driver material, M(1), is reduced following the ramp variation indicated.



The asymptotic reactivity corresponding to the full ramp is:

$$\rho = 0.517\% \quad (\text{using equation (15)}).$$

5.4.1 Temporal step length, δt

The temporal step length, δt , used is:

	<u>TWIGL</u>	<u>LUMAC</u>	<u>MITKIN</u>
$\delta t, s$	10^{-2}	8×10^{-4}	$10^{-3}; 10^{-2}, t \geq 0.25$
	<u>SADI</u>	<u>POW</u>	
$\delta t, s$	2.5×10^{-4}	10^{-2}	

POW convergence for the kinetics section corresponds to the condition;

$$S_{\text{err}}^{(n)}(t) \leq 10^{-3} \quad (\text{see equation (11)}),$$

which is recommended by Pollard [1974] to save machine time, although the IBM360/50 computational time for the job of 22 CPU-minutes is hardly excessive.

TABLE 4
NORMALISED CENTRAL THERMAL FLUX

Time, (s)	TWIGL	LUMAC	MITKIN	SADI	POW
0.	1.000	1.000	1.000	1.000	1.000
0.02			1.030	1.033	1.037
0.05	1.120		1.122	1.116	1.121
0.10	1.298	1.297	1.299	1.293	1.298
0.15	1.550		1.549	1.543	1.550
0.20	1.933	1.940	1.929	1.919	1.931
0.25	2.033		2.037	2.064	2.036
0.30	2.044	2.079	2.004	2.047	2.045
0.35	2.053		1.984	2.056	2.052
0.40	2.062		1.981	2.066	2.062
0.45	2.070*				2.069
0.50	2.079*				2.079

* estimated from published total thermal flux

5.5 Kinetic Calculation, $t \geq 0$

Results of the calculation are presented for the normalised central thermal flux (Table 4)

$$\phi_2(0,0,t)/\phi_2(0,0,0) \quad ,$$

that is $\phi_{2,1,2}(t)/\phi_{2,1,2}(0)$ in terms of flux at mesh point (1,1), group 2.

5.6 Observation on Results

Unlike the problem given in Section 4, the time variation does not introduce asymmetric flux behaviour, and mostly the time variation at all points is similar. For the simpler problem given in the present section, POW again achieves acceptable accuracy whereas LUMAC, MITKIN and SADI do not.

6. 2D TWIGL TEST-POSITIVE STEP OF $\frac{1}{2}\$$

The present calculation is also for the reactor described in Section 5, but with the ramp of Section 5.4 changed to a step of the same size.

6.1 Time Dependent Data

The thermal removal cross section of the driver material, $M(1)$, is reduced instantaneously from 0.15 cm^{-1} to 0.1465 cm^{-1} at $t=0$. The asymptotic reactivity is again $\rho = 0.517\$$.

6.1.1 Temporal step length

The temporal step length, δt , is as indicated:

	<u>TWIGL</u>	<u>LUMAC</u>	<u>MITKIN</u>	<u>SADI</u>	
$\delta t, s$	10^{-3}	8×10^{-5}	2×10^{-4}	2.5×10^{-4}	
	<u>POWa</u>	<u>POWb</u>	<u>POWc</u>	<u>POWd</u>	<u>POWe</u>
$\delta t, s$	10^{-2}	10^{-2}	10^{-3}	10^{-3}	10^{-4}

POW convergence during the kinetics calculation is taken as:

$$S_{\text{err}}^{(n)}(t) \leq 10^{-3} \quad \text{POWa and POWc} \quad , \quad \text{and}$$

$$S_{\text{err}}^{(n)}(t) \leq 10^{-4} \quad \text{POWb, POWd and POWe} \quad ,$$

in order to verify the adequacy of the lower accuracy convergence limit.

6.2 Kinetics Calculation, $t \geq 0$

Results of the calculation are presented in Table 5 for the normalised central thermal flux, $\phi_2(0,0,t)/\phi_2(0,0,0)$.

TABLE 5
NORMALISED CENTRAL THERMAL FLUX

Time (s)	TWIGL	LUMAC	MITKIN	SADI	POWa	POWb	POWc	POWd	POWe
0.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.01	1.594	1.629	1.631	1.621	1.627	1.627	1.623	1.624	1.623
0.02	1.837	1.879	1.881	1.983	1.923	1.924	1.870	1.872	1.872
0.03	1.934	1.973	1.978	2.026	1.953	1.954	1.964	1.966	
0.04	1.979		2.028	2.024	2.029	2.031	2.000	2.002	
0.05	2.002		2.067	2.023	2.000	2.000	2.015	2.017	
0.10	2.030			2.032		2.044		2.033	
0.20	2.048			2.050		2.053		2.051	

It is interesting to note that dropping the POW region rebalance scheme [Pollard 1975] makes little difference to the POWb computational time, whereas dropping group rebalance for the early time steps increases the computational time by more than a factor of 6 and convergence is not completely achieved. This result is in contrast with the asymmetric 1D calculations given in Section 4 where group rebalance is only of marginal help.

6.3 Observation on Results

As one would expect, the step change is harder to calculate than the equivalent ramp discussed in Section 5.

From the POW results, we note the insignificant loss of accuracy using the convergence condition

$$S_{err}^{(n)}(t) \leq 10^{-3}$$

and that a step length of a least $\delta t = 10^{-3}$ s is needed for early times as the $\delta t = 10^{-2}$ s results show a slight 'wobble'. Comparison of POW with the other codes suggests that the TWIGL step length of $\delta t = 10^{-3}$ s is inadequate, but otherwise reasonable agreement is obtained.

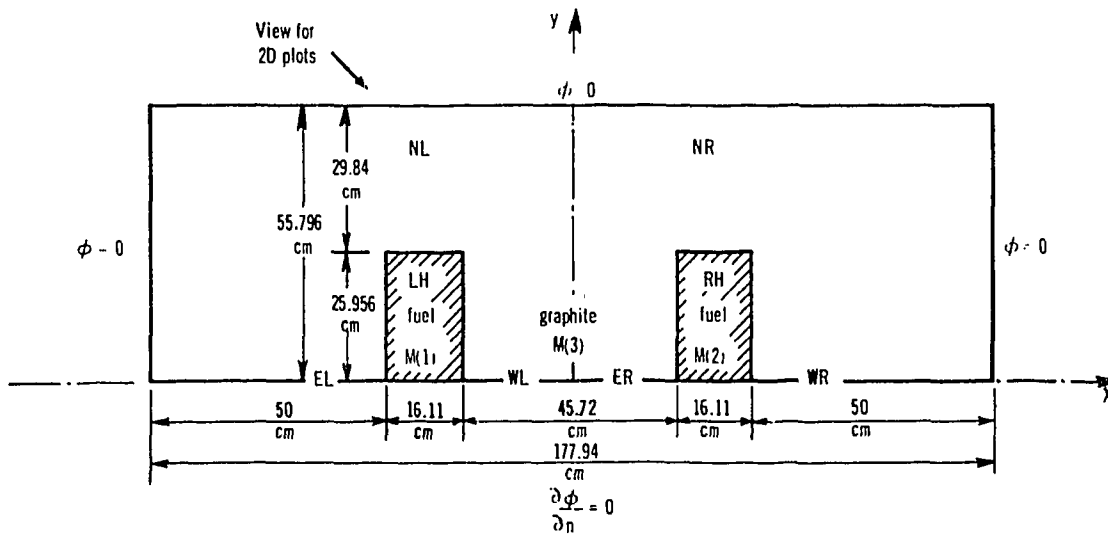
7. 2D MOATA - SPATIALLY ASYMMETRIC NEGATIVE STEP OF 1\$

The present calculation concerns the estimation of neutron flux following the dropping of a safety rod in the AAEC research reactor Moata (a 100 kW Argonaut type reactor of highly enriched uranium in aluminium plates, cooled and moderated by light water with graphite

reflector). The drop in one tank only of the two-tank reactor causes flux asymmetries as calculated for the earlier 1D benchmark (Section 4). A comparison is made with an experiment conducted on Moata as part of a Summer School run at the Research Establishment for final year high school students [Barry et al. 1974]. The main purpose of the present calculation is to provide a possible 'benchmark' problem for use by others.

7.1 Problem Description

A 2D model of Moata, which is an approximation of that of J. Connolly (AAEC private communication), is subject to a negative reactivity step in the left hand fuel tank. The half reactor layout is as follows:



NL, etc., are detector positions detailed later.

POW input data for the calculation is listed in Appendix A.

7.2 Initial Data Required

7.2.1 Mesh spacing

The following mesh spacing is used:

$$\begin{aligned} \delta x, \text{ left and right hand} &= 50/7 = 7.1428571 \text{ cm,} \\ &\text{graphite} \\ \text{left and right hand} &= 16.11/5 = 3.222 \text{ cm, and} \\ &\text{fuel} \end{aligned}$$

central graphite = 45.72/7 = 6.5314285 cm;
 δy , fuel = 25.956/5 = 5.1912 cm, and
 top graphite = 29.84/5 = 5.968 cm.

The result of this choice of large mesh is a set of 32 x 11 solution points for POW. The boundary condition $\phi = 0$ is taken close to the actual boundary with $c = 10^{-6}$ for ease of calculation by others using codes having this requirement.

7.2.2 Initial nuclear data

The 2-group data for the problem are indicated below:

	M(1)	M(2)	M(3)	
$\sigma_{tr} \text{ cm}^{-1}$, g=1	0.2436	same	0.279	D=1.36836 1.19474 for M(1),M(3)
=2	1.558	as	0.3646	0.21395 0.914244 for M(1),M(3)
$\sigma_{rem} \text{ cm}^{-1}$, g=1	3.9655-2	M(1)	5.6669-3	rem = absorption plus axial leakage
=2	5.5971-2	†	2.081-3	plus outscatters
$v\sigma_f \text{ cm}^{-1}$, g=1	2.13-3		0	
=2	8.074-2		0	
$\sigma_{sg' \rightarrow g} \text{ cm}^{-1}$, g'=1	0,3.532-2		0,3.461-3	
=2	6.302-4,0		1.651-4,0	note the slight upscatters
χ_p , g=1	1		same	
=2	0		as	
$v \text{ cm s}^{-1}$, g=1	9.+6		M(1)	
=2	2.3+5		†	

7.2.3 Delayed group data

Effective 6 delayed group data used are derived from data of Keepin [1965] and Tomlinson [1972].

Family d	Effective delay fraction β_d	Decay constant $\lambda_d \text{ s}^{-1}$
1	2.511-4	1.24-2
2	1.664-3	3.05-2
3	1.495-3	1.11-1
4	2.975-3	3.01-1
5	8.571-4	1.14
6	3.113-4	3.01
$\beta = 7.5535-3$		

The delayed group fission spectra are taken to be the same as the prompt spectrum, χ_p .

7.3 Steady State Calculation, $t \ll \beta$

The POW steady state calculation flux results are displayed in Figures 2 and 8. The multiplication is $k = 1.01227$ and the normalisation adopted is

$$\sum_g \int_{\text{reactor}} \frac{v}{k} \sigma_{fg}(\mathbf{x}) \phi_g(\mathbf{x}) d\mathbf{x} = 1 \quad (16)$$

7.4 Time Dependent Data

As a very approximate representation of the safety rod drop in the near vicinity of the left hand fuel tank, the thermal removal cross section of M(1) is instantaneously increased by $9.74887 \times 10^{-4} \text{ cm}^{-1}$. The asymptotic reactivity corresponding to this change is $\rho = -0.798\%$. The cross section change for M(1) was determined by a SEARCH calculation in POW to give $\rho \approx -80\%$, which is the reactivity of the safety rod determined at an AAEC Summer School [Barry et al. 1974] and elsewhere [T. Wall, private communication].

7.4.1 Temporal step length δt

The temporal step length, δt , is chosen in a stepwise constant fashion as indicated

t , (s)	δt , (s)	e-folding time, (s)
$0 \leq t < 10^{-2}$	10^{-3}	-3.3×10^{-2}
$10^{-2} \leq t < 10^{-1}$	9×10^{-3}	-9.9×10^{-1}
$10^{-1} \leq t < 1$	9×10^{-2}	-9.1
$1 \leq t < 10$	0.9	-17
$10 \leq t < 100$	9	-38
$100 \leq t < 200$	10	-60

This type of stepwise change was chosen so that δt was always much less than the magnitude of the overall e-folding time observed during the calculation. The 0.9 factors were chosen so that every tenth output corresponded to 10^{-2} , 10^{-1} , 1, 10, 100 s as required for display of results given in Figures 2 to 9.

The convergence test used in the POW calculation is taken as:

$$S_{\text{err}}^{(n)}(t) \leq 10^{-3} .$$

7.5 Kinetics Calculation, $t \geq 0$

In terms of the spatial mesh indices numbered from the bottom left hand corner,

$$i=1,2,\dots,32; \quad j=1,2,\dots,11,$$

thermal flux results are given for the following detector positions:

EL(6,1)

NL(10,9) NL is very approximately the detector position ;
for experimental results given in Figure 9.

WL(15,1)

ER(18,1)

NR(23,9)

WR(27,1)

The flux results are recorded in Table 6 and, in addition, an asymmetry tilt indicator is given:

$$T = \frac{\phi_{\text{NR},2}(t) - \phi_{\text{NL},2}(t)}{\phi_{\text{NR},2}(t) + \phi_{\text{NL},2}(t)} . \quad (17)$$

For completeness Appendix B details some output of the POW run.

As the last time step length, $\delta t = 10$ s, may be somewhat excessive, the calculation is repeated from $t = 100$ s with $\delta t = 5$ s. Results are indicated in Table 7.

7.6 Observation on Results

The results beyond about 10 s show a 'wobble' indicative of an excessively large step length. A strict 'benchmark' calculation would need to employ a step length of probably no more than one second in order to give results to four significant figures. This can be left as an exercise for those with fast computing equipment, since the IBM360/50 took about 180 CPU minutes to produce the entries in Table 6.

8. FLUX TILT IN SYMMETRIC SYSTEMS ASYMMETRICALLY DISTURBED

From results given for the 1D benchmark calculation (Figure 1) and the 2D Moata calculation (Figure 8), we notice the extreme tilting that can result from an asymmetric disturbance to a symmetrical system. Wade & Rydin [1972], using perturbation analysis, produced a relationship connecting the tilt indicator, T (equation (17)), with the reactivity, ρ (equation (15)) and the dominance ratio (ratio of k -eigenvalues). Here

TABLE 6
THERMAL FLUX AT DIFFERENT DETECTOR POSITIONS $\times 10^8$

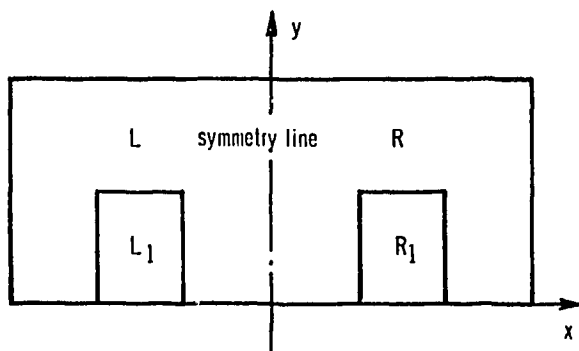
time (s)	EL	NL	WL	ER	NR	WR	T
0	935800	355600	1432000	1432000	355700	935900	0
10^{-2}	576800	228400	990600	1133000	299700	806200	0.135
10^{-1}	417900	164600	713400	815100	215100	580800	0.133
1	370600	146100	635300	729000	192700	520900	0.138
10	213500	84540	370700	431200	114600	310500	0.151
55	65160	25900	114300	134500	35900	97410	0.162
100	29410	11700	51750	60890	16270	44190	0.163
150	11980	4772	21120	25070	6713	18220	0.169
200	5882	2341	10400	12190	3258	8865	0.164
asymptotic							0.182

TABLE 7
THERMAL FLUX FOR DIFFERENT STEP LENGTHS $\times 10^8$

time (s)	EL	NL	WL	ER	NR	WR	T
150 ($\delta t=10$)	11980	4772	21120	25070	6713	18220	0.169
150 ($\delta t=5$)	12490	4972	22050	25950	6938	18860	0.165
200 ($\delta t=10$)	5882	2341	10400	12190	3258	8865	0.164
200 ($\delta t=5$)	5847	2326	10340	12120	3239	8815	0.164

we pursue a simpler approach to the asymmetric disturbance to produce a somewhat similar result.

Given a symmetric reactor, such as the one below



with boundary condition $\partial\phi/\partial n = 0$ or $\phi = 0$, the steady state equation is (compare with equation (5b))

$$\begin{aligned} & \{-\nabla \cdot D_g(\mathbf{x})\nabla + \sigma_{\text{remg}}(\mathbf{x})\} \phi_{g(i)}(\mathbf{x}) = \\ & = \sum_{g'} [\sigma_{sg' \rightarrow g}(\mathbf{x}) + \chi_g \frac{\nu}{k_{(i)}} \sigma_{fg'}(\mathbf{x})] \phi_{g'(i)}(\mathbf{x}) \quad , \quad (18) \end{aligned}$$

where $k_{(1)} = k$, the normal multiplication, $\phi_{g(1)}$ the corresponding eigenfunction, and $k_{(2)} = dk_{(1)}$, the second highest magnitude eigenvalue, $\phi_{g(2)}$ the eigenfunction, with $d =$ the dominance ratio.

Consider an asymmetric disturbance in L_1 corresponding to a pure absorption effect, $\sigma'_{\text{remg}} = \sigma_{\text{remg}} + \delta\sigma_{\text{remg}}$, then the steady state is retained by changing the multiplication to k' , the first eigenvalue of the equation

$$\begin{aligned} & \{-\nabla \cdot D_g(\mathbf{x})\nabla + \sigma_{\text{remg}}(\mathbf{x}) + \delta\sigma_{\text{remg}}(\mathbf{x})\} \phi'_g(\mathbf{x}) = \\ & = \sum_{g'} [\sigma_{sg' \rightarrow g}(\mathbf{x}) + \chi_g \frac{\nu}{k'} \sigma_{fg'}(\mathbf{x})] \phi'_{g'}(\mathbf{x}) \quad . \quad (19) \end{aligned}$$

Let us assume that the disturbance does not encourage high harmonics, so that

$$\phi'_g(\mathbf{x}) = a\phi_{g(1)}(\mathbf{x}) + b\phi_{g(2)}(\mathbf{x}) \quad (20)$$

is a reasonable approximation with a and b yet to be determined.

Introducing the half source terms

$$F^{(i)}(V) = \sum_g \int_V \frac{v}{k_{(i)}} \sigma_{fg}(\mathcal{X}) \phi_{g(i)}(\mathcal{X}) d\mathcal{X} \quad (V = L \text{ or } R) \quad (21)$$

$$\text{and} \quad F'(V) = \sum_g \int_V \frac{v}{k'} \sigma_{fg}(\mathcal{X}) \phi'_g(\mathcal{X}) d\mathcal{X} \quad , \quad (22)$$

we adopt the arbitrary normalisations (as in equation (16))

$$F^{(1)}(V) = \frac{1}{2} \quad \text{since the 1st harmonic flux} \\ \text{is symmetric,} \quad (23)$$

$$F^{(2)}(L) = -\frac{1}{2} \quad , \quad F^{(2)}(R) = \frac{1}{2} \quad \text{since the 2nd harmonic} \\ \text{flux is asymmetric, and} \quad (24)$$

$$F'(L) + F'(R) = 1 \quad ; \quad (25)$$

equation (20) then gives

$$k' = ak_{(1)} \quad . \quad (26)$$

If equation (20) held strictly, we would have for the right half of the reactor (R)

$$aF^{(1)}(R) \left(\frac{k_{(1)}}{k'} - 1 \right) + bF^{(2)}(R) \left(\frac{k_{(2)}}{k'} - 1 \right) = 0 \quad , \quad (27)$$

which is obtained by subtracting a and b proportions of equation (18) from equation (19) and integrating. Hence

$$\frac{1}{2}a \left(\frac{k_{(1)}}{k'} - 1 \right) + \frac{1}{2}b \left(\frac{k_{(2)}}{k'} - 1 \right) = 0 \quad . \quad (28)$$

Now the reactivity, ρ' , ($=\beta\rho$ of equation (15)) is

$$\rho' = \frac{k' - k_{(1)}}{k'} \quad , \quad (29)$$

$$\text{therefore } a = 1/(1-\rho') \quad (30)$$

$$\text{and} \quad b = -\rho' / \{ (1-\rho') [1-d(1-\rho')] \} \quad . \quad (31)$$

An overall, normalisation independent, tilt indicator is taken similar to equation (17), namely

$$T' = \frac{F'(R) - F'(L)}{F'(R) + F'(L)} \quad , \quad (32)$$

$$\text{giving} \quad T' = db/a \quad (33)$$

and finally

$$T' = -\rho'd/[1-d(1-\rho')] \quad (34)$$

which is the required result.

The connection given by equation (34) differs slightly from that of Wade & Rydin [1972] because of the appearance of the additional factor $(1-\rho')$ in the denominator, which is normally quite close to one. Wade & Rydin demonstrate the applicability of equation (34) over a wide range of variation of ρ' and d by a series of 1D calculations.

Using the Moata calculations of Section 7 as an example, we determine $k_{(2)} = 0.97863 = k$ of an isolated right hand half of the reactor, hence $d = 0.9668$ which compares well with the result required for Chebyshev extrapolation and estimated during the full symmetric reactor calculation (starting from an asymmetric trial flux corresponding to the left hand safety rod inserted), $d = 0.9664$. We then have T' (equation (34)) = 0.213 (0.176 without the additional $(1-\rho')$ factor), which is in very good agreement with the directly estimated quantity T' (equation (32)) = 0.214.

It is perhaps not surprising that the thermal flux local tilt indicator at position NR compared with NL, T (equation (17)) = 0.182 is only in approximate agreement with the overall tilt indicator of equation (32). Accepting the estimates of a and b made available through equations (30) and (31), $a = 0.9940$ and $b = 0.2189$, with the local flux estimate of ϕ'_g given by equation (20), we calculate for the local tilt $T = 0.184$. Again very good agreement is obtained, suggesting the adequacy of the approximation (20).

Equation (34) ultimately breaks down since for

$$\rho'_L = -\frac{(1-d)}{d} \quad (\text{corresponding to completely removing } L), \quad (35)$$

we have $T' \rightarrow \infty$ instead of the actual limit $T' \rightarrow 1$. This suggests that results from equation (34) should be ignored if T' exceeds 1, that is if

$$-\rho' > -\rho'_L/2 \quad . \quad (36)$$

As an example close to the limit expressed by equation (36), we return to the asymmetrically disturbed, extremely loosely coupled 1D system of Section 4. We have $\rho' = -0.000707$, $d = 0.99844$, $\rho'_L/2 = -0.000781$, therefore $a = 0.99929$ and $b = 0.82709$, giving an estimate for the thermal peak flux tilt (mesh position 99 compared with 23) of $T = 0.895$ (0.453 without the $(1-\rho')$ factor), which is within reasonable agreement

of the directly estimated quantity $T = 0.870$.

9. CONCLUSIONS

The checkout of POW with results of other multidimensional kinetic codes for benchmark type calculations indicates that POW is correctly solving the steady state and kinetic equations. It is to be hoped that further benchmark calculations of the type undertaken here will be tackled by laboratories with fast computing equipment as confidence in the results of running such a code needs to rest on adequate code checkout.

10. ACKNOWLEDGEMENT

Thanks are expressed to Mr Robert Knott, Physics Division, AAEC, for setting up the Moata experiments used by the Summer School to yield results reported in Figure 9.

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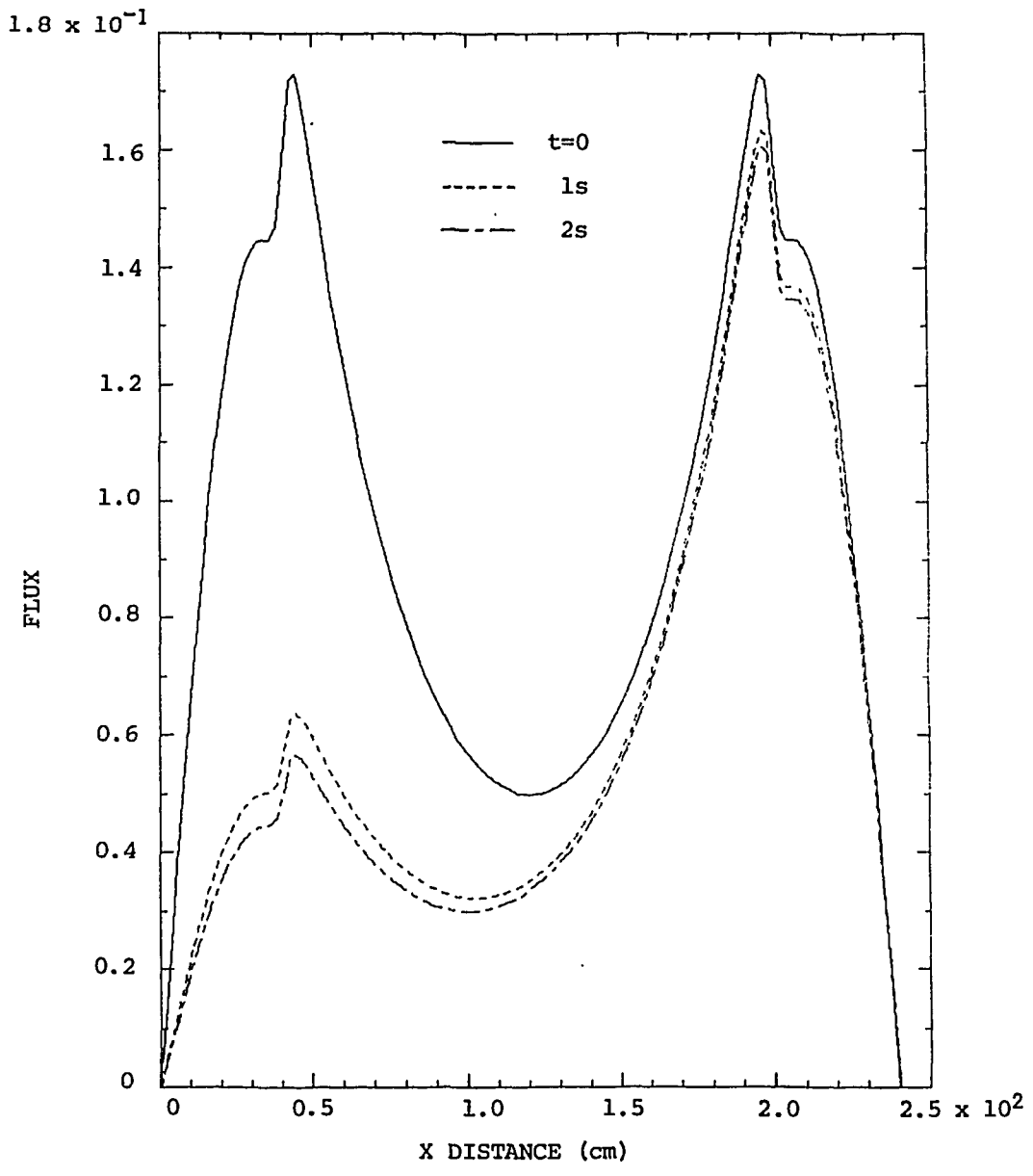


FIGURE 1. 1D BENCHMARK FLUX PLOT

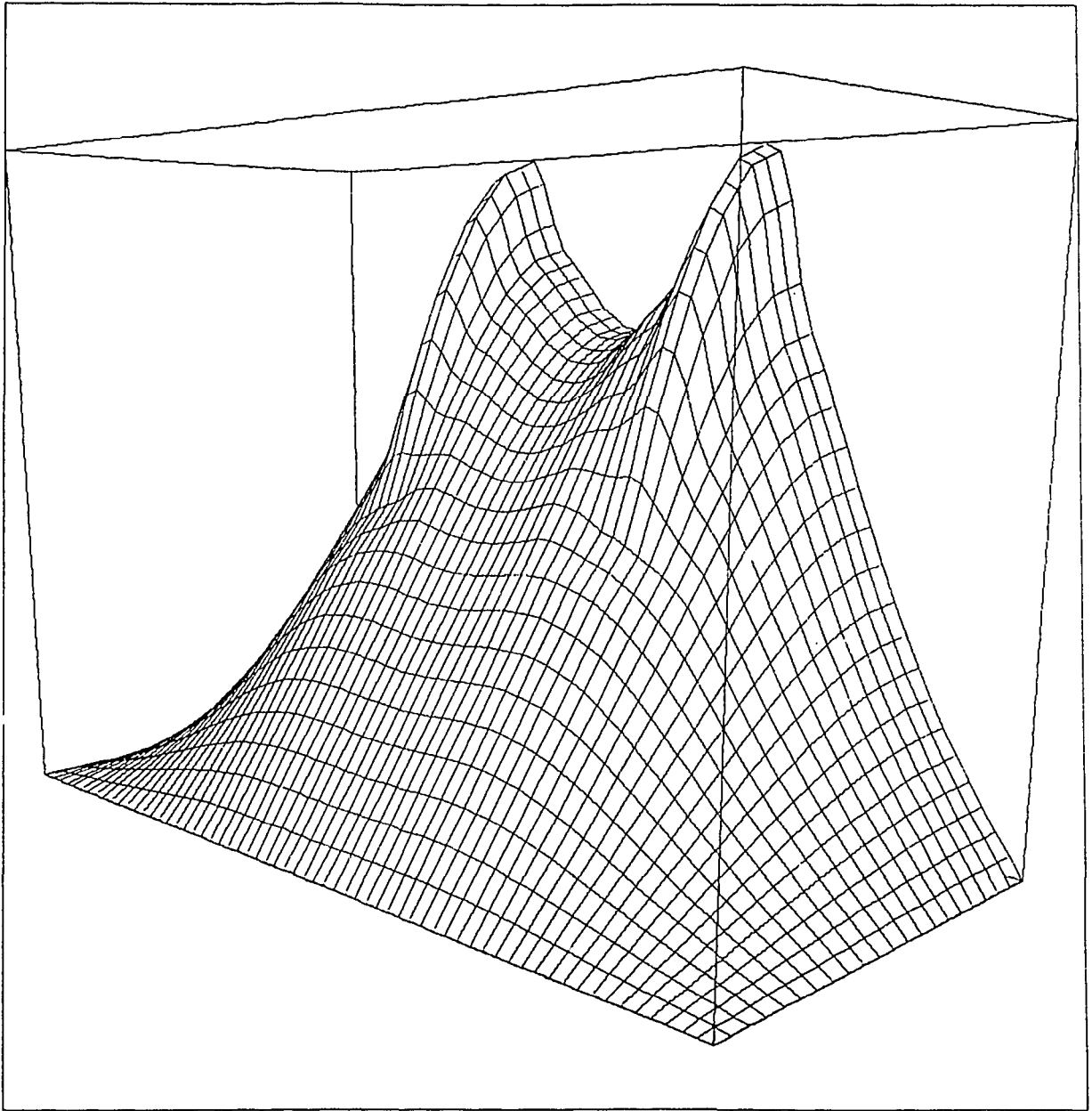


FIGURE 2. 2D MOATA FLUX PLOT, $t = 0$

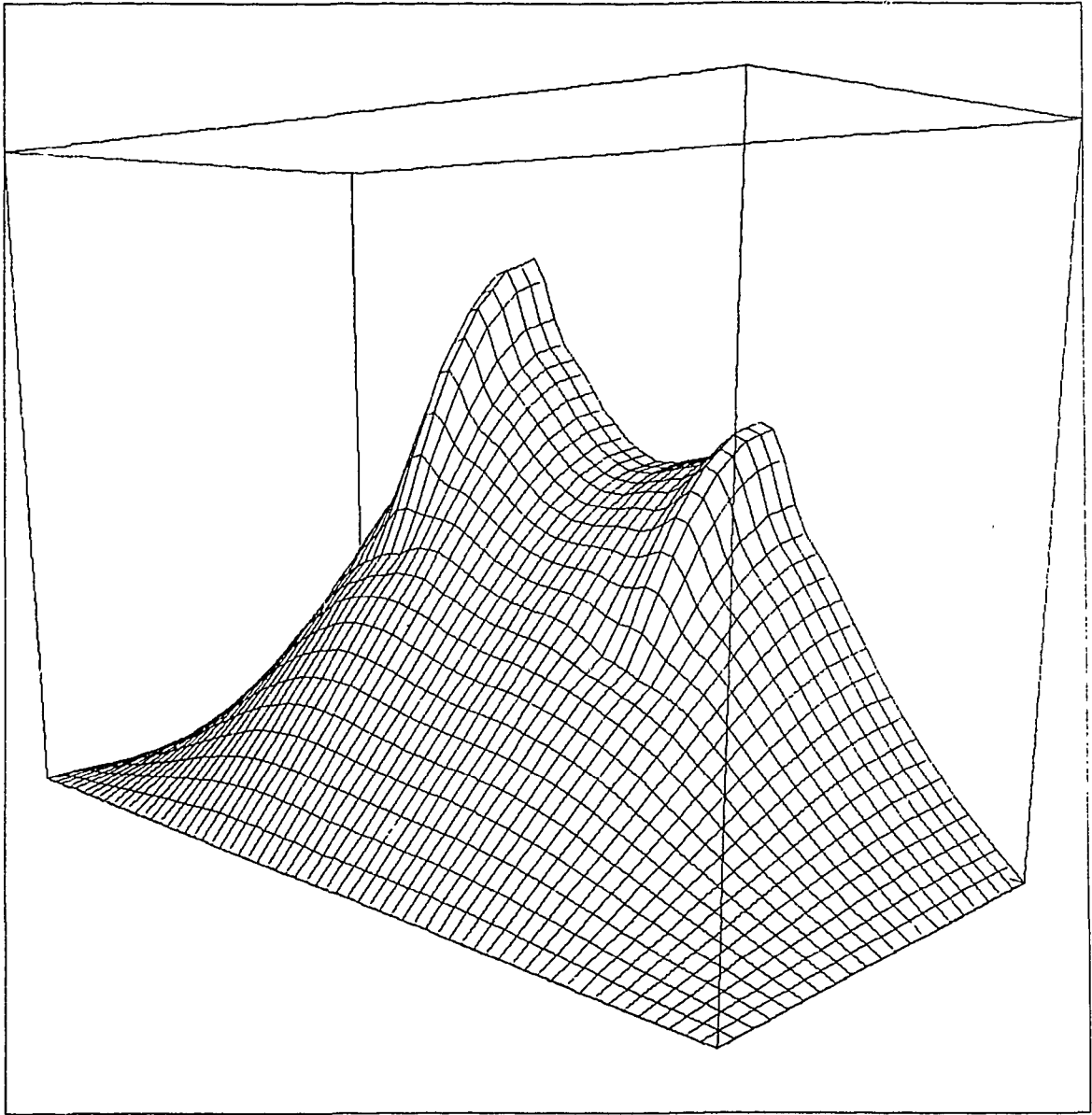


FIGURE 3. 2D MOATA FLUX PLOT, $t = 10^{-2}$ s

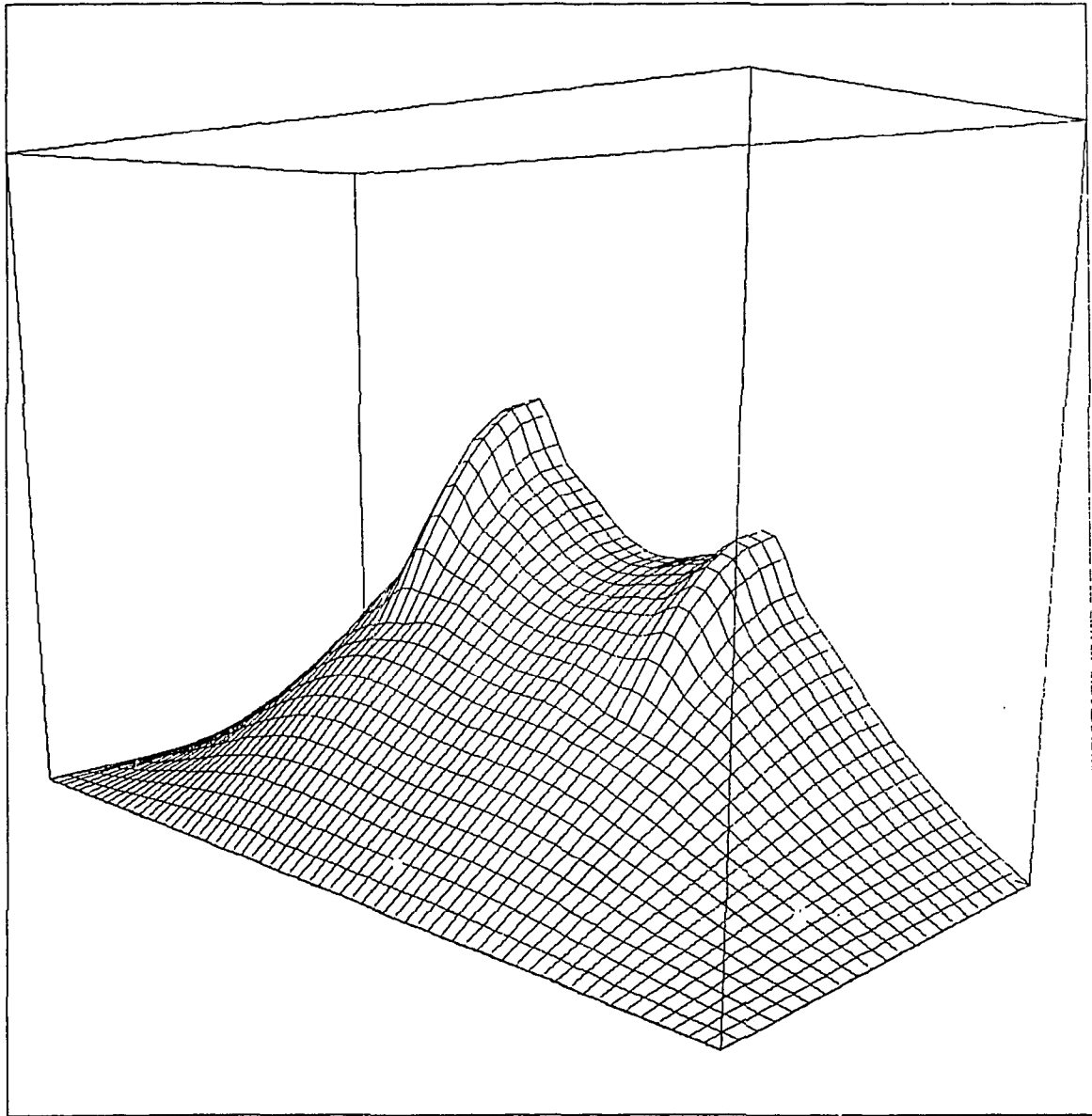


FIGURE 4. 2D MOATA FLUX PLOT, $t = 10^{-1} s$

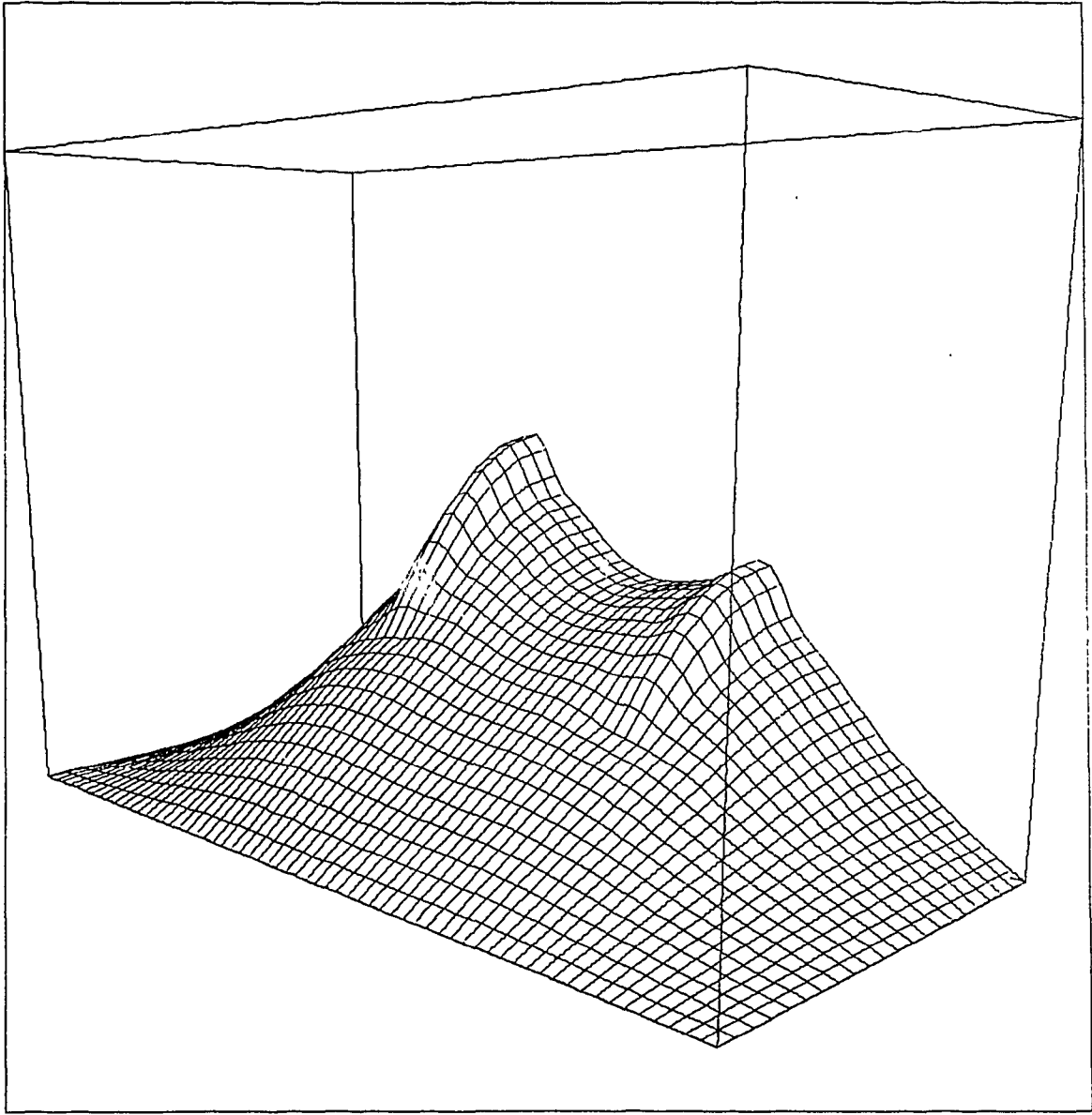


FIGURE 5. 2D MOATA FLUX PLOT, $t = 1$ s

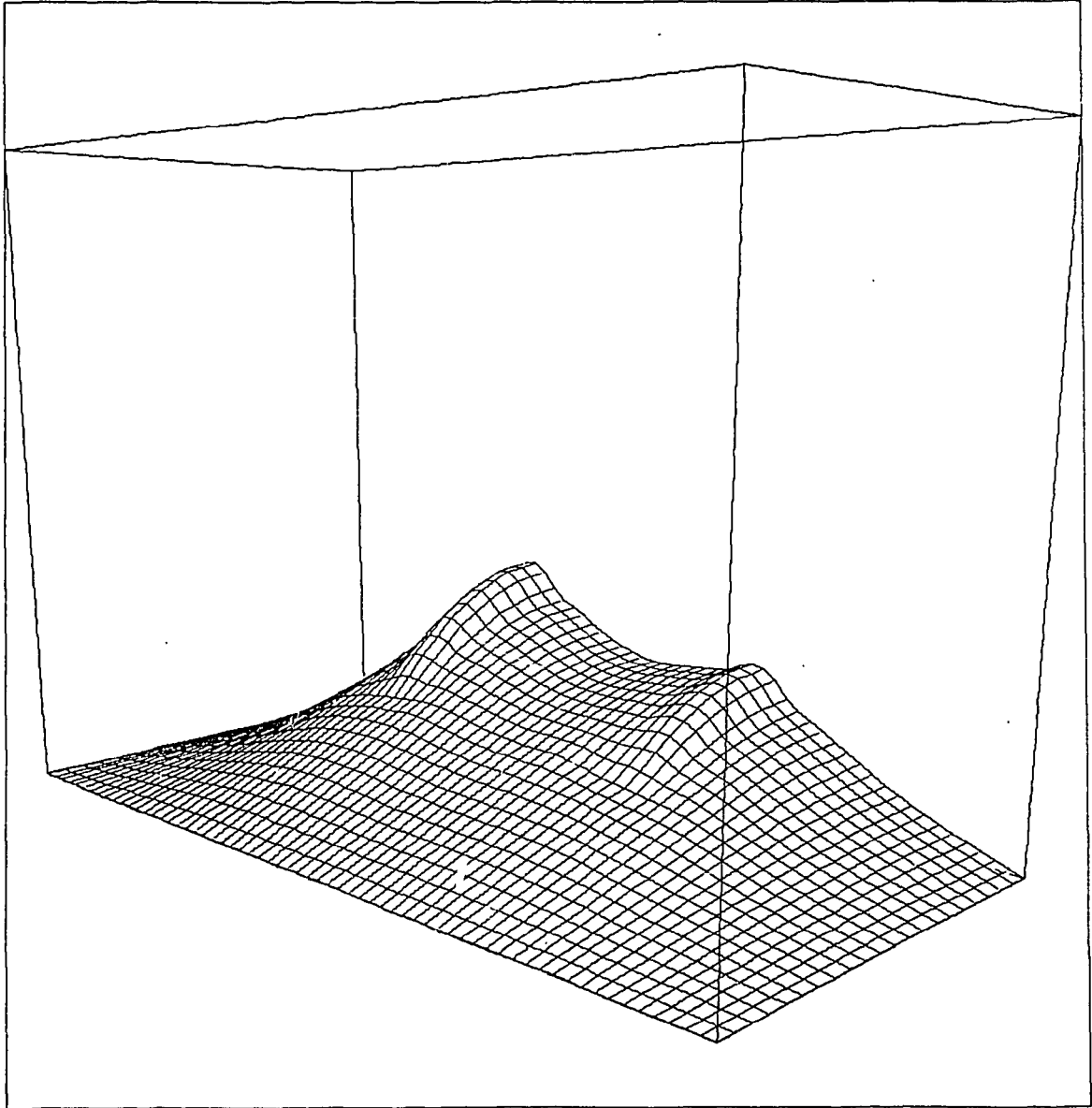


FIGURE 6. 2D MOATA FLUX PLOT, $t = 10$ s

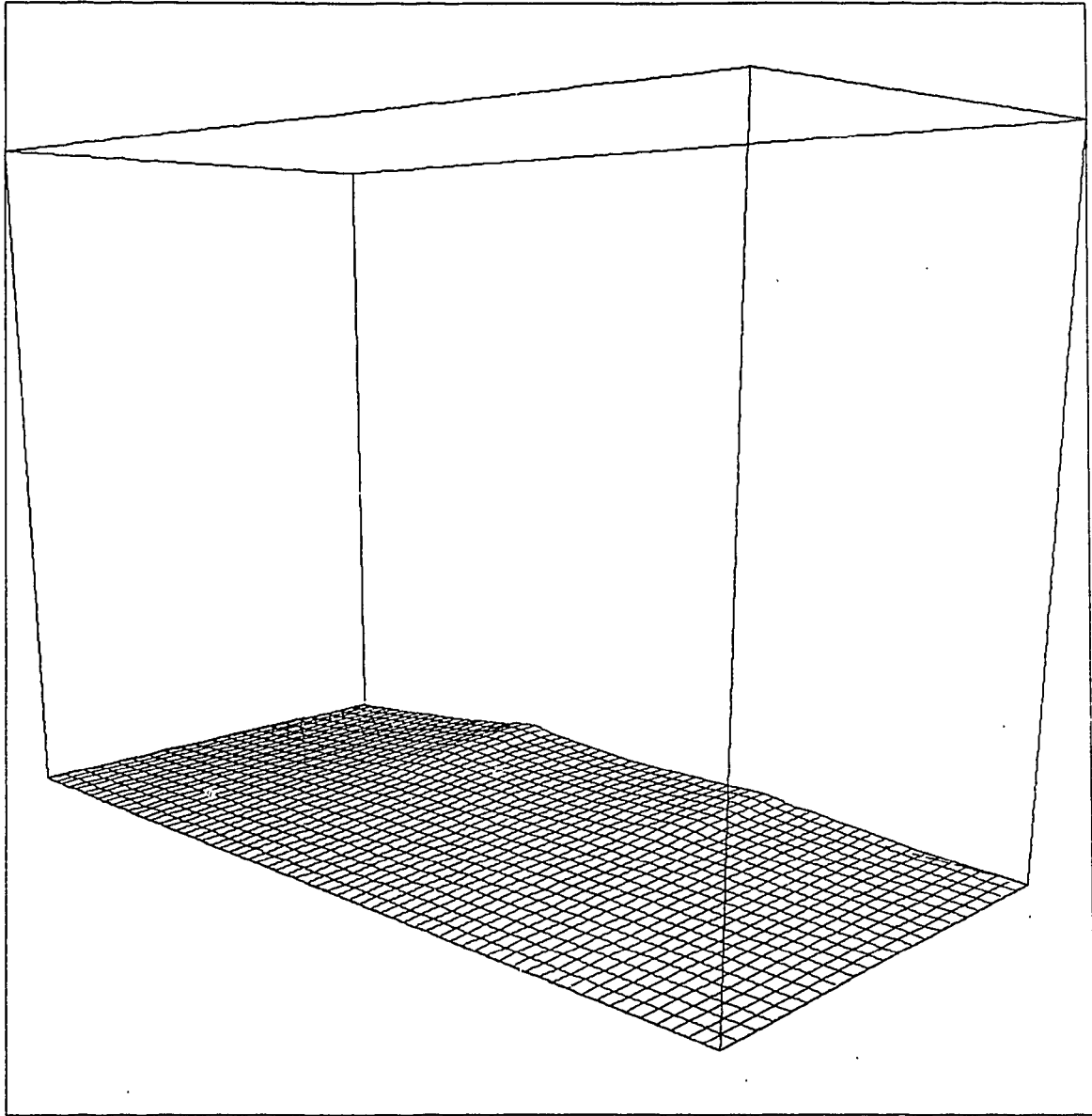


FIGURE 7. 2D MOATA FLUX PLOT, $t = 100$ s

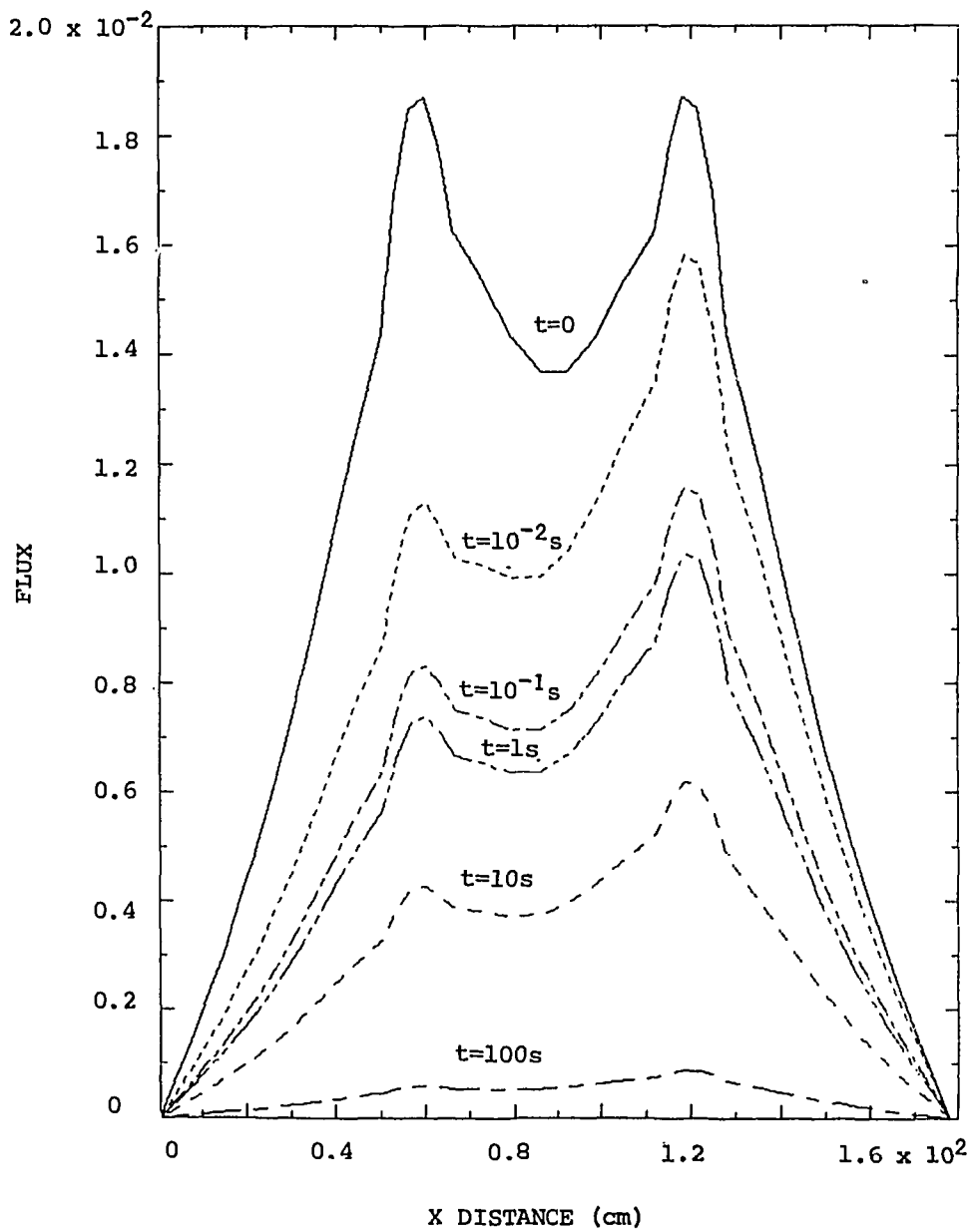


FIGURE 8. MOATA FLUX ALONG X-AXIS

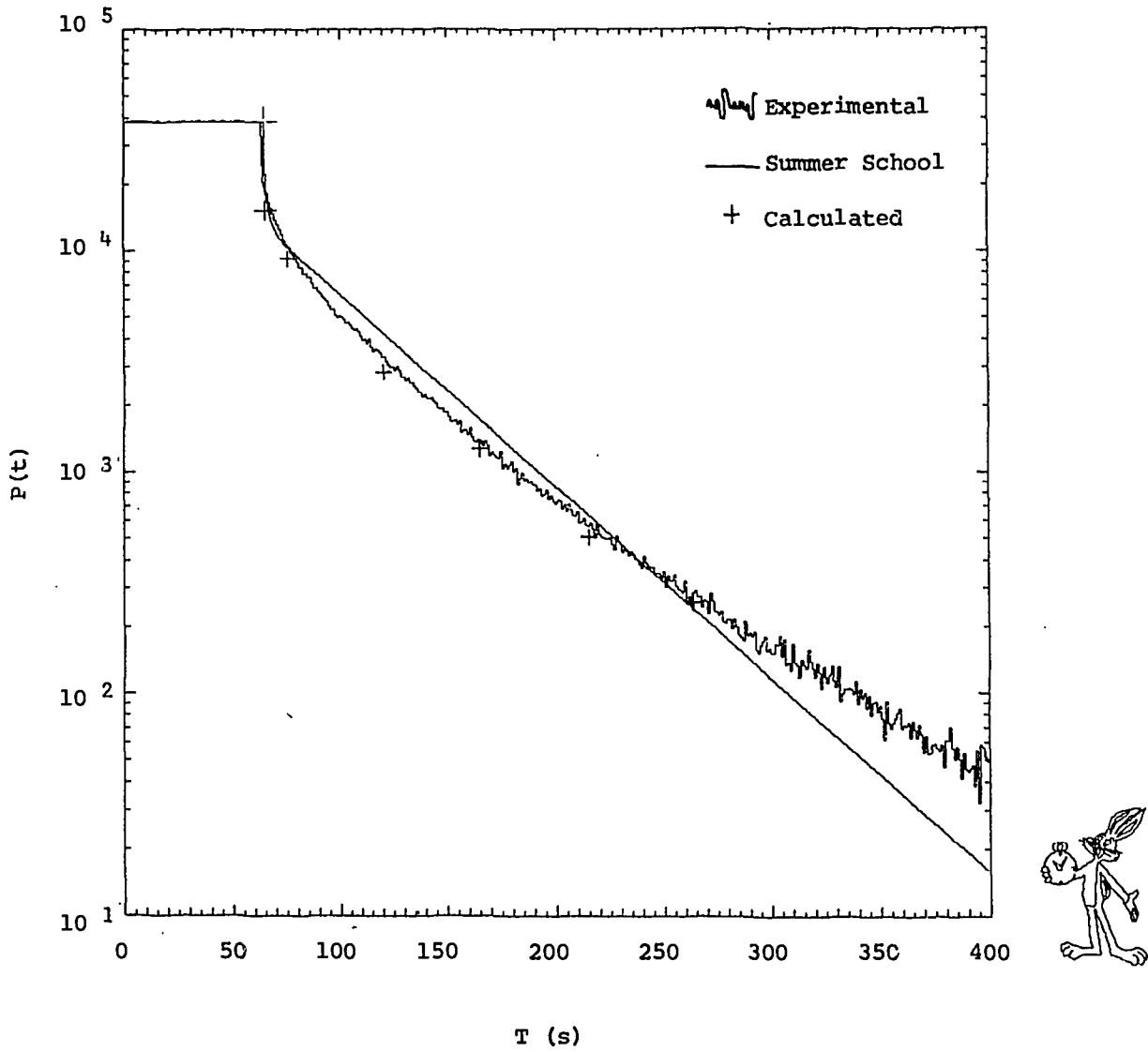


FIGURE 9. MOATA FLUX AT POSITION N1 AGAINST TIME

APPENDIX A

POW INPUT DATA LISTING FOR MOATA CALCULATION

```

//JPPMOAT2 JOB ('PH100930/P22PHNRA',N1),J.P.POLLARD,
//          CLASS=J,
//          TIME=180
// EXEC BUFFPROG,PRG=VPLOT
// EXEC AUS,FL1='AUS,JPPOWFLX',DISPFL1=OLD
//GO.SYSIN DD *
*DD1
STEP *
      LINK POW(1,2)
      END
STOP
*DD2
PRELUDE MAXX=32,MAXY=11,MAXG=2,MAXF=1,MAXGD=6,MAXM=5 END
POW MOATA2G MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)
* 2GP XS DATA FOLLOWS
* COLLAPSED FROM 4GP DATA OF CONNOLLY (SEE POW REPORT,PG E3)
* REM INCLUDES LEAKAGE BASED ON EXPTL AXIAL BSQ OF 0.00184
NG=2
XSD FUELLT M(1)
2.436-1,1.558
3.9655-2,5.5971-2
2.130-3,8.074-2
0,3.532-2
6.302-4,0
DEFN FUELRT M(2)=M(1)
XSD C M(3)
2.790-1,3.646-1
5.6669-3,2.0810-3
2*0
0,3.461-3
1.651-4,0
* VEL ADJUSTED FROM FUEL CELL CALC TO ALLOW FOR C REFL
VEL 9.+6,2.3+5
SP 1,0
* 6GP EFFECTIVE DELAYED DATA FOLLOWS
* DERIVED FROM DATA OF KEEPIN (1965) AND TOMLINSON (1972)
IGD=6
SD
DLAMDA
1.24-2,3.05-2,1.11-1,3.01-1,1.14,3.01
BETAD
2.511-4,1.664-3,1.495-3,2.975-3,8.571-4,3.113-4
XM=1.-6,7*7.1428571,5*3.222,7*6.5314285,5*3.222,7*7.1428571,1.-6
YM=0,5*5,1912,5*5.968,1.-6
REG MX=1(1)31 MY=1(1)10 M(3)
REG MX=8(1)12 MY=1(1)5 M(1) MX=20(1)24 M(2)
OUTPUT DATA PRINT FLUX
RESTART
GROUPS 1 2,2
MREG -1=1,2,3
PLOT -3,135
PUREJY 0 3 3 10 10 0.7 0.8 3 1.-20 -1
EDIT -1,0,0,0
* ADJUST ABSORBER TO GIVE REQD REACTIVITY
* AKEFF=AKEFF*0.993934 IE -80 CENTS WITH BETA=7.5535-3
LIST AKEFF END
COMPUTE (0,1,0)=(0,1,0)(2,0,0.993934) 9
SEARCH(-0,01) #AKEFF 1,-4
DEFN M(5)=M(1) XSD MODIFY M(5)=3*1.-20,0.012,6*1.-20
WSEA 4*0,1 DEFN FUELLT WSEA M(1)=M(2) M(5)

```



```
OUTPUT DATA PRINT
START
* RESTORE SYMMETRIC T=0 FLUX
SEARCH(0)
DEFN FUELLT M(1)=M(2)
RESTART
SEARCH(-0.01)
WSEA 4*0,1 DEFN FUELLT WSEA M(1)=M(2) M(5)
CALC=REAL,KINETICS
OUTPUT DATA PRINT FLUX
IOUT 0 5 10
PULSE(21)=(0,1),(1000,1)
REACP -21,1,0,0
PULSE(22)=(0,1.-3),(0.99999-2,1.-3),(0.99999-2,9.-3),(0.99999-1,9.-3),
(0.99999-1,9.-2),(0.99999,9.-2),(0.99999,0.9),(9.9999,0.9),
(9.9999,9.),(99.999,9.),(99.999,10.),(1000,10.)
DTS -22,1,200
ACCF0=1,-3,ACCLAM=1,-3
RESTART
TLIM=0.9
NIL=0 OUTPUT
AFTER 1 RESTART
PLOT -3 EDIT -1 0 0 0
AFTER 2 RESTART
PLOT -3 EDIT -1 0 0 0
AFTER 3 RESTART
PLOT -3 EDIT -1 0 0 0
AFTER 4 RESTART
PLOT -3 EDIT -1 0 0 0
AFTER 5 RESTART
PLOT -3 EDIT -1 0 0 0
* COLLECTION OF PLOTS OF THERMAL FLUX ALONG X-AXIS
XYPLT -6 -8
MSEA 2 1 1 0(1)5 CALL SUB6
STOP
/*
//
//
```

APPENDIX B

SOME USEFUL OUTPUT FROM MOATA CALCULATION

LAYOUT INCLUDING DETECTOR POSITIONS

POW MOATA2G MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)

LAYOUT OF XY REACTOR SEGMENT

0.00

Y****

EXTRAPOLATION DISTANCES 0.00 Y * 0.00 IN TR.M.F-PATHS (0.0 IMPLIES ZERO CURRENT CONDITION)

*XXXX

0.0

NOTE THAT I=M(1),A=M(10),LJ=M(36) AND THAT THE FISSILE MATS ARE IN HEAVY TYPE

		1											2											3									
		1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	
10		3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
9		2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
8		3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
7		3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
6		3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
5		3	3	3	3	3	3	1	1	1	1	1	1	3	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3	3	
4		3	3	3	3	3	3	1	1	1	1	1	1	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3	3		
3		3	3	3	3	3	3	1	1	1	1	1	1	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3	3		
2		3	3	3	3	3	3	1	1	1	1	1	1	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3	3		
1		3	3	3	3	3	3	1	1	1	1	1	1	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3	3		

SILK METHOD TO BE USED FOR 2D DIFFUSION EQN

SLOP WITH LINE= 2 , I.E. * * ,GROUP AND REGION REBALANCE WITH *

LLLLL

*LX= 5 SEG BET LINES 1 8 13 20 25 32

*LY= 3 SEG BET LINES 1 3 6 11

POW MDATA2G MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1)

MESH POINT COORDINATES

*** DESIGNATES BOUNDARY CONDITION -TR.M.F.PATHS (0.0 IMPLIES ZERO CURRENT CONDITION)

X MESH POINT	MESH WIDTH (CMS)	POSITION (CMS)	Y MESH POINT	MESH WIDTH (CMS)	POSITION (CMS)
***	1.00000E-06		***	0.0	
1	7.14286E+00	0.0	1	5.19120E+00	0.0
2	7.14286E+00	7.14286E+00	2	5.19120E+00	5.19120E+00
3	7.14286E+00	1.42857E+01	3	5.19120E+00	1.03824E+01
4	7.14286E+00	2.14286E+01	4	5.19120E+00	1.55736E+01
5	7.14286E+00	2.85714E+01	5	5.19120E+00	2.07648E+01
6	7.14286E+00	3.57143E+01	6	5.96800E+00	2.59560E+01
7	7.14286E+00	4.28571E+01	7	5.96800E+00	3.19240E+01
8	3.22200E+00	5.00000E+01	8	5.96800E+00	3.78920E+01
9	3.22200E+00	5.32220E+01	9	5.96800E+00	4.38600E+01
10	3.22200E+00	5.64440E+01	10	5.96800E+00	4.98280E+01
11	3.22200E+00	5.96660E+01	11	1.00000E-06	5.57960E+01
12	3.22200E+00	6.28880E+01	***		
13	6.53143E+00	6.61100E+01			
14	6.53143E+00	7.26414E+01			
15	6.53143E+00	7.91729E+01			
16	6.53143E+00	8.57043E+01			
17	6.53143E+00	9.22357E+01			
18	6.53143E+00	9.87671E+01			
19	6.53143E+00	1.05299E+02			
20	3.22200E+00	1.11830E+02			
21	3.22200E+00	1.15052E+02			
22	3.22200E+00	1.18274E+02			
23	3.22200E+00	1.21496E+02			
24	3.22200E+00	1.24718E+02			
25	7.14286E+00	1.27940E+02			
26	7.14286E+00	1.35083E+02			
27	7.14286E+00	1.42226E+02			
28	7.14286E+00	1.49369E+02			
29	7.14286E+00	1.56511E+02			
30	7.14286E+00	1.63654E+02			
31	7.14286E+00	1.70797E+02			
32	1.00000E-06	1.77940E+02			

OUTPUT DATA FOR PUW MDATA2G MDATA KINETICS CALC FOLLOWING ROD DROP IN M(1)

```
XSC FUELLT M( 1)
2.43600E-01 1.55800E+00
3.56550E-02 5.59710E-02
2.13000E-03 8.07400E-02
0.0 3.53200E-02
6.30200E-04 0.0
DCX( 1)
1.36836E+00 2.13950E-01
CCY( 1)
1.36836E+00 2.13950E-01

XSC FUELRT M( 2)
2.43600E-01 1.55800E+00
3.96550E-02 5.59710E-02
2.13000E-03 8.07400E-02
0.0 3.53200E-02
6.30200E-04 0.0
DCX( 2)
1.36836E+00 2.13950E-01
CCY( 2)
1.36836E+00 2.13950E-01

XSC C M( 3)
2.79000E-01 3.64600E-01
5.66690E-03 2.08100E-03
0.0 0.0
0.0 3.46100E-03
1.65100E-04 0.0
DCX( 3)
1.19474E+00 9.14244E-01
CCY( 3)
1.19474E+00 9.14244E-01

FER
3.17200E-11 3.17200E-11 0.0 0.0 0.0
SP
1.00000E+00 0.0
```

OUTPUT PRINT FOR PWR MOATA2G MOATA KINETICS CALC FOLLOWING ROD DROP IN M(1) CONVERGED

EDGE PEAL FLX (MULTIPLIED BY 1.E+08) FROM SOURCE OF IN/SEC FULLJWS

GROUP 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	7327	15957	27202	42246	61628	84135	105311	111907	116132	117687	116590	113190	101109	89004
9	0	14883	32576	56026	88156	130910	182716	233918	250018	266175	263463	257880	250391	218409	168038
8	0	22790	50266	87611	140648	214331	311175	415346	449065	469991	475531	465509	442311	369425	306249
7	0	30976	68939	122073	200785	317318	484595	692789	765300	809337	818602	792228	735081	571960	449483
6	0	39120	87846	158013	266282	437403	708069	1129627	1305084	1401172	1418620	1354677	1201152	833434	614509
5	0	45727	103367	188055	322318	542583	911097	1543146	1822745	1975538	1999515	1891502	1642631	1070500	758900
4	0	51465	118536	214558	372146	635174	1089097	1876105	2230271	2425976	2454309	2311477	1993427	1275578	886786
3	0	55914	127498	235254	411102	708854	1222891	2121390	2527492	2752770	2784030	2617018	2250614	1430990	985759
2	0	58733	134199	245415	435806	754650	1306773	2271781	2708854	2951592	2984562	2803248	2408009	1527462	1048086
1	0	59699	136495	252922	444260	770266	1335209	2322467	2769957	3018336	3051921	2865862	2461020	1560133	1069311

	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	81322	81524	89010	101118	113202	116604	117701	116146	111921	105325	84146	61636	42252	27206
9	0	170580	170584	188050	218429	250420	259311	263496	260208	250050	233948	182740	130928	88168	56034
8	0	271792	271799	306270	369402	442362	465586	475590	470051	449123	415401	311217	214860	140667	87623
7	0	387453	387464	449516	572019	735158	792325	818706	809442	765401	692882	484660	317851	200813	122090
6	0	513146	513181	614557	833523	1201307	1354847	1418803	1401358	1305260	1129781	708167	437463	266319	158035
5	0	620542	620551	758901	1070617	1642833	1891745	1999781	1975806	1822996	1543360	911224	542658	322363	188081
4	0	715376	715898	886859	1275720	1993676	2311779	2454640	2426311	2230582	1876368	1088249	636263	372198	214588
3	0	790314	790340	985881	1431151	2250897	2617362	2784408	2753153	2527848	2121689	1223063	708954	411160	235297
2	0	837479	837506	1048174	1527635	2408314	2803619	2984970	2952004	2709236	2272103	1306957	754757	435868	248450
1	0	853612	853640	1069401	1500310	2461332	2866242	3052338	3018808	2770248	2322795	1335398	770375	444323	252957

	31	32
11	0	0
10	7328	0
9	14885	0
8	22793	0
7	30980	0
6	39125	0
5	45733	0
4	51472	0
3	55922	0
2	58742	0
1	59707	0

B4

GR CUP 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	20491	42021	85331	90527	116703	141743	162725	170063	175959	180368	183331	184969	184810	182384
9	0	40931	84053	130975	182014	235349	286465	328980	343772	355624	364452	370349	373561	372852	367234
8	0	61152	125853	196810	274766	356975	435778	499720	521729	539412	552615	561557	566737	565811	555706
7	0	30780	166712	231902	367861	430952	588838	669545	696278	718930	736050	748514	758978	762482	746550
6	0	99185	205328	324250	458726	604701	743271	820873	838082	866591	884752	896189	933436	960075	935631
5	0	113493	235568	373703	532281	708453	883577	1006175	1128877	1212250	1230030	1188272	1147835	1138546	1092819
4	0	125564	261231	416097	596311	800861	1012879	1183024	1371244	1485695	1506050	1439261	1345098	1300662	1231585
3	0	134751	280847	448737	646099	873548	1115424	1321073	1550260	1684772	1707114	1624701	1497349	1427995	1340002
2	0	140507	293171	469335	677688	919882	1180767	1407699	1659972	1806036	1829562	1738243	1592498	1508709	1408827
1	0	142468	297376	476376	688511	935782	1203157	1437149	1696932	1846791	1870710	1776478	1624802	1536305	1432400
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	180340	180343	182392	184823	184985	183349	180307	175978	170083	162745	141761	116718	90539	65340	42026
9	362559	362565	367251	372889	373596	370387	364492	355654	343813	329020	286501	235380	182038	130993	84064
8	547264	547273	555733	565853	566791	561625	552677	539475	521792	499782	435835	357023	274803	196837	125870
7	732632	732645	746587	762540	759052	748594	736134	719017	696365	669631	588917	481017	367911	261938	166735
6	913524	913540	935679	960151	933531	896291	884859	866701	838192	820982	743371	604783	458789	324294	205356
5	1059808	1059828	1092877	1138639	1147954	1188419	1230191	1212414	1129031	1006313	883699	708550	532354	373755	235601
4	1186929	1186951	1231652	1300771	1345242	1439442	1506251	1485899	1371434	1183189	1013020	800972	596393	416155	261267
3	1285555	1285624	1340077	1428117	1497512	1624908	1707344	1685005	1550477	1321258	1115580	873669	646189	448799	280886
2	1348118	1348144	1408906	1508839	1592672	1738466	1829810	1806287	1650205	1407897	1180932	920010	677782	469400	293212
1	1369524	1369551	1432480	1536438	1624980	1776707	1870963	1847048	1697171	1437352	1203326	935912	688607	476442	297417
	31	32													
11	0	0													
10	20494	0													
9	40936	0													
8	61160	0													
7	80791	0													
6	99198	0													
5	113508	0													
4	125581	0													
3	134770	0													
2	140526	0													
1	142488	0													

B5

APPENDIX C

2D FAST REACTOR BENCHMARK WITH FEEDBACK

Since the main study reported here, a further series of benchmark studies has come to the author's attention [Buffoni, et al. 1975]. Benchmark problem 1 of this series, with the descriptive title, 'Superprompt critical transient with feedback, 2 group neutron diffusion problem in fast reactor', was undertaken with POW. The fast reactor benchmark is well documented, and affords POW the opportunity to tackle a simple single temperature feedback model. In addition, four laboratories have submitted four different sets of solutions, and so a further contribution is warranted.

Rather than specify all of the data used by POW, we will be content to indicate only those data which are not detailed as part of the benchmark problem, namely:

- (1) r-mesh spacing in cm (5* means 5 of the numbers that follow)
5*3.4, 7*3., 10*3.5374, 10*4.,
- (2) z-mesh spacing in cm
10*4., 10*3.85, 4*3.25, 10*3.85, 10*4.,
- (3) number of spatial solution points is 1485, and
- (4) temporal step length is $\delta t = 2.5 \times 10^{-4}$ s for $t < 20 \times 10^{-3}$ s,
otherwise $\delta t = 5 \times 10^{-3}$ s.

Results of the POW calculation are shown in Tables C1 and C2 and Figures C1 to C3. These results are considered acceptable compared with those of the other contributors, namely

UK-RIS, Fletcher using the code SPARK,
CRNL, McDonnell using the code ADEP,
CSN, Buffoni using the code NADYP-A, and
GfK, V \ddot{a} th using the code KINTIC-1.

POW gave a steady state multiplication of $k=1.000093$, with flux almost the same as the plotted GfK results and used 130 minutes of CPU time (now on an IBM360/65) to calculate the reactor behaviour to $t=100 \times 10^{-3}$ s.

REFERENCE

Buffoni, A., Fletcher, J.K., Galati, A., McDonnell, F.N., Musco, A. & V \ddot{a} th, L. [1975] - Review of kinetics benchmark calculations. In 'Proceedings of the Joint NEACRP/CSNI specialists' meeting on new developments in three-dimensional neutron kinetics and

review of kinetics benchmark calculations'. Technische
University ät München, MRR 145, pp.505-551.

TABLE C1

CASE 1. 2-GROUP SPECTRA (%) AT $z = 88.25$ cm FOR START AND END
OF TRANSIENT IN CENTRE OF REACTOR AND AT POSITION OF MOVABLE ABSORBER

t (s)	Calculated by	r = 0		r = 20 cm	
		Group 1	Group 2	Group 1	Group 2
0	UK-RIS	86.0	14.0	86.1	13.9
	CRNL	86.0	14.0	86.0	14.0
	CSN	85.8	14.2	85.8	14.2
	GfK	86.1	13.9	86.0	14.0
	AAEC	86.0	14.0	85.9	14.1
100×10^{-3}	UK-RIS	86.2	13.8	84.5	15.5
	CRNL	-	-	-	-
	CSN	86.5	13.5	85.1	14.9
	GfK	86.2	13.8	84.5	15.5
	AAEC	86.2	13.8	84.5	15.5

TABLE C2

CASE 1. TIME AND GROUP DEPENDENT FLUX SHAPE INDICATOR,
 $\phi_G (r = 20, z = 88.25, t) / \phi_G (r = 38, z = 88.25, t)$

t (s)	Calculated by	Group 1	Group 2
0	UK-RIS	0.87	0.85
	CRNL	0.90	0.90
	CSN	0.88	0.93
	GfK	0.88	0.89
	AAEC	0.87	0.90
100×10^{-3}	UK-RIS	0.96	1.12
	CRNL	-	-
	CSN	0.92	1.05
	GfK	0.93	1.12
	AAEC	0.93	1.11

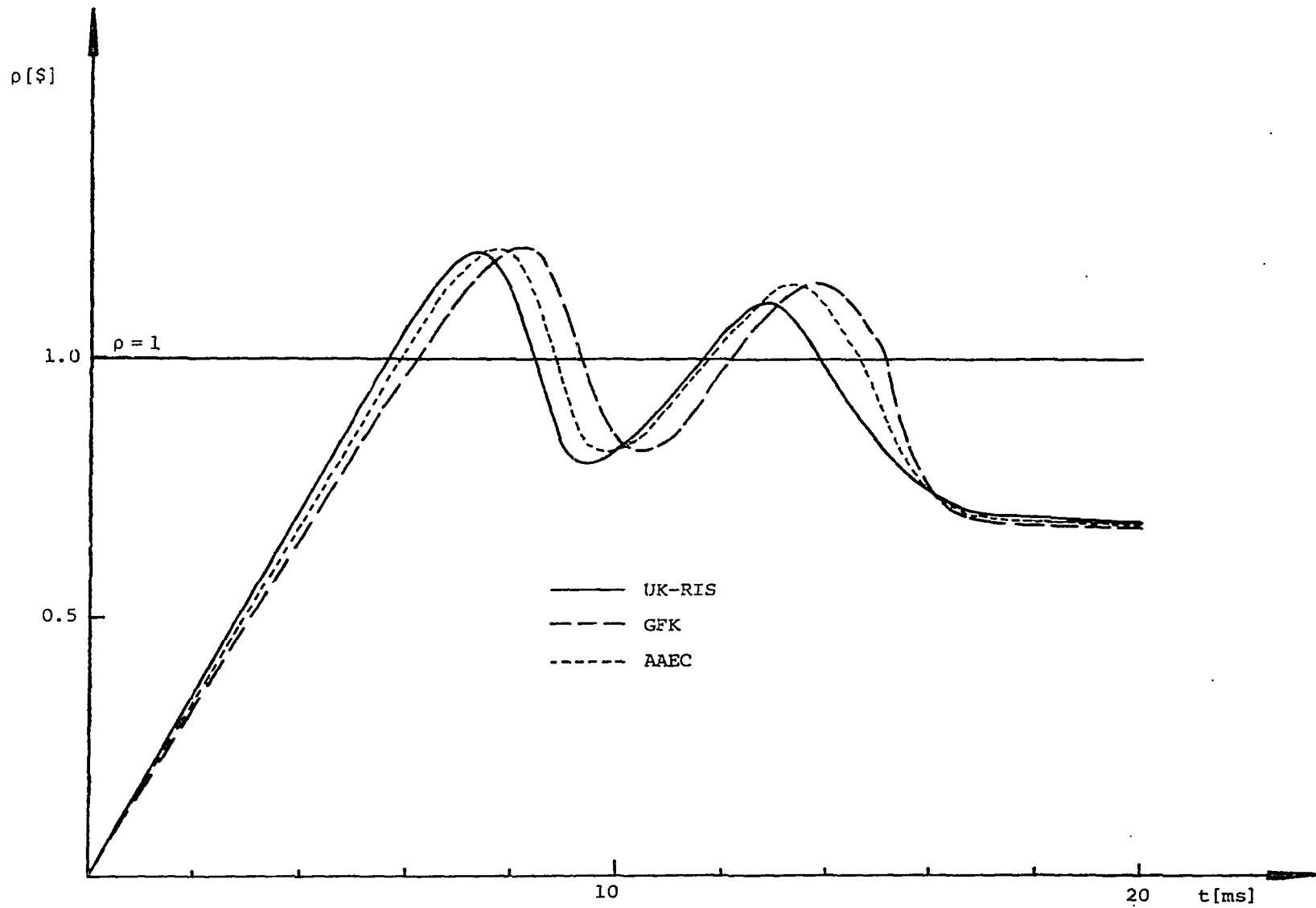


FIGURE C1. 2D FAST REACTOR BENCHMARK 1, $\rho(t)$

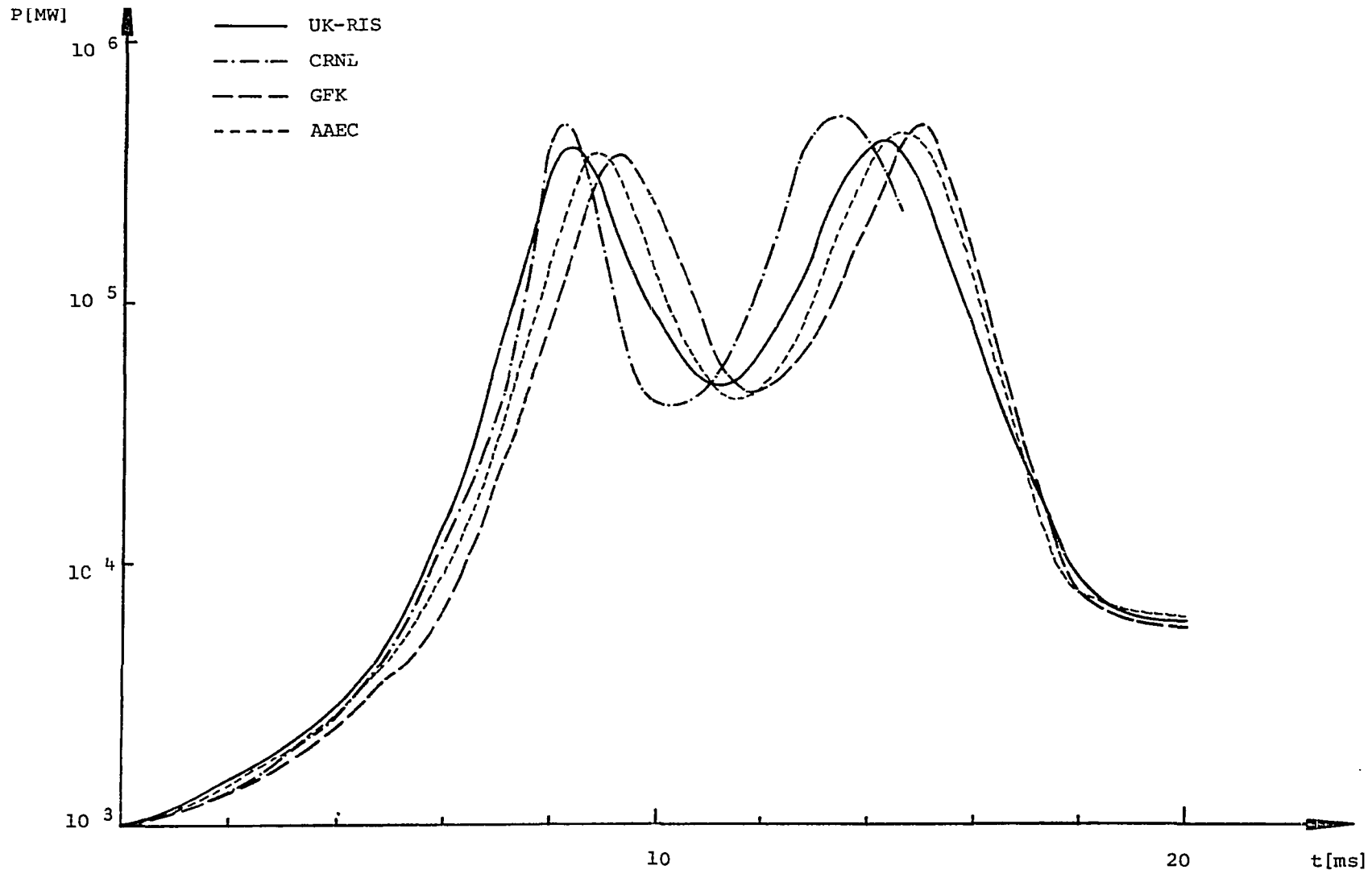


FIGURE C2. 2D FAST REACTOR BENCHMARK 1, POWER(t)

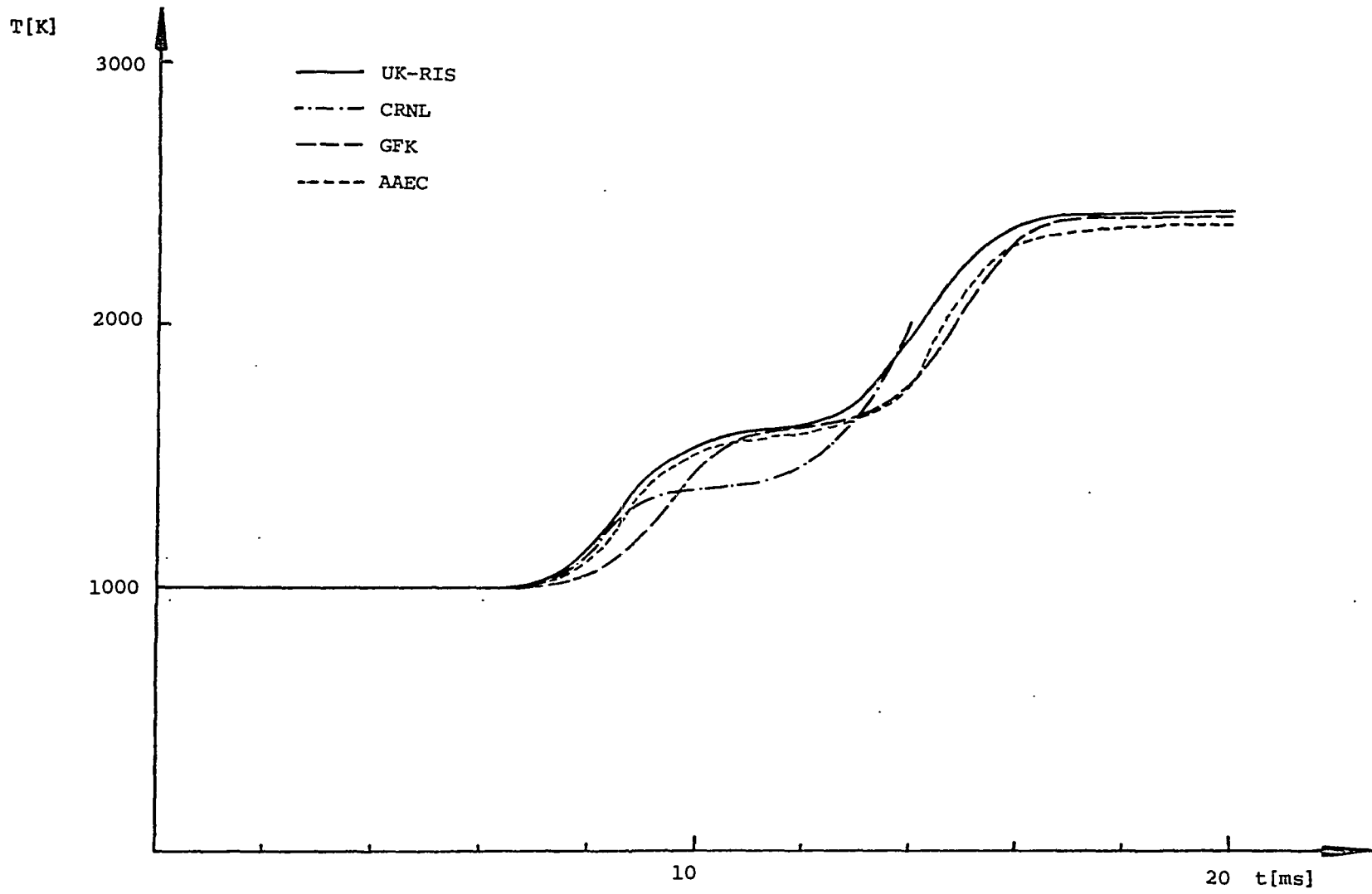


FIGURE C3. 2D FAST REACTOR BENCHMARK 1, TEMP.(t)

