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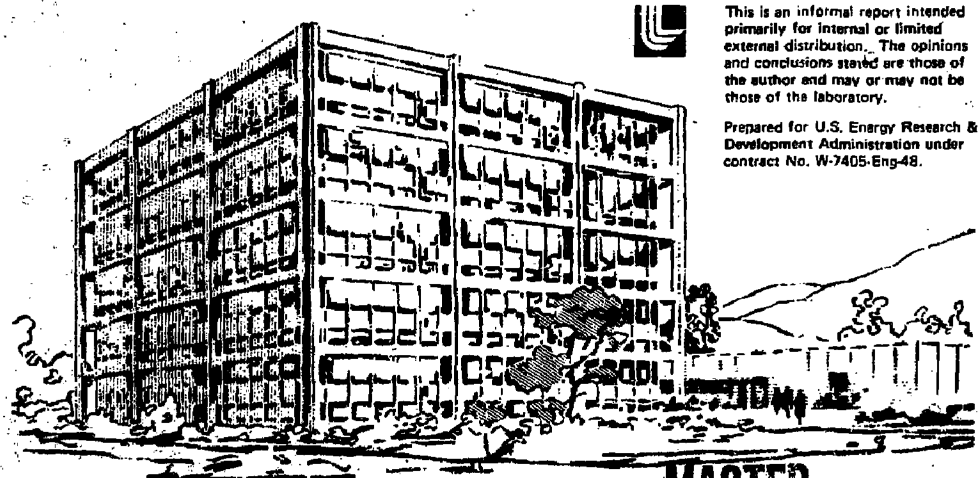

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Beam-Plasma Instability in Ion Beam Systems Used in Neutral Beam Generation

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ABSTRACT

The beam-plasma instability is analyzed for the ion beams used for neutral beam generation. Both positive and negative ion beams are considered. Stability is predicted when the beam velocity is less than the electron thermal velocity; the only exception occurs when the electron density accompanying a negative ion beam is less than the ion density by nearly the ratio of electron to ion masses. For cases in which the beam velocity is greater than the electron thermal velocity, instability is predicted near the electron plasma frequency.

I. Introduction

The ion beams used in the production of neutral beams are highly directed and have small random components of velocity relative to the directed velocity. They propagate in a background plasma, usually generated by the beam, which plays the essential role of space charge neutralization. This beam-plasma system is thus potentially subject to the two-stream instability.

For neutral beam systems based on positive ions the instability could increase the angular spread of the beam. One would expect the spread to be small, however, unless the instability becomes virulent. Indeed, there is no evidence that the present generation of neutral beams are effected by the instability. Systems based on negative ions are potentially more sensitive as they require the acceleration of the beam after it has propagated some distance. The acceleration process is very sensitive to the space charge density in the negative ion beam, and thus would amplify any density perturbations. The beams based on double charge-exchange start at a relatively low energy (<5 keV) and propagate relatively far (~100 cm) and thus could be strongly effected by an instability.

In this note the beam plasma mode is analyzed (neglecting collisional processes) and shown to be stable for beam velocities below the electron thermal velocity and for the parameters of interest. We conclude that the two-stream instability is not important in neutral beam systems using double charge-exchange in cesium.

For higher velocity beams (beam velocity > electron thermal velocity) instability is predicted near the electron plasma frequency. A rough argument concerning nonlinear processes suggests that the instability will not have serious consequences, although considerably more analysis is required to verify this conclusion.

II. Formulation of the Problem

Consider a homogenous and infinate beam plasma system. In the fluid approximation, the dispersion relation for waves propagating at an angle θ to the beam is

$$1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2 - k^2 v_e^2} - \frac{\omega_b^2}{(\omega - kv_b \cos\theta)^2 - k^2 v_t^2} = 0 \qquad (1)$$

The ions have been assumed to be cold. For later purposes we define an effective ion mass by

$$\omega_{pi}^2 = \sum_{\text{all species}} \frac{4\pi n q^2}{m} = \frac{4\pi n_i e^2}{M_i}$$

If there is only one species of singly charged ions, M_i is the ion mass.

The electrons have been assigned a random velocity $v_e = \sqrt{2kT_e/m_e}$, and kinetic corrections are assumed negligible. The beam (mass M_b) propagates at a velocity v_b with a thermal spread v_t.

We are particularly interested in negative ion beams and thus define:

$$n_b = \alpha n_i, \; n_e = (1-\alpha)n_i \text{ with } 0 < \alpha < 1,$$

$$y = kv_b, \; W_e = v_e^2/v_b^2, \;\; W_b = v_t^2/v_b^2 .$$

Thus, Eq. (1) becomes

$$\frac{1}{\omega_{pi}^2} = \frac{1}{\omega^2} + \frac{\alpha M_i/M_b}{(\omega - y\cos\theta)^2 - y^2 W_b} + \frac{(1-\alpha)M_i/m_e}{\omega^2 - y^2 W_e} \qquad (1a)$$

We initially neglect the beam thermal spread, although later we will show it to have an important stabilizing effect. For $W_b = 0$, the right-hand-side of Eq. (1a) has the forms shown in Fig. 1. Instability occurs when the minimum in the curve becomes greater than ω_{pi}^{-2}. We see that the system behaves differently for the cases of $v_e > v_b \cos\theta$ and $v_e < v_b \cos\theta$. In the 1 keV, D^- beam used in our double-charge exchange work, $v_b = 3 \times 10^7$ cm/sec, and $T_e \simeq 0.5$ eV as measured, $v_e \simeq 4.2 \times 10^7$ cm/sec, so $v_e > v_b$. At higher energy the opposite case occurs. The two cases are analyzed separately below.

IV. Negative Ion Beam: Electron Thermal Velocity Greater than Beam Velocity

For $v_e > v_b$, the minimum in the r.h.s. of Eq. (1a) occurs below that for the limit $y^2 W_e >> \omega^2$. We will find below that this is also a good approximation in the dispersion relation. We thus write

$$\frac{1}{\omega_{pi}^2} + \frac{(1-\alpha)M_i/M_b}{y^2 W_e} = \frac{1}{\omega^2} + \frac{\alpha M_i/M_b}{(\omega - y\cos\theta)^2} \tag{2}$$

To find a necessary condition for instability of Eq. (1), we analyze Eq. (2). The minimum of the r.h.s. of (2) occurs at

$$\omega_m = y\cos\theta/[1 + (\alpha M_i/M_b)^{1/3}]. \tag{3}$$

The component of phase velocity along the beam is thus somewhat less than the beam velocity.

The necessary condition for instability follows from Eqs. (2) and (3) as

$$\frac{\left[1 + (\alpha M_i/M_b)^{1/3}\right]^3}{y^2 \cos^2\theta} > \frac{1}{\omega_{pi}^2} + \frac{(1-\alpha)M_i}{y^2 W_e m_e}$$

This is most easily satisfied for $y \to 0$, so the system is unstable for progagation at angles

$$\cos^2\theta < \frac{\left[1 + (\alpha M_i/M_b)^{1/3}\right]^3}{(1 - \alpha)\, W_e} \left(\frac{m_e}{M_i}\right) \qquad (4)$$

This angle is very close to $\pi/2$ unless $(1-\alpha) \sim m_e/M_i$; that is, $n_e \sim m_e n_i/M_i$. This is a stringent condition and far stronger than required in a beam to eliminate electrons before acceleration. We wish to accelerate as few electrons as possible, and so require $n_e v_e \ll n_b v_b$; this, however, leads to $n_e \sim 10^{-2} n_b$, rather than $n_e \sim 10^{-4} n_b$ as required to permit the wave to propagate along the beam.

We thus find from Eq. (4) that the dielectric effect of the electrons will prevent instability unless $\theta \approx \pi/2$; in particular, instability will occur only for

$$\theta > \frac{\pi}{2} - \frac{\left[1 + (\alpha M_i/M_b)^{1/3}\right]^{3/2}}{(1-\alpha)^{1/2}\, W_e^{1/2}} \left(\frac{m_e}{M_i}\right)^{1/2} \qquad (5)$$

Some numerical examples are of interest. Assume $\alpha = 1/2, W_e = 2$. Then for instability

(a) $M_i/M_b = 2$ (D_2^+ plasma)

$\theta > 88.1°$

(b) $M_i/M_b = 66.5$ (C_s^+ plasma)

$\theta > 88.3°$

At these angles of propagation, even a small beam temperature is significant. For example, at $\cos\theta = 10^{-2}$, a temperature of 10^{-4} times the beam energy will dominate the beam response. Physically, a small angular spreading of the beam will cause particles to move transverse rapidly enough to destroy the coherence of the beam.

The r.h.s. of Eq. (1a) is plotted for the case $W_b > \cos\theta$; note that $W_b \ll W_e$ for all cases of interest. It is clear that all the roots of the dispersion relation are stable.

We conclude that for $v_e > v_b$ the only possible two-stream instability propagates almost across the beam, and that it is stabilized by a very small beam temperature. This conclusion will breakdown if the electron density is smaller than the beam density by a factor of m_e/M_i, but this extreme case is unlikely to be interesting in a real experiment.

V. Negative Ion Beam: Electron Thermal Velocity less than Beam Velocity

For systems such as those based on positive ions or double charge-exchange formation of negative ions using sodium, one may have $v_e < v_b$. For the case of zero beam temperature the dispersion relation

$$\frac{1}{\omega_{pi}^2} = \frac{1}{\omega^2} + \frac{\alpha M_i/M_b}{(\omega - y\cos\theta)^2} + \frac{(1-\alpha)\, M_i/m_e}{\omega^2 - y^2 W_e} \qquad (6)$$

is plotted in Fig. 1; note that either plot may apply, depending on the value of $\cos\theta$. The case $v_b \cos\theta < v_e < v_b$ has already been analyzed; as before, the system is predicted to be stable. The case of interest is thus $v_e < v_b \cos\theta$; an instability is possible for $\sqrt{W_e} < \omega < y\cos\theta$.

In this case the large mass ratio in the last term of Eq. (6) can be balanced either by $n_e/n_i \sim m_e/M_i$ or the resonance in the beam term. Neglecting the first possibility, we need $\omega - y\cos\theta = \Omega \ll \omega$; Eq. (6) yields

$$\Omega^2 \simeq \frac{\alpha M_i}{M_b}\left[\frac{1}{\omega_{pi}^2} - \frac{1}{y^2\cos^2\theta} - \frac{(1-\alpha)M_i/m_e}{y^2(\cos^2\theta - W_e)}\right]^{-1} \qquad (7)$$

Equation (7) is plotted in Fig. (3); instability is predicted for

$$y^2\cos^2\theta = k^2v_b^2\cos^2\theta < \omega_{pi}^2\left(1 + \frac{1-\alpha}{1 - W_e/\cos^2\theta}\ \frac{M_i}{m_e}\right) - \omega_{pe}^2 .$$

(The approximation breaks down near the singular point.)

The instability is not particularly sensitive to beam temperatures; $\Omega^2 > W_b = v_t^2/v_b^2$ permits the mode to be unstable. We thus predict that for $v_e < v_b$ there will be an instability with $\cos\theta < v_e^2/v_b^2$, frequency $\sim\omega_{pe}$, and phase velocity along the field $= v_b$.

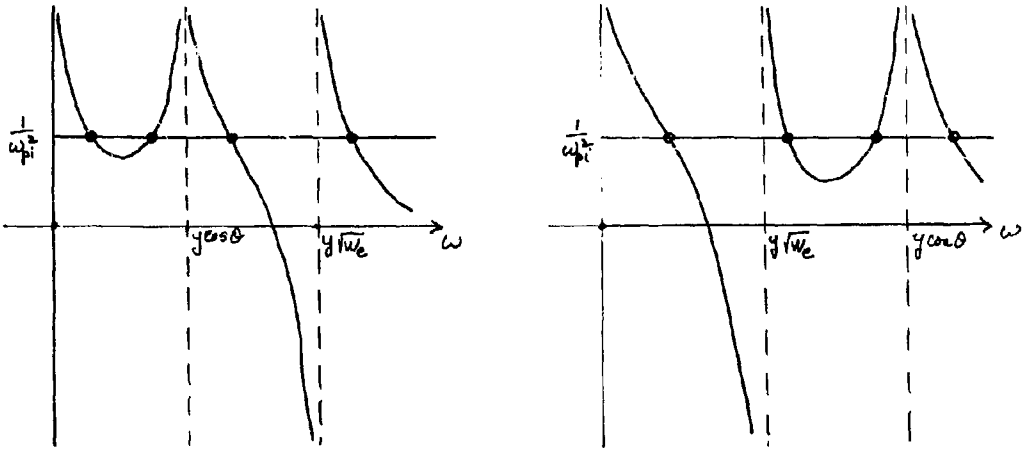
The mode is also unstable at low electron densities. At $\alpha = 1$, for example, Eq. (6) is the dispersion relation of the standard beam-plasma which is well known to be unstable at $\omega = \omega_{pi}(M_i/M_b)^{1/3}$.

For reasonable electron densities the oscillation at $\omega = \omega_{pe}$ should couple quite strongly to the electrons. A nonlinear consequence of the instability will then be to heat the electrons; the instability would saturate when v_e reaches $\sim v_b$. When the beam velocity is only slightly larger than the electron thermal velocity the instability should thus be weak, but for higher velocity beams the electron heating could be significant. Even in that case the fractional energy lost by the beam will be $\sim m_e/M_b$, so the instability may not have serious consequences unless ionization effects become important.

A more detailed analysis is needed, of course, to verify these conclusions.

VI. Positive Ion Beams

For positive ion beams, define β by $n_b = \beta n_i$, so $n_e = (1 + \beta)n_i$. The dispersion relation is thus identical with Eq. (1a), except that the first factor of α is replaced by β, and the factor $(1-\alpha)$ is replaced by $(1+\beta)$. The analysis then follows precisely as in the previous sections. We conclude that for $v_e > v_b$ the beam is stable for all values of β: unlike the negative ion case electrons cannot be eliminated from the beam. Similarly, for $v_e < v_b$ the beam is unstable as concluded in Section V.



$v_e > v_b \cos\theta$ $\qquad\qquad$ $v_e < v_b \cos\theta$

Fig 1. $(W_b = 0)$

The roots at $\omega < 0$ are not shown.

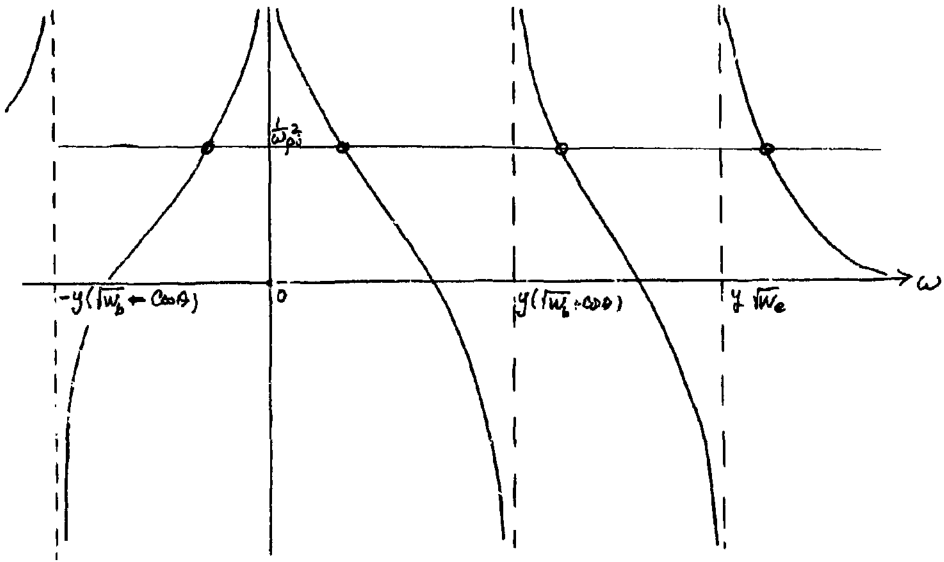


Fig 2

$y\cos\theta < \sqrt{W_b} < \sqrt{W_e} - y\cos\theta$

The roots at $\omega < -\sqrt{W_b}+\cos\theta$ are not shown.

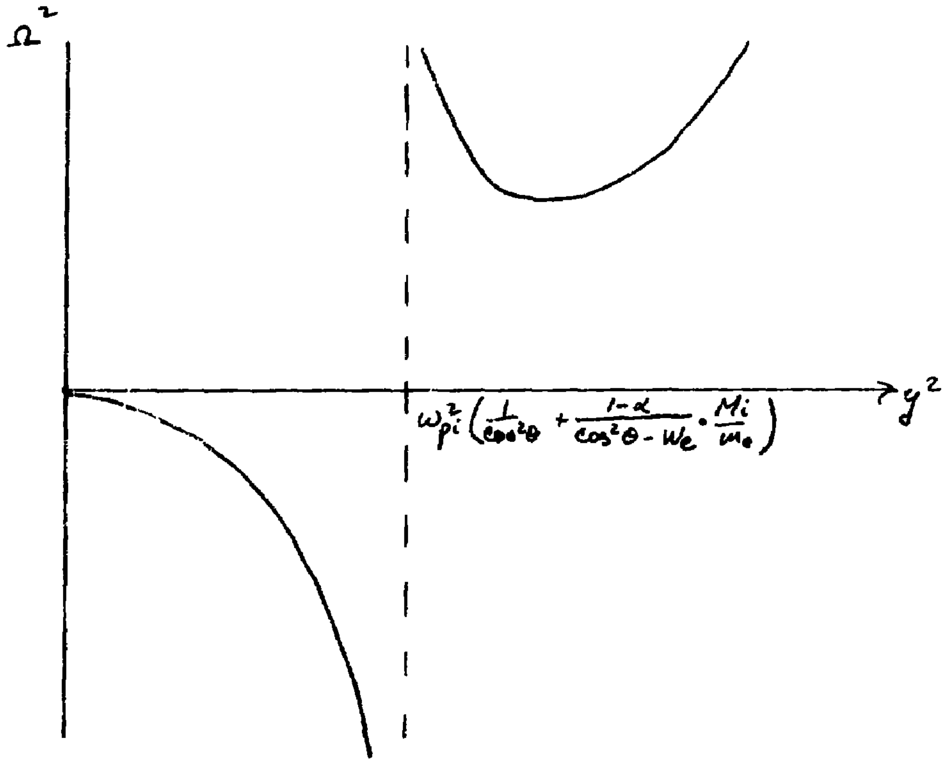


Fig 3

$v_e < v_b \cos\theta$

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