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ON THE BARYON MAGNETIC MOMENTS

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ABSTRACT

In the context of quark confinement ideas, the baryon magnetic moments are calculated by assuming a SU(3) breaking due to the inequalities of the quark masses ($m_p \neq m_n \neq m_\lambda$).

The modified SU(6) result for the ratio of the magnetic moments of the neutron and proton, viz

$$\frac{\mu(N)}{\mu(P)} \approx -\frac{2}{3}\left(1 - \frac{5}{9}\,\frac{m_n - m_p}{m_p}\right)$$

is obtained. The p-quark is found heavier than the n-quark by circa 15 MeV.

An alternative way of evaluating the baryon magnetic moments by means of simple physical considerations based on the properties of the SU(6) baryon S-wave functions is given.

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1. INTRODUCTION

In a recent paper, De Rújula, Georgi and Glashow[1] have discussed interesting implications for hadron spectroscopy of the gluon gauge model of strong interactions.

The model involves four types of fractionally charged quarks, each appearing in three colors and coupled with and octet of massless, neutral gluons. The quarks are confined within colorless hadrons by a long-range spin-independent force, whereas the short-range part of the quark-quark interaction is Coulomb-like. It originates from one-gluon exchange with a fine-structure α replaced by $-\frac{4}{3}\alpha_s$ for a $\bar{q}$ q pair in a meson and $-\frac{2}{3}\alpha_s$ for a q q pair in a baryon One gluon exchange also gives rise to two-body Fermi-Breit interaction terms, generalized for quarks of different masses.

Among the several consequences of the model, De Rújula, Georgi and Glashow considered the magnetic moment of the S-wave, ordinary baryons. They modified the well-known SU(6) treatment[2] by taking into account the breaking of SU(3) assumed to be exclusively due to the different masses of the various types of quarks $(m_p = m_n \neq m_\lambda)$.

In this way, they obtained the formula

$$\frac{\mu(\Lambda)}{\mu(P)} = -\frac{1}{3}\frac{m_p}{m_\lambda} \quad . \tag{1.1}$$

relating the magnetic moments of the Λ and proton (P). On the other hand, their linear treatment of the baryon masses gave the result[1]

$$\frac{m_p}{m_\lambda} = \frac{2\,(\Sigma^* - \Sigma)}{2\,\Sigma^* + \Sigma - 3\Lambda} = 0.622 \quad . \tag{1.2}$$

Eq (1) generalizes the well-known SU(3) result $\frac{\mu(\Lambda)}{\mu(P)} = -\frac{1}{3}$,bringing it closer, due to Eq.(2), to the experimental value.

In the present note, we investigate the consequences of having $m_p \neq m_n \neq m_\lambda$ in the SU(6) treatment of the baryon magnetic moments, having in mind to estimate the mass difference of the non-strange quarks p and n in a confining scheme. It comes out that the well-known SU(6) result

$$\frac{\mu(N)}{\mu(P)} = -\frac{2}{3} \qquad (1.3)$$

is modified by the inequality of the p and n quark masses, in a good approximation, as follows

$$\frac{\mu(N)}{\mu(P)} \approx -\frac{2}{3}\left(1 - \frac{5}{9}\delta_{np}\right) \qquad (1.4)$$

with $\delta_{np} = \frac{m_n - m_p}{m_p}$.

By means of the experimental values of $\mu(P)$ and $\mu(N)$, one can estimate the masses of the non-strange quarks p and n, corresponding to a value $\delta_{np} = -0.047$. Consequently, the p-quark is heavier than the n-quark , with a mass difference of circa 15 MeV.

We also calculated the magnetic moments of the other S-wave octet baryons. The influence of the factor δ_{np} also appears in the Σ^-,Σ^0 and Ξ^- magnetic moments, giving a contribution of the order of a few per cent with respect to De Rújula, Georgi and Glashow result.

This paper is organized as follows. Section 2 is devoted to the SU(6) treatment for the baryon magnetic moments and to a discussion of the results obtained from the assumption of unequal quark masses.

In Section 3, it is shown that the results obtained follows directly , in a transparent way, from simple physical considerations based on the properties of the S-wave baryon SU(6) states.

2. A SLIGHTLY MODIFIED SU(6) CALCULATION

In this Section, we calculate the baryon magnetic moments assuming a SU(3) breaking in the baryon magnetic moment operator by taking differen masses for the constituent quarks.

The baryon magnetic moment operator M is assumed to be additive, each of the three quarks contributing to it as a normal particle (g_i=2).One has, in an abvious notation (with $\hbar = c = 1$):

$$M = \sum_{i=1}^{3} g_i \frac{e_i}{2m_i} S_3^{(i)}$$

$$= e \sum_{i=1}^{3} \frac{Q_i}{2m_i} \sigma_3^{(i)} \qquad (2.1)$$

$$= \frac{e}{2m_p} \sum_{i=1}^{3} \frac{Q_i}{1 + \delta ip} \sigma_3^{(i)} ,$$

where

$$\delta ip = \frac{m_i - m_p}{m_p} . \qquad (2.2)$$

The magnetic moment of the baryon B, in units of the nuclear magneton $\frac{e}{2m_p}$ is then given by

$$\mu(B) \frac{e}{2m_p} = < B\uparrow \mid M \mid B\uparrow > . \qquad (2.3)$$

Therefore, the number $\mu(B)$, expressing the magnetic moment of B in nuclear magnetons is

$$\mu(B) = \frac{m_p}{m_p} < B\uparrow \mid \sum_{i=1}^{3} \frac{Q_i}{1 + \delta ip} \sigma_3^{(i)} \mid B\uparrow >. \qquad (2.4)$$

In (2.3) and (2.4) the matrix elements are taken between SU(6) baryon states. The explicit expressions for these states are well known[3]. With the help of them, the computation of (2.4) is a straightforward exercice.

We confine ourselves to the octet $(\underset{\cdots}{8}, \frac{1}{2})$ baryons of the 56-dimensional representation of SU(6), the only ones that have been accessible to magnetic moment experiments. The results, in nuclear magneton units, are explicitly the following:

$$\mu(p) = \frac{m_p}{m_p}\ \frac{1}{9}\left[\,8 + \frac{1}{1+\delta_{np}}\right] \tag{2.5}$$

$$\mu(N) = \frac{m_p}{m_p}\ \frac{-2}{9}\left[\,1 + \frac{2}{1+\delta_{np}}\right] \tag{2 6}$$

$$\mu(\Sigma^+) = \frac{m_p}{m_p}\ \frac{1}{9}\left[\,8 + \frac{1}{1+\delta_{\lambda p}}\right] \tag{2.7}$$

$$\mu(\Sigma^0) = \frac{m_p}{m_p}\ \frac{1}{9}\left[4 - \frac{2}{1+\delta_{np}} + \frac{1}{1+\delta_{\lambda p}}\right] \tag{2.8}$$

$$\mu(\Sigma^-) = \frac{m_p}{m_p}\ \frac{1}{9}\left[\frac{1}{1+\delta_{\lambda p}} - \frac{4}{1+\delta_{np}}\right] \tag{2.9}$$

$$\mu(\Xi^0) = \frac{m_p}{m_p}\ \frac{-2}{9}\left[1 + \frac{2}{1+\delta_{\lambda p}}\right] \tag{2.10}$$

$$\mu(\Xi^-) = \frac{m_p}{m_p}\ \frac{1}{9}\left[\frac{1}{1+\delta_{np}} - \frac{4}{1+\delta_{\lambda p}}\right] \tag{2.11}$$

$$\mu(\Lambda) = \frac{m_p}{m_p}\ \frac{-1}{3}\ \frac{1}{1+\delta_{\lambda p}}\ . \tag{2.12}$$

where

$$\delta_{np} = \frac{m_n - m_p}{m_p} \quad \text{and} \quad \delta_{\lambda p} = \frac{m_\lambda - m_p}{m_p}\ .$$

A few remarks are now in order.

i) Since δ_{np} is a small number, one gets, from (2.5) and (2.6), in a good approximation, the following expression for the ratio of the magnetic moments of the neutron (N) and proton (P):

$$\frac{\mu(N)}{\mu(P)} \approx -\frac{2}{3}\left(1-\frac{5}{9}\delta_{np}\right) \qquad (2.13)$$

ii) If we use the experimental values for $\mu(N)$ and $\mu(P)$[4], one gets from (2.5) and (2.6) the values of m_p and m_n

$$m_p = 338 \text{ MeV} \qquad (2.14)$$

$$m_n = 322 \text{ MeV} \qquad (2.15)$$

Thus, the p-quark is heavier than the n-quark by circa 15 MeV.

In the following Table, we give the numerical results for $\mu(B)$, calculated with the quark masses of m_p and m_n given by (2.14) and (2.15) . For the m_λ mass, we used the value

$$m_\lambda = 543 \text{ MeV} \qquad (2.16)$$

obtained from (1.2) and (2.14).

Table. Baryon magnetic moments, in nuclear magneton units.

Baryon	P	N	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-	Λ
Theory	Imput	Imput	2.66	0.78	-1.10	-1.38	-0.44	-0.58
Experiment[5]	2.793	-1.913	2.62±0.41	?	-1.48 ± 0.37	?	-1.93±0.75	-0.67±0.06

It is seen that the contribution of δ_{np} for $\mu(\Sigma^-)$ is of about 5 per cent with respect to the value calculated in Ref. 1.

3. AN ALTERNATIVE WAY

In this final section, we wish briefly to show that the results for the baryon magnetic moments $\mu(B)$, obtained in the previous section, follow directly from simple physical properties of the SU(6) baryon S-wave states.

In order to illustrate the ideas involved, we consider the cases of the Λ and Σ^0 particles, both of which are composed of p, n and λ quarks.

By simple inspection of the corresponding SU(6) states[3], it is easily seen that the $|\Lambda\uparrow\rangle$ state has the peculiar feature that the λ-quark appears with spin up 100% of the time. As $|\Lambda\uparrow\rangle$ is a symmetric state, the $\lambda\uparrow$ appears accompanied with an n-p pair in a symmetric combination, namely $n\uparrow p\downarrow + n\downarrow p\uparrow$. It is then clear that the contribution of the n-p pair to the magnetic moment of the Λ cancels out. Therefore, the magnetic moment of the Λ is equal to the magnetic moment of the λ-quark, a conclusion in entire agreement with Eq.(2.12).

On the other hand, in the $|\Sigma^0\uparrow\rangle$ state, the λ-quark appears with spin down ($\lambda\downarrow$) two thirds of the time and with spin up ($\lambda\uparrow$) one third of the time. When the λ is $\lambda\uparrow$, it comes accompanied with $n\uparrow p\downarrow + n\downarrow p\uparrow$ and in this configuration the pair n-p clearly gives a vanishing contribution to the Σ^0 magnetic moment and so, only the $\lambda\uparrow$ contributes to the Σ^0 magnetic moment.

When the λ is $\lambda\downarrow$, it appears necessarily together with a pair in a configuration $n\uparrow\ p\uparrow$, whose contribution to the Σ^0 magnetic moment is $\frac{1}{2m_n}\left(-\frac{1}{3}\right) + \frac{1}{2m_p}\cdot\frac{2}{3}$. Hence, taking into account the fractions of the time for the $\lambda\uparrow$ and $\lambda\downarrow$ in $|\Sigma^0\uparrow\rangle$, one has immediatly:

$$\mu(\Sigma^0) = \frac{2}{3}\left[\frac{1}{2m_\lambda}\left(\frac{1}{3}\right) + \frac{1}{2m_p}\left(\frac{2}{3}\right) + \frac{1}{2m_n}\left(-\frac{1}{3}\right)\right] + \frac{1}{3}\,\frac{1}{2m_\lambda}\left(-\frac{1}{3}\right) ,$$

a result coinciding with Eq. (2.8).

In much the same way one can deduce the magnetic moments of the other S-wave baryons confirming the results of the direct calculation.

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