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DYNAMICS OF A COAXIAL PLASMA GUN

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Introduction

Experimental results on the propagation of an ionizing wave into a neutral gas are presented by Eninger (1966) and Axnäs (1972, 1976). The apparatus used is a coaxial plasma gun with an azimuthal magnetic bias field. A significant feature is that magnetic field lines are closed within the plasma produced and hence that the governing transport processes are those across the magnetic field. The studies are concerned with both the dynamics and the processes of ionization within the propagating ionized region. The experimental conditions are chosen so that the plasma produced is only partially ionized. The results differ from what would be expected in the limiting theoretical model, the snow-plow, in which all the neutral gas is swept up by the propagating wave.

The results confirm the existence of a velocity limitation observed in other $\underset{\sim}{E} \wedge \underset{\sim}{B}$ discharges. These velocity limits are found to be relatively insensitive to the precise choice of experimental parameters over a considerable range of values and to be determined by the type of gas used in the experiment. The limits are in rather close agreement with the critical velocity postulated by Alfvén (1954, 1960). Critical ionization velocity effects have been found in a variety of experimental situations, either as a directly observed velocity or indirectly as a voltage limitation. The mechanism envisaged is a rapid increase in ionization when the velocity of neutral gas u_c, with respect to a plasma is such that the atoms have a corresponding kinetic energy equal to the ionization energy, eV_i. Thus if the atomic mass is m_a,

$$\frac{1}{2} m_a u_c^2 = eV_i \qquad (1)$$

Although such a process is energetically possible the energy cannot be released in a binary collision between an ion and a neutral atom. Also the ionization probability is commonly very small for ion-neutral collisions in experimental situations. There is clear experimental evidence that energetic electrons can be produced in a critical velocity interaction and that these can account for the observed ionization (Danielsson 1970; Danielsson and Brenning, 1975). An extensive review of critical velocity experiments is given by Danielsson (1973).

Many theories have been developed to explain voltage limitation and the critical ionization velocity. A useful survey is given by Sherman (1973). As indicated by the definition of critical velocity in Equation 1 the essential problem is that of energy balance and the way in which the kinetic energy associated with neutral atoms can be used to produce ionization. An essential link appears to be the production of energetic electrons. Sherman (1969, 1972) argues that the modified two-stream instability provides a suitable mechanism for this and can form part of a circular process in which relative motions between ions and electrons resulting from ionization lead to the acceleration of electrons and further ionization. Raadu (1975) shows that in such a process the electrons may be thought of as having an effectively increased mass. Unstable electrostatic modes for which the effective mass is comparable to the mass of the neutral atom can lead to electron acceleration of the right order of magnitude. A basic requirement is that the electrons are restricted to move parallel to the magnetic field. This is the case for the modes considered when the electron gyrofrequency excedes the plasma frequency. For the standard discharge parameters used by Axnäs (1972, 1976) these two frequencies are comparable, and the collision time is larger than the theoretically estimated growth time (the inverse ion plasma frequency) of the unstable electrostatic modes which are effective in accelerating the electrons. Thus there are theoretical reasons for expecting ionization by electrons to be important in the coaxial gun experiments.

The way in which the critical velocity mechanism operates must depend on the particular experiment under discussion. Thus in evaluating an experiment it is also important to consider the consequences of particle and momentum balance given that the critical velocity mechanism is operating. Such analyses are given by Eninger (1966) and Axnäs (1972) in connection with their experimental studies. Axnäs gives separate treatments of particle and momentum balance. Here it is intended to treat the balance equations simultaneously for ions and electrons taking into account the discharge geometry and boundary conditions appropriate for the coaxial plasma gun.

Discharge Geometry and Boundary Conditions

The ratio between the inner and outer radial distances of the electrodes in the coaxial plasma gun is 0.347. The inner electrode (radius 1.25 cm) acts as the cathode and the outer electrode (radius 3.6 cm) as the anode. An azimuthal bias magnetic field $\underset{\sim}{B}$ which depends inversely on the radial distance is provided by an axial current. The magnetic field due to the discharge current is negligible. The standard discharge parameters used by Axnäs (1972, 1976) are referred to here for numerical estimates. Thus for the filling pressure of 0.1 torr the collision times are 2.7×10^{-8} s for ions and 2.1×10^{-9} s for electrons (Eninger, 1966). For a magnetic field of 0.66 Tesla at the mean radial distance the ion and electron Hall parameters at the outer electrode are then $(\omega\tau)_i = \Lambda = 1.15$ and $(\omega\tau)_e = 165$ respectively. The ion Hall parameter Λ is significant for the theoretical analysis. The large electron Hall parameter implies that the electrons move with the E/B drift velocity. The radial drift velocity of both ions and electrons is certainly less than the critical velocity and for hydrogen the time for radial drift is therefore greater than 5.10^{-7} s. Comparing this with the collision times it follows that ions and electrons undergo many collisions before reaching the walls. Thus the initial motion of newly produced particles is unimportant and a mean motion can be expected determined by a balance between electromagnetic forces and neutral drag arising from collisions with the background neutral gas.

In the present analysis it is assumed that the structure of the discharge depends only on the radial distance. Since the electrons move with the E/B drift velocity the electric field must have a component, E_o, parallel to the axis to provide a radial electron drift. Assuming steady conditions within the discharge there can be no induced electric field and it follows directly from Maxwell's equations that the axial electric field component, E_o, is a constant independent of the radial distance. For the standard discharge conditions, the axial electric field is close to one tenth of the radial electric field at the mean radial distance and directed in such a way as to give an outward electron drift (Axnäs, 1972). However the electrode surfaces must be equipotentials and thus it is necessary to assume boundary sheaths to match the internal plasma condition with these surfaces. The potential across these sheaths must depend on the position and be comparable in magnitude to the internal potential over the length of the discharge due to the constant axial electric field E_o.

The discharge geometry, plasma equipotentials and wall sheaths are indicated in Figure 1. A further consequence of the constant axial field and of the large electron Hall parameter is that throughout the discharge the electrons have a finite outward radial drift velocity. There must therefore be a source of electrons at the inner electrode or in the cathode sheath. The contribution of this source to the total electron production is taken to be an initially free parameter and may dominate the contribution from internal ionization. However all the ions within the plasma are assumed to be produced by internal ionization. There can therefore be no ion current out of the walls and this places an important restriction on the ion current within the discharge. In general there will be an outer region of the discharge where ions drift to the anode and an inner region where they drift to the cathode. Limiting cases where there is only one such region follow if the ion current is zero at either one of the electrodes.

Basic Equations

The treatment developed here is intended to follow through the consequences of particle and momentum balance for ions and electrons in a weakly ionized magnetized plasma where collisions with neutral particles dominate. Charge neutrality is assumed. The electron ionization rate coefficient is α. In a steady state the particle conservation equation for electrons is

$$\mathrm{div}(\underset{\sim}{j}_e) = -e\alpha n_a n_e \qquad (2)$$

and for the ions we have

$$\mathrm{div}(\underset{\sim}{j}_i) = e\alpha n_a n_e \qquad (3)$$

where n_a is the background neutral particle density. The total current density is source free as may be seen by taking the sum of Equations 2 and 3. In particular this means that the total radial current is a constant and the total radial current density depends inversely on the radial distance.

In addition to particle conservation, momentum balance must be satisfied for both ions and electrons. However the electron Hall parameter $(\omega\tau)_e$ is so large that the electron momentum equation reduces to the condition that the electrons move with the E/B drift velocity and hence their current density is given by

$$\underset{\sim}{j}_e = -n_e e \frac{\underset{\sim}{E} \wedge \underset{\sim}{B}}{|\underset{\sim}{B}|^2} \qquad (4)$$

Since the potential energy of a particle associated with the voltage applied over the plasma volume is here very much greater than the thermal energy, the effects of pressure gradients on the momentum balance and associated diffusion can be neglected in the plasma interior. This applies to both electrons and ions. Thus assuming steady conditions the momentum balance equation for ions can be written as a generalised Ohm's law (cf. A.fvén and Fälthammar, 1963) as follows:

$$\underset{\sim}{j}_i/\sigma = \underset{\sim}{E}_{\parallel} + \{1 + (\omega\tau)_i^2\}^{-1}\{\underset{\sim}{E}_{\perp} - (\omega\tau)_i \, \underset{\sim}{b} \wedge \underset{\sim}{E}\} \qquad (5)$$

where the electric field $\underset{\sim}{E}$ is separated into parallel, $\underset{\sim}{E}_{\parallel}$, and perpendicular, $\underset{\sim}{E}_{\perp}$, components to the unit vector $\underset{\sim}{b}$ in the magnetic field direction. The scalar conductivity σ is given by,

$$\sigma = \frac{n_i e^2 \tau_i}{m_i}$$

where τ_i is the momentum exchange time for the ions with the neutral gas. The ion Hall parameter $(\omega\tau)_i$ is given by

$$(\omega\tau)_i = \frac{eB\tau_i}{m_i}$$

In the present situation with axisymmetry and purely azimuthal magnetic field the parallel electric field, $\underset{\sim}{E}_{\parallel}$, is zero. The electron, e^-, and ion, X^+, motions are indicated in Figure 1. As a direct consequence of the generalised Ohm's law for the ions and E/B drift motion of the electrons there is a simple relation between the radial ion current density, $j_{i,r}$, and the axial component of the total current density, j_z, namely

$$j_{i,r} = (\omega\tau)_i j_z \qquad (6)$$

From this relation we see that the radial ion current reverses direction at the point where the total current density becomes purely radial.

The natural way to complete the system of equations would be to include equations for energy balance and the ionization rate. However, the present analysis is to be applied to a situation where the critical ionization velocity mechanism is operating. The ionization rate is then expected to become strongly dependent on the local E/B value in the plasma (c.f. Axnäs, 1976) and a large range of ionization rates thus correspond to values of E/B close to the critical velocity. For this reason we consider case A in which the electrons are assumed to have a drift velocity equal to the critical velocity and the ionization rate coefficient, α, is a function of position consistent with the requirements of particle and momentum balance. The structure of the discharge in this case is derived in Appendix A. In principle the deduced values of the ionization rate should be shown to correspond to values close to the critical velocity, so that only negligible corrections would be required to produce an exact solution. The reason for assuming that it is the electron component of the plasma which has the critical velocity is that the electrons are apparently responsible for producing the ionization in the critical velocity mechanism.

For comparison we also consider case B in which the critical velocity mechanism does not occur and the ionization rate coefficient α is a given constant. The structure of the discharge in this case is derived in Appendix B.

In both cases it is assumed that the neutral gas density, n_a, and the ion collision time, τ_i, are constants. Equations derived in Appendix A are labelled A and those in Appendix B are labelled B.

Results

In deriving the structure of the discharge region from the set of basic equations it is useful to introduce dimensionless parameters. Thus the parameter η is the axial electric field component in units of the total electric field at the outer electrode for critical velocity conditions,

$$\eta = E_o/u_c B_o$$

The ionization rate compared with a typical drift rate for electrons across the discharge is given by λ where

$$\lambda = \frac{B_o R \alpha n_a}{E_o}$$

And R is the radial distance of the outer electrode. For small values of λ electrons drift across the discharge with only a small contribution to their density from internal ionization and as can be seen from case B the radial dependence of the density is then approximately an inverse square of the distance (Equation B1). From the definition of η and λ the ionization rate per electron may be written,

$$\alpha n_a = \eta\lambda u_c/R \approx 1.4 \times 10^6 \eta\lambda$$

using the critical velocity for hydrogen. Thus from experimental estimates, $\alpha n_a \sim 10^6 s^{-1}$ (Axnäs, 1972), the product $\eta\lambda$ should be of the order of unity.

Certain general results follow directly from considering the basic equations independently of the detailed calculations given in the appendices. The condition that there are no ion sources at the walls means that somewhere within the discharge the radial ion current must change direction. At this point the radial ion current is zero and from the generalized Ohm's law, Equation 5,

$$(\omega\tau)_i E_o = -E_r \qquad (7)$$

Since this equation must be satisfied at some point within the plasma the range of values possible for the axial electric field are restricted if the behaviour of the Hall parameter and radial electric field are known. This result is quite general. In special cases such as the critical velocity case such considerations lead to a specific range of values for the axial field parameter η (Equation A2).

For case A, the critical velocity case, the ratio between axial and radial electric field at the mean radius measured by Axnäs (1972) implies $\eta \approx 0.15$. The radial electric field (Equation A1) then approaches the inverse radial dependence found by Eninger (1966). For constant τ_i both $(\omega\tau)_i$ and E_r have a similar radial dependence and from Equation 7, E_o is almost completely specified. Thus from Equation A2, $\Lambda \approx \eta^{-1}$ and hence $\Lambda \approx 6.7$. The measured axial field in this way leads to an estimate of Λ greater than that calculated from standard parameters, 1.15.

A second general argument follows from considering the radial electron current density $j_{e,r}$ and the total radial current density j_r. From the generalised Ohm's law for ions and the E/B electron drift motion we find that for large values of the ion Hall parameter, $(\omega\tau)_i$,

$$j_{e,r} \approx - \frac{(\omega\tau)_i \, E_o}{E_r} j_r \qquad (8)$$

Now if the axial drift velocity of the electrons is equal to the critical velocity (a constant) and since the axial electric field component is a constant, the ratio between the electron and total radial currents is simply proportional to the ion mean collision time τ_i and is therefore a constant if τ_i is a constant. Since the total current is a constant throughout the discharge it then follows that in this case the electron current must also be a constant. Thus for large values of the ion Hall parameter $(\omega\tau)_i$ the contribution to the electron current from ionization within the discharge must be negligible and the parameter λ must be very small. In case A, the critical velocity case, the axial electric field becomes small for large values of the ion Hall parameter and the axial electron drift velocity approaches the critical velocity. Also τ_i is assumed to be a constant and it follows from the detailed calculations that the ionization rate (Equation A5) does become small as expected from the general argument. However from this general argument it is clear that this only applies if the collision time τ_i is a constant and that if τ_i varies over the discharge it is possible to have significant ionization rates even in the limit of a large ion Hall parameter.

For the critical velocity case, case A, the structure of the discharge region is considered for different choices of the pair of parameters η and Λ. The ions tend more to drift outwards for increasing η or Λ. Plotted curves for different choices are labelled a, b and c with subscripts 0 or 1. A subscript 0 indicates that the pair is chosen so that the ion current vanishes at the outer electrode (anode) and all the ions drift in towards the cathode. Given one of the pair the other parameter is in this case the minimum compatible with the condition that there are no ion wall sources (Equation A2). A subscript 1 corresponds to zero ion current at the cathode. Given one of the parameter pairs the other is now a maximum. There is an absolute limit for the axial electric field parameter η given by the condition that the radial electron drift at the anode is less than the total critical velocity, $\eta \lesssim 1$.

The radial dependence of the normalised ionization rate λ given by Equation A5 is shown in Figure 2. The curves a have $\eta = 0.96$, close to the absolute limit, and give large values of the ionization rate near the anode. Here we note that by comparison with case B the electron density should increase outwards if $\lambda > 2$ (Equation B1). For $\eta = 0.96$, however, the ion Hall parameter must be rather small, $0.292 \lesssim \Lambda \lesssim 0.982$. If instead we fix $\Lambda = 1.15$, the standard value, the curves b are found with $\lambda \lessapprox 0.72$. For this choice the ionization rate varies by less than an order of magnitude over the discharge which is favorable to the basic assumption that critical velocity conditions apply throughout the discharge. Finally for curves c we take $\eta = 0.5$ which is greater than the experimental estimate, 0.15, and leads to small values of the ionization rate, $\lambda \lessapprox 0.27$. In general for large values of Λ and corresponding small values of $\eta \approx \Lambda^{-1}$ the approximate expression

$$\lambda \approx \Lambda^{-2} x^2$$

where the radial distance x is normalised to the anode radius R, may be used (c.f. Equation A5).

The electron density for <u>case A</u>, Equation A3, is given in Figure 3. For a constant ionization rate, <u>case B</u>, we would have (Equation B1)

$$n_e = n_o x^{\lambda-2}$$

and the inverse square dependence which follows for negligible internal ionization, $\lambda = 0$, is included as a dashed curve in Figure 3. Curves b_o, c_o are rather close to this curve as might be expected since the ionization rate indicated in Figure 2 is on average small. The corresponding curves b_1, c_1 are not included as they lie rather close to those plotted. The curve a_1 becomes horizontal close to the anode where $\lambda(x) = 2$.

In a steady situation the total ion current out of the discharge i.e. the sum of the ion currents into the electrodes, must be equal in magnitude to the difference between the electron currents into the anode and out of the cathode i.e. the contribution to the total electron current from internal ionization. This may be calculated for the critical velocity case using the electron density which follows from Equation A3 and the radial E_o/B drift motion. The ratio between the absolute values of the total ion current I_i and the electron current at the cathode I_e calculated in this way is shown in Figure 4 as a function of the ion Hall parameter Λ. The lower curve is for the minimum corresponding value of η which follows if the ion current at the anode is zero. The upper curve is for the maximum possible η which for $\Lambda \lesssim 0.938$ is 1 and is otherwise given by the condition that the ion current at the cathode is zero. Values of η are indicated along the two curves. The labels a, b and c with subscripts indicate the same pairs of values for η and Λ used above. For large Λ we have approximately

$$|I_i|/|I_e| \approx \frac{1}{2} \Lambda^{-2}(1-x_1^2)$$

where x_1 (=0.347) is the ratio between the anode and cathode radial distances, and the ion current is relatively small. This limiting approximation is indicated by the dotted curve. From Figure 4 it is clear that the ion current can never be much greater than the electron current and is small for the standard value $\Lambda = 1.15$ (points b_o and b_1).

The total current lines given by Equation A7 for the critical velocity case are shown in Figure 5. Label a without a subscript is for a representative intermediate case for which the radial ion current is zero at a normalised radial distance $x = 0.7$. The points at which the radial ion current is zero are indicated by a small circle. From Equation 6 these are also the points at which the total current density is purely radial and the direction of the current line slope changes. Such a point always exists if there are no ion sources at the walls. For the standard ion Hall parameter the current lines (b_o and b_1) do not depart very much from the radial direction. The drift motion is to the right.

Perpendicular to the current lines the electrons and ions have a common drift velocity u_d. In view of the coaxial plasma gun geometry it is useful to think of the particle motions as vector sums of individual drift motions parallel to the total current density and a common axial drift velocity u_s. The perpendicular drift velocity u_d is then to be taken as a component of this axial drift velocity. The relation between u_s and u_d is indicated in Figure 5. For a discharge region of finite length the axial drift velocity u_s would have to be a constant, so that in a reference frame moving with this velocity all particle motions would be parallel to the current lines and there would be no tendency to create charges on boundaries within the plasma formed by current lines. The axial drift velocity u_s for the critical velocity case is shown in Figure 6 which follows from Equation A8. For a standard ion Hall parameter (b_o and b_1) u_s does not vary greatly over the discharge and is not far from the critical velocity u_c. For large Λ we have from Equation A8

$$u_s \approx (1 - \frac{1}{2} \Lambda^{-2} x^2)\ u_c$$

and the axial drift velocity approaches the critical velocity throughout the plasma.

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Appendix A

Here we derive the structure of the discharge region assuming that the electron drift velocity is equal to the critical ionization velocity. The neutral gas density, n_a, and the ion collision time, τ_i, are taken to be constants. We use the particle conservation equation for the electrons and assume they have the E/B drift velocity. The ion current density follows from the total current, knowing the electron current. The generalised Ohm's law for ions is used.

Given that the electron E/B drift velocity is equal to the critical velocity, u_c, the total electric field must be

$$|\underset{\sim}{E}| = u_c B_o x^{-1}$$

x is the radial distance normalised to the anode radius, R. From the normalised axial electric field $\eta = E_o/u_c B_o$ the radial electric field is

$$E_r = -u_c B_o (x^{-2} - \eta^2)^{1/2} \qquad \text{(A1)}$$

and depends approximately on the inverse radial distance for small values of the axial field. Clearly η is never greater than 1. A further restriction on the normalised electric field η follows from the radial component of the generalised Ohm's law for the ions,

$$j_{i,r} = \sigma\{1 + (\omega\tau)_i^2\}^{-1} \{E_r + (\omega\tau)_i E_o\}$$

and the condition that there are no ion currents out of the electrodes. Thus we have

$$(\Lambda^2 + 1)^{-1/2} \lesssim \eta \lesssim (\Lambda^2 + x_1^2)^{-1/2} \quad \text{and } \eta \lesssim 1 \qquad \text{(A2)}$$

where x_1 (= 0.347) is the normalised inner electrode radius. For large values of the ion Hall parameter, Λ, the normalised axial electric field η must be approximately Λ^{-1}.

Knowing the electric field components the components of the electron drift velocity may also be found. Using the condition that the total current density, which depends inversely on the radial distance, is the sum of the ion and electron current densities the ion current density may be written

$$j_{i,r} = j_o x^{-1} - j_{e,o} \frac{n_e x}{n_o}$$

where j_o and $j_{e,o}$ are the values taken by the total and electron radial current densities at the outer electrode. We now write

$$j_o = \beta j_{e,o}$$

Now the radial ion current density also follows from the generalized Ohm's law. For consistency between the two equations the ion density n_i, which is equal to the electron density n_e, must be

$$n_i = \beta n_o \frac{\eta(1+\Lambda^2 x^{-2})}{\eta x^2+\Lambda(1-\eta^2 x^2)^{1/2}} \qquad \text{(A3)}$$

From the condition that $n_i = n_o$ at the outer electrode β can be determined.

Since the electron density is known the electron conservation equation may be used to find the necessary dependence of the ionization rate coefficient, α. If $\lambda(x)$ is the normalised ionization rate defined by

$$\lambda(x) = \frac{B_o R \alpha n_a}{E_o} = \frac{1}{n_e x} \frac{d}{dx} (n_e x^2) \qquad \text{(A4)}$$

where R is the outer electrode radius, we then have

$$\lambda(x) = \frac{\Lambda x^2}{(\Lambda^2+x^2)} \left(\frac{1+\{\eta\Lambda-(1-\eta^2 x^2)^{1/2}\}^2}{\Lambda(1-\eta^2 x^2)+\eta x^2(1-\eta^2 x^2)^{1/2}}\right) \qquad \text{(A5)}$$

Thus λ is always positive as it should be physically since it is an ionization rate. For large values of the ion Hall parameter, Λ, the normalised ionization rate λ is approximately $\Lambda^{-2}x^2$.

The total current density is given using the generalised Ohm's law for the ions and the E/B drift velocity for the electrons,

$$\underset{\sim}{j} = -\frac{\beta n_o e E_o}{B_o}\left(x^{-1},\ 0,\ \frac{(1-\eta^2x^2)^{1/2}-\eta\Lambda}{\eta x^2+\Lambda(1-\eta^2x^2)^{1/2}}\right) \qquad \text{(A6)}$$

in cylindrical coordinates. If z is the axial coordinate normalised to the anode radius, R, the current lines can now be found by integrating

$$\frac{dz}{dx} = \frac{j_z}{j_r}$$

which gives

$$\eta z = (1-\eta^2x^2)^{1/2} + (4+\eta^2\Lambda^2)^{-1/2}\log_e\frac{A-(1-\eta^2x^2)^{1/2}}{A(1-\eta^2x^2)^{1/2}-1} \qquad \text{(A7)}$$

where

$$2A = \eta\Lambda + (4+\eta^2\Lambda^2)^{1/2}$$

The common drift velocity of the ions and electrons perpendicular to the total current may also be found, using the electron drift velocity, and is

$$u_d = (\Lambda^2+x^2)^{-1/2}\,\Lambda u_c$$

If the particle motions are expressed as the vector sum of individual drift motions parallel to the total current density and a common axial drift velocity, u_s, the perpendicular drift velocity, u_d, must be taken to be a component of the axial velocity which is then

$$u_s = \{\eta x^2 + \Lambda(1-\eta^2x^2)^{1/2}\}^{-1}\,\Lambda u_c \qquad \text{(A8)}$$

The axial drift velocity thus depends on the radial distance.

Appendix B

Here we derive the structure of the discharge region assuming a constant ionization rate coefficient, α. The neutral gas density, n_a, and the ion collision time, τ_i, are taken to be constants. We use the particle conservation equation for the electrons and assume they have the E/B drift velocity. The ion current density follows from the constant total current, knowing the electron current. The generalised Ohm's law for ions is used.

The radial drift velocity of the electrons is determined by the axial electric field component E_o and the particle conservation equation,

$$\frac{1}{r}\frac{d}{dr}\left\{\frac{n_e r^2 E_o}{B_o R}\right\} = \alpha n_a n_e$$

may be integrated to give the electron density,

$$n_e = n_o x^{\lambda-2} \tag{B1}$$

where x = r/R, the radial distance normalised to R the outer electrode radius and

$$\lambda = \frac{B_o R \alpha n_a}{E_o} \tag{B2}$$

is the normalised ionization rate. The radial electron current follows directly,

$$j_{e,r} = j_{e,o}\, x^{\lambda-1}$$

The ion current density may now be derived using the condition that the total radial current density, with value j_o at the outer electrode, depends inversely on the radial distance. Thus we have

$$j_{i,r} = j_o x^{-1} - j_{e,o}\, x^{\lambda-1}$$

It should be noted that the electron and total current densities are negative. We now write

$$j_o = \beta j_{e,o}$$

and from the condition that there can be no ion currents out of the electrodes, β must satisfy the condition

$$x_1^{\lambda} \leqslant \beta \leqslant 1 \qquad \text{(B3)}$$

where x_1 (= 0.347) is the normalised inner electrode radius.

The radial electric field E_r may be derived from the ion current density using the generalised Ohm's law for the ions,

$$j_{i,r} = \sigma\{1+(\omega\tau)_i^2\}^{-1}\{E_r + (\omega\tau)_i E_o\}$$

which after some manipulation gives

$$E_r = -\Lambda^{-1} E_o \{x(\beta x^{-\lambda}-1) + \Lambda^2 \beta x^{-(\lambda+1)}\} \qquad \text{(B4)}$$

where Λ is the ion Hall parameter at the outer electrode.

The axial component of the total current density may be found from the axial component of the generalised Ohm's law for the ions and the electron drift motion. Taking cylindrical coordinates the total current density is

$$\underset{\sim}{j} = -\frac{n_o e E_o}{B_o}(\beta x^{-1},\ 0,\ \frac{\beta - x^{\lambda}}{\Lambda}) \qquad \text{(B5)}$$

Perpendicular to this current the ions and electrons have a common drift velocity u_d which may be derived from the electron drift motion or directly from the balance between magnetic forces and neutral drag acting on the plasma,

$$|\underset{\sim}{j}|B = m_i n_i u_d \tau_i^{-1}$$

Using the expression for the total current we find

$$u_d = \frac{\beta\Lambda E_o}{B_o} x^{-\lambda}\{1 + \frac{x^2(1-\beta^{-1}x^\lambda)^2}{\Lambda^2}\}^{1/2} \qquad \text{(B6)}$$

In view of the coaxial plasma gun geometry it is more useful to consider the particle motion as being the vector sum of individual drift motions parallel to the total current density and a common drift velocity, u_s, parallel to the axis. Then the drift velocity, u_d, must be interpreted as a component of the axial drift velocity, u_s, which is then

$$u_s = \frac{\beta\Lambda E_o}{B_o} \cdot x^{-\lambda}\{1 + \frac{x^2(1-\beta^{-1}x^\lambda)^2}{\Lambda^2}\} \qquad \text{(B7)}$$

and depends on the radial distance.

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Figure Captions

Fig. 1. The coaxial plasma gun discharge geometry indicating the position of the wall sheaths and the form of the internal equipotentials. The electron, e^-, and ion, X^+, motions are indicated. The electrons have the E/B drift motion on the equipotentials. The ions collide with neutral particles and have also a tendency to drift in the direction of the electric field $\underset{\sim}{E}$. The magnetic field $\underset{\sim}{B}$ is produced by an axial bias current.

Fig. 2. The normalised ionization rate $\lambda(x)$ given as a function of the normalised radial distance x. The curves labelled a_o to c_1 are for the following values of the parameters η, the normalised axial electric field, and Λ, the ion Hall parameter at the outer electrode:

a_o: $\eta = 0.96$, $\Lambda = 0.292$
a_1: $\eta = 0.96$, $\Lambda = 0.982$
b_o: $\eta = 0.656$, $\Lambda = 1.15$
b_1: $\eta = 0.832$, $\Lambda = 1.15$
c_o: $\eta = 0.5$, $\Lambda = 1.732$
c_1: $\eta = 0.5$, $\Lambda = 1.970$

The radial ion current vanishes at the outer electrode for subscript o and at the inner one for subscript 1.

Fig. 3. The normalised electron density $n_e(x)/n_e(1)$ given as a function of the normalised radial distance x. The upper dashed curve is for the case of no internal ionization. The curves a_o, a_1, b_o, c_o are for the following values of the parameters η, the normalised axial electric field, and Λ, the ion Hall parameter at the outer electrode:

a_o: $\eta = 0.96$, $\Lambda = 0.292$
a_1: $\eta = 0.96$, $\Lambda = 0.982$
b_o: $\eta = 0.656$, $\Lambda = 1.15$
c_o: $\eta = 0.5$, $\Lambda = 1.732$

The radial ion current vanishes at the outer electrode for subscript o and at the inner one for subscript 1.

Fig. 4. The ratio between the absolute values of the total ion current into the electrodes, I_i, and the electron current at the cathode, I_e, given as a function of the ion Hall parameter at the anode, Λ. The upper and lower curves are for the corresponding maximum and minimum possible values of the axial field parameter, η. Values of η are indicated on both curves. For $\Lambda \lesssim 0.938$ on the upper curve $\eta = 1$. The points labelled a_o to c_1 correspond to the specific pairs of values for η and Λ referred to in the text. The dotted curve is for the limiting expression when Λ is large.

Fig. 5. The lines followed by the total current density $\underset{\sim}{j}$. The small circles indicate the points at which the radial ion current is zero and the total current is purely radial. The perpendicular drift velocity, u_d, and the axial drift velocity, u_s, *defined in the text are indicated.* The current lines labelled a_o to c_1 are for the following values of the normalised axial electric field, η, and the ion Hall parameter at the anode, Λ:

a_o: $\eta = 0.96$, $\Lambda = 0.292$
a : $\eta = 0.96$, $\Lambda = 0.771$
a_1: $\eta = 0.96$, $\Lambda = 0.982$
b_o: $\eta = 0.656$, $\Lambda = 1.15$
b_1: $\eta = 0.832$, $\Lambda = 1.15$
c_o: $\eta = 0.5$, $\Lambda = 1.732$
c_1: $\eta = 0.5$, $\Lambda = 1.970$

The radial ion current vanishes at the outer electrode for subscript o and at the inner one for subscript 1.

Fig. 6. The ratio between the axial drift velocity, u_s, and the critical velocity, u_c, as a function of the normalised radial distance x. The curves labelled a_o to c_1 are for the following values of the normalised axial electric field, η, and the ion Hall parameter at the anode, Λ:

a_o: $\eta = 0.96$, $\Lambda = 0.292$
a : $\eta = 0.96$, $\Lambda = 0.771$
a_1: $\eta = 0.96$, $\Lambda = 0.982$

b_o: $\eta = 0.656$, $\Lambda = 1.15$

b_1: $\eta = 0.832$, $\Lambda = 1.15$

c_o: $\eta = 0.5$, $\Lambda = 1.732$

The radial ion current vanishes at the outer electrode for subscript o and at the inner one for subscript 1.

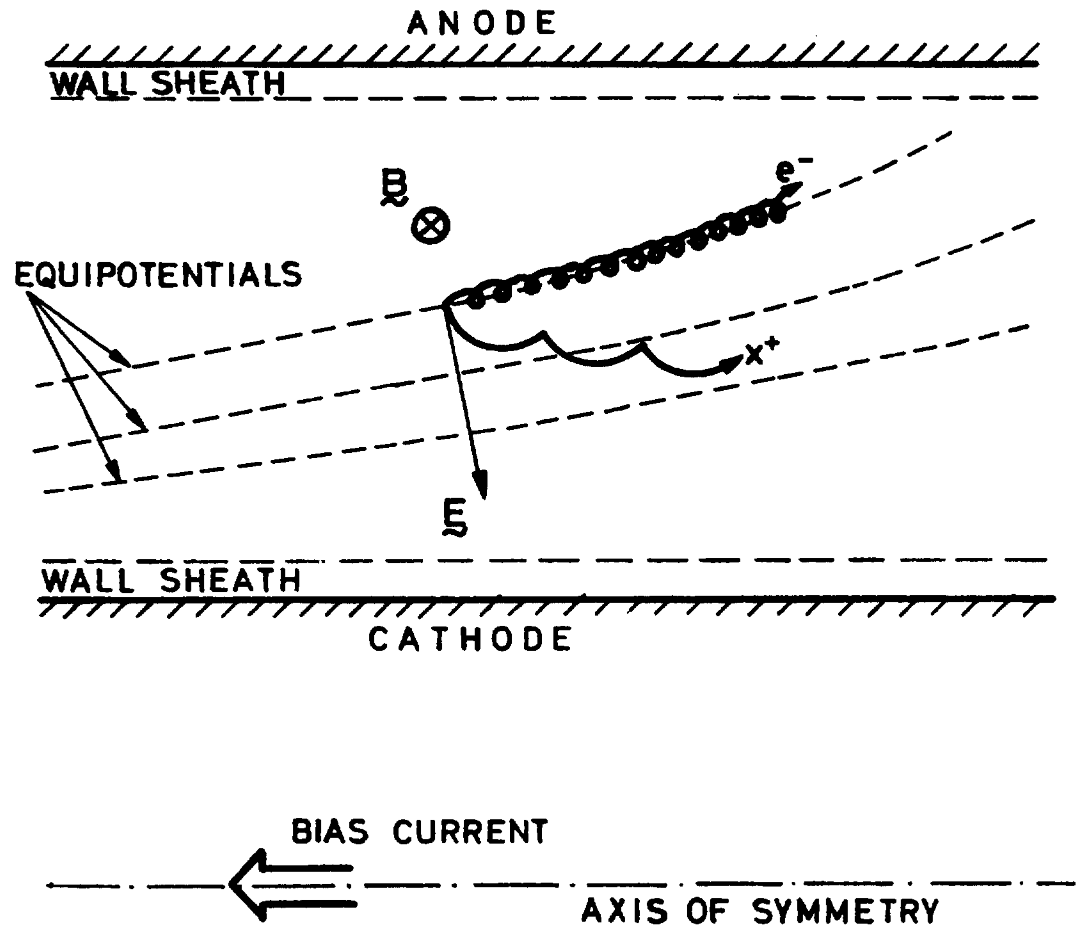
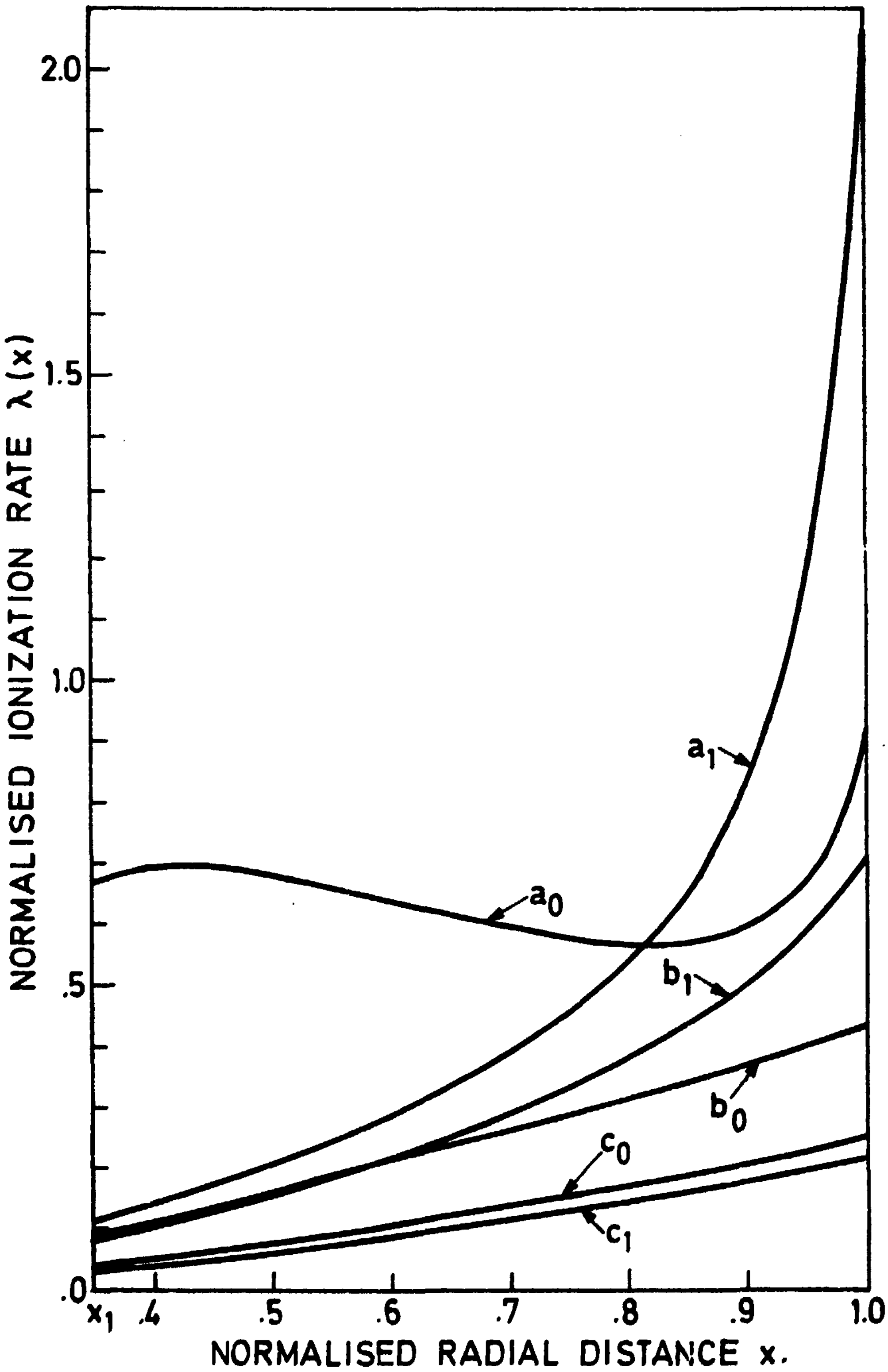

ANODE
WALL SHEATH
B
e^-
EQUIPOTENTIALS
x^+
E
WALL SHEATH
CATHODE
BIAS CURRENT
AXIS OF SYMMETRY

Fig. 1



2.0
1.5
1.0
.5
.0
NORMALISED IONIZATION RATE λ(x)
a1
a0
b1
b0
c0
c1
x1
.4
.5
.6
.7
.8
.9
1.0
NORMALISED RADIAL DISTANCE x.

Fig. 2

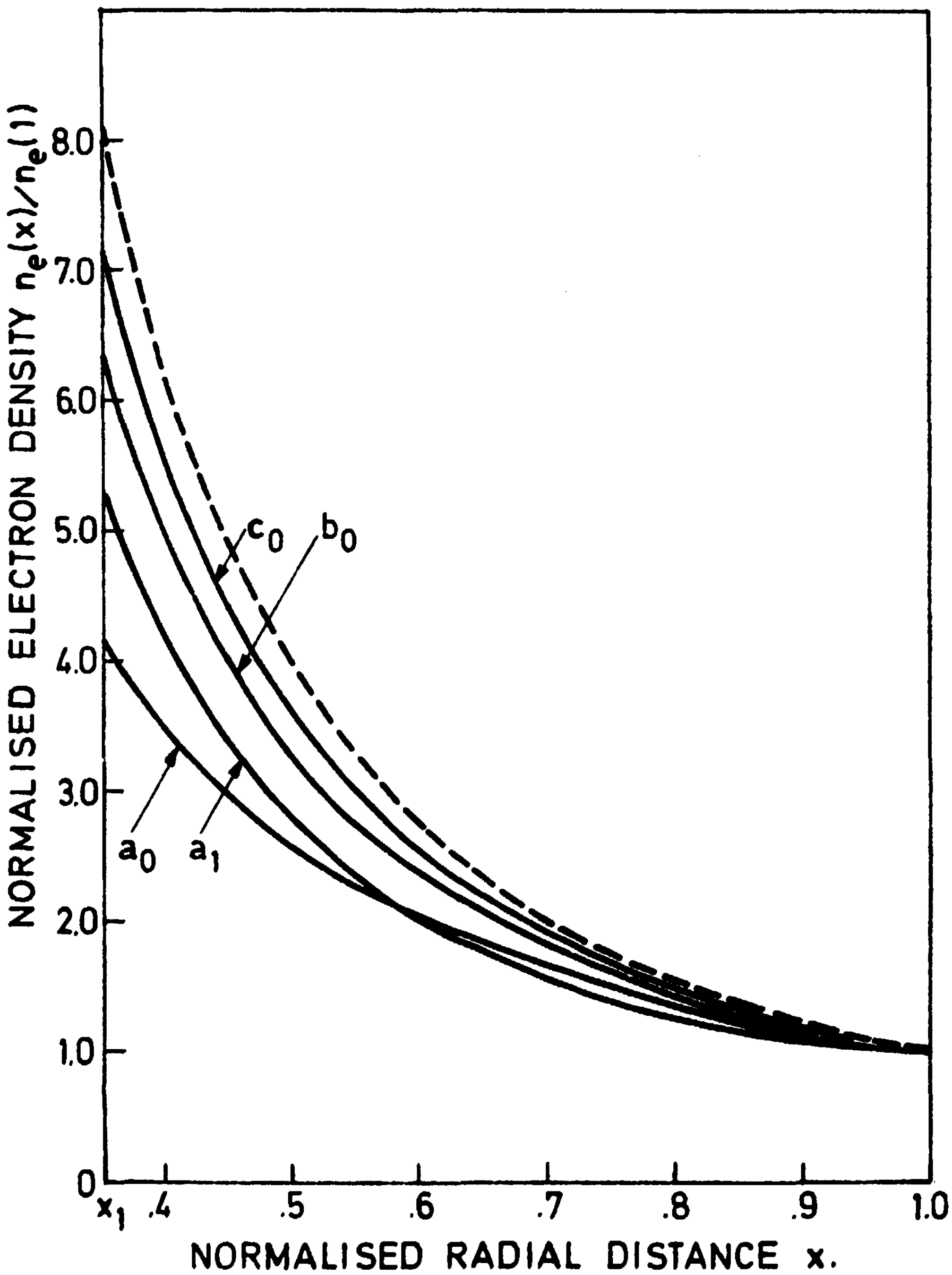
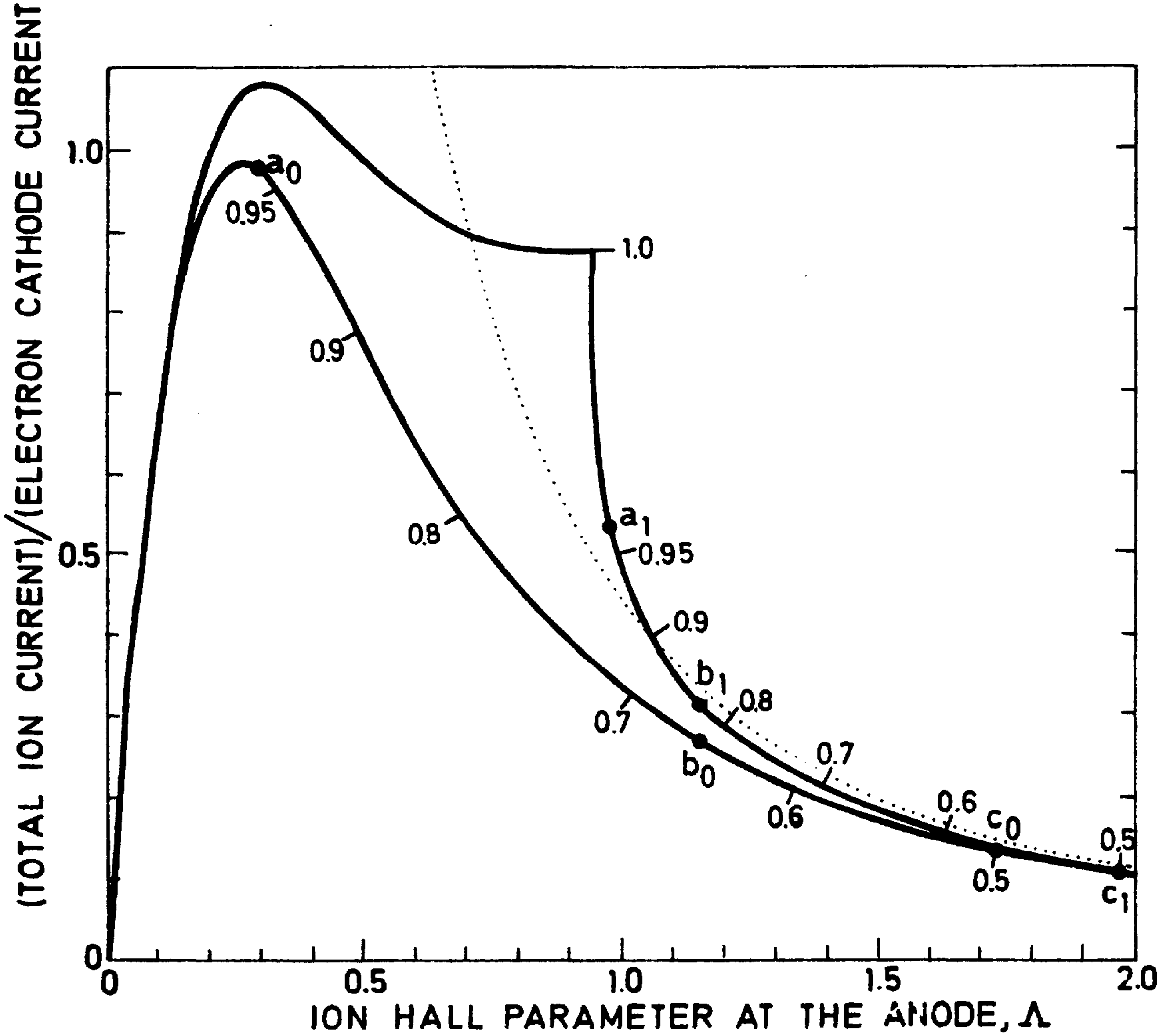

NORMALISED ELECTRON DENSITY $n_e(x)/n_e(1)$
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
c_0
b_0
a_0
a_1
x_1
.4
.5
.6
.7
.8
.9
1.0
NORMALISED RADIAL DISTANCE x.

Fig. 3



(TOTAL ION CURRENT)/(ELECTRON CATHODE CURRENT)
ION HALL PARAMETER AT THE ANODE, Λ
0
0.5
1.0
1.5
2.0
a0
a1
b0
b1
c0
c1
0.95
0.9
0.8
0.7
0.6
0.5

Fig. 4

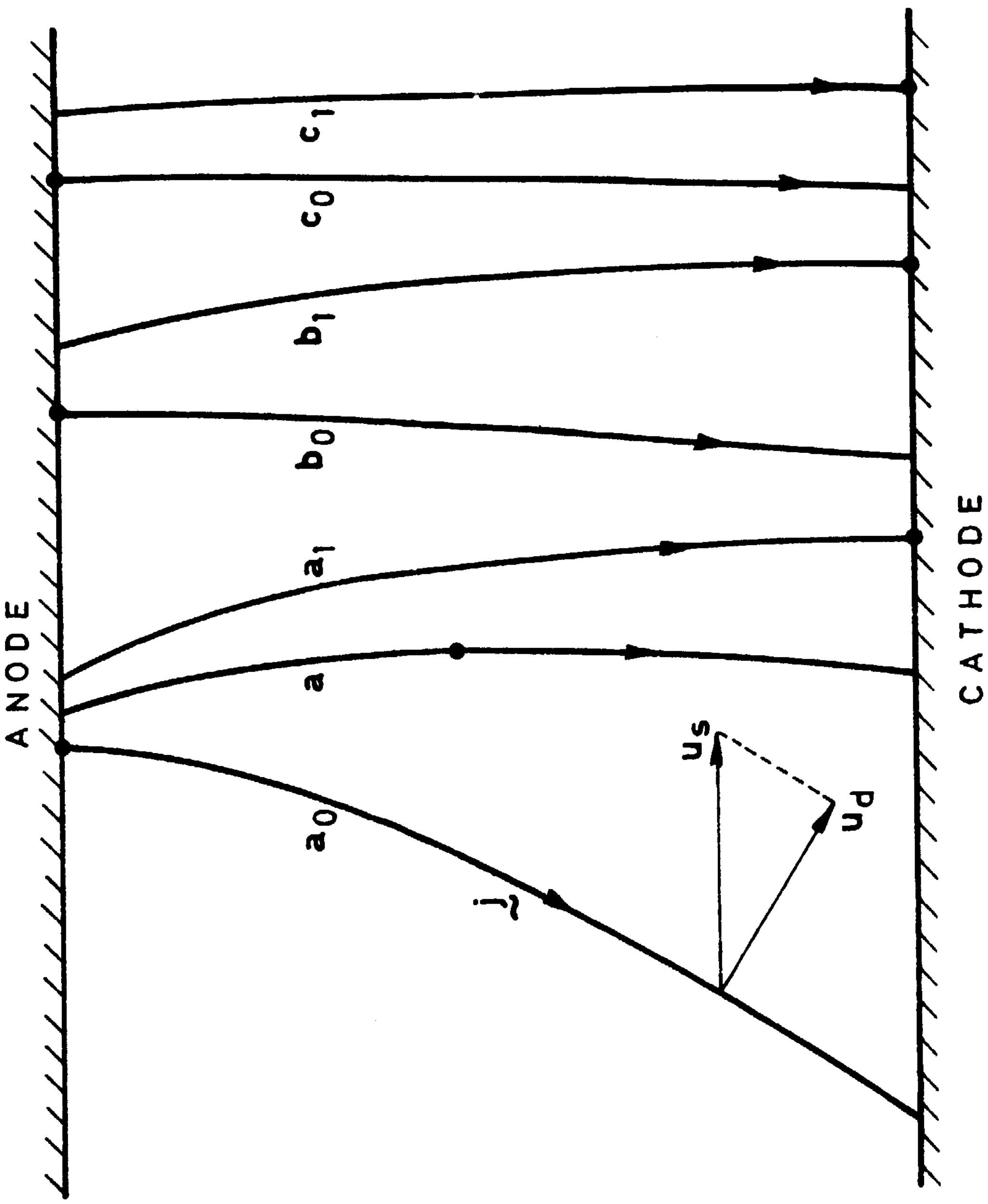
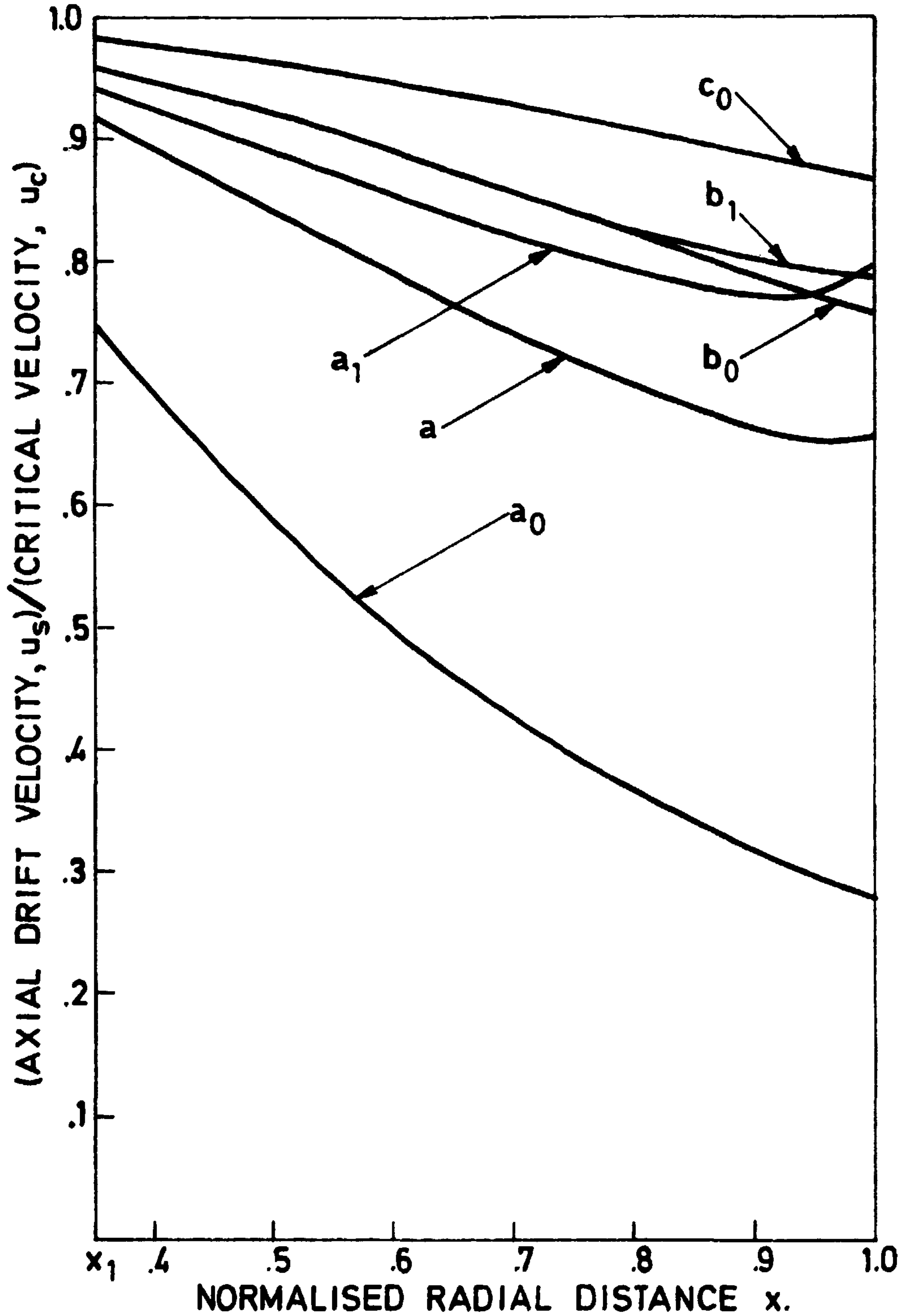


Fig. 5



1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
(AXIAL DRIFT VELOCITY, u_s)/(CRITICAL VELOCITY, u_c)
c_0
b_1
a_1
b_0
a
a_0
x_1
.4
.5
.6
.7
.8
.9
1.0
NORMALISED RADIAL DISTANCE x.

Fig. 6

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DYNAMICS OF A COAXIAL PLASMA GUN

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The dynamics of an ionizing wave in a coaxial plasma gun with an azimuthal bias magnetic field is analysed in a theoretical model. Only the radial dependence is treated and instead of including a treatment of the energy balance two separate physical assumptions are made. In the first case it is assumed that the total internal electric field is given by the critical ionization velocity condition and in the second that the ionization rate is constant. For consistency wall sheaths are assumed to match the internal plasma potential to that of the walls. On the basis of momentum and particle balance the radial dependence of the electron density, current density, electric field and drift velocity are found. An electron source is required at the cathode and the relative contribution from ionization within the plasma is deduced. The assumption that there are no ion sources at the electrodes leads to a restriction on the possible values of the axial electric field.

Key words: Critical ionization velocity, Critical velocity, Ionization, Plasma dynamics, Electron density, Crossed field discharge, Coaxial plasma gun.