SOME REMARKS ON COHERENT NONLINEAR COUPLING OF WAVES IN PLASMAS

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INTRODUCTION

During the last decades a vivid and increasingly important research has been carried out in the field of nonlinear plasma physics, like in the field of nonlinear optics.

To interpret new experiments, carried out at high intensities of radiation, which have currently become available, many nonlinear processes have been analysed, and methods developed, which may be regarded as unifying theoretical approaches to describe various nonlinear processes.

A detailed description of the nonlinear evolution of a wave in its interaction with other waves should account for the changes in the amplitude as well as in the phase of the wave, generally in space and time. For many situations of practical interest, such as those where the plasma exhibits turbulent features, the description of the plasma becomes so complicated that such detailed phase-analysis is generally not possible, and certain simplifying assumptions are needed. The random-phase method, which has been used extensively to describe such situations, thus introduces an ensemble averaging over the phases, as considered motivated by the physical situation. On the other hand experiments on nonlinear interaction of waves in plasmas indicate that in many cases coherent features are preserved, at least to some extent, in the nonlinear interaction. Detailed studies of the basic system of equations describing coherent nonlinear evolution of waves are therefore of primary interest to modern plasma physics and its applications, for example to astrophysics, space science or fusion-plasma research.

The purpose of the present contribution is to discuss various possibilities of analysing the basic set of coupled equations and to point out inherent difficulties associated with the description of the nonlinear interaction between different types of waves, and for general coupling situations, including deviations from perfect matching of frequencies and wave-numbers. We apply our considerations to specific examples in the field of stimulated excitation of waves, namely to parametric excitation of hybrid resonances in hot magnetized multi-ion component plasma and to laser-plasma interaction.

BASIC SYSTEM OF EQUATIONS AND APPROACHES OF SOLUTION

The basic equations for the complex amplitudes a_i of three coupled waves, propagating in the x-direction, are

$$\frac{\partial a_0}{\partial t} + \nu_0 a_0 + v_0 \frac{\partial a_0}{\partial x} = c_{12} a_1 a_2^* \exp(-i\Delta\Phi) \ , \quad (1)$$

$$\frac{\partial a_1}{\partial t} + \nu_1 a_1 + v_1 \frac{\partial a_1}{\partial x} = c_{02} a_0 a_2^* \exp(i\Delta\Phi) \ , \quad (2)$$

$$\frac{\partial a_2}{\partial t} + \nu_2 a_2 + v_2 \frac{\partial a_2}{\partial x} = c_{01} a_0 a_1^* \exp(i\Delta\Phi) \ , \quad (3)$$

where

$$\nu_i = \mathrm{Im}(\omega_i)$$

is the linear damping or growth of a wave with group velocity

$$v_i = \frac{\partial \omega}{\partial k} \ ,$$

and

$$\Delta\Phi = (\Delta\omega)t - \int^{x} \Delta k(x')dx' \ ,$$

with

$$\Delta\omega = \mathrm{Re}(\omega_0) - \mathrm{Re}(\omega_1) - \mathrm{Re}(\omega_2) ,$$

$$\Delta k = k_0 - k_1 - k_2 ,$$

and where c_{ij} are the coupling coefficients.

Here $\Delta\omega$ represents a frequency mismatch, which we regard as fixed, determined by the frequencies ω_i, whereas Δk represents a wave-number mismatch, which in an inhomogeneous medium (including also long scale-length random inhomogeneities) depends on x.

A complete study of this system of waves, considering space and time variations of the amplitudes is connected with serious difficulties, in particular with regard to those introduced by differences in the parameters describing damping and group-velocity of the waves. Such differences do indeed exist in physical systems. Even the technique of inverse scattering does not allow for a solution of the full system. It is therefore customary to consider simplified model equations, where

i) One of the waves is sufficiently strong as compared to the others, so that its amplitude can be considered as constant and one can use a parametric approach. Useful information about the thresholds and growth-rates for various processes can be obtained from such an approach, which has been used frequently to describe stimulated scattering and decay processes, e.g. in laser-plasma interaction. Considering a case where, for example, two of the modes are electromagnetic waves with frequencies ω_0 (the pump wave) and ω_2, whereas the third wave, corresponds to a collective excitation in the medium with a characteristic frequency ω_L, one obtains as is well known (Nishikawa 1968) an equation of the form

$$(\omega-\omega_L)^2 - [i(\Gamma_1+\Gamma_2) + \Delta\omega](\omega-\omega_L) + i\Delta\omega\Gamma_1 - \Gamma_1\Gamma_2 + \frac{G}{4\omega_L\omega_2} = 0 , \tag{4}$$

where the frequency mismatch

$$\Delta\omega = \omega_0 - \omega_L - \omega_2$$

and where Γ_1 and Γ_2 refer to linear damping rates of the parametrically excited waves. The quantity G is proportional to the power of the pump wave (see example below).

Considering first the case, where $\Gamma_1 = \Gamma_2 = 0$, we obtain by solving Eq. (4) for the root corresponding to growth (time factor

$\exp(i\omega t)$, and assuming $G/(\omega_L\omega_2) > (\Delta\omega)^2$) the expression

$$\omega_1 = \omega_L + \frac{\Delta\omega}{2} - \frac{i}{2}\left[G/(\omega_L\omega_2) - (\Delta\omega)^2\right]^{1/2}, \tag{5}$$

which clearly exhibits the important and well known fact that the threshold value for the pump power to surmount the effect of mismatch is determined by

$$G = \omega_L\omega_2(\Delta\omega)^2 \tag{6}$$

whereas there is no growth for

$$|\Delta\omega| > 2\gamma_0 \tag{7}$$

where

$$\gamma_0 = \frac{1}{2}\left(G/(\omega_L\omega_2)\right)^{1/2} \tag{8}$$

is the growth-rate in the absence of mismatch (the maximum growth-rate), i.e. for $\Delta\omega = 0$.

Taking into account the damping rates Γ_1 and Γ_2 in Eq. (4) the condition for threshold becomes (Nishikawa 1968),

$$G = 4\omega_L\omega_2\Gamma_1\Gamma_2\left[1 + (\Delta\omega)^2/(\Gamma_1+\Gamma_2)^2\right], \tag{9}$$

and the maximum growth-rate

$$\gamma_0 = \frac{1}{2}\left\{\left[G/(\omega_L\omega_2) + (\Gamma_1-\Gamma_2)^2\right]^{1/2} - (\Gamma_1+\Gamma_2)\right\} \tag{10}$$

We have reviewed here, some of the basic formulas because we need them in the following, when we are going to use them for some specific applications.

ii) The mismatch or damping of one of the waves is so large that the corresponding term in one of the equations dominates the linear part of the equation, which becomes algebraic and allows for direct solution for one of the unknown amplitudes in terms of the others. By substitution of this amplitude into the remaining equations, one is left with two nonlinear coupled equations, more suited for detailed discussion.

Introducing the following expressions for the normalized intensities

$$I_0 = s_0 \mathrm{Re}\left(\frac{c_{01}^* c_{02}}{\nu_2 - i(\Delta\omega - v_2\Delta k)}\right)\frac{2}{v_1 - v_0}\,|a_0|^2 , \qquad (11a)$$

$$I_1 = s_1 \mathrm{Re}\left(\frac{c_{12}^* c_{01}}{\nu_2 + i(\Delta\omega - v_2\Delta k)}\right)\frac{2}{v_0 - v_1}\,|a_1|^2 , \qquad (11b)$$

where s_0 and s_1 account for the signs of the Re( ) factors in (11a,11b), so that I_0 and I_1 are positive quantities, and furthermore introducing the variables

$$\xi = x - v_0 t ,$$

$$\eta = x - v_1 t ,$$

and

$$\alpha_0 = \frac{2\nu_0}{v_0 - v_1} ,$$

$$\alpha_1 = \frac{2\nu_1}{v_1 - v_0} ,$$

we obtain the following coupled equations

$$\frac{\partial}{\partial\eta} I_0 + \alpha_0 I_0 = s_1 I_0 I_1 , \qquad (12a)$$

$$\frac{\partial}{\partial\xi} I_1 + \alpha_1 I_1 = s_0 I_0 I_1 . \qquad (12b)$$

The system (12a,12b) is by far not trivial as a result of the presence of α_0 and α_1 and in view of the two independent variables ξ and η. Let us here only note that for $s_0 = s_1 = 1$, and considering a stationary case, the eqs.(12a,12b) exhibit solutions of the explosive type for which $I_0 \sim I_1 \sim (\xi_0 - \xi)^{-1}$. This dependence is characteristic for an incoherent interaction, and corresponds to the results of a random phase approach, for which the corresponding three coupled equations are of the form

$$\frac{\partial N_0}{\partial t} + \alpha_0 N_0 = N_1 N_2 + N_0 N_2 + N_0 N_1 ,$$

where N_i are the quasi-particle densities, and where signs refer to a nonlinearly (explosively) unstable case. A solution of Eqs. (1-3) for $\Delta\Phi = 0$ and considering the phase-dynamics of the complex amplitudes would correspond to $I_0 \sim I_1 \sim (\xi_0-\xi)^{-1/2}$. Apparently the result mirrors the destruction of coherent interaction accompanying the assumption of large $\Delta\Phi$. For I_0 and I_1 corresponding to electromagnetic waves, coupled by plasmons. Such an explosive state should be possible in a medium, containing molecules with inverted populations.

iii) Only the dependence on one independent variable (x or t) is considered when studying the coupled system of the three (slowly) varying amplitudes, and mismatches are neglected. Even so, the system can be given analytic solutions in terms of elliptic functions <u>only</u> when all the coefficients of dissipation ν_i are equal. The nonlinear wave interaction may lead to either nonlinearly stable or unstable solutions depending on the properties of the interacting waves (in the case of real coupling constants nonlinear (explosive) instabilities may occur for interaction between negative - and positive - energy waves).

It has been demonstrated recently (Nakach and Wilhelmsson 1976) that, when two of the coefficients of dissipation are equal, the set of three coupled equations can be reduced to a single equivalent equation, which in the nonlinearly unstable case, where one wave is undamped, asymptotically takes the form of an equation defining the third Painlevé transcendent. The general solution of even the stationary form of the system, and for perfect matching, i.e. $\Delta\Phi = 0$, therefore belongs to a higher class of functions which reduces to the Jacobian elliptic type only when all coefficients of dissipation are equal.

iv) Use of computer is necessary to study the full system of equations (1-3), which may then also be supplemented by higher order terms (cf. Oraevskii et al 1973, Weiland and Wilhelmsson 1973, 1976). As a transition between coherent and incoherent dynamics of the system coherent interaction between partial subsystems of three wave-packets have been studied (Weiland et al 1975, Weiland and Wilhelmsson 1976). The stationary form of the Eqs. (5a,5b) have also been studied by means of computer, including effects of higher order terms (White et al 1972, Anderson and Bondeson 1976).

After this survey on the possibilities of studying wave-wave coupling from a coherent interaction point of view let us briefly discuss some specific examples which are related to experiments in magnetized plasmas and in laser-plasma interaction.

PARAMETRIC EXCITATION OF A LOWER-HYBRID RESONANCE IN A MULTI-ION COMPONENT PLASMA

As an example of parametric interaction let us consider the coupling between a strong ordinary electromagnetic wave with a lower-hybrid wave (Stix 1965) and an other ordinary electromagnetic wave (Nguyen The Hung 1975), for the specific case where all waves propagate perpendicularly to the magnetic field. We pay particular attention to the presence of multi-ion components in the plasma, corresponding to experimental situations in present fusion-plasma devices (Wilhelmsson 1976).

Denoting by C_k the fractional concentration and by Z_k the charge of the impurity ions of the kind denoted by suffix k, and assuming that in the presence of the impurity ions, we have charge conservation the ion and electron number densities of a hydrogen plasma are related by

$$N_{oi} = N_{oe}(1 + \Sigma\, C_k Z_k)^{-1} \tag{13}$$

The characteristic frequencies of a fusion plasma fulfil the following condition

$$\omega_{Hi}^2 \ll \omega_{pi}^2 \approx \omega_{LH}^2 \ll \omega_{He}^2 \simeq \omega_{pe}^2 \,, \tag{14}$$

where ω_{LH} denotes the lower hybrid frequency, and the other notations refer to Larmor and plasma frequencies of the protons and electrons.

From the hot plasma dielectric tensor (See for example Ichimaru 1973) and considering the multi-ion species (Wilhelmsson 1976) we obtain the following relation for the lower-hybrid frequency

$$\omega_{LH}^2 = \frac{\omega_{He}^2 + k^2u_e^2}{\omega_{pe}^2 + k^2u_e^2 + \omega_{He}^2} \cdot \frac{m_e}{m_i} \cdot \omega_{pe}^2 (1 + \Sigma\, C_k Z_k)^{-1} \cdot \left[1 + \frac{k^2u_i^2}{(\omega_{LH}^o)^2} + \Sigma\, C_k Z_k^2 \frac{m_i}{m_k}\left(1 + \frac{k^2u_k^2}{(\omega_{LH}^o)^2}\right)\right] , \tag{15}$$

where we introduced the notation ω_{LH}^o for the lower-hybrid frequency in the absence of impurity and temperature corrections, i.e.

$$(\omega_{LH}^{o})^2 = \frac{\omega_{pi}^2 \cdot \omega_{He}^2}{\omega_{He}^2 + \omega_{pe}^2}$$

From the quantity $G/(\omega_{LH}\omega_2)$ which enter the threshold and growth-rate equations (9,10) we obtain ($\omega_L = \omega_{LH}$),

$$\frac{G}{\omega_{LH}\omega_2} = k^2\left(\frac{e}{m_e\omega_0}\right)^2 \cdot E_0^2 \cdot \frac{\omega_{LH}\omega_{pe}^4}{\omega_2(\omega_{pe}^2 + k^2u_e^2 + \omega_{He}^2)(\omega_{He}^2 + k^2u_e^2)} \qquad (16)$$

where E_0 is the electric field amplitude of the pump wave, and where other notations are standard. Introducing collision frequencies ν_e and ν_i we have $2\Gamma_1 = \nu_i$ and $2\Gamma_2 = \nu_e(\omega_{pe}^2/\omega^2)$. For realistic impurity concentrations of hot fusion plasmas it turns out that the value of the lower-hybrid frequency may be decreased by about 20% as a result of the presence of impurities. As a consequence the threshold of the process here considered will increase by about 20% whereas the maximum growth-rate will decrease by about 10%. It should be added that different impurities may be located in different parts of the plasma, adding their contributions to $\Delta k(x)$ and thereby to the mismatches in the nonlinear couplings.

For vanishing static magnetic field the results apply to the stimulated excitation of ion-acoustic waves, i.e. to stimulated Brillouin scattering in the presence of multi-ion specia.

EFFECT OF FREQUENCY BROADENING IN LASER-PLASMA INTERACTION

For the nonlinear coupling processes in laser-plasma interaction it is important to consider spectral broadening effects (Valeo and Oberman, 1973, Anderson and Wilhelmsson 1976).

We notice that the threshold condition (9) of parametric instability can be written

$$(\Delta\omega)^2 + (\Gamma_1+\Gamma_2)^2 = \frac{(\Gamma_1+\Gamma_2)^2}{\Gamma_1\Gamma_2}\gamma_0^2 \qquad (9a)$$

with γ_0 defined by relation (12). When "blurring" effects on the left hand side become larger than the expression on right hand side the coupling process is no longer efficient.

Let us represent the influence of the coupling process on the frequency width of the backscattered wave, due to stimulated scattering, by a normalized Gaussian function $K(|\omega-\omega'|)$ of width σ_{NL}. We furthermore assume that the spectra of incident and reflected radiation can be represented by Gaussian functions of widths D_I and D_R. The frequency broadening process can then be described as follows in terms of the powers of the reflected and incident radiation

$$P_R(\omega) = \int K(|\omega-\omega'|)P_I(\omega')d\omega' \tag{17}$$

(cf. Wilhelmsson 1963)

and

$$D_R^2 = D_I^2 + \sigma^2_{NL} \tag{18}$$

where we take

$$\sigma^2_{NL} = (\Delta\omega_{cr})^2 + (\Gamma_1+\Gamma_2)^2 = \frac{(\Gamma_1+\Gamma_2)^2}{\Gamma_1\Gamma_2}\gamma_0^2 , \tag{19}$$

with $\Delta\omega_{cr}$ denoting the critical frequency deviation for marginal fulfilment of relation (17). We then have

$$\left(\frac{D_R}{D_I}\right)^2 = 1 + \frac{\sigma^2_{NL}}{D_I^2} , \tag{20}$$

or if in relation (17) we indentify Γ_1, the damping rate of the reflected wave, with the width of the reflected spectrum and furthermore assume $\Gamma_2 > \Gamma_1$ we obtain (Anderson and Wilhelmsson 1976)

$$\frac{D_R}{D} = \left(1 + \alpha\frac{W_L}{D_R/D}\right)^{1/2} , \tag{21}$$

where W_L is the energy of the incident laser pulse and α a proportionality constant.

In figure 1 we depict the relative bandwidth D_R/D_0 as a function of αW_L. One of the points (W_L = 1,4 J, $D_R/D_I \approx$ 2,5) has been determined from available experimental data (Baldis, Pepin and Johnston 1975), and as could be seen in the figure the theoretical curve then follows closely the other experimental points.

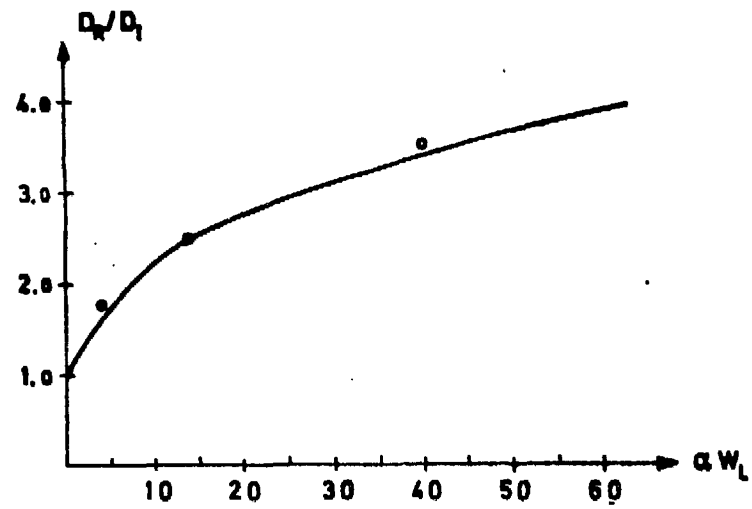

Fig. 1. The theoretical curve (Anderson and Wilhelmsson 1976) corresponding to relation (21), normalized by and compared with the experimental results on laser reflection of Baldis, Pepin and Johnston 1975.

It should furthermore be mentioned that recent measurements on finite bandwidth effects on the parametric Decay Instability in the radiofrequency and HF-domain (Oberschain, Luhmann, and Greiling 1976) give results which for wide-bandwidth pumps ($\Delta\omega >> \gamma$) are consistent with theoretical predictions.

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