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STRUCTURAL ANALYSIS OF SALT CAVITIES FORMED BY SOLUTION MINING:

I. METHOD OF ANALYSIS AND PRELIMINARY RESULTS FOR SPHERICAL CAVITIES

Technical Memorandum Report RSI-0043

Arlo F. Fossum

January 15, 1976

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STRUCTURAL ANALYSIS OF SALT CAVITIES FORMED BY SOLUTION MINING: I. METHOD OF ANALYSIS AND PRELIMINARY RESULTS FOR SPHERICAL CAVITIES

Submitted To

Oak Ridge National Laboratory
Oak Ridge, Tennessee

operated by

Union Carbide Corporation
Nuclear Division
for the
Energy Research and Development Administration

By

Arlo F. Fossum

RE/SPEC INC.
P. O. BOX 725 - RAPID CITY, S. D. 57701 - 605/343-7868

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P. O. Box 725
Rapid City, South Dakota

January 15, 1976

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FOREWORD

The technical contents of this report have been reviewed by Dr. Paul F. Gnirk and Mr. Joe L. Ratigan of RE/SPEC Inc.

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(RSI/0043)

RE/SPEC INC.

P. O. Box 725 • Rapid City, S D 57701 • 605/343-7868

January 15, 1976

TECHNICAL MEMORANDUM REPORT RSI-0043

TO: Dr. William C. McClain
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, TN 37830

FROM: Dr. Arlo F. Fossum
RE/SPEC Inc.
P. O. Box 725
Rapid City, SD 57701

SUBJECT: Structural Analysis of Salt Cavities Formed by Solution Mining: I. Method of Analysis and Preliminary Results for Spherical Cavities (Union Carbide Corporation, Nuclear Division Subcontract No. 4269; RSI/001000/FY76).

1. INTRODUCTION

1.1. Statement of Objectives

The primary objective of this effort is an analysis of the structural stability of cavities formed by solution mining in salt domes. In particular, the effects of depth (i.e. initial state of in situ stress), shape, volume (i.e. physical dimensions of the cavity), and sequence of salt excavation/fluid evacuation on the timewise structural stability of a cavity are of interest. It is anticipated that an assessment can be made of the interrelation between depth, cavern size, and cavern shape or of the practical limits therewith. In general, the cavity shape is assumed to be axisymmetric and the salt is assumed to exhibit nonlinear creep behavior.

In this report, the primary emphasis is placed on the methodology of the finite element analysis, and the results of preliminary calculations for a spherically-shaped cavity. It is common practice for engineers to apply elasticity theory to the behavior of rock in order to obtain near field stresses and displacements around an underground excavation in an effort to assess structural stability. Rock masses, particularly at depth, may be subjected to a rather complex state of initial stress, and may be nonhomogeneous and anisotropic. If one also includes complex geometrical excavation shape, the use of analytical techniques as an analysis tool is practically impossible. Thus, it is almost a necessity that approximate solution techniques be employed. In this regard, the finite element method is ideal as it can handle complex geometries and nonlinear material behavior with relative ease. An unusual feature of the present study is the incorporation into the finite element code of a procedure for handling the gradual creation or excavation of an underground cavity. During the excavation sequence, the salt is permitted to exhibit nonlinear stress-strain-time dependence. The bulk of this report will be devoted to a description of the analysis procedures, together with a preliminary calculation for a spherically-shaped cavity.

1.2. Method of Approach

The decision was made at the outset of this study to develop a code both streamlined and dedicated specifically to the analysis of cavities formed by solution mining. Based on certain aspects of the thermo/viscoelastic code RSI/TEVCO and of the creation and intended usage of solution cavities in salt, the computer code RSI/SCAMP (Solution Cavity Analysis with Mining Procedures) was developed. As regards the particular aspects of creation and intended usage of the solution cavities for storage of radioactive waste, the aspects of sequential cavity construction and possible complex initial in situ stress states due to the formation of the dome at large were viewed as integral for the analysis procedure.

Intuition indicates that important facets of stress relaxation take place in the salt around the cavity early in the time scale, i.e. within a day or so after the formation of the cavity. Thus, it was decided that the construction effects or the sequential excavation of the cavity would be significant in the evolution of the stress and displacement fields in the vicinity of the cavity. In addition, and based on the same line of reasoning, the evacuation of the washing fluid from the cavity after the desired cavity dimensions had been achieved was also deemed important. At the outset, the assumption was made that the most critical state of in situ stress would be that calculated on the basis of a gravitating elastic medium, and the least critical would be that based on the supposition of lithostatic loading, i.e. a uniform or hydrostatic stress state due to the weight of the overburden. For the case of a gravitating elastic medium and a Poisson's ratio of 0.333, the in situ horizontal stress would be exactly one-half of the vertical stress. This thinking was somewhat revised as a consequence of the meeting in New Orleans, LA on December 9, 1975 between personnel of RE/SPEC Inc. and the Oak Ridge National Laboratory. In view of the tectonics of salt dome formation, it appears plausible that the horizontal stresses could be greater in magnitude than the vertical stresses. If this is indeed the case in salt domes, the horizontal closure of a salt cavity may be greater than that arising from either an elastic gravitating medium or a lithostatic condition. From the standpoint of the storage of radioactive waste material and the timewise structural stability of the cavity, the horizontal closure may be more critical than the vertical closure. As a consequence, an option has been incorporated into RSI/SCAMP for the specification of any arbitrary initial premining stress field. Hence, a parameter defined by the ratio of the initial horizontal to vertical stress has been included, in addition to the particular parameters of cavity depth, shape, and size.

For the sake of documentation, this report includes an appendix in which a description is given of the finite-element procedure utilized in RSI/SCAMP. In particular, the essential elements of the cavity excavation/fluid evacuation procedure, the method of prescribing initial premining stresses, and the method of utilizing a nonlinear stress-strain-time material behavior law are described.

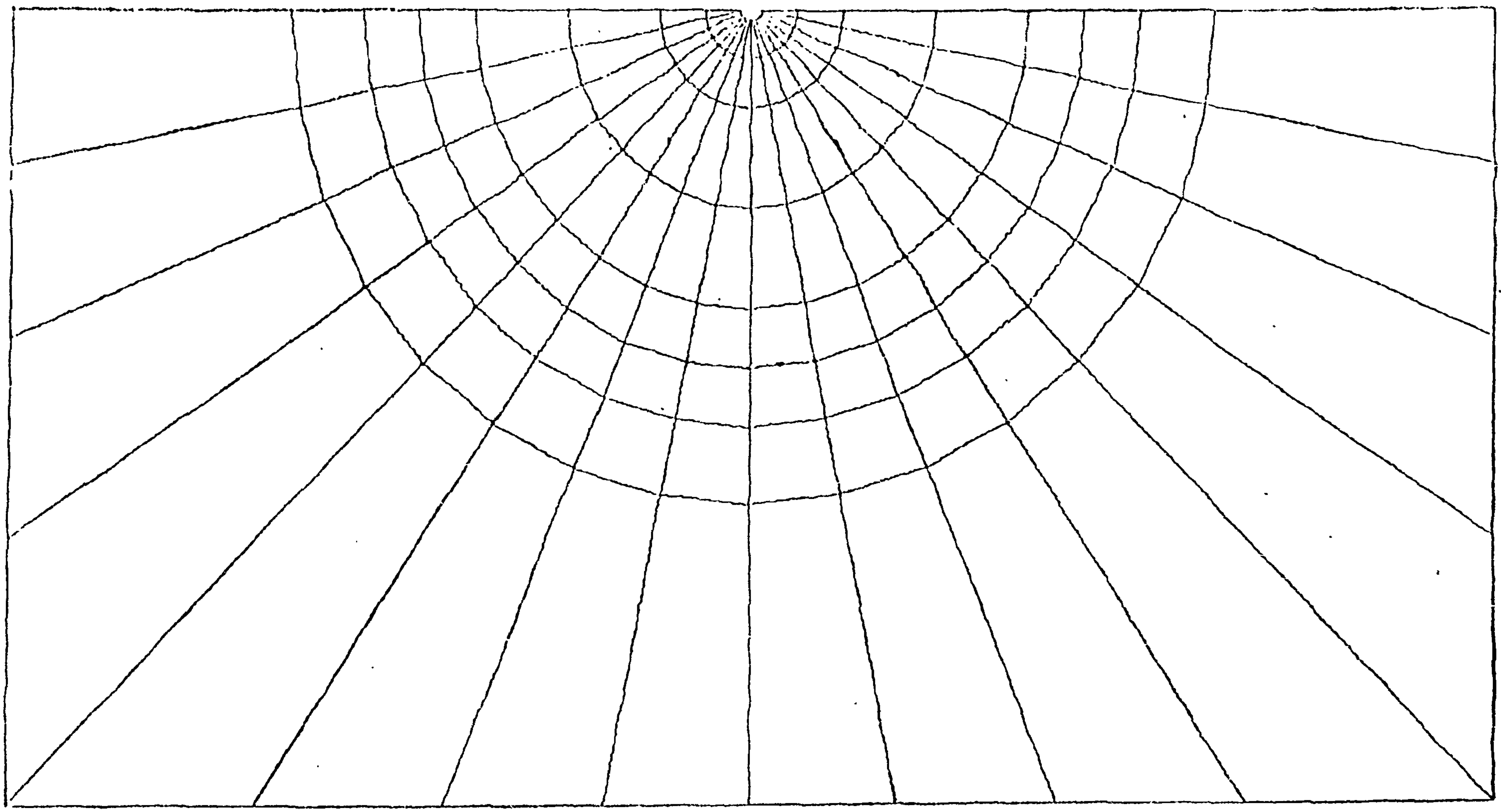
2. PRESENTATION AND DISCUSSION OF PRELIMINARY RESULTS

2.1. Spherical Cavity Model

In order to assess the computational accuracy of RSI/SCAMP, a spherical cavity was chosen for the initial model. The cavity was assumed to be situated at a depth of 2,000 feet (610 meters) in salt, and to have a diameter of 300 feet (91 meters). The cavity size would be adequate for a liquid volume storage of approximately 2.52 million barrels (400,000 cubic meters).

Apart from the stability implications, the spherical model of a cavity in salt is of interest from the standpoint of an assessment of modeling accuracy. In particular, since a closed-form analytical elastic solution is available for a spherical cavity in an infinite solid subjected to a homogeneous stress field, a means is therefore available for assessment of whether or not the mesh selection for the model is adequate. Included in this assessment are the two main considerations of whether or not the mesh grid is fine enough in areas of high stress gradient, and of whether or not boundaries of the model are sufficiently removed from the wall of the cavity. Sufficiency of the latter consideration is established if the creation of the cavity does not appreciably affect the pre-mining stress state at the boundary. In other words, the stress and displacement states at the boundaries of the finite-element model should be essentially identical with those states that existed prior to the simulated excavation of the cavity. The mesh gradation in regions of high stress gradients is determined by comparison of the finite-element calculation with stress concentration factors at selected locations on the wall of the spherical cavity, where the stress concentration factors are determined from the exact solution. By means of the above consideration, it was found that the rather coarse mesh of eight-noded isoparametric elements, as illustrated in Figure 1, yielded reasonable results for comparison with the closed-form analytical results. The finite-element mesh contains 128 biquadratic elements and 433 nodes, with 866 degrees of freedom.

The sequence of simulated construction and usage operations for the spherical cavity is illustrated in Figure 2. The center of the spherical cavity is situated at a depth of 2,000 feet. The top and bottom of the model are located at depths of 1,600 and 2,400 feet, respectively. The model may be thought of as a right circular cylinder with a height of 800 feet and a diameter of 800 feet. The boundary conditions on the vertical sides of the model are prescribed as ones of zero radial displacement and unrestricted vertical displacement; this is equivalent to a condition of rollers on the vertical sides. The bottom of the model may undergo unrestricted radial displacement,



(RSI-0043)

Figure 1. Spherical Cavity Finite Element Mesh of 8-Noded Isoparametric Ring Elements.

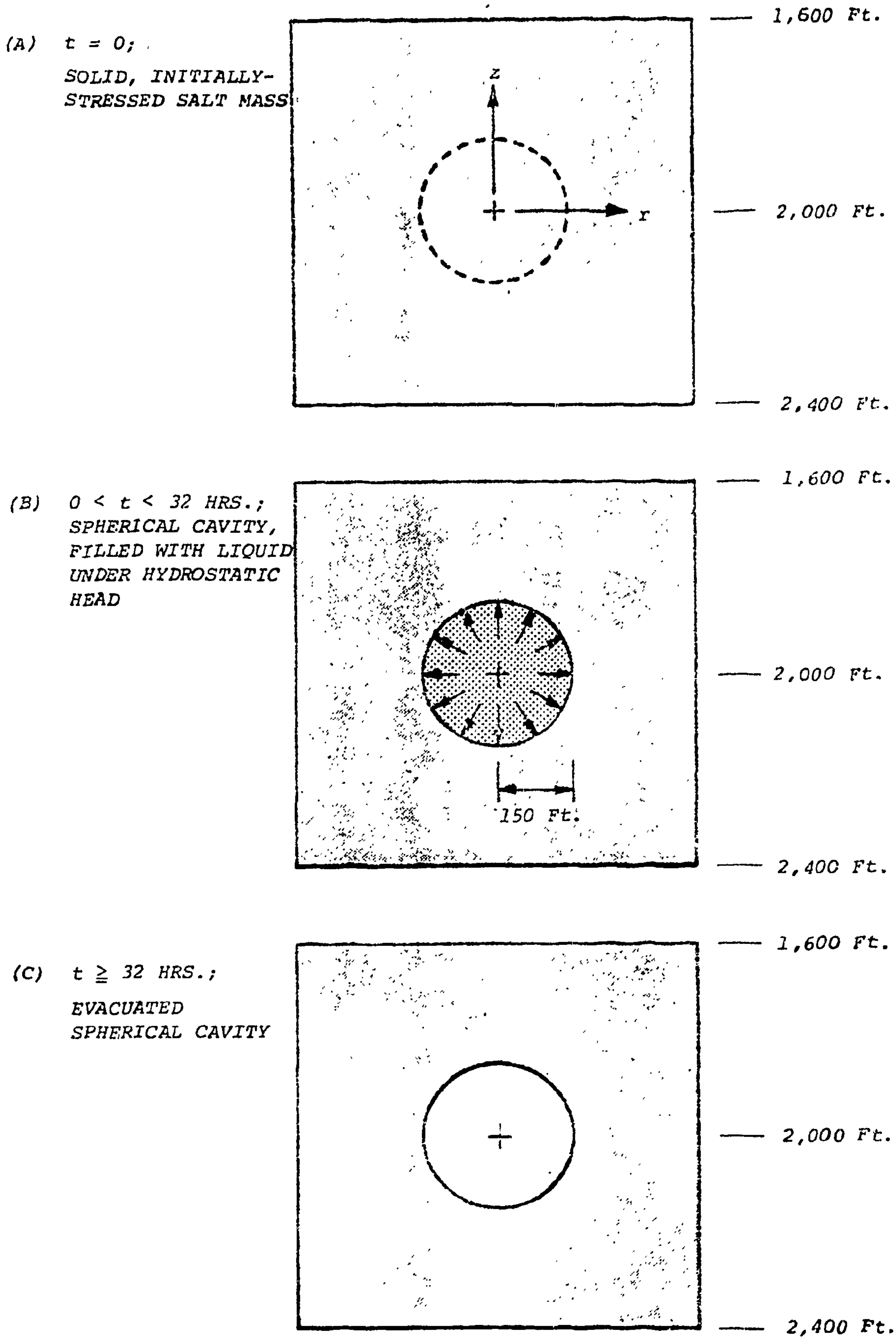


Figure 2. Sequence of Simulated Construction and Usage Operations for a Spherical Cavity Having a Diameter of 200 Ft. and Situated at a Depth of 2,000 Ft. in Salt.

but zero vertical displacement, except at the point on the vertical centerline through the center of cavity at which both the radial and vertical displacements are prescribed to be zero. The top of the model is subjected to an equivalent force of 1,600 feet of overburden salt. At time $t = 0$, the initial state of stress in the model is calculated on the basis of a gravitating elastic medium. Essentially, the vertical stress is directly proportional to the weight of the overburden, and the radial and tangential (or horizontal) stresses are identical and equal to $\nu/(1-\nu)$ times the vertical stress. The salt was assumed to have a modulus of elasticity of 1.5×10^6 psi and a Poisson's ratio of 0.35. At time $t = 0+$, a cavity with a diameter of 300 feet is excavated at the 2,000 foot level and immediately filled with water under hydrostatic head. The cavity is permitted to creep for a time period of 32 hours under these conditions. At time $t = 32$ hours, the water is evacuated from the spherical cavity and creep is permitted to continue. It should be emphasized that this sequence of simulated construction and usage operation is only a test situation for assessment of the code calculation capability. Clearly, as will be detailed in a later report, the simulated operational sequence would involve sequential excavation of the cavity to a final diameter, and sequential withdrawal of the liquid within the cavity, each involving much longer periods of time.

2.2. Rheological Model

The rheological model of the salt is based on data obtained previously (1,2), and its incorporation into the finite-element code RSI/SCAMP is discussed in Appendix A. Essentially, the nonlinear stress-strain-time incremental equation is given by:

$$d\varepsilon_{ij} = \frac{3}{4}\left[\frac{\sigma_{eq}}{24,300}\right]^3 \frac{dt}{2\sqrt{t}} \; \frac{(\sigma_{ij} - \delta_{ij}\,\sigma_m)}{\sigma_{eq}} \qquad (1)$$

where:

$d\varepsilon_{ij}$ = incremental strain tensor

σ_{ij} = total stress tensor (psi)

δ_{ij} = Kronecker delta

t = total time (hrs.)

dt = incremental time (hrs.)

$\sigma_m = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$

$\sigma_{eq} = \sqrt{3II_D}$

II_D = second invariant of deviatoric stress

2.3. Time-Dependent Stress Behavior

In order to make an initial assessment of the need to include the sequence of construction or excavation by solution mining, it is sufficient to mine out the cavity to the final configuration at time $t = 0$ and examine the rate of stress relaxation. If the stress relaxation is significant early in the operational history of the cavity, then one may conclude that construction effects will probably be significant in the evolution of the stress and displacement fields in the vicinity of the cavity wall. For ease of discussion, we will direct our attention to the vertical stress component at the wall of the cavity and plot this stress component as a function of the azimuthal angle θ, as graphically illustrated in Figure 3. Since a liquid is employed to solution mine the cavity, the liquid pressure on the cavity wall must be included in the analysis. Once the cavity is completely excavated to its final configuration in a field situation, the liquid is allowed to remain for at least 24 hours, during which time leaks are assessed. Liquid is then evacuated or displaced with another liquid or gas. For the current example trial situation, the cavity is instantaneously and completely evacuated of the fluid 32 hours after excavation to its final configuration.

According to Figure 3, the vertical stress distribution at the wall of the cavity is almost symmetrical with respect to a horizontal axis through the center of the cavity. One will note a slight variation in the symmetry due to the change in gravity loading over the axial length of the cavity and to a differential in the liquid pressure in the cavity from top to bottom. The vertical stress at the top of the cavity is equal to the radial stress, and is equal to the static head of the liquid at a depth of 1,850 feet. The maximum vertical stress due to stress concentration occurs at the equator of the cavity, i.e., at $\theta = 90^{o}$ and is exactly equal to the tangential stress at that point. After 32 hours, the static head of the liquid in the cavity is instantaneously removed and as a consequence the vertical stress at the top and bottom of the cavity drop effectively to zero. The point of major interest here is the stress relaxation with time. As illustrated in Figure 3, the maximum change in the vertical stress due to relaxation occurs at the equator, i.e. at $\theta = 90^{o}$ of the cavity, and is seen to occur within 32 hours after completion of the cavity excavation and during which time the wall of the cavity is subjected to the static head of the liquid.

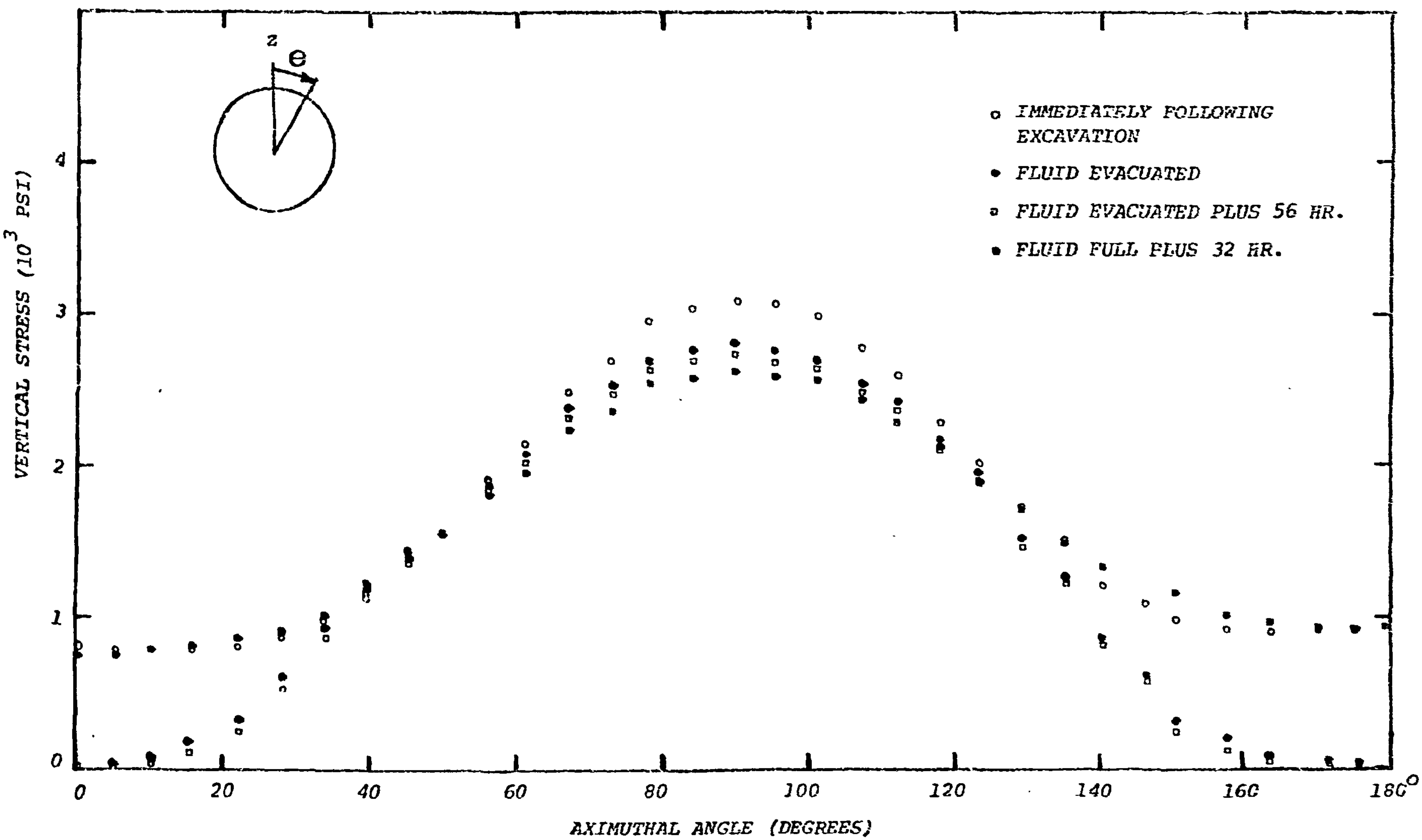


Figure 3. Vertical Stress at Face of Cavity Vs. Azimuth.

Although not illustrated in Figure 3, the vertical stresses after one year are essentially the same as at time t = 56 hours following evacuation of the liquid. Thus, one may conclude at this point that the effects of construction or sequential excavation may be significant in the evolution of the stress field. It should be mentioned that at the depth under consideration, the salt in the cavity wall remains elastic; i.e. for the usual range of salt strengths, significant plastic failure is not indicated.

Figure 4 is a plot of the vertical stress as a function of radial distance away from the wall of the cavity in the horizontal plane that passes through the center of the cavity, i.e. at $\theta = 90^{o}$. *The spatial distribution of the vertical stress is given at time t = 0+, i.e. immediately following excavation of the cavity, and after one year following the excavation. Approximately 90 percent of the increase in the vertical stress due to stress concentration above the in situ premining vertical stress occurs within one radius of the cavity wall. Thus, the effect of the cavity on the stress state in the country salt is very localized. This information will be useful in assessing the necessary separation distance between caverns in the event that a sequence of caverns is excavated at a given depth.*

Plots of the principal stresses at time t = 0+ and t = 32+ hours are presented in Figures 5 and 6, respectively. As indicated previously, these times are representative of the cavity immediately after excavation and immediately after evacuation of the liquid.

The concept of the traction-free surface in finite-element analysis deserves some clarifying remarks at this time. In order to have a traction-free surface, it is necessary to require that the normal and shear stresses vanish on the tangent plane at any point on the proposed free surface. This requirement will be impossible to satisfy in general if constant-strain finite elements are used. It is possible to refine the mesh near a free surface to such a degree that the magnitudes of the elemental stresses (normal and shear) will be quite small; however, the analyst must decide how small is acceptable. Many times the analyst will gloss over this point by arguing that the stresses are calculated at the center of the element, which is located a finite distance from the free face, and therefore the stresses should not be zero. This is of course true, but would hold for any size of element and does not establish how small the element should be since the analyst does not know a priori the magnitude of the stresses at the center of the element under consideration.

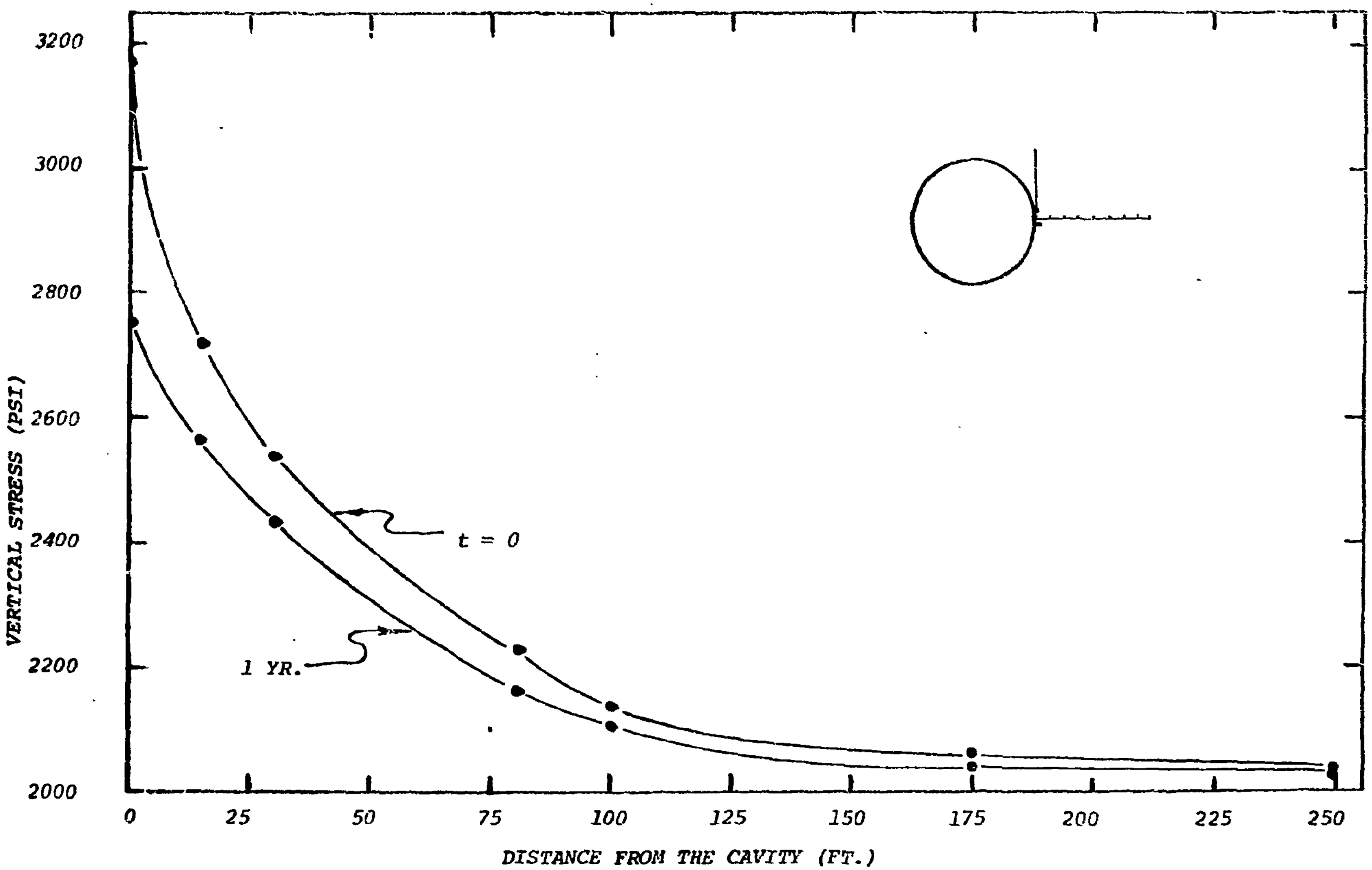


Figure 4. Vertical Stress as a Function of Radial Distance Away from the Cavity Wall.

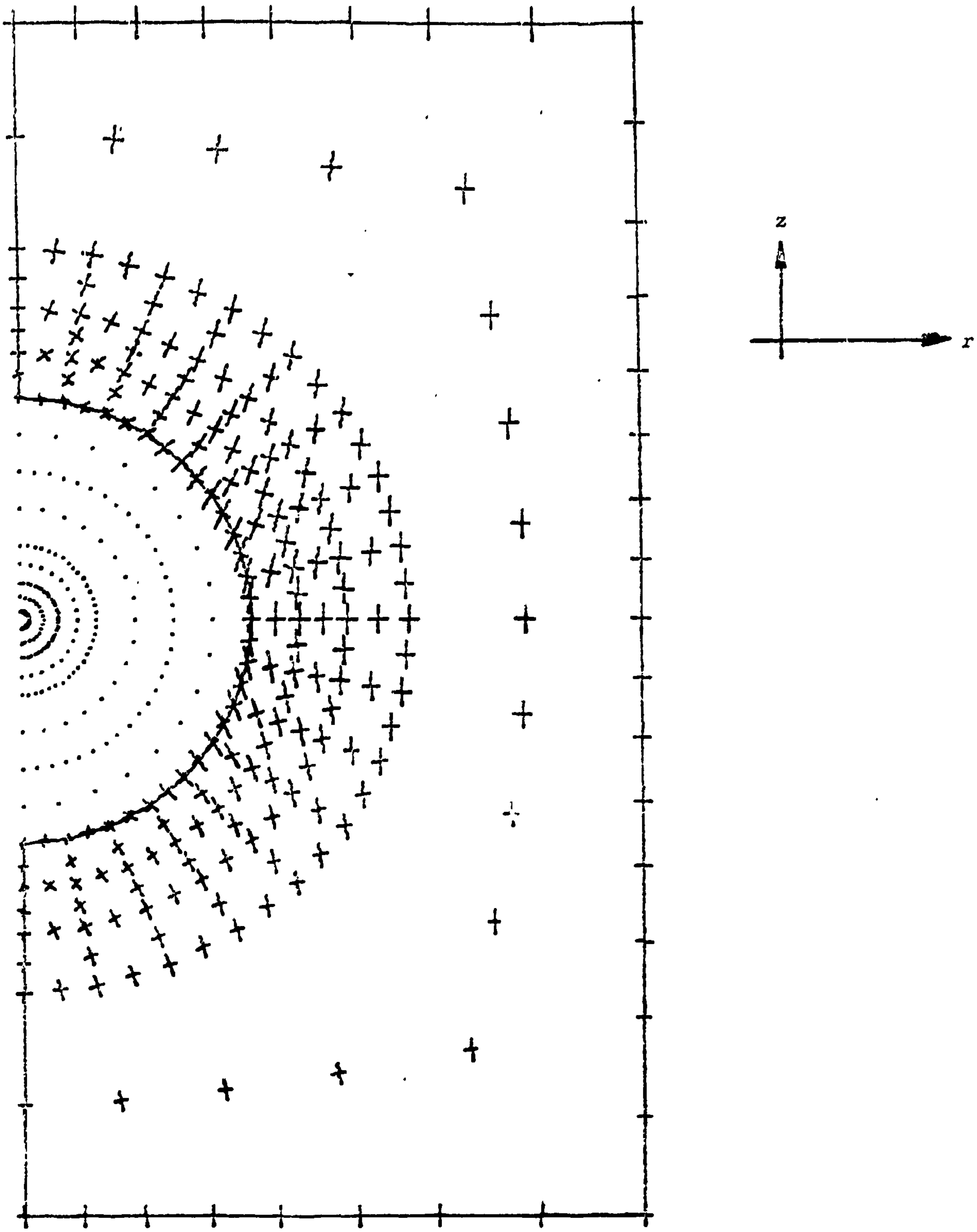


Figure 5. In-Plane Principal Stress Plot Immediately Following Excavation.

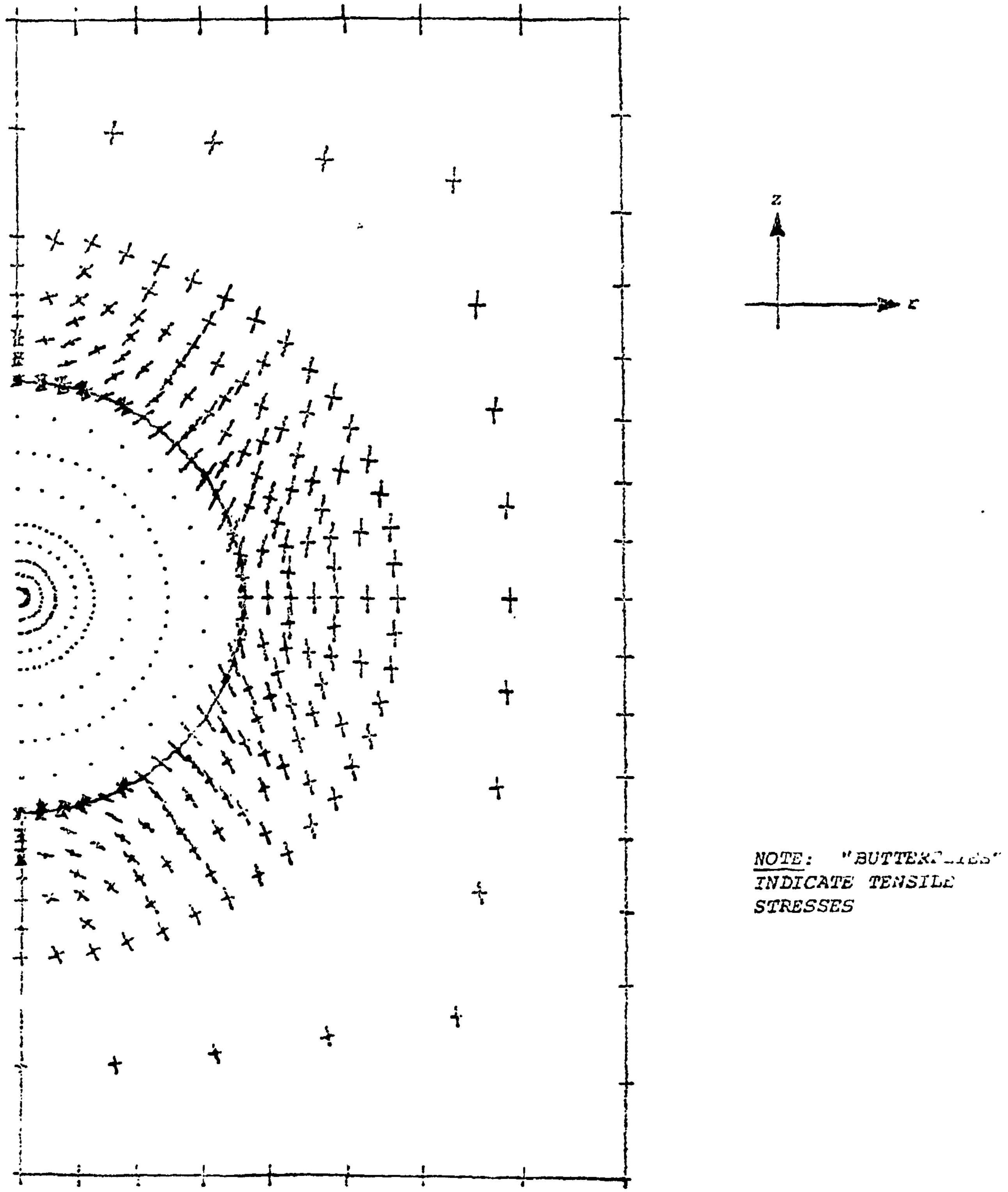


Figure 6. In-Plane Principal Stress Plot Immediately Following Fluid Evacuation.

For higher order elements, such as linear strain elements, nodal stresses are usually recorded and the average stress or mid-point stress argument breaks down. The mesh used in this preliminary analysis is relatively coarse and the non-vanishing of the normal and shear stresses on the free face is indicated via the in-plane principal stress plots of Figures 5 and 6.

The opinion of this author is that the traction-free condition at a free surface should be enforced, because at or near the boundary, failure or yielding is most probable. The traction-free condition can be achieved by considerable refinement of the mesh near the free surface or by a relaxation procedure in the finite-element analysis, or by a combination thereof. Steps are currently being taken to incorporate the latter into RSI/SCAMP.

2.4. Time-Dependent Displacement Behavior

In general, all displacements increase with time as one might anticipate. For an initial stress state corresponding to a gravitating elastic medium, the largest deflections occur at the top of the cavity at the vertical axis of symmetry. The smallest deflections occur at the bottom of the cavity on the vertical axis of symmetry. Plots of the vertical downward displacement at the top of the cavity along the vertical axis of symmetry as functions of time are given in Figures 7 and 8. In order to emphasize the displacement phenomenon for early times, the vertical displacement is plotted as a function of logarithmic time in Figure 7. The effect of the fluid evacuation is clearly seen as an abrupt discontinuity in the timewise displacement behavior at 32 hours. Figure 8 illustrates more clearly the displacement behavior at later times. Specifically, the rate at which the displacement occurs is decreasing with time, a fact which may not be immediately apparent from Figure 7.

The inward displacement of the cavity wall at the equator of the cavity is illustrated in Figures 9 and 10. In Figure 9, it is seen that the jump or "instantaneous" displacement due to the evacuation of the fluid is not as great as observed at the top of the cavity. We note the possibility exists that a different initial premining stress state might very well reverse the comparative magnitudes of the inward displacements of the cavity wall at the top and equator. The initial premining stress state that comes immediately to mind is one in which the horizontal stresses are greater than the vertical stress, which may be the case in salt domes.

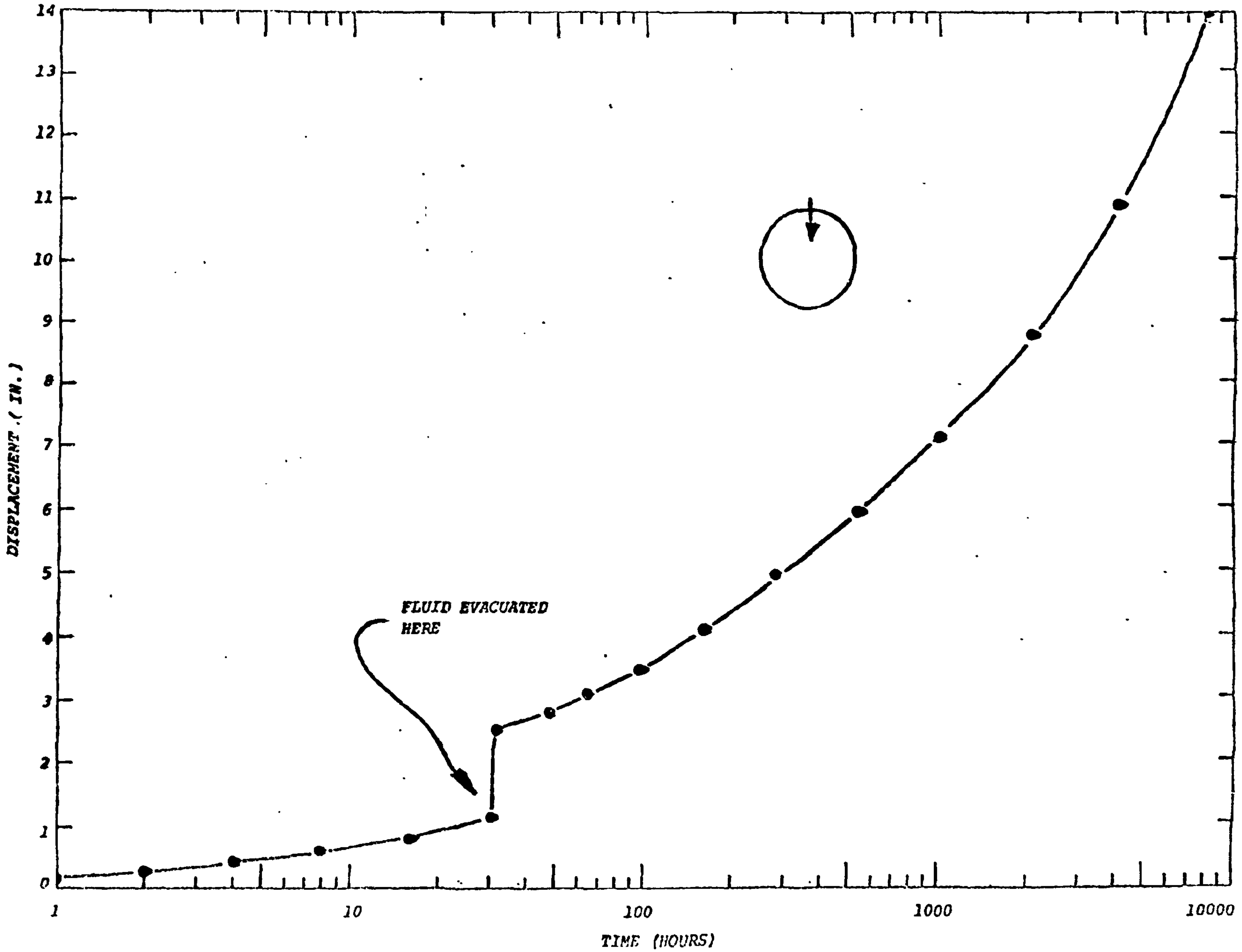


Figure 7. Downward Displacement at Axis of Symmetry as a Function of Logarithmic Time.

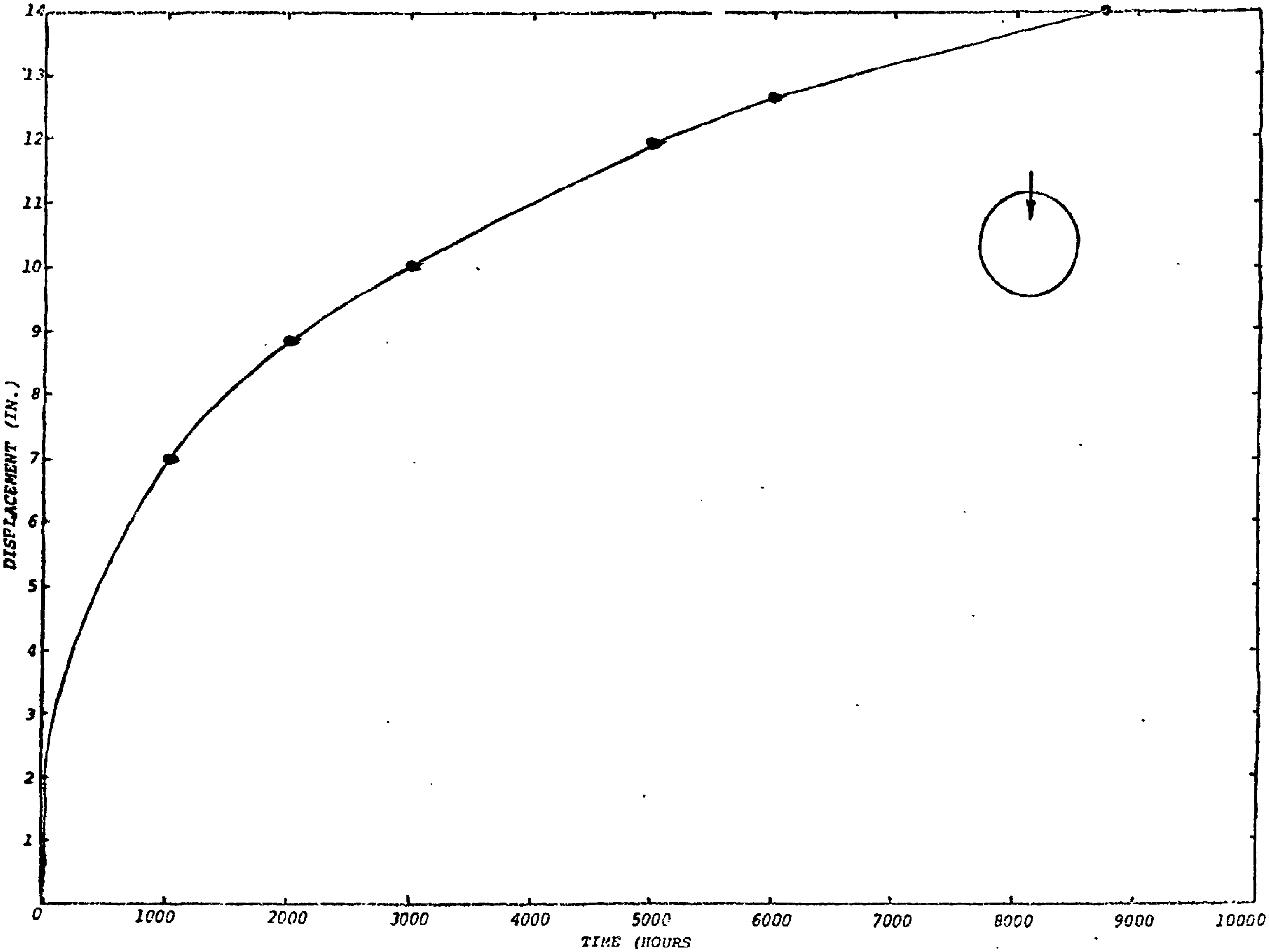


Figure 8. Downward Displacement at Axis of Symmetry as a Function of Time.

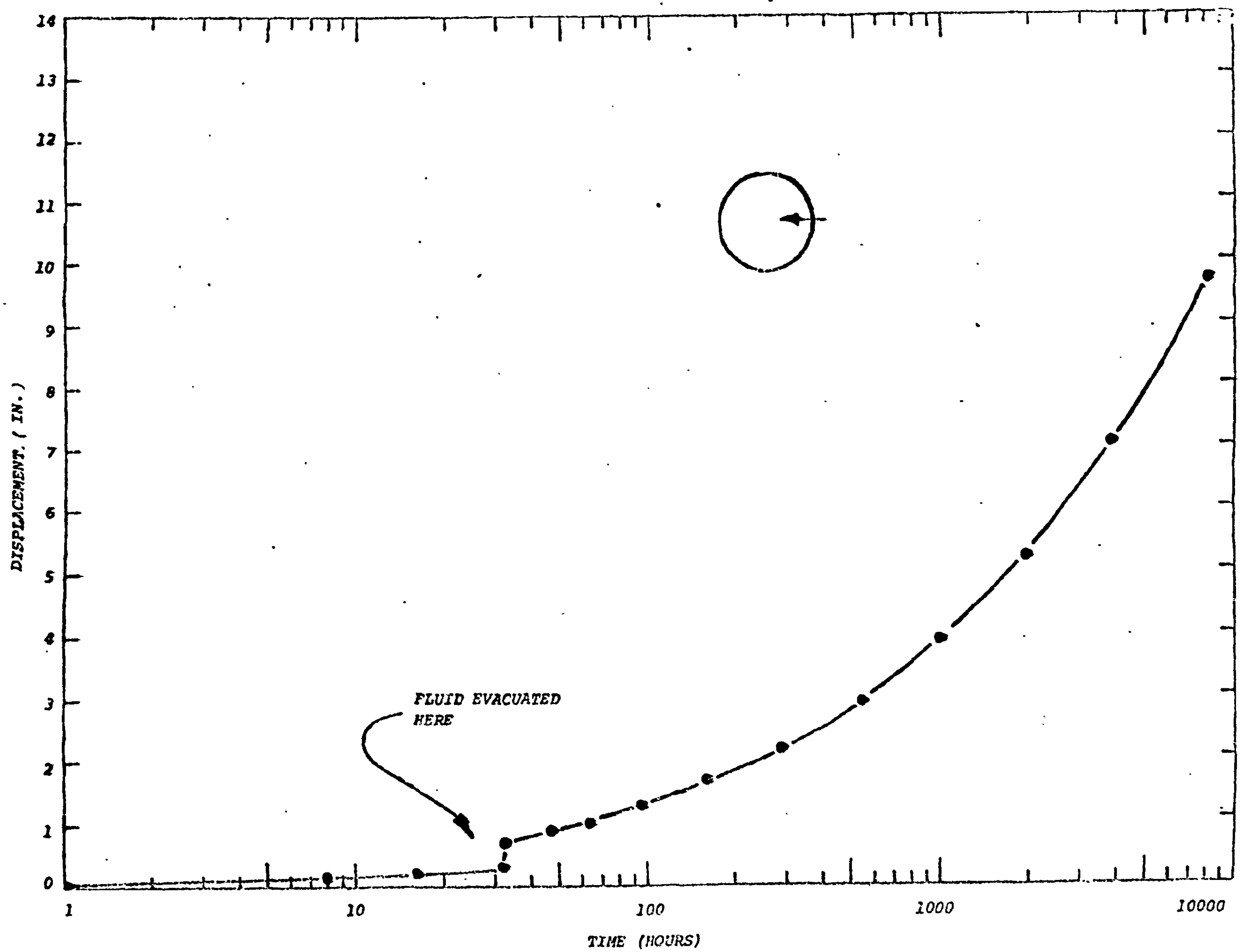


Figure 9. Inward Displacement at Equator as a Function of Logarithmic Time.

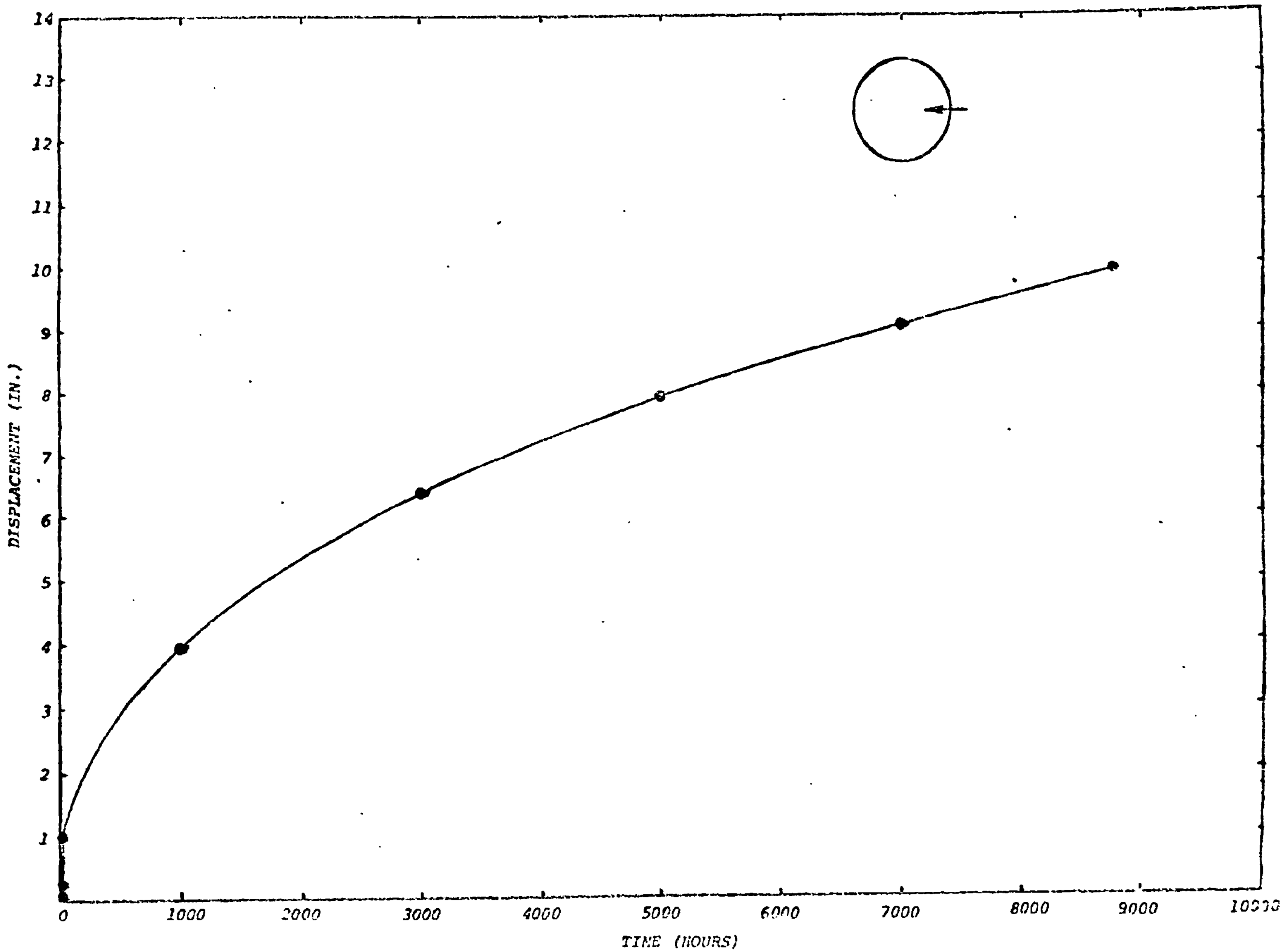


Figure 10. Inward Displacement at Equator as a Function of Time.

The upward displacement of the bottom of the cavity along the vertical axis of symmetry is shown in Figures 11 and 12. As indicated in Figure 11, the vertical upward displacement at the bottom of the cavity is minimal until the liquid is evacuated. After one year, the upward vertical displacement of the bottom of the cavity is equal to approximately one-half of the downward vertical displacement at the top of the cavity.

Figure 13 is an exaggerated illustration of the overall time-dependent closure of the cavity, with arrows depicting the direction of flow at various points on the wall of the spherical cavity. Although the magnitudes of the displacements have been exaggerated, the directions are accurate as are the relative positions of the dots on the arrows that indicate three different times. Following excavation of the cavity, these times correspond to one week, three months, and one year. The dots on the periphery of this sphere depict its position immediately following excavation. One will note that the shape of the cavity is altered from that of the sphere to that of an ellipsoid with increasing time.

The decay of the radial displacements away from the wall of the cavity on the horizontal plane that intersects the center of the cavity, i.e. at $\theta = 90^{o}$, is illustrated in Figure 14 for a range of time. As with the vertical stress in this plane, the influence of the cavity on the displacement field in the country salt is essentially localized. For all practical purposes, the country salt beyond one diameter away from the wall of the cavity is relatively uninfluenced by the existence of the cavity. A similar behavior is observed both above and below the cavity on the vertical axis of symmetry.

As noted previously, the liquid volume of the cavity immediately after excavation is approximately 2.52 million barrels. One year after completion of the excavation, approximately 100,000 barrels of storage space has been lost by closure due to creep.

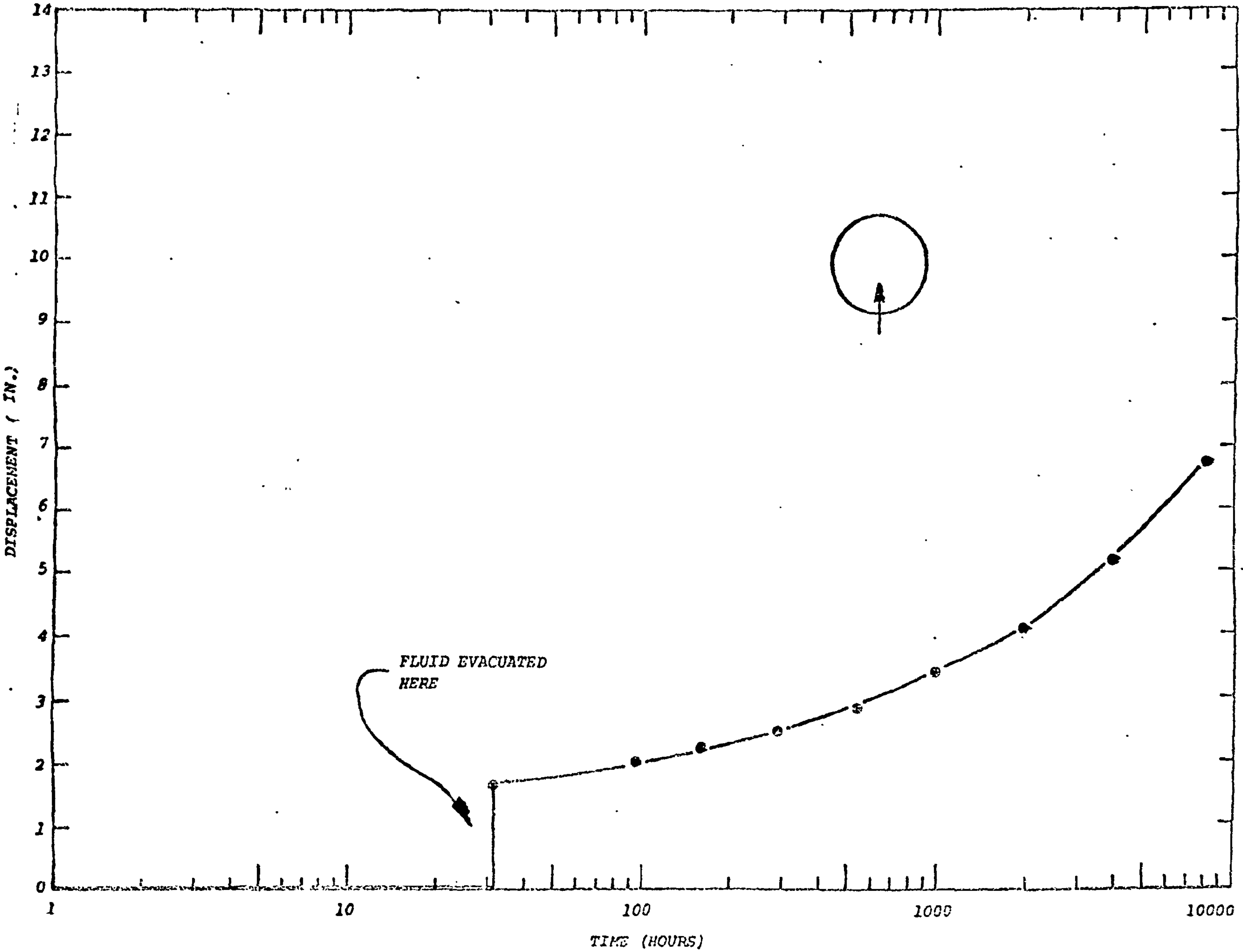


Figure 11. Upward Displacement at Axis of Symmetry as a Function of Logarithmic Time.

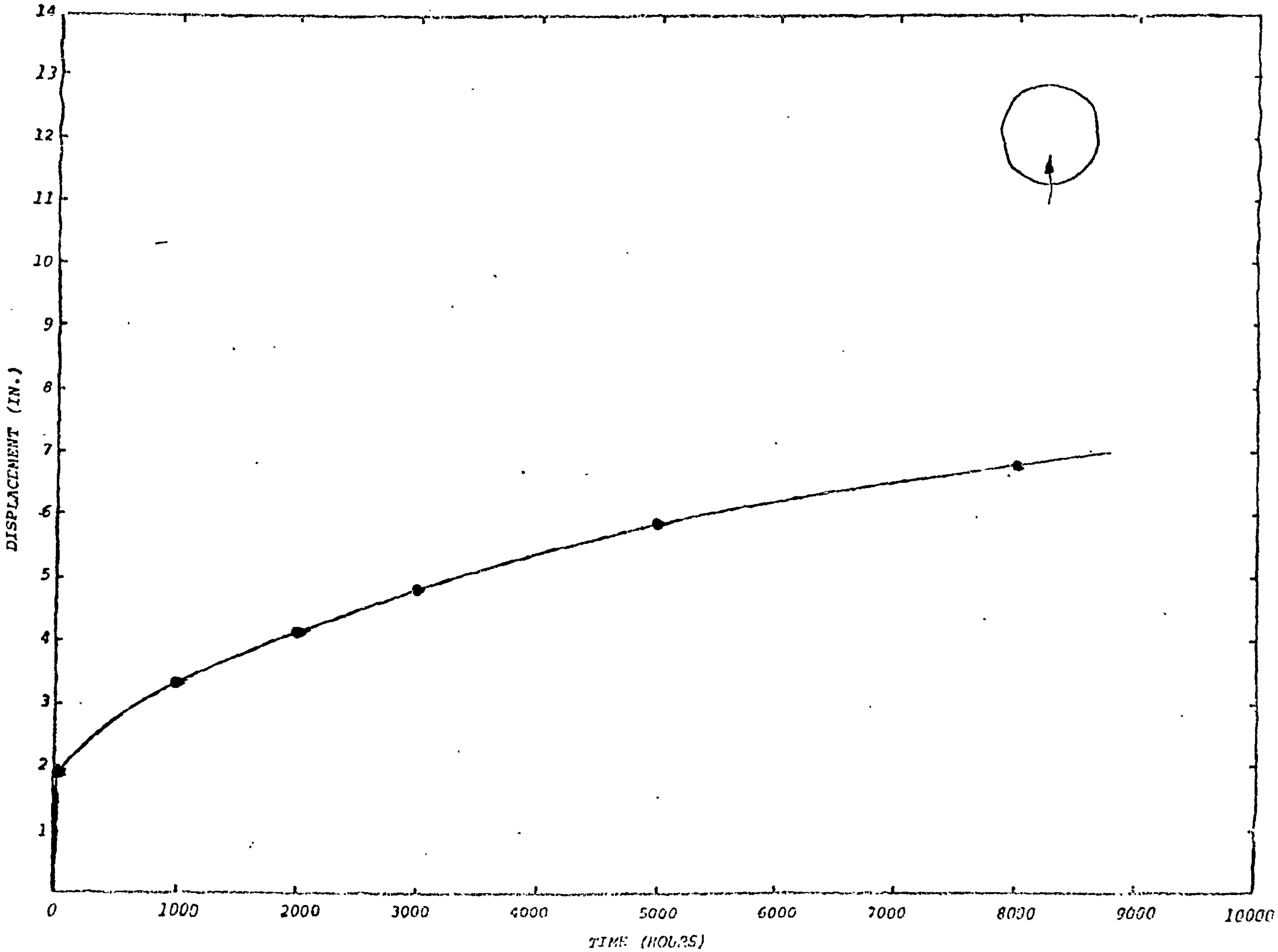


Figure 12. Upward Displacement at Axis of Symmetry as a Function of Time.

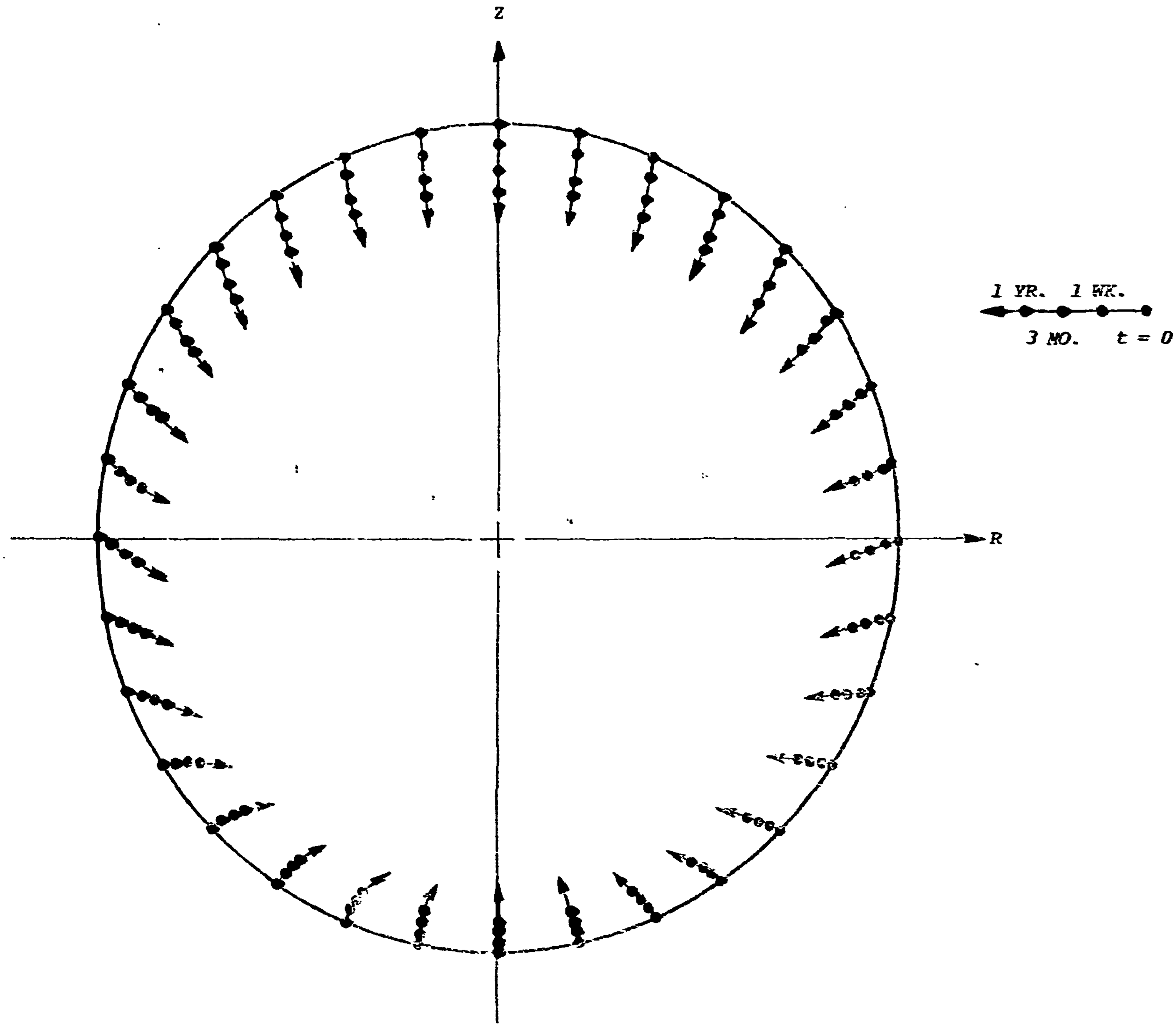


Figure 13. Exaggerated Illustration of the Overall Time-Dependent Closure of the Cavity.

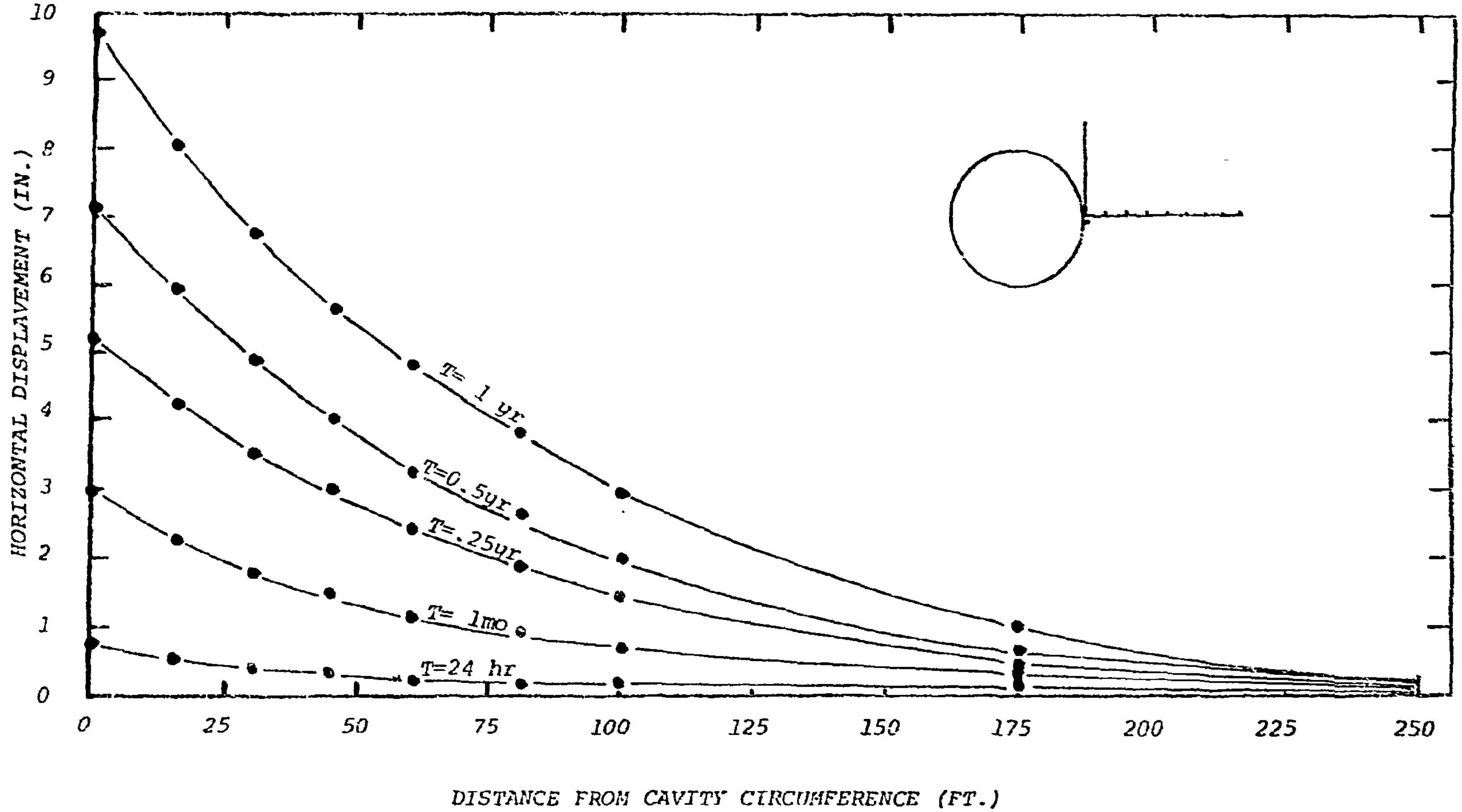


Figure 14. Decay of Radial Displacements Away from the Cavity Wall at Different Times.

3. CONCLUDING REMARKS

As a consequence of these preliminary results for a spherical solution cavity, we feel that considerable insight has been gained into the overall aspects of the structural behavior of solution cavities in general. In particular, it has been established that gradual construction effects, i.e. sequential excavation, may be significant in the evolution of the near field stresses and displacements in the cavity wall. The influence of the cavity on the country salt is relatively of a local nature, with most of the perturbation occurring within one radii of the cavity wall. Most of the stress relaxation occurs very early, essentially in the first day following an excavation. The displacement of the cavity wall increases with time at a decreasing rate for the state of initial premining stress and the rheological law considered in this study. For a spherical cavity that is excavated in a salt mass with an initial premining stress state corresponding to a gravitating elastic medium, the vertical closure of the cavity is greater than the horizontal closure.

The investigation to date has also raised a number of questions which will provide additional direction in the continued analyses of the structural stability of solution cavities. In particular, we may sight the following questions:

(1) What influence does the ratio of the premining horizontal stress to the premining vertical stress have on the stress and displacement fields after excavation of the cavity? In the literature (3), this ratio is sometimes referred to as the coefficient of lateral earth pressure. Since the creep law is nonlinear, it is probable that the stress concentrations in the wall of the cavity are magnified. Is this effect more prominent for increasing values of the lateral earth pressure? Also, what value of the coefficient is most critical from the point of view of the overall structural stability of the cavity in relation to its shape? If it is found that the most critical value of the coefficient is greater than the one corresponding to a lithostatic premining state of stress, the question of what is the state of premining stress in a salt dome becomes imperative. Obviously, this latter question can only be answered by in situ measurements, but an upper limit could probably be estimated for a finite element analysis.

(2) What influence does the shape of the cavity have on the distribution of the stresses in the cavity wall? Obviously, the stress concentration will be higher for cavity shapes with rather abrupt corners, or with innerbedded layers of say anhydrite or shale that dissolve at a much slower rate than the salt. A spherical cavity may not necessarily be the most favorable shape in the event that the premining state of stress is nonuniform. What shape in conjunction with the premining state of

stress provides the least variation of stresses along the wall of the cavity? The governing stress in the creep law is written in terms of the second invariant of the deviatoric stress tensor. It is also possible that this invariant is not affected to a great extent by different shapes. Another question involves the effects over the long term. Does stress relaxation with time decrease the influence of the shape on stability?

(3) What cavity is preferred with regard to displacements? It is possible that one shape may be preferred to minimize vertical closure and another to minimize horizontal closure. In other words, depending upon the intended usage of the cavity for storage, it may be desirable by adjustment of the cavity shape to minimize the displacements at certain locations on the cavity wall.

(4) At what depth does the dome salt cease to behave in a purely visco-elastic fashion, such that a viscoelastic/plastic constitutive law representation of the material would be more appropriate?

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APPENDIX A

VISCOELASTIC STRESS ANALYSIS BY THE FINITE-ELEMENT METHOD AS UTILIZED IN THE COMPUTER CODE RSI/SCAMP

The finite-element computer code RSI/SCAMP (Solution Cavity Analysis with Mining Procedures)as used in this study was developed by RE/SPEC Inc. The type of element that is utilized by the code is the eight-noded isoparametric ring element. An incremental procedure, similar to that employed by Greenbaum and Rubinstein (4), is utilized and allows for use of arbitrary forms of the creep constitutive law. The basic axioms of behavior for creep are generally the same as those for plasticity with the creep strain rate replacing the plastic strain increment. The basic axioms may be itemized as follows:

(1) The principal directions of stress and creep strain rate are coincident and remain so during straining;

(2) The sum of the principal strain rates is constant;

(3) The principal strain rates are proportional to the deviatoric principal stresses.

With regard to a mathematical formulation of multi-axial stress-strain relationships, it is useful to define an equivalent stress σ_{eq} as:

$$\sigma_{eq} = \sqrt{\frac{3}{2}S_{ij}S_{ij}} = \sqrt{3II_D} \qquad \text{(A-1)}$$

Similarly, the equivalent strain rate increment $d\varepsilon^c_{eq}$ is defined by:

$$d\varepsilon^c_{eq} = \sqrt{\frac{2}{3}\, d\varepsilon^c_{ij} d\varepsilon^c_{ij}} \qquad \text{(A-2)}$$

The assumptions that the principal axes of the creep strain increment coincide with the principal stress axes and that the principal strain rates are proportional to the deviatoric principal stresses allow one to relate the creep strain increment to the deviatoric stress components by means of the equations:

$$d\varepsilon^c_{ij} = S_{ij}d\lambda \qquad (i,j = 1,2,3) \qquad \text{(A-3)}$$

where S_{ij} is the deviatoric stress tensor and $d\lambda$ is a factor of proportionality.

Note that incremental creep strains are related to the finite stress components. Thus, the factor $d\lambda$ appears in incremental form. With the use of equations (A-1) through (A-3), the factor $d\lambda$ can be determined to be:

$$d\lambda = \frac{3}{2} \frac{d\varepsilon^c_{eq}}{\sigma_{eq}} \qquad \text{(A-4)}$$

Hence, equation (A-3) can be written as:

$$d\varepsilon^c_{ij} = \frac{3}{2} \frac{d\varepsilon^c_{eq}}{\sigma_{eq}} S_{ij} \qquad \text{(A-5)}$$

The creep constitutive equation (A-5) is completely general at this point. All that remains is to determine $d\varepsilon^c_{eq}$ experimentally. If σ_i, where $i = 1,2,3$, denote principal stresses, then for a triaxial compression test in which the confining pressure $P = \sigma_2 = \sigma_3$, the equivalent stress σ_{eq} defined by equation (A-1) becomes:

$$\sigma_{eq} = \sigma_3 - \sigma_1 \qquad \text{(A-6)}$$

From the assumption that the sum of the principal creep strain rates is constant and choosing this constant as zero, one finds that:

$$d\varepsilon^c_2 = d\varepsilon^c_3 = -\frac{1}{2} d\varepsilon^c_1 \qquad \text{(A-7)}$$

The choice of this constant is arbitrary, but if the material undergoes no volume change, then this constant must be zero. Equation (A-2) for the equivalent creep strain increment then becomes:

$$d\varepsilon^c_{eq} = d\varepsilon^c_1 \qquad \text{(A-8)}$$

The incremental creep law, as determined from the creep law based on experimental evidence (1,2) is given by the following relationship:

$$d\varepsilon^c_1 = \left[\frac{\sigma_1 - \sigma_m}{16,200}\right]^3 \frac{dt}{2\sqrt{t}} \qquad \text{(A-9)}$$

where:
- σ_1 = axial principal stress (psi)
- σ_m = mean stress (psi)
- t = total time (hrs.)
- dt = time increment (hrs.)

In the triaxial compression test discussed here, the mean stress becomes:

$$\sigma_m = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) = \frac{1}{3}(\sigma_1+2\sigma_3) \qquad \text{(A-10)}$$

and hence:

$$\sigma_1 - \sigma_m = \frac{2}{3}(\sigma_1-\sigma_3) = \frac{2}{3}\ \sigma_{eq} \qquad \text{(A-11)}$$

Thus, equation (A-9) becomes upon substitution of equation (A-11):

$$d\varepsilon_1^c = \left[\frac{\sigma_{eq}}{24,300}\right]^3 \frac{dt}{2\sqrt{t}} \qquad \text{(A-12)}$$

From equations (A-5), (A-8), and (A-12), the generalized incremental creep constitutive equation is given by:

$$d\varepsilon_{ij}^c = \frac{3}{4}\left[\frac{\sigma_{eq}}{24,300}\right]^3 \frac{dt}{\sqrt{t}} \ \frac{S_{ij}}{\sigma_{eq}} \qquad \text{(A-13)}$$

where the deviatoric stress tensor is defined as $S_{ij} = \sigma_{ij} - \delta_{ij}\,\sigma_m$.

The main assumption which is made in the incremental solution procedure used in the finite-element code is that the stress is considered constant within each increment of time. In the class of solution cavity problems currently under consideration, the stresses may change appreciably after each step of a sequence of salt excavation and liquid evacuation from the cavity. Hence, relatively small time steps must be used immediately following an excavation or liquid evacuation, with a gradual increase in the size of the time increment thereafter. The significance of the assumption that the stresses remain constant during a time increment is that one is permitted to solve the nonlinear problem as a sequence of successive linear problems, i.e. the problem is taken to be linear within each time increment. The incremental creep strain from a previous time increment is used as an initial strain for the current time increment. This is accomplished by expressing the elastic stress-strain relations as follows:

$$\{\sigma\} = [C]\ (\{\varepsilon\} - \{\varepsilon_c\}) \qquad \text{(A-14)}$$

where $[C]$ is the elasticity matrix.

The effect of the initial strains on the finite-element equations is simply an additional load term which permits the initial strains to be included in the formulation. The notion of initial strain allows the sequence of excavation to be incorporated into the finite-element computer code.

After completion of the preliminary study for the spherical cavity as given in this report, the computer code RSI/SCAMP was upgraded to allow for specification of an arbitrary initial stress field. This is accomplished by solving for the equivalent nodal load vector and applying it in the manner in which body forces are applied. This option will allow for a more realistic approximation of anticipated states of in situ stress.