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KHARKOV PHYSICAL TECHNICAL INSTITUTE
OF THE ACADEMY OF SCIENCES OF UkrSSR**

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Pashnev A.I., Volkov D.V., Zheltukhin A.A.

**VACUUM TRANSITIONS IN DUAL MODELS. II.
QUARK STRUCTURE OF RESONANCES IN THE
NEVEU-SCHWARZ DUAL MODEL**

Kharkov 1976

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Physical interpretation of results obtained in the first part I is studied. The presence of an additional degeneracy of resonance states in the Neveu-Schwarz dual model is shown. To consider spontaneous vacuum transitions analytical continuation of the rearranged dual amplitude in the IVT constant is performed. It's shown that exact $SU(2)$ -symmetry and Higgs effect take place here. The presence of $SU(2) \times SU(2) \times U(1) \times U(1) \times \dots$ -symmetry and relations of the algebraic realization of chiral symmetry in the rearranged Neveu-Schwarz dual model are deduced.

In the first part^[1] of the present work an integral representation method of summing induced vacuum transitions (IVT) in the Neveu-Schwarz dual model (NSDM) has been formulated and a solution of a system of integral equations for the rearranged Neveu-Schwartz dual amplitudes has been obtained.

The present part of the work deals with physical interpretation of the results obtained in the first part^[1].

It will be shown that the spectrum of resonance states of the NSDM contains an additional degeneration which in the presence of IVT's manifests in splitting of all the π - and ρ -daughter trajectories on the genuine daughters and on the so called 'adopted' ones. Such a degeneration of the resonance spectrum is analogous to the degeneration of spectrum found recently by the authors in the Veneziana dual model^[2].

After the degeneration of spectrum is properly taken into account the resulting NS dual amplitudes exhibit an internal structure with an infinite number of conserving hyper charges together with an implicit spin structure.

Another important property of the degeneration due to the presence of the IVT's in case of the NSDM is that it contains both G-odd and G-even parity states. As a consequence the dual amplitudes with any number of G-even external particles are contained in the solution of the integral representation method.

Analytical continuation of the rearranged dual amplitudes in the IVT constant on the other Riemann sheets gives possibility to consider spontaneous vacuum transitions (SVT). As a consequence of the SVT's the intercepts of π and ρ -trajectories acquire new values. In the case when $\alpha_0 = 0$ which corresponds to the original NSDM the intercept of the ρ -trajectory after SVT becomes equal to zero, and the ρ -meson acquires a non-zero mass value. This testifies to the presence of the Higgs effect in the NSDM. For the same value of intercept $\alpha_0 = 0$ the π - and ρ -meson trajectories as a

consequence of SVT trajectories become degenerate. So an exact SU(2) symmetry takes place. For the masses of particles belonging to different trajectories relations of the algebraic realization of chiral symmetry are satisfied. That shows the presence, in the rearranged NSDM of the SU(2) x SU(2) x U(1) symmetry group.

The content of the paper is disposed as follows.

The structure of separate terms in the integral representation, of the rearranged dual amplitudes is studied in Sec.1. In Sec.2 the physical meaning of the rearrangement of the dual amplitudes due to IVT is discussed. The connection of the resulting trajectory splitting with an inherent quark structure is established. The explicit form of the rearranged 4-point functions is given in Sec.3. A connection of the quark structure of the NSDM with the σ -model and with the Adler principle is discussed. In Sec.4. SVT's are considered. The presence of the mass relations analogous to those of the algebraic realization of the chiral symmetry is demonstrated.

For convenience we list here the main formulas received in Part I.

The initial NSDM amplitude has the form

$$B_n(p_1, p_2, \dots, p_n) = \frac{1}{\Omega} \int \frac{\prod_{j=1}^n dz_j d\varphi_j}{\prod_{j=1}^n (z_{j+1} - z_j - \varphi_{j+1} \varphi_j)^{1/2}} \hat{Z}_{jk}^{-\alpha_{jk}-1} \quad (1)$$

where

$$\hat{Z}_{jk} = \frac{(z_k - z_j - \varphi_k \varphi_j)(z_{k+1} - z_{j-1} - \varphi_{k+1} \varphi_{j-1})}{(z_{k+1} - z_j - \varphi_{k+1} \varphi_j)(z_k - z_{j-1} - \varphi_k \varphi_{j-1})} \quad (2)$$

and φ_j 's are anticommuting variables with the integration rule $\int d\varphi_j = 0$, $\int \varphi_j d\varphi_i = \delta_{ji}$

The spurious summation between the i -th and $i+1$ particles results in the appearance of the additional factor

in the integrand of the representation (1)

$$R(Z_{i-1}, Z_i, Z_{i+1}, Z_{i+2}; d_i, d_{i+1}, d_0; \beta) = -\frac{1}{2\pi i} \frac{1}{\Gamma(-d_i - \frac{1}{2})\Gamma(-d_{i+1} - \frac{1}{2})} \int_{\gamma} ds K(s, d_0, \beta) \times$$

$$\times \left\{ \frac{1}{\Gamma(\frac{1}{2} - s)\Gamma(s - d_0 - \frac{1}{2})} \sum_{n=0}^{\infty} \frac{(\tilde{Z}_{i+1})^{s+n}}{\Gamma(2s - d_0 + n)} \left[\Gamma(-d_i + s + n - \frac{1}{2}) - \beta \Phi_i \frac{\Gamma(\frac{1}{2} - s)\Gamma(s - d_0 - \frac{1}{2})\Gamma(-d_i + s + n)}{\Gamma(-d_0 - \frac{1}{2})} \right] \right\} \quad (3)$$

$$\times \left[\Gamma(-d_{i+1} + s + n - \frac{1}{2}) - \beta \Phi_{i+1} \frac{\Gamma(\frac{1}{2} - s)\Gamma(s - d_0 - \frac{1}{2})\Gamma(-d_{i+1} + s + n)}{\Gamma(-d_0 - \frac{1}{2})} \right] - (s \rightarrow s + \frac{1}{2}) \Big\}$$

where

$$K(s, d_0, \beta) = \frac{2^{2d_0+3} \pi \Gamma(-2s)\Gamma(2s - 2d_0 - 1)}{1 - 2^{2d_0+3} \pi \beta^2 \frac{\Gamma(-2s)\Gamma(2s - 2d_0 - 1)}{\Gamma^2(-d_0 - \frac{1}{2})}} \quad (4)$$

$$\Phi_i = \varphi_{i-1} \sqrt{\frac{Z_{i+1} - Z_{i-1}}{(Z_{i+1} - Z_i)(Z_i - Z_{i-1})}} - \varphi_i \sqrt{\frac{Z_{i+1} - Z_{i-1}}{(Z_{i+1} - Z_i)(Z_i - Z_{i-1})}} +$$

$$+ \varphi_{i+1} \sqrt{\frac{Z_i - Z_{i-1}}{(Z_{i+1} - Z_i)(Z_{i+1} - Z_{i-1})}} + \frac{1}{2} \frac{\varphi_{i-1} \varphi_i \varphi_{i+1}}{\sqrt{(Z_{i+1} - Z_i)(Z_{i+1} - Z_{i-1})(Z_i - Z_{i-1})}} \quad (5)$$

and the integration contour γ is shown in Fig.1. (Fig.2 shows the analogous integration contour in the VDM).

The function (3) may also be represented in the form of the sum of the residues at poles of the integrand defined by the condition

$$1 - 2^{2d_0+3} \pi \beta^2 \frac{\Gamma(-2s)\Gamma(2s - 2d_0 - 1)}{\Gamma^2(-d_0 - \frac{1}{2})} = 0 \quad (6)$$

As a result of all spurion summations the rearranged amplitude acquires the additional factor

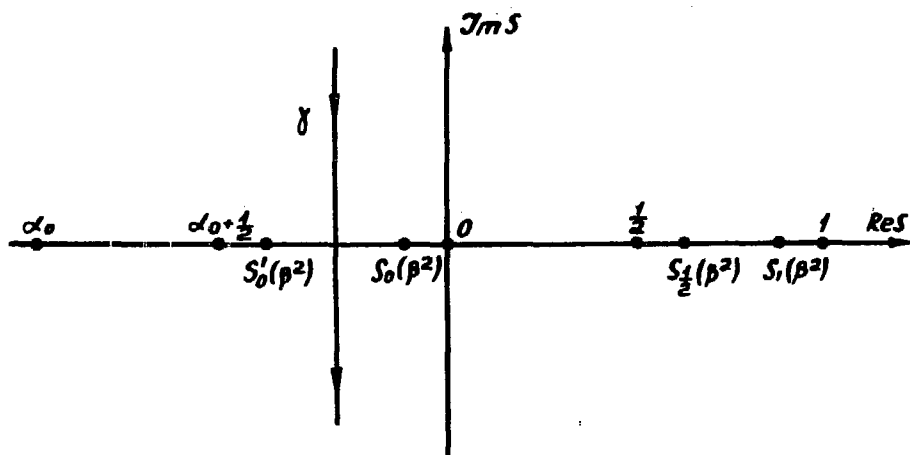


Fig.1.
The integration contour in the NSDM

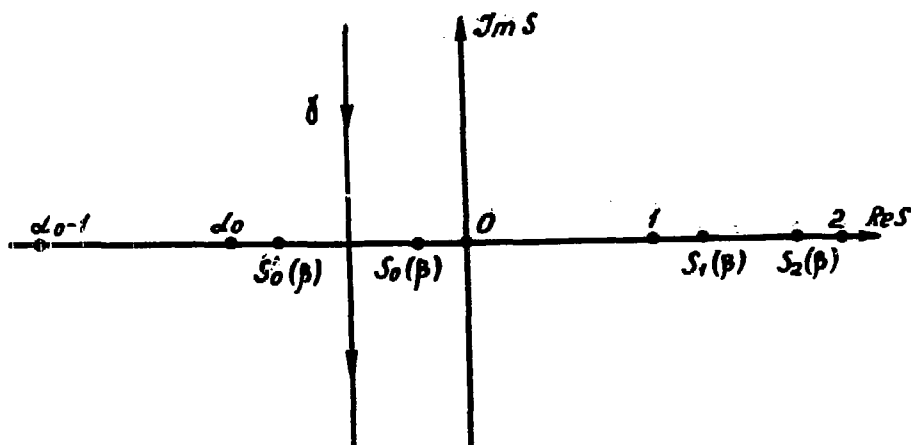


Fig.2.
The integration contour in the VDM.

$$R(Z_j; \alpha_j; \alpha_0; \beta) = \left(-\frac{1}{2\pi i}\right)^n \int_{\gamma} \prod_{i=1}^n dS_i K(S_i, \alpha_0, \beta) \times \quad (7)$$

$$\times \sum_{n_1, \dots, n_n=0}^{\infty} \left\{ A_1(S_n, S_1) B_1(S_1) A_2(S_1, S_2) B_2(S_2) \dots A_n(S_{n-1}, S_n) B_n(S_n) - \right.$$

$$\left. - (S_1 \rightarrow S_1 + \frac{1}{2}) - \dots + (S_1 \rightarrow S_1 + \frac{1}{2}, S_2 \rightarrow S_2 + \frac{1}{2}) + \dots \right\}$$

where

$$B_i(S_i) = \frac{1}{\Gamma(\frac{1}{2} - S_i) \Gamma(S_i - \alpha_0 - \frac{1}{2}) \Gamma(n_i + 2S_i - \alpha_0) n_i!} \times \quad (8)$$

$$\times \left[\frac{(Z_i - Z_{i-1} - \varphi_i \varphi_{i-1})(Z_{i+2} - Z_{i+1} - \varphi_{i+2} \varphi_{i+1})}{(Z_{i+1} - Z_i - \varphi_{i+1} \varphi_i)(Z_{i+2} - Z_i - \varphi_{i+2} \varphi_i)} \right]^{S_i + n_i}$$

and

$$A_i(S_{i-1}, S_i) = \frac{1}{\Gamma(-d_i - \frac{1}{2})} \left\{ \left[1 - \beta^2 \frac{\Gamma(\frac{1}{2} - S_i) \Gamma(S_i - \alpha_0 - \frac{1}{2}) \Gamma(\frac{1}{2} - S_{i-1}) \Gamma(S_{i-1} - \alpha_0 - \frac{1}{2})}{\Gamma^2(-\alpha_0 - \frac{1}{2})} \right] \Gamma(d_i + S_{i-1} + S_i + n_{i-1} + n_i) \right. \quad (9)$$

$$\left. + (-1)^{n_{i-1}} \left[\frac{\Gamma(\frac{1}{2} - S_{i-1}) \Gamma(S_{i-1} - \alpha_0 - \frac{1}{2})}{\Gamma(-\alpha_0 - \frac{1}{2})} + \frac{\Gamma(\frac{1}{2} - S_i) \Gamma(S_i - \alpha_0 - \frac{1}{2})}{\Gamma(-\alpha_0 - \frac{1}{2})} \right] \Gamma(-d_i + S_{i-1} + S_i + n_{i-1} + n_i) \beta \varphi_i \right\}$$

1. We begin with the discussion of the physical meaning of the expressions (3) and (7). For this purpose we shall at first remind the physical meaning of spurion summation procedure in the VDM emphasising those properties which are general for the both models under consideration. For simplicity we shall restrict ourselves with the consideration of the first step of spurion summation procedure (Fig.3).

As it has been already mentioned the n-point β -function as a result of the first step summation acquires the additional

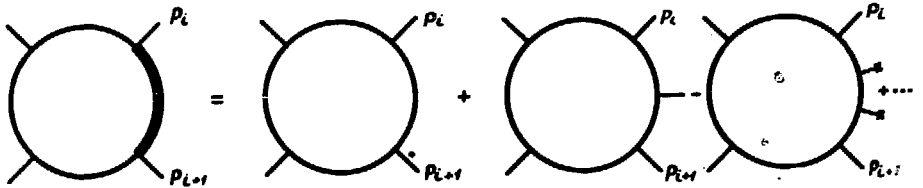


Fig.3.

The graphical representation of the B_n -function rearranged in the result of the spurion summation between the i -th and $i+1$ st particles.

function $R^i(Z_{i,i+1}; d_i, d_{i+1}; d_0, \beta)$ in the integrand of the Koba-Nielsen representation. In the VDM the corresponding function $R^i(Z_{i,i+1}; d_i, d_{i+1}; d_0, \beta)$ is given by the following expression

$$R^i(Z_{i,i+1}; d_i, d_{i+1}; d_0, \beta) = \frac{1}{2\pi i} \frac{1}{\Gamma(-d_i)\Gamma(-d_{i+1})} \int_{\gamma} ds K(s, d_0, \beta) \times \sum_{n=0}^{\infty} \frac{\Gamma(-d_i + s + n)\Gamma(-d_{i+1} + s + n)}{\Gamma(2s - d_0 + n + 1) n!} (1 - Z_{i,i+1})^{s+n} \quad (10)$$

where

$$K(s, d_0, \beta) = \frac{(d_0 - 2s)\Gamma(-s)\Gamma(s - d_0)}{1 - \beta B(-s, s - d_0)} \quad (11)$$

The integration contour γ in the representation (10) encloses all the roots $S_m(\beta)$ of the equation (see Fig.2)

$$1 - \beta B(-s, s - d_0) = 0 \quad (12)$$

which coincide with the roots

$$S_m(0) = m \quad (m = 0, 1, 2, \dots) \quad (13)$$

when $\beta = 0$ and change continuously

When β is varying along a contour starting at $\beta = 0$.

In order to give some physical interpretation of the redefined β_n -function we shall use the following formal procedure [2]. Let us define dual amplitude as a residue at $\alpha_k = 0$ of the expression

$$\prod_{k=0}^n \frac{1}{\alpha_k} \beta_n^{R_k}$$

where $\alpha_k = \alpha' p_k^2 + \alpha_0$.

This procedure fixes on the mass shell condition for all the external particles.

Transform the integral (10) to the sum of the residues at the poles $S_m(\beta)$. Partial terms of this sum up to unessential renormalization factors have the following form

$$\frac{\Gamma(-d_i + S_m(\beta) + n) \Gamma(-d_{i+1} + S_m(\beta) + n)}{d_i \Gamma(-d_i) d_{i+1} \Gamma(-d_{i+1})} \prod_{\ell=2}^{n-2} Z_{i\ell, i\ell}^{S_m(\beta) + n} \quad (14)$$

In order to get the expression (14) we used the well-known property of anharmonic ratios

$$1 - Z_{i, i+1} = \prod_{\ell=2}^{n-2} Z_{i\ell, i\ell} \quad (15)$$

The remarkable property of the expression (14) is that it has no poles at $d_i = 0$ and at $d_{i+1} = 0$ when $\beta \neq 0$. At the same time due to the presence of $\Gamma(-d_i + S_m(\beta) + n)$ and $\Gamma(-d_{i+1} + S_m(\beta) + n)$ in the numerator of the expression (15) it has poles in the variables d_i and d_{i+1} at

$$\alpha(p^2) \approx m + n + r + \frac{(-1)^m}{m!} \frac{\Gamma(m - \alpha_0)}{\Gamma(-\alpha_0)} \beta \quad (16)$$

Passing from the expression (14) to the formula (16) only

the terms linear in the parameter β has been retained

$$S_m(\beta) \approx m + \frac{(-1)^m}{m!} \frac{\Gamma(m-d_0)}{\Gamma(-d_0)} \beta \quad (17)$$

When the conditions (16) with $m+n+v=0$ are fulfilled the residues at the poles of $\alpha(p^2)$ correspond to the external particles lying on the shifted parent trajectory.

The trajectories $\alpha_{i,i+l} (l=2, \dots, n-2)$ corresponding to the internal particles undergo the same shifting. This results from the factors $Z_{i,i+l}^{S_m(\beta)+n}$ in the expression (14) and the factors $Z_{i,i+l}^{-\alpha_{i,i+l}-1}$ in the integrand of the Veneziano amplitude.

So we may conclude that every term (14) leads to the same trajectory shift simultaneously for the i -th and $i+1$ st external particles and for all the internal trajectories

$\alpha_{i,i+l} (l=2, \dots, n-2)$, corresponding to the diagram diagonals shown in Fig.4.

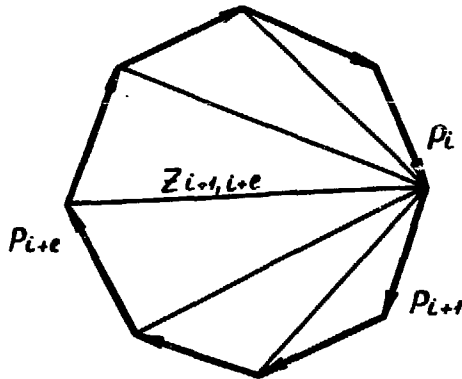


Fig.4

Momentum polygon for the Veneziano n -point amplitude. Shown diagonals correspond to the $Z_{i,i+l}$ variables.

This remarkable property of the expression (14) allows to form a quark interpretation of the rearranged dual amplitudes. To get such an interpretation let us consider the dual diagram pictured on Fig.5.

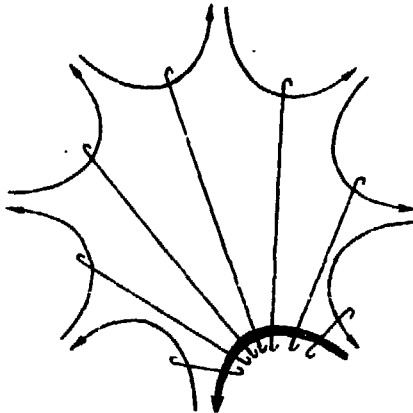


Fig.5

Quark diagram for n-point Veneziano amplitude. Subtending the quark lines corresponds to the polygon diagonales shown on Fig. 4.

As it can be seen from this diagram all the shifted trajectories $\alpha_i, \alpha_{i+1}, \alpha_{i+2}, \dots$ ($i = 2, \dots, n-2$) correspond to tying together the external lines in such a way that the quark line between the i -th and $i+1$ -th external particles always participates. That is why the effect of taking into account spurion emission on the first step of summation may be interpreted as change of the only quark mass on the line between the i -th and $i+1$ -th particles. In accordance with an infinite number of the terms having the form (14) in the residue sum the quark line between the i -th and $i+1$ -th particles is ∞ -fold split so that each m in the expression (14) corresponds to its own variety of quark. (see Fig.6).

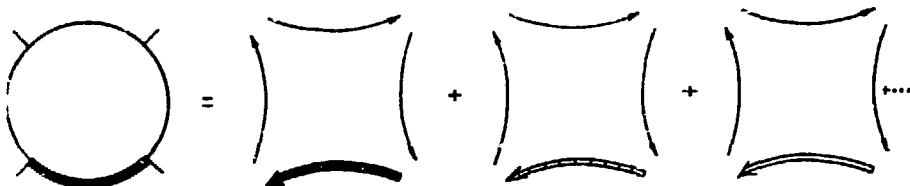


Fig.6

External quark line splitting for the Veneziano dual amplitude corresponding to the first step of spurion summation.

To study the splitting of the trajectories we shall consider the expression (16). This expression defines the shifts of the main trajectory ($m+n+r=0$) and of its daughters ($m+n+r=J$). The value of the shift is given by the last term of the expression (16). The dependence of this term on m leads to $J+1$ -fold splitting of the squared masses of the external particles belonging to daughter trajectories. All the split daughter trajectories are divided into two groups. The first group contains the split trajectories which are shifted by the same value as the parent trajectory. They are called genuine daughter trajectories. The trajectories of the second group are shifted by the value which differs from the parent trajectory shift. They are not situated at integer distances from the parent trajectory and thus are not its true daughters. These trajectories we call adopted daughter trajectories. Each adopted daughter trajectory is in its turn the parent of the family of its own daughter trajectories. The scheme of trajectory splitting is shown on Fig.7.

Each split trajectory is characterized by its value of

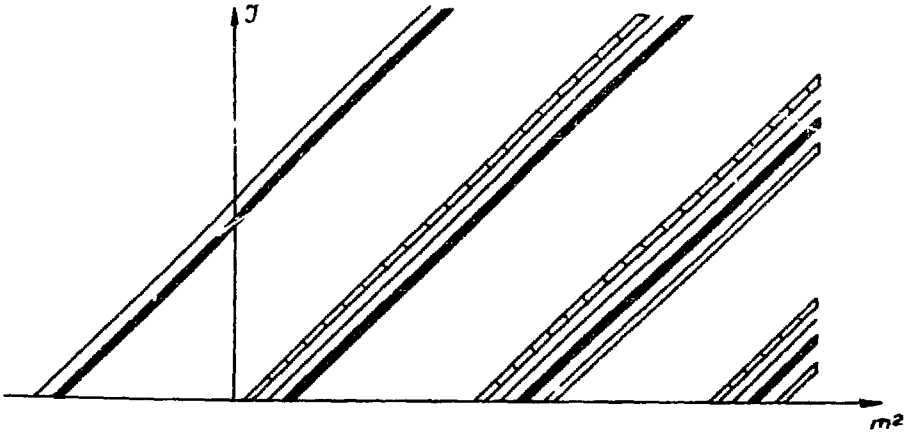


Fig.7.

The scheme of trajectory splitting in the VDM
 - trajectories of the initial Veneziano model

- (m=0) the shifted parent and genuine daughter trajectories
- (m=1) adopted daughter trajectories
-** (m=2) and their daughters
-** (m=3)

Each split trajectory is characterized by its value of m ($m=0,1,\dots$). As it has been shown above each value of m is connected with its variety of quark. Therefore resonance state splitting is uniquely defined by the mass spectrum of the corresponding quark states. The presence of above discussed splitting proves an additional degeneration of resonance spectrum in the VDM. Spurious emission plays the role of a perturbation removing the degeneracy inherent in the model.

Note that variety of quark is connected with conservation of some hypercharge Y_i . Indeed, fulfilling the subsequent steps of the summation between the $(i+1)$ st and $(i+2)$ nd particles and so on we shall get the quark line splitting between the $(i+1)$ st and $(i+2)$ nd particles, etc., analogous to the first step splitting. As a result of the summation at all the steps all the quark lines are split and the rearranged Veneziano

amplitude is represented in the form of a sum with an infinite number of terms corresponding to the diagrams shown on Fig.8.

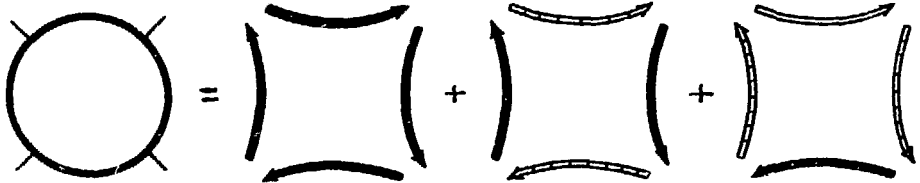


Fig.8

Quark line splitting in the Veneziano dual amplitude corresponding to the summation of all the spurions.

As it may be easily seen from the quark diagrams (Fig.8) the number of quarks of each variety is conserved for any n -point function and this results in the conservation of the corresponding hypercharge Y_i .

Thus the dual model under consideration contains an infinite number of additive conservation laws, and a definite type of hypercharge corresponds to each of them. As a consequence the symmetry of the resulting quark model is that of the $U(1) \times U(1) \times \dots$ group.

2. The quark interpretation of the rearranged Veneziano dual amplitude considered in the previous section may be generalized with some variations for the rearranged Neveu-Schwarz dual amplitudes. To study the rearranged amplitude structure in the NSDM we shall use an analogous method. For this purpose we shall begin with the consideration of the structure of the factor function $R^\circ(\tilde{Z}_{i,i+1}; \Phi_i, \Phi_{i+1}; d_i, d_{i+1}, \alpha_0, \beta)$ (3) appearing at the first step of spurion summation (see Fig.3)

Transforming the integral (3) into a sum of an infinite number of residues at the poles $S_m(\beta^2)$ ($m = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$) we may consider separate terms of this sum. Unlike the VDM the NSDM has two different types of terms. The first type contains only even powers of the constant β . The corresponding terms have the following form

$$\frac{\Gamma(-\alpha_i - \frac{1}{2} + S_m(\beta^2) + n) \Gamma(-\alpha_{i+1} - \frac{1}{2} + S_m(\beta^2) + n)}{(\alpha_i + \frac{1}{2}) \Gamma(-\alpha_i - \frac{1}{2}) (\alpha_{i+1} + \frac{1}{2}) \Gamma(-\alpha_{i+1} - \frac{1}{2})} \prod_{\ell=2}^{n-2} \hat{Z}_{i+1, i+\ell}^{S_m(\beta^2)+n} \quad (18)$$

and

$$\frac{\Gamma(-\alpha_i + S_m(\beta^2) + n) \Gamma(-\alpha_{i+1} + S_m(\beta^2) + n)}{(\alpha_i + \frac{1}{2}) \Gamma(-\alpha_i - \frac{1}{2}) (\alpha_{i+1} + \frac{1}{2}) \Gamma(-\alpha_{i+1} - \frac{1}{2})} \phi_i \phi_{i+1} \prod_{\ell=2}^{n-2} \hat{Z}_{i+1, i+\ell}^{S_m(\beta^2)+n} \quad (19)$$

The second type contains only the odd powers of the constant β . Such terms have the form

$$\beta \frac{\Gamma(-\alpha_i - \frac{1}{2} + S_m(\beta^2) + n) \Gamma(-\alpha_{i+1} + S_m(\beta^2) + n)}{(\alpha_i + \frac{1}{2}) \Gamma(-\alpha_i - \frac{1}{2}) (\alpha_{i+1} + \frac{1}{2}) \Gamma(-\alpha_{i+1} - \frac{1}{2})} \phi_{i+1} \prod_{\ell=2}^{n-2} \hat{Z}_{i+1, i+\ell}^{S_m(\beta^2)+n} \quad (20)$$

and

$$\beta \frac{\Gamma(-\alpha_i + S_m(\beta^2) + n) \Gamma(-\alpha_{i+1} - \frac{1}{2} + S_m(\beta^2) + n)}{(\alpha_i + \frac{1}{2}) \Gamma(-\alpha_i - \frac{1}{2}) (\alpha_{i+1} + \frac{1}{2}) \Gamma(-\alpha_{i+1} - \frac{1}{2})} \phi_i \prod_{\ell=2}^{n-2} \hat{Z}_{i+1, i+\ell}^{S_m(\beta^2)+n} \quad (21)$$

To receive the expressions (18-21) we have used the following relations

$$\tilde{Z}_{i, i+n} \approx \prod_{\ell=2}^{n-2} \hat{Z}_{i+1, i+\ell} \quad (22)$$

which are analogous to the equation (15). We also have introduced the poles $\frac{1}{\alpha_i + \frac{1}{2}}$ and $\frac{1}{\alpha_{i+1} + \frac{1}{2}}$ corresponding to the external π -mesons. As it follows from the expres-

sions (18-21) when $\alpha_i + \frac{1}{2} = 0$ these expressions have no poles at $\alpha_i + \frac{1}{2} = 0$ and $\alpha_{i+1} + \frac{1}{2} = 0$. At the same time they have poles in the variables α_i and α_{i+1} at the values

$$\alpha(P_k^2) = -\frac{1}{2} + m + n + r - (-1)^{2m} \frac{\Gamma(2m-1-2d_0)}{2(2m)! \Gamma^2(-d_0-\frac{1}{2})} \beta^2 \quad (23)$$

and

$$\alpha(P_k^2) = m + n + r - (-1)^{2m} \frac{\Gamma(2m-1-2d_0)}{2(2m)! \Gamma^2(-d_0-\frac{1}{2})} \beta^2 \quad (24)$$

here $k = i, i+1$; $m = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$.

While making the transition from the expressions (18-21) to the expressions (23,24) only the square terms in the parameter β have been retained. The residues at the poles (23) ($m+n+r=0$) correspond to the external π -meson lying on the shifted π -meson parent trajectory. At the same time the residues at the poles (24) correspond to the external scalar particles lying on the first shifted ρ -meson daughter trajectory. The appearance of new additional external states which do not lie on the shifted π -trajectory or on its daughters is an essentially new phenomenon for the rearranged Nambu-Schwarz amplitude and it may seem rather strange. But the appearance of such particles in the role of external particles is a direct consequence of G -parity conservation in the NSDM. Indeed as it has been pointed above a contribution into the rearranged B_n function is given only by the processes involving an even total number of π -mesons and spurions. If an odd number of spurions is emitted in a process the rearranged amplitude must describe the process involving odd number of external particles. Owing to G -parity conservation only even number of the particles with negative G -parity can be present. That means that the process of spurion emission is follo-

wed by a substitution of some of the particles with negative G -parity by the particles with positive G -parity. The physical interpretation of this substitution may be the following (see Fig.9).

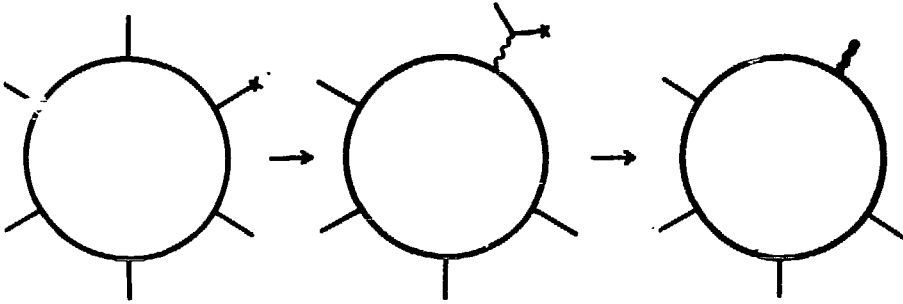


Fig.9

The scheme describing the transition to the G -even sector of the NSDM

A spurion interacts with the neighbouring π -meson and this results in the appearance of a positive G -parity state lying on the shifted trajectory. As a result of this an external π -meson of the rearranged amplitude is substituted by positive G -parity state.

The terms (20), (21) contain odd powers of the constant β and this corresponds to emission of an odd number of spurions. As a consequence the states having positive G -parity are generated. Owing to the presense of the factors ϕ_i and ϕ_{i+1} , these forms give an unequal to zero contribution into β_n^k . Note that there exists an attracting possibility for the particles corresponding to the ϕ -operators to be fermions. However it is difficult to extract the spin content immediately from the scattering amplitudes. So possibility of such interpretation is now unclear.

The terms of the type (18) and (19) correspond to processes involving emission of an even number of spurions. The terms (19) corresponds to the generation of a pair of the particles

with positive G-parity, the terms (18) correspond to the shift of the external π -meson trajectories (see Fig.10).

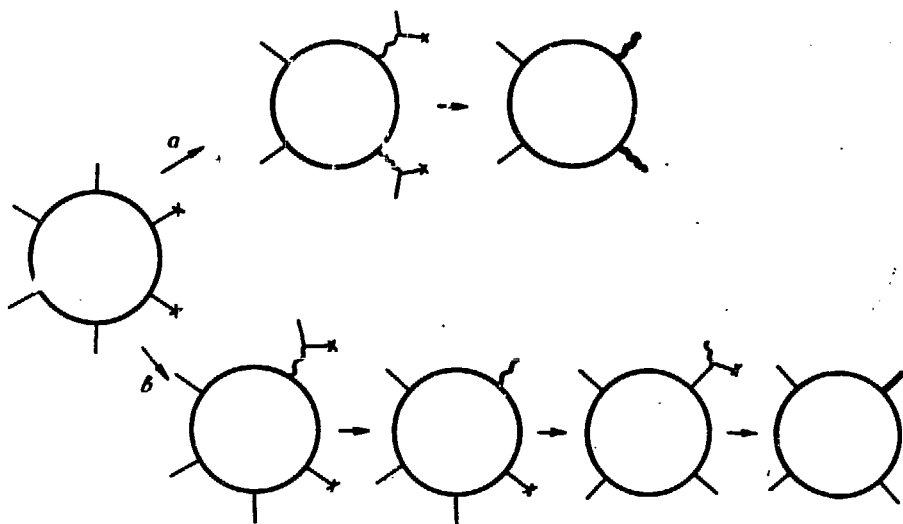


Fig.10.

Possible emission schemes of an even number of spurions.

- a) generation of two scalar states
- b) generation of a renormalized G-odd state

Note that the effect of the generation of the particles with positive G-parity shows that spontaneous breakdown in the NSDM is connected with a nonzero vacuum expectation value of the field operators of these particles. G-parity conservation prohibits the vacuum expectation value being nonzero for π -meson.

Having the physical meaning of the expressions (18-21) with $m+n+r=0$ we shall represent the rearranged B_n -function by means of the graphs shown on Fig.11, 12.

The graphes (Fig.11) correspond to the rearranged amplitudes, containing an even number of external particles and are described by the terms (18,19).

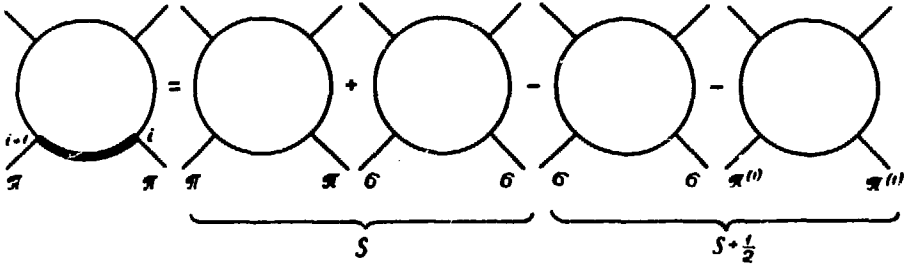


Fig.11.

A result of the emission of an even number of spurions. Two last terms correspond to the substitution $S \rightarrow S + \frac{1}{2}$ in the formulas (18,19)

The graphs (Fig.12) correspond to the rearranged amplitudes containing an odd number of external particles and are described by the terms (20,21).

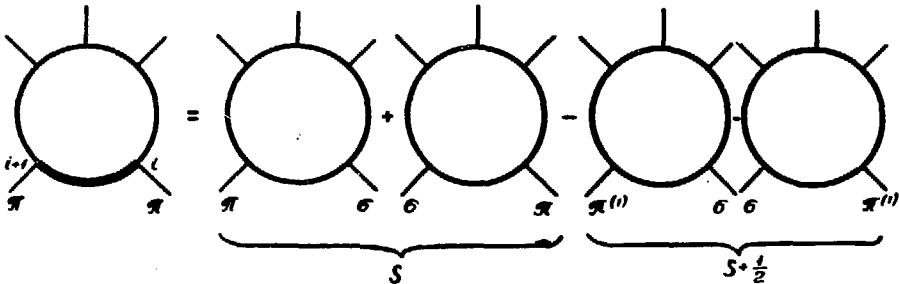


Fig.12

A result of the emission of an odd number of spurions

As it may be seen from Fig.11,12 the rearranged B_n -function is split into the sum of terms each of which corresponds to a renormalized amplitude in a definite sector of the NSDM.

For example, the renormalized amplitude describing \mathbb{N} -meson sector corresponds to the first graph shown on Fig.11 and the renormalized amplitude describing \mathbb{N} and σ -meson, sector corresponds to the second and third graphs shown on Fig.11 and to the first and second graphs on Fig.12.

Let us consider now the factors $\frac{z^{S_m(\beta^i)+n}}{z_{i+1, i \in \mathcal{C}}}$ and the invariant functions ϕ_i and ϕ_{i+1} in the expressions (18-21). The appearance of the two last functions is a consequence of the transition to G-even sector. The presence of the terms $\frac{z^{S_m(\beta^i)+n}}{z_{i+1, i \in \mathcal{C}}} (\ell=2, \dots, n-2)$ leads to the shift of the internal trajectories corresponding to diagram diagonals shown on Fig.4.

From the above mentioned correspondence between quark diagram diagonals and shifted trajectories $d_i, d_{i+1}, d_{i+1, i \in \mathcal{C}}$ we again necessarily get the quark interpretation of the rearranged amplitudes now for the case of the NSDM.

Thus taking into account spurion emission in the NSDM results in two effects. Firstly, we come to the G-even sector of the model and, secondly, we reveal the NSDM quark structure. The latter testifies to the presence of an additional degeneration in the resonance state spectrum of the NSDM similar to that of the VDM.

To study this degeneration and the corresponding daughter trajectory splitting we shall consider the expressions (23) and (24). In these expressions the \mathbb{N} and ρ -meson daughter trajectories have the values $m+n+r=J$ ($J = \frac{1}{2}, 1, \frac{3}{2}, \dots$). The dependence of the last term in the formulas (23) and (24) on m leads to the squared mass splitting of the external particles lying on the daughter trajectories. Let us consider this splitting in more detail. The \mathbb{N} -meson daughter trajectory splitting is given by integer values of m ($m=1, 2, \dots$) in the expression (23) and by half-integer values of m ($m = \frac{1}{2}, \frac{3}{2}, \dots$) in the expression (24). When $m+n+r=J$ the formula (23) gives the $J+1$ -fold splitting of the J -th

π -meson daughter trajectory, the expression (24) gives the J -fold splitting of the same trajectory. So the total splitting of π -meson daughter trajectory is $2J+1$ -fold.

Analogously it follows from the expressions (23,24) that the splitting of the ρ -meson trajectory is $2J$ -fold.

All the considerations dealing with the genuine and adopted daughter trajectories in the VDM may be carried out without any changes to the case of the NSDM. The trajectory splitting scheme is presented on Fig.13.

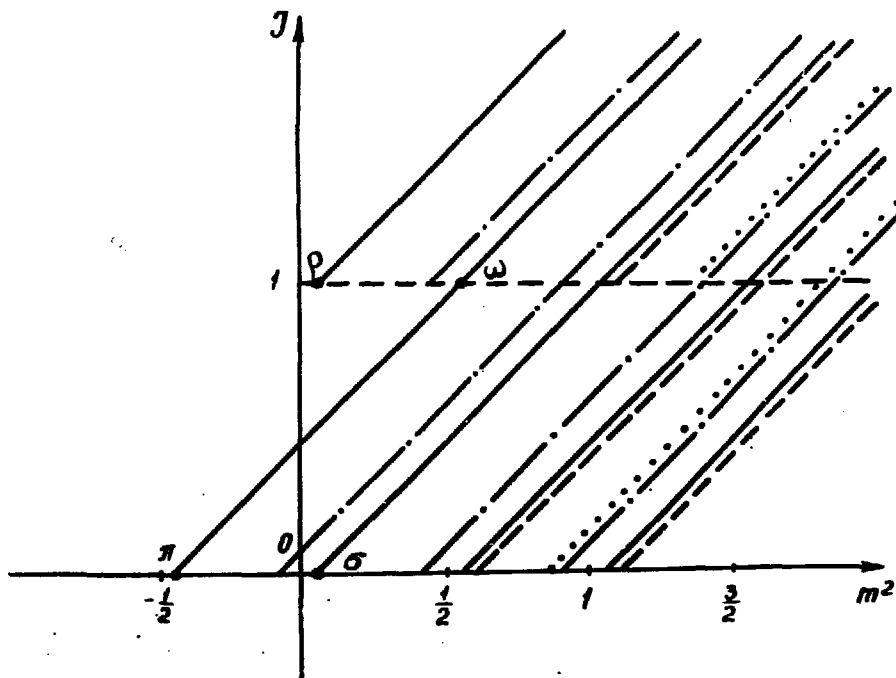


Fig.13

Trajectory splitting scheme on the NSDM

Analyzing the poles of Γ -function corresponding to the external particles in the expressions (18-21) we have considered the effect of the shift and splitting of the squared mass of the π -meson and the scalar particles lying on ρ and π -meson daughter trajectories. From the structure of the residues at the poles corresponding to the internal trajectories of the rearranged dual amplitudes we shall get analogous splitting of the states with higher spins. We shall show that the ρ -meson trajectory is shifted and the states with spin ≥ 1 lying on the π -parent trajectory and π and ρ -daughter trajectories acquire an additional splitting compared with the splitting of the scalar states belonging to the same trajectories (see Fig.12). The effect of ρ -meson trajectory shift testifies to the presence of the Higgs mechanism connected with spontaneous breakdown in the NSDM. We shall also show that the presence of an additional splitting of vector mesons, leads to the Weinberg algebraic realization of chiral symmetry. To illustrate the above-mentioned features of the rearranged amplitudes in the NSDM together with some other properties such as the Adler condition, the similarity to the σ -model and so on shall consider the 4-point B -functions.

3. The rearranged 4-point B -functions are given by the expression (1,7) when $n=4$

$$B_4^R(p_1, p_2, p_3, p_4) = \int \frac{\mathcal{D}\hat{Z}^{(4)}}{d\Omega} \hat{Z}_{12}^{-\alpha_{12}-1} \hat{Z}_{23}^{-\alpha_{23}-1} R(\hat{Z}_j; \alpha_j; \alpha_0, \beta) \quad (25)$$

where the invariant differential volume $d\Omega = \frac{\mathcal{D}\hat{Z}^{(3)}}{d\Phi_{1,3,4}}$ and $\Phi_{1,3,4}$ is the 3-point invariant function defined by the relation (5).

Let us write the expression for $d\Omega$ in a simpler form. For this purpose we make the substitution of the variables

$$\varphi_4 \rightarrow \varphi_4' = \Phi_{1,3,4}$$

in the invariant volume $d\hat{Z}^{(3)}$ (3)

$$D\hat{Z}^{(3)} = \frac{dZ_1 dZ_3 dZ_4 d\varphi_1 d\varphi_3 d\varphi_4}{\sqrt{(Z_4 - Z_3 - \varphi_3 \varphi_4)(Z_3 - Z_1 - \varphi_3 \varphi_1)(Z_4 - Z_1 - \varphi_4 \varphi_1)}} \quad (26)$$

to get the following expression:

$$D\hat{Z}^{(3)} = \frac{dZ_1 dZ_3 dZ_4 d\varphi_1 d\varphi_3 d\varphi_4}{\sqrt{(Z_4 - Z_3 - \varphi_3 \varphi_4)(Z_3 - Z_1 - \varphi_3 \varphi_1)(Z_4 - Z_1 - \varphi_4 \varphi_1)}} \left(1 - \frac{1}{2} \sqrt{\frac{Z_4 - Z_1}{(Z_3 - Z_1)(Z_4 - Z_3)}} \varphi_3 \varphi_1\right) \left(1 - \frac{1}{2} \sqrt{\frac{Z_4 - Z_3}{(Z_3 - Z_1)(Z_4 - Z_1)}} \varphi_4 \varphi_1\right) \quad (27)$$

Dividing both parts of the relation (27) on the differential

$d\Phi_{1,3,4}$ we get the expression for the invariant volume $d\Omega$.

It is convenient to carry out the calculations of the amplitudes in a fixed coordinate system where

$$Z_1 = 0, Z_3 = 1, Z_4 = \infty, \varphi_1 = \varphi_3 = 0 \quad (28)$$

In this coordinate system we have

$$\Phi_{1,3,4}/Z_4 \equiv \Phi = \frac{\varphi_4}{Z_4} \quad (29)$$

and the following expression for the volume element

$$\frac{D\hat{Z}^{(4)}}{d\Omega} = \frac{dZ_2 d\varphi_2 d\Phi}{1 - \Phi \varphi_2} \quad (30)$$

In the considered coordinate system the power factors

$$\frac{\hat{Z}_{1,2}^{-d_{12}-1}}{Z_{1,2}} \quad \text{and} \quad \frac{\hat{Z}_{3,4}^{-d_{23}-1}}{Z_{3,4}} \quad \text{have the form} \quad (31)$$

$$\frac{\hat{Z}_{1,2}^{-d_{12}-1}}{Z_{1,2}} = [Z_2(1 - \varphi_2 \Phi)]^{-d_{12}-1}$$

$$\frac{\hat{Z}_{2,3}^{-d_{23}-1}}{Z_{2,3}} = [(1 - Z_2)(1 - \varphi_2 \Phi)]^{-d_{23}-1} \quad (32)$$

Taking into account the relations (30-32) we get for the expression (25)

$$\begin{aligned}
 B_i^R(p_1, p_2, p_3, p_4) &= \left(-\frac{1}{2i}\right)^4 \int_{i=1}^4 K(s_i, d_0, \beta) ds_i \sum_{n_i=0}^{\infty} \left\{ \int dZ_1 Z_1^{-d_{12} + S_1 + S_2 + n_1 + n_2 - 1} \right. \\
 &\times (1-Z_2)^{-d_{23} + S_1 + S_2 + n_1 + n_2 - 1} \int d\Phi d\psi_2 (1-\Phi\psi_2)^{d_{12} + d_{23} + 1 - \sum_{\alpha=1}^2 S_\alpha + n_\alpha} \\
 &\times \left. \prod_{i=1}^4 B_i(s_i) A_i(s_{i-1}, s_i) - (s_1 \rightarrow s_1 + \frac{1}{2}) - \dots + (s_1 \rightarrow s_1 + \frac{1}{2}, s_2 \rightarrow s_2 + \frac{1}{2}) + \dots \right\}
 \end{aligned} \tag{33}$$

where

$$B_i(s_i) = \frac{1}{\Gamma(\frac{1}{2} - s_i) \Gamma(s_i - d_0 - \frac{1}{2}) \Gamma(n_i + 2s_i - d_0) n_i!} \tag{34}$$

To simplify the further calculation let us introduce the following notation

$$A_i(s_{i-1}, s_i) = E_i + \Phi_i Q_i \tag{35}$$

where the functions $E_i(s_{i-1}, s_i, -d_i - \frac{1}{2}, d_0, \beta)$ and $Q_i(s_{i-1}, s_i, -d_i - \frac{1}{2}, d_0, \beta)$ and denote the terms containing respectively even and odd powers of the parameter β . We shall also write the relation (33) in the form

$$B_i^R(p_i) = \left(-\frac{1}{2i}\right)^4 \int_{i=1}^4 ds_i K(s_i, d_0, \beta) \sum_{n_i=0}^{\infty} \left\{ B_i(s_i) B_i^R(s_i + n_i; p_i) - (s_1 \rightarrow s_1 + \frac{1}{2}) - \dots \right\} \tag{36}$$

We shall study the function $B_i^R(s_i + n_i; p_i)$ in the formula (36) which corresponds to the first term of the sum over the substitutions $s_\alpha \rightarrow s_\alpha + \frac{1}{2}$ in the expression (36). To receive the total sum over all such substitutions it is sufficient to symmetrise the function $B_i^R(s_i + n_i; p_i)$ according to the following Young scheme

S_1	S_2	...	S_n
$S_1 + \frac{1}{2}$	$S_2 + \frac{1}{2}$...	$S_n + \frac{1}{2}$

he functions $A_i(S_{i-1}, S_i)$ have the form in the coordinate system (28)

$$A_1(S_0, S_1) = E_1 + \beta Q_1 \left(\Phi \sqrt{z_2} + \frac{\Psi_2}{\sqrt{z_2}} \right) \quad (37)$$

$$A_2(S_1, S_2) = E_2 - \beta \frac{Q_2}{\sqrt{z_2(1-z_2)}} \Psi_2$$

$$A_3(S_2, S_3) = E_3 + \beta Q_3 \left(\Phi \sqrt{1-z_2} + \frac{\Psi_2}{\sqrt{1-z_2}} \right)$$

$$A_4(S_3, S_4) = E_4 - \beta Q_4 \Phi$$

Using the relation (37) we may fulfil the integration over Ψ_2 and Φ in the expression (33)

$$\begin{aligned} & \int d\Phi d\Psi_2 (1-\Phi\Psi_2)^{d_{12}+d_{23}+1-\sum_{i=1}^n (S_i+n_i)} \prod_{i=1}^n A_i(S_{i-1}, S_i) = \\ & = (d_{12}+d_{23}+1-\sum_{i=1}^n (S_i+n_i)) E_1 E_2 E_3 E_4 - \beta^2 (1-z_2)^{\frac{1}{2}} E_1 E_2 Q_3 Q_4 + \\ & + \beta^2 (1-z_2)^{-\frac{1}{2}} Q_1 Q_2 E_3 E_4 - \beta^2 (z_2)^{-\frac{1}{2}} E_1 Q_2 Q_3 E_4 - \beta^2 (z_2)^{\frac{1}{2}} Q_1 E_2 E_3 Q_4 + \\ & + \beta^2 [z_2(1-z_2)]^{\frac{1}{2}} E_1 Q_2 E_3 Q_4 - \beta^2 \left[\left(\frac{z_2}{1-z_2} \right)^{\frac{1}{2}} - \left(\frac{z_2}{1-z_2} \right)^{-\frac{1}{2}} \right] Q_1 E_2 Q_3 E_4 \end{aligned} \quad (38)$$

Substituting the relation (38) into the expression (33) we get the following expression for the function $R_n^R(S_i+n_i; p_i)$

$$\begin{aligned}
B_i^R(\alpha_i, n_i, p_i) = & - \frac{\Gamma(-\alpha_{i2} + S_2 + S_4 + n_2 + n_4) \Gamma(-\alpha_{23} + S_1 + S_3 + n_1 + n_3)}{\Gamma(-\alpha_{i2} - \alpha_{23} + \sum_{i=1}^4 (S_i + n_i) - 1)} E_1 E_2 E_3 E_4 - \\
& - \beta^2 B(-\alpha_{i2} + S_2 + S_4 + n_2 + n_4, -\alpha_{23} - \frac{1}{2} + S_1 + S_3 + n_1 + n_3) (E_1 E_2 Q_3 Q_4 - Q_1 Q_2 E_3 E_4) - \\
& - \beta^2 B(-\alpha_{i2} - \frac{1}{2} + S_2 + S_4 + n_2 + n_4, -\alpha_{23} + S_1 + S_3 + n_1 + n_3) (E_1 Q_2 Q_3 E_4 + Q_1 E_2 E_3 Q_4) + (39) \\
& + \beta^2 B(-\alpha_{i2} - \frac{1}{2} + S_2 + S_4 + n_2 + n_4, -\alpha_{23} - \frac{1}{2} + S_1 + S_3 + n_1 + n_3) E_1 Q_2 E_3 Q_4 - \\
& - \beta^2 [B(-\alpha_{i2} + \frac{1}{2} + S_2 + S_4 + n_2 + n_4, -\alpha_{23} - \frac{1}{2} + S_1 + S_3 + n_1 + n_3) - \\
& - B(-\alpha_{i2} - \frac{1}{2} + S_2 + S_4 + n_2 + n_4, -\alpha_{23} + \frac{1}{2} + S_1 + S_3 + n_1 + n_3)] Q_1 E_2 Q_3 E_4
\end{aligned}$$

As it has been already pointed above different terms in the expression (39) correspond to different sectors of the NSDM.

Let us consider the first term in the expression (39) when $n_i = 0$ and all the variables S_i are equal to $S_0(\beta^2)$ which is the root of the equation (6) satisfying $S_0(0) = 0$.

We may represent the first term of the expression (39) in the form:

$$\frac{\Gamma(1 - \alpha_{i2}^{\mathcal{P}} + 2S_0) \Gamma(1 - \alpha_{23}^{\mathcal{P}} + 2S_0)}{\Gamma(1 - \alpha_{i2}^{\mathcal{P}} - \alpha_{23}^{\mathcal{P}} + 4S_0)} \prod_{i=1}^4 \Gamma_i(-\alpha_i^{\mathcal{P}} + 2S_0) C_i \quad (40)$$

where the relation

$$\alpha_0 = \alpha_0^{\mathcal{P}} - 1 = \alpha_0^{\mathcal{P}} - \frac{1}{2} \quad (41)$$

is used, and the normalizing constants corresponding to the i -th particle are denoted by C_i . The poles of the $\Gamma(-\alpha_i^{\mathcal{P}} + 2S_0)$ -function correspond to the external particles lying on the \mathcal{P} -meson shifted parent trajectory and its daughters. Having determined the residues at the poles in the variables $\alpha_i^{\mathcal{P}}$ of the expression (40) when $\alpha_i^{\mathcal{P}} - 2S_0 = 0$

We get the expression for the Lovelace 4-point amplitude with the ρ -meson trajectory intercept shifting equal to $2S_0$. Note that the spin zero states are less degenerated than the states with higher value of spin belonging to the same trajectory. That is a generalization of the well known fact that in the NSDM the scalar meson on the ρ -trajectory is decoupled. To prove the above statement let us consider the first term of the expression (39)

$$- \frac{\Gamma(-\alpha_{12} + S_2 + S_4 + n_2 + n_4) \Gamma(-\alpha_{23} + S_1 + S_3 + n_1 + n_3)}{\Gamma(-\alpha_{12} - \alpha_{23} + \sum_{i=1}^4 (S_i + n_i) - 1)} E_1 E_2 E_3 E_4 \quad (42)$$

The nearest pole in the variable α_{23} of this expression is at the point $\alpha_{23} = S_1 + S_3 + n_1 + n_3$ and its residue is equal to

$$\alpha_{12} - S_2 - S_4 - n_2 - n_4 + 1 \quad (43)$$

This shows that the exchange of a vector particle takes place. A simple calculation shows that the total number of vector states at the level $\mu^2 = -\alpha_0 + K$ equals to the number of zero spin states at the level $\mu^2 = -\alpha_0 - \frac{1}{2} + K$. This number is larger than that of the states with zero spin belonging to the same trajectory (see Fig.13).

As it will be shown below the presence of the additional vector state splitting leads to the algebraic realization of the chiral $SU(2) \times SU(2)$ symmetry in the NSDM.

We have briefly discussed the physical consequences connected with the first term in the expression (39). The symmetrization of this term according the Young scheme results in transition to other sectors of the NSDM. For example, making the substitution $S_3 \rightarrow S_3 + \frac{1}{2}$ in the formula (40) we get the $\Pi\Pi\sigma\sigma$ scattering amplitude in the following form

$$\frac{\Gamma(1 - \alpha_{12}^0 + 2S_0) \Gamma(1 - \alpha_{23}^{\frac{1}{2}} + 2S_0)}{\Gamma(1 - \alpha_{12}^0 - \alpha_{23}^{\frac{1}{2}} + 4S_0)} \quad (44)$$

where σ is a G-even spin zero particle.

Carrying out the substitution $S_i \rightarrow S_i + \frac{1}{2}$ in the formula (44) we get the $\sigma\sigma\sigma\sigma$ scattering amplitude

$$\frac{\Gamma(1-\alpha_{12}^{\sigma}+2S_0)\Gamma(2-\alpha_{23}^{\sigma}+2S_0)}{\Gamma(2-\alpha_{12}^{\sigma}-\alpha_{23}^{\sigma}+4S_0)} \quad (45)$$

Note that the amplitude (45) shows the presence of the 3-point $\sigma\sigma\sigma$ vertex proportional to $(1-\alpha^{\sigma}+2S_0)$. The expression (44), (45) give only a part of all the possible 4-particle amplitudes. In order to obtain all the amplitudes we must consider other terms in the expression (39). These terms describe the interactions of the types $\pi\pi\sigma$, $\pi\sigma\pi$ etc.

Let us compare various terms in (39) of the amplitudes with the same external particles. As an example we consider $\pi\pi\sigma$ interaction. Putting in the second term of (39) all the variables S_i equal to $S_0(\beta^2)$ and all the n_i 's equal to zero we find the second term of the $\pi\pi\sigma$ scattering amplitude:

$$-\frac{\beta^2}{-\alpha_{23}^{\pi}+2S_0} \frac{\Gamma(1-\alpha_{12}^{\sigma}+2S_0)\Gamma(1-\alpha_{23}^{\pi}+2S_0)}{\Gamma(1-\alpha_{12}^{\sigma}-\alpha_{23}^{\pi}+4S_0)} \quad (46)$$

The representation (46) has the π -meson pole. The first pole in the $\pi\sigma$ channel of the amplitude (44) corresponds to ω -meson and to its first daughter scalar state. The presence of the π -meson pole in the amplitude (46) shows the appearance of the additional 3-point $\pi\pi\sigma$ vertex. This testifies that amplitude rearrangement in the NSDM is analogous to the rearrangement appearing in the spontaneously broken σ -model. This analogy is not accidental. While considering SVT we shall show below that the chiral $SU(2) \times SU(2) \times U(1) \times U(1) \times \dots$ symmetry group takes place in the NSDM. Another important property of the rearranged amplitudes testifying to the connection with chiral symmetry and PCAC consists

in the fulfilment of the Adler condition for the particles belonging to the π -meson trajectory and its daughters. To prove this property we again consider the expression (39) for the rearranged B_4 -function. As an example we consider the first term in the expression (39) which is equal to

$$\frac{\Gamma(1-\alpha_{12}^g + S_2 + S_4) \Gamma(1-\alpha_{23}^g + S_1 + S_3)}{\Gamma(1-\alpha_{12}^g - \alpha_{23}^g + S_1 + S_2 + S_3 + S_4)} \quad (47)$$

with the external particle squared momentum

$$P_i^2 = -\alpha_0^g + S_{i-1}(\beta^2) + S_i(\beta^2) \quad (48)$$

Putting the 4-momentum $P_4 = 0$ and using the mass shell condition for the 4-momenta P_1, P_2, P_3 we find

$$\alpha_{12}^g = \alpha_3^g = \alpha_0^g + P_3^2 = \frac{1}{2} + S_2 + S_3 \quad (49)$$

$$\alpha_{23}^g = \alpha_1^g = \alpha_0^g + P_1^2 = \frac{1}{2} + S_1 + S_4 \quad (50)$$

Substituting the relations (49) and (50) into the expression (48) we get

$$\frac{\Gamma(\frac{1}{2} + S_4 - S_3) \Gamma(\frac{1}{2} + S_3 - S_4)}{\Gamma(0)} \quad (51)$$

Owing to the presence of the factor $\Gamma(0)$ in the denominator this expression becomes equal to zero when

$$S_4(\beta^2) - S_3(\beta^2) \neq \pm \frac{1}{2}, \pm \frac{3}{2}, \dots \quad (52)$$

Note that $S_m(0)$ may acquire only integer and half-integer values. The condition (52) restricts the square mass values of the external particle with the zero momentum. The expression (48) gives the square mass value equal to

$$M_i^2 = -\alpha_0^g + n \quad (53)$$

when $n = 0, 1, 2, \dots$.

Analogous restrictions for the square mass values of the external particles with zero momentum appear while considering

other terms in the expression (39). To prove this we shall consider the second term in the expression (39) equal to

$$B(1 - \alpha_{12}^{\rho} + S_2 + S_4, -\alpha_{23}^{\pi} + S_1 + S_3) \quad (54)$$

and having the external particle squared momentum

$$p_i^2 = -\alpha_0^{\pi} + S_{i-1} + S_i \quad (i = 1, 2) \quad (55)$$

$$p_j^2 = 1 - \alpha_0^{\rho} + S_{j-1} + S_j \quad (j = 3, 4) \quad (56)$$

Putting the 4-momentum $p_4 = 0$ and using the mass shell condition for the 4-momentum 1, 2, 3

$$\alpha_{12}^{\rho} = \alpha_3^{\rho} = \alpha_0^{\rho} + p_3^2 = 1 + S_3 + S_2 \quad (57)$$

$$\alpha_{23}^{\pi} = \alpha_1^{\pi} = \alpha_0^{\pi} + p_1^2 = S_4 + S_1 \quad (58)$$

Substituting the relations (57) and (133) into the expression (54) we shall obtain the following expression

$$B(S_4 - S_3, S_3 - S_4) \quad (59)$$

The function B (59) becomes equal to zero when

$$S_4(\beta^2) - S_3(\beta^2) \neq 0, \pm 1, \pm 2, \dots \quad (60)$$

On the base of the condition (60) and the relation (56) we conclude that the Adler condition is fulfilled for the particles having the squared mass equal to

$$\mu_4^2 = 1 - \alpha_0^{\rho} + \frac{1}{2} + n \quad (61)$$

when $\beta = 0$ ($n = 0, 1, 2, \dots$)

Comparing the conditions (53) and (61) for the squared masses of the external particles we conclude that these particles must belong to $\tilde{\pi}$ -meson trajectory or its degenerated daughters.

Further we shall prove the Adler condition in a general form for an arbitrary n -point B_n^R function emphasizing the quark formulation of the Adler condition in dual models.

3. The presence of a quark structure and of a related $U(1) \times U(1) \times \dots$ symmetry group essentially conditions fulfillment of the Adler condition in the NSDM. In order to stress the presence of a connection between quarks and the Adler condition we firstly consider the rearranged VDM where the IVT's are taken into account and formulate the Adler condition in terms of quarks.

The rearranged $(n+1)$ -point B_{n+1}^R function has the following form in the VDM:

$$B_{n+1}^{R_i}(\beta, p_1, \dots, p_{n+1}) = \frac{1}{\int \mathcal{Z}} \int dZ^{(n+1)} \prod_{j,k} Z_{j,k}^{-\alpha_{jk}-1} \times R(Z_{12}, Z_{23}, \dots, Z_{i,i+1}, \dots, Z_{nn+1}; d_i; d_0, \beta) \quad (62)$$

When the momentum p_j of the amplitude (62) goes to zero an additional rebuilding of the integrand follows. This rebuilding is fully analogous to that taking place in the process of spurion summation and consists in the appearance of the additional function R_1

$$R_1(Z_{i,i+1}; d_{iy} - S_{i-1} - S_y; d_{y,i+1} - S_i - S_{i+1}; d_0, \beta) = \frac{1}{Z_{i,i+1}} \int_{Z_i}^{Z_{i+1}} \frac{dy (Z_{i+2} - Z_{i-1})}{(Z_{i+2} y (y - Z_{i-1}))} \times (Z_i, Z_{i-1}, y, Z_{i+1})^{-d_{iy} + S_{i-1} + S_y - 1} (y, Z_i, Z_{i+1}, Z_{i+2})^{-d_{y,i+1} + S_i + S_{i+1} - 1} \times (Z_{i+1}, y, Z_{i+2}, Z_{i-1})^{S_y - S_i} \quad (63)$$

in the integrand of the function B_n^R . The variable y corresponds to a soft particle and its position is between the variables Z_i and Z_{i+1} .

Using the mass shell condition for the momenta of the i -th and $(i+1)$ -th particles we get

$$-d_{iy} + S_{i-1}(\beta) + S_y(\beta) = S_y(\beta) - S_i(\beta) \quad (64)$$

$$-d_{y,i+1} + S_i(\beta) + S_{i+1}(\beta) = S_i(\beta) - S_{i+1}(\beta) \quad (65)$$

Substituting the relations (64) and (65) into the expression (63) we get the following form of the function R_1

$$R_1(Z_{i,i+1}; S_i - S_y; S_y - S_i; d_0, \beta) = \left(\frac{Z_{i+2} - Z_i}{Z_{i+2} - Z_{i+1}} \right)^{S_i - S_y} B(S_i - S_y, S_y - S_i) \quad (66)$$

The expression (66) goes to zero when $S_i(\beta) - S_y(\beta) \neq 0, \pm 1, \pm 2, \dots$. Taking into account that the particle with zero momentum is composed of the quark $S_y(\beta)$ and the antiquark $S_i(\beta)$ we get the quark formulation of the Adler condition in the VDM.

When the momentum of the particle composed of the quark with the hyper charge Y_e and the antiquark with the hyper charge \bar{Y}_k goes to zero the dual amplitude also goes to zero if $Y_e \neq Y_k$ and all the other particles are on the mass shell. This means that in the VDM the Adler condition takes place for the particles which hyper charges are different from zero.

Similarly we may prove that the Adler condition is fulfilled in the rearranged NSDM. Repeating the above considerations we shall find that the additional function R_1 appearing in the β_n^R integrand has the following form when the momentum p_y goes to zero

$$R(\bar{Z}_{i,i+1}; \Phi_i, \Phi_{i+1}; d_{iy} - S_{i-1} - S_y; d_{y,i+1} - S_i - S_{i+1}; d_0, \beta) = \quad (67)$$

$$= \int_{Z_i}^{Z_{i+1}} dy d\psi \frac{1}{(\bar{Z}_{i,i+1})^{\frac{1}{2}}} \left[\frac{Z_{i+2} - Z_{i+1} - \psi_{i+2} \psi_{i+1}}{(Z_{i+2} - y - \psi_{i+2} \psi)(y - Z_{i+1} - \psi \psi_{i+1})} \right]^{\frac{1}{2}} Z_{iy}^{-d_{iy} + S_{i-1} + S_y - 1} Z_{y,i+1}^{-d_{y,i+1} + S_i + S_{i+1} - 1} \times$$

$$\times (\bar{Z}_{i,i+1}, y, Z_{i+2}, Z_{i+1})^{S_y - S_i} A_i(S_{i-1}, S_y) A_y(S_i, S_y) A_{i+1}(S_y, S_{i+1})$$

We have taking into account the contribution of the first term to the Neveu-Schwarz amplitude. Taking into account the other terms leads to the analogous contributions.

The functions $A_j(S_{j-1}, S_j)$ in the integrand of the formula (67) in general case depend on the integration variable y . Using the definition (35) we represent the product $A_i A_y A_{i+1}$ in the form

$$A_i A_y A_{i+1} = (E_i + \beta \Phi_i Q_i)(E_y + \beta \Phi_y Q_y)(E_{i+1} + \beta \Phi_{i+1} Q_{i+1}) \quad (68)$$

Various sectors of the rearranged NSDM correspond to various terms in the expression (68).

For example, we consider the integral corresponding to the term $E_i E_y E_{i+1}$ which does not depend on the variable y . For this term the mass shell condition for the momenta p_i and p_{i+1} leads to the relations

$$-\alpha_{iy} + S_{i-1} + S_y = S_y(\beta^2) - S_i(\beta^2) + \frac{1}{2} \quad (69)$$

$$-\alpha_{yi+1} + S_i + S_{i+1} = S_i(\beta^2) - S_y(\beta^2) + \frac{1}{2} \quad (70)$$

To derive these relations we have used the following expressions

$$p_i^2 = -\alpha_0^{\mathcal{H}} + S_{i-1} + S_i + n_{i-1} + n_i \quad (71)$$

$$p_{i+1}^2 = -\alpha_0^{\mathcal{H}} + S_y + S_{i+1} + n_{i+1} + n_i \quad (72)$$

where for simplicity all the n_j are put equal to zero. Substituting the relations (69) and (70) into the expression (67) we obtain

$$R_1 = \left[\frac{z_{i+2} - z_i - \varphi_{i+2} \varphi_i}{z_{i+2} - z_{i+1} - \varphi_{i+2} \varphi_{i+1}} \right]^{S_i - S_y} \frac{(\varphi_i + \varphi_{i+1})}{(z_{i+1} - z_i)^{\frac{1}{2}}} \frac{\Gamma(\frac{1}{2} + S_i - S_y) \Gamma(\frac{1}{2} + S_y - S_i)}{\Gamma(0)} E_i E_y E_{i+1} \quad (73)$$

The function R_1 equals to zero when

$$S_i(\beta^2) - S_y(\beta^2) \neq \pm \frac{1}{2}, \pm \frac{3}{2}, \dots \quad (74)$$

Using the expression for the squared mass of the particle with zero momentum, i.e.

$$M_y^2 = -\alpha_0^{\mathcal{H}} + S_y + S_{i-1} + n_y + n_{i-1} \quad (75)$$

we again conclude that the Adler condition takes place only for the particles belonging to the \mathcal{H} -meson trajectory or its daughters. The consideration of the integral (73) corresponding to other terms of the expression (68) results in the same conclusion. The particles belonging to the split \mathcal{H} -meson

daughter trajectories may be composed either of a quark and an antiquark of the same type or of a quark corresponding to an integer (half-integer) value of $S_m(0)$ and an antiquark corresponding to a half-integer (integer) value of $S_m(0)$. Having considered the SVT's in the NSDM we shall show that the \mathbb{N} -meson daughter trajectories composed of a quark and an antiquark of the same type contain the particles having a negative G-parity whereas the trajectories composed of a quark and an antiquark of different types contain the particles with a positive G-parity. For latter particles the Adler condition is provided by the same mechanism as in the case of the VDM. For the particles with quarks of the same type the Adler condition is provided in an analogy with chiral symmetry theories.

4. For studying the SVT's in the NSDM we consider an analytical continuation of the function R along an arbitrary contour beginning at $\beta = 0$ on the β -plane. The analytical continuation procedure is analogous to that considered by the authors in the VDM and is defined by the representation (3). In this representation the integrand poles are the functions of the parameter β^2 defined by the condition:

$$1 - \mathbb{N} \beta^2 2^{2\alpha_0+3} \frac{\Gamma(-2S)\Gamma(2S-2\alpha_0-1)}{\Gamma^2(-\alpha_0-\frac{1}{2})} \quad (76)$$

When $\beta = 0$ this equation leads to two root series:

$$S_m(0) = m \quad (m = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots) \quad (77)$$

and

$$S'_m(0) = -\alpha_0 + \frac{1}{2} - m \quad (m = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots) \quad (78)$$

which are separated by the integration contour γ in the S -plane. When the parameter β^2 is varied the roots $S_m(\beta^2)$ and $S'_m(\beta^2)$ are moving continuously in the S -plane.

Let us consider in the β^2 -plane a closed contour C passing through the point $\beta = 0$. After going around the contour C and returning to the point $\beta = 0$ each of the roots $S_m(\beta^2)$ and $S'_m(\beta^2)$ acquires some of the

possible values (77) or (78). However, the values of some of the roots after enclosing the contour C may not coincide with their initial values.

In general case the enclosement of an arbitrary closed contour in the β^2 plane corresponds to some permutation of the roots (77) and (78).

For the phenomenon of SVT essential are such movements of the roots $S_m(\beta^2)$ and $S'_m(\beta^2)$ in the result of which some roots of the $S_m(0)$ series (77) change their places with some roots of the $S'_m(0)$ series (78). When the parameter β^2 is continuously varying such roots collide with the integration contour γ . As a consequence the integration contour γ is deformed and the integral (3) receives some additional contribution as compared with its initial value. So if a contour C accompanied by interchange of the roots (78) and (79) exists then the integral (3) is a multivalued function of the parameter β^2 and the values it acquires on the other Riemann sheets when $\beta = 0$ correspond to SVT's.

In order to study the connection of various contour enclosements with the permutations of the roots $S_m(0)$ and $S'_m(0)$ we consider the Riemann surface of the function $S(\beta^2)$ defined by the equation (76)

For the particular case $\alpha_0 = -1$ the equation (76) has the form

$$1 - \frac{2\pi\beta^2}{\sin 2\pi S} = 0 \quad (79)$$

and its solution corresponds to arcsinus function

$$S(\beta^2) = \frac{1}{2\pi} \text{Arcsin } 2\pi\beta^2 \quad (80)$$

The graphical representation of the roots of the equation (79) is shown on Fig.14.

Fig.15. shows the Riemann surface of the function $S(\beta^2)$ (80). This Riemann surface has an infinite number of leaves. Each pair of neighbouring leaves is sewn together along the left (or right) cut going from the branch point $\beta^2 = -\frac{1}{2\pi} \left(+\frac{1}{2\pi} \right)$

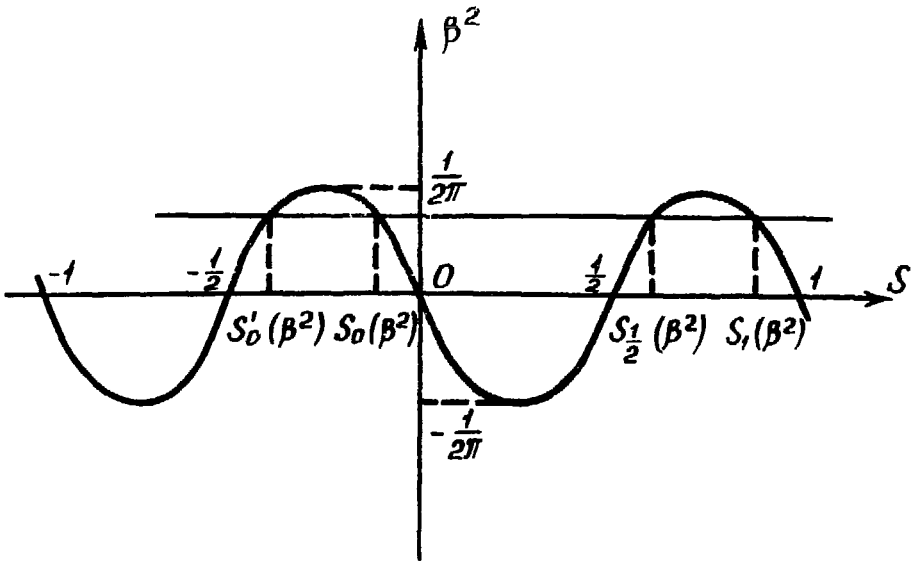


Fig.14

The graphical solution of the equation $\beta^2 = \frac{\sin 2\pi S}{2\pi}$

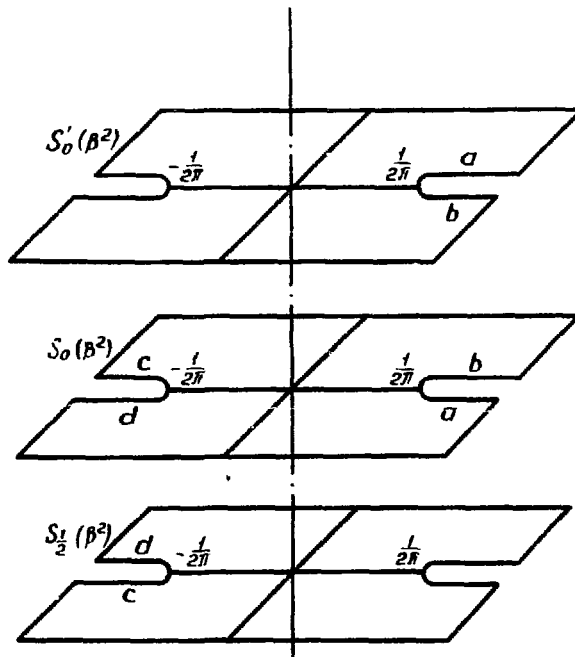


Fig.15.

The Riemann surface of the function

$$S(\beta^2) = \frac{1}{2\pi} \text{Arcsin } 2\pi\beta^2$$

to infinity. When $\beta = 0$ each leaf corresponds to the definite root $S_m(0)$ or $S'_m(0)$ shown on Fig.1. When $\alpha_0 = -1$, the branch point positions coincide on all the leaves (see Fig.12). As it is shown on Fig.(15) the enclosement of a branch point corresponds to the transition from the leaf designated by a half-integer number to the neighbouring lower (upper) leaf designated by an integer number. Fig.16 shows the corresponding movement of the equation (76) roots in the S -plane.

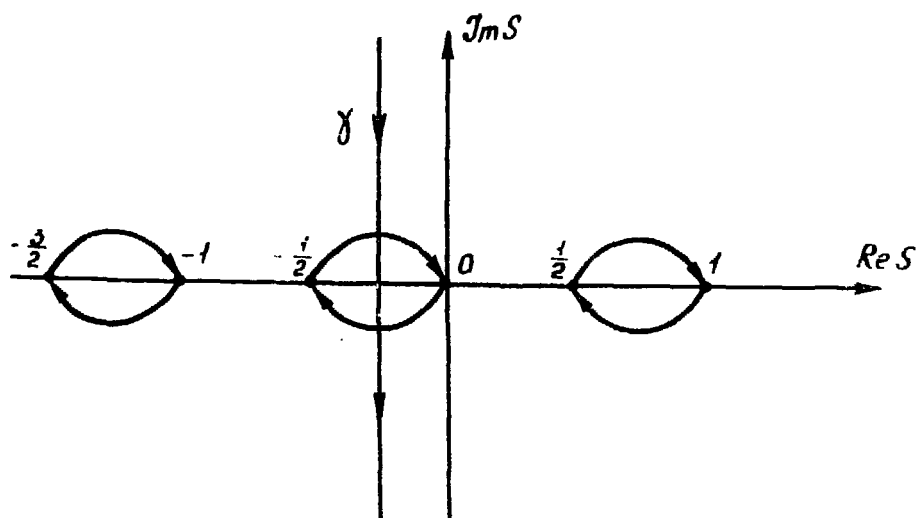


Fig.16.

The motion of the equation $\beta^2 = \frac{1}{2\pi} \sin 2\pi S$ roots while enclosing the branch point.

Fig.17,18 present the graphical determination of the equation (76) roots and the Riemann surface of the function

$S(\beta^2)$ when $-1 \leq \alpha_0 < -\frac{1}{2}$.

When $\beta^2 = 0$ the function $S(\beta^2)$ has the roots

$$S_m(0) = m \quad \text{and} \quad S'_m(0) = \alpha_0 + \frac{1}{2} - m \quad (m = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots)$$

When β^2 varying the roots are shifted and coincide in pairs at the minimum and maximum points $\beta_0^2, \beta_{\frac{1}{2}}^2, \dots$ corresponding to the branch points in the complex β^2 plane.

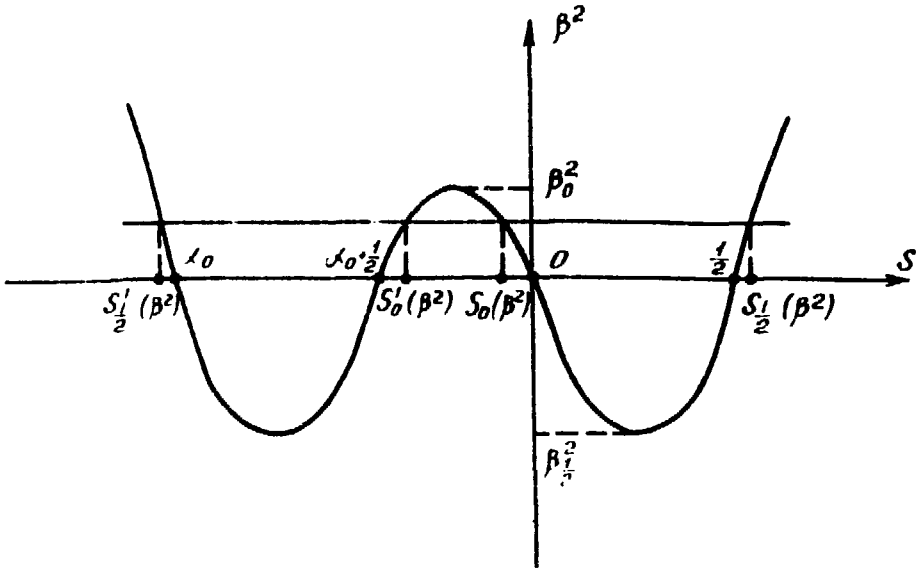


Fig.17

The graphical solution of the equation (76)
when $-1 \leq d_0 < -\frac{1}{2}$

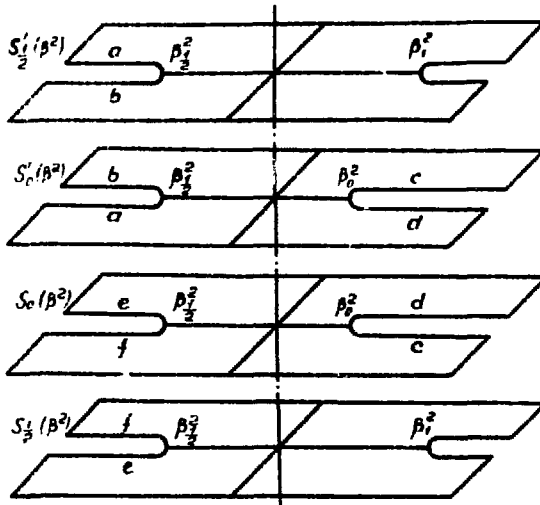


Fig.18

The Riemann surface of the function $S(\beta^2)$ for the equation (76). (The sides of cuts which are sewn together are denoted by the same letters).

Unlike the sinus function the function $\left[\frac{\Gamma(-2S)\Gamma(2S-2\alpha_0-1)}{\Gamma^2(-\alpha_0-\frac{1}{2})} \right]^{-1}$ maximum and minimum have various relative value except the maximums and minimums located at the points symmetrical with respect to the transformation $S \rightarrow -S + \alpha_0 + \frac{1}{2}$. In the result of this the branch point positions on various leaves are different. The enclosement of the nearest branch point $\beta_0^2 = -2^{-2\alpha_0-3}\beta_1^{-1}$ corresponds to the motion of the equation (76) roots in the S-plane as shown in fig.(19). Thereupon the roots $S_0(0) = 0$ and $S'_0(0) = \alpha_0 + \frac{1}{2}$ change their positions.

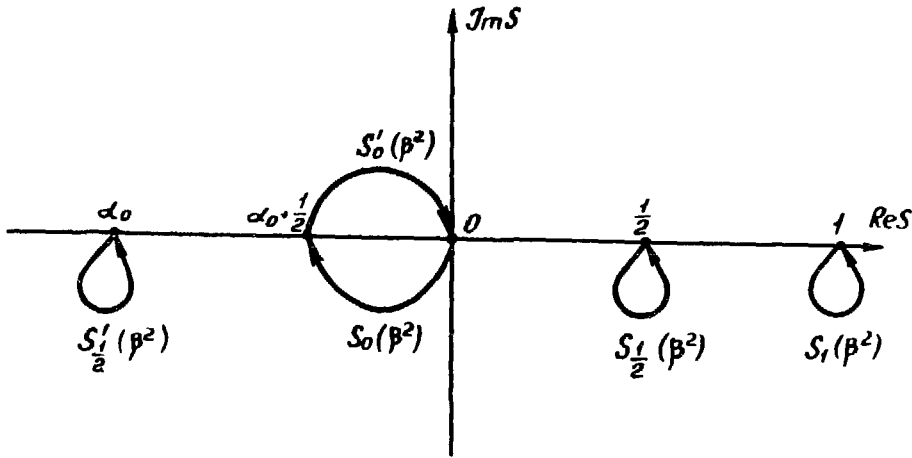


Fig.19

The motion of the roots $S_m(\beta^2)$ and $S'_m(\beta^2)$ while enclosing the branch point β_0^2 on the β^2 plane

As it follows from Fig.19 an arbitrary enclosement in the β^2 -plane corresponds to some permutation of the equation (76) roots. Owing to the equation (76) symmetry under the substitution $S \rightarrow -S + \alpha_0 + \frac{1}{2}$ every such a permutation reduces (to a precision of a root numeration) to the transition of some roots $S_m(0)$ (77) into the points symmetrical relatively to the point $\frac{1}{2}(\alpha_0 + \frac{1}{2})$. In other words each

closed contour corresponds to a permutation of some $S_m(0)$ roots with $S'_m(0)$ roots with the same value of m .

The $\mu_m^2 \rightarrow -\mu_m^2$ rule.

The values of the intercepts of the π and ρ -meson parent and all adopted daughter trajectories redefined in the result of the SVT's may be easily obtained using the root permutation rule formulated above for an arbitrary closed contour.

As it has been pointed above the consideration of spurion emission makes possible to discover resonance state degeneration connected with their quark structure.

When the parameter β^2 is small the trajectory intercepts for the resonances composed of the m -th quark and n -th antiquark have the following form

$$\alpha_0^\pi(m, n) = \alpha_0 + \frac{1}{2} - S_m(\beta^2) - S_n(\beta^2) \quad (81)$$

and

$$\alpha_0^\rho(m, n) = \alpha_0 + 1 - S_m(\beta^2) - S_n(\beta^2) \quad (82)$$

when $\beta = 0$

$$\alpha_0^\pi(m, n) = -\mu_m^2 - \mu_n^2 \quad (83)$$

and respectively

$$\alpha_0^\rho(m, n) = \frac{1}{2} - \mu_m^2 - \mu_n^2 \quad (84)$$

where

$$\mu_m^2 = -\frac{1}{2}(\alpha_0 + \frac{1}{2}) + m$$

is the m -th quark square mass. While enclosing the contour on the β^2 plane corresponding to a definite SVT some roots

$S_m(0) = m$ transit into the values symmetrical relatively to

the point $S = \frac{1}{2}(\alpha_0 + \frac{1}{2})$, i.e. $m \rightarrow \alpha_0 + \frac{1}{2} - m$.

As a result of this transition the quark effective square mass corresponding to given m change its sign, i.e. $\mu_m^2 \rightarrow -\mu_m^2$.

Similar effect takes place in the $\lambda\psi^3$ -theory where SVT result in changing the sign of scalar particle squared mass

Tachyon elimination

If $2n - \frac{1}{2} < \alpha_0 \leq 2n + \frac{1}{2}$ ($n = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$), then $\mu_m^2 < 0$ and when $m \leq n$ ($m = 0, \frac{1}{2}, 1, \dots$) there are tachyons in the initial NSDM. Let us choose the enclosement contour in the β^2 -plane in such a way that the first $2n+1$ quarks change the sign of the squared mass $\mu_m^2 \rightarrow -\mu_m^2 = \frac{1}{2}(\alpha_0 + \frac{1}{2}) - m > 0$, ($m \leq n$)

As the result we get the following values for the \mathbb{N} and ρ parent and their adopted daughter trajectory intercepts.

$$\begin{aligned} \alpha_0^{\mathbb{N}}(k, \ell) &= \alpha_0 + \frac{1}{2} - k - \ell & (k, \ell > n) \\ \alpha_0^{\mathbb{N}}(k, \ell) &= -k + \ell & (k > n, \ell \leq n) \\ \alpha_0^{\mathbb{N}}(k, \ell) &= k - \ell & (k \leq n, \ell > n) \\ \alpha_0^{\mathbb{N}}(k, \ell) &= -\alpha_0 - \frac{1}{2} + k + \ell & (k, \ell \leq n) \end{aligned} \quad (85)$$

and

$$\begin{aligned} \alpha_0^{\rho}(k, \ell) &= \alpha_0 + 1 - k - \ell & (k, \ell > n) \\ \alpha_0^{\rho}(k, \ell) &= -k + \ell + \frac{1}{2} & (k > n, \ell \leq n) \\ \alpha_0^{\rho}(k, \ell) &= k - \ell + \frac{1}{2} & (k \leq n, \ell > n) \\ \alpha_0^{\rho}(k, \ell) &= -\alpha_0 + k + \ell & (k, \ell \leq n) \end{aligned} \quad (86)$$

i.e. in all the cases $\alpha_0^{\mathbb{N}}(k, \ell) \leq 0$ and $\alpha_0^{\rho}(k, \ell) \leq \frac{1}{2}$. The latter guarantees tachyon elimination in the rearranged dual amplitudes as the spin zero state lying on the ρ -trajectory is decoupled.

$\alpha_0 = 0$. $SU(2) \times SU(2) \times U(1) \times U(1) \times \dots$ symmetry

The value of $\alpha_0 = 0$ corresponds to the NS original model.

When $\alpha_0 = 0$ we have

$$\alpha_0^{\mathbb{N}}(0, 0) = \alpha_0^{\mathbb{N}}(0, \frac{1}{2}) = \alpha_0^{\mathbb{N}}(\frac{1}{2}, 0) = \alpha_0^{\mathbb{N}}(\frac{1}{2}, \frac{1}{2}) = -\frac{1}{2} \quad (87)$$

$$\alpha_0^{\rho}(0, 0) = \alpha_0^{\rho}(0, \frac{1}{2}) = \alpha_0^{\rho}(\frac{1}{2}, 0) = \alpha_0^{\rho}(\frac{1}{2}, \frac{1}{2}) = 0 \quad (88)$$

for the trajectories defined by the conditions (85) and (86).
 When $\kappa, \ell = 0, \frac{1}{2}$ the trajectories $\alpha_0^{\tilde{\pi}}(\kappa, \ell)$ ($\alpha_0^{\tilde{\rho}}(\kappa, \ell)$)
 which had different positions before the SVT have the same position after the SVT and therefore are degenerated. (see Fig.20).

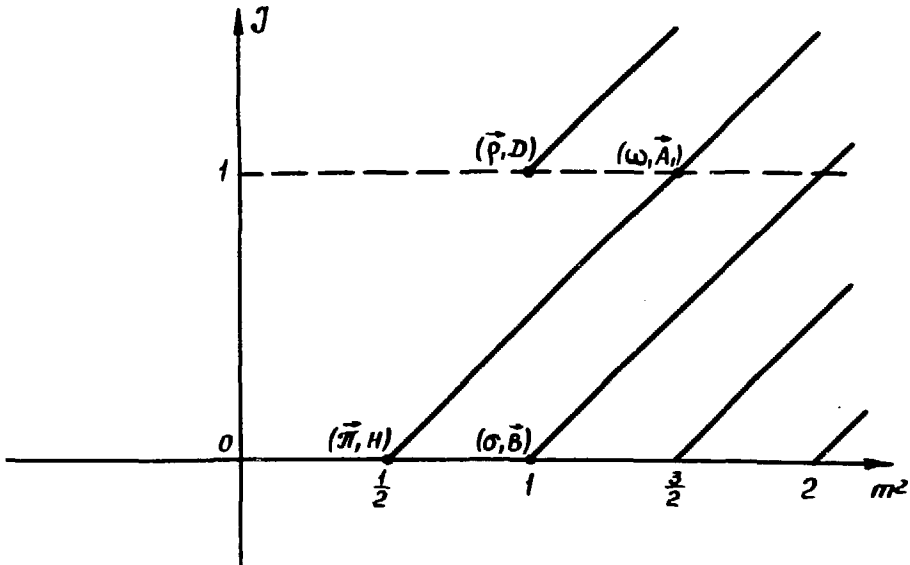


Fig.20

Trajectory positions after the SVT's

In this case the ρ -meson trajectory intercept becomes zero and the ρ -meson square mass equals to 1Gev^2 . The appearance of a non-zero ρ -meson mass as a consequence of the SVT shows the presence of the Higgs effect. The degeneration of the trajectories (87) and (88) is a result of the coincidence of the square masses of the quarks with $m = 0$ and $m = 1/2$ after SVT. The last circumstance is connected with the behavior of the roots $S_0(\beta^2)$ and $S_{\frac{1}{2}}(\beta^2)$ in the S -plane. As it is shown on Fig.19 when $\alpha_0 = 0$ the final positions of the roots $S_0(\beta^2)$ and $S_{\frac{1}{2}}(\beta^2)$ coincide.

That results in the coincidence of all the properties of the quark states corresponding to these roots and in the increase of the dual amplitude symmetry, up to the symmetry of the $SU(2) \times U(1) \times U(1) \times \dots$ -group.

On this case the 0-th and $1/2$ -th quarks compose the doublet $SU(2)$ representation and the particles belonging to the trajectories (87) and (88) correspond to the $SU(2)$ -singlet and triplet states. Note that the charged particles are situated on the trajectories with the intercepts equal to $\alpha_0^{RQ}(0, \frac{1}{2})$ and $\alpha_0^{RQ}(\frac{1}{2}, 0)$, and the particles with zero charges are situated on the trajectories with the intercepts $\alpha_0^{RQ}(0, 0)$ or $\alpha_0^{RQ}(\frac{1}{2}, \frac{1}{2})$. In the case when the external particles belong to the trajectories (87) or (88) dual amplitude dependence on the internal quantum numbers factorizes in the form of the spur of the isospin matrices T_i ($i = 0, 1, 2, 3$) and corresponds to the equal introduction of isospin states into dual amplitudes by means of the Chan-Paton method.

Having stated the $SU(2)$ symmetry for the states belonging to the trajectories (87), (88) and their daughters we shall consider which particles correspond to these states. For this purpose note that all the states situated on the trajectories with the intercepts equal to $\alpha_0^S(m, m)$ have a positive G-parity and all the states lying on the trajectories with the intercepts $\alpha_0^T(m, m)$ have a negative G-parity. This follows from consideration of the structure of separate terms in the expression (39). Analogously we may show that the states belonging to the trajectory $\alpha_0^S(m, n)$ ($\alpha_0^T(m, n)$) have the G-parity opposite to the G-parity of the states belonging to the trajectories with the intercepts $\alpha_0^T(m, n)$ ($\alpha_0^S(m, n)$). The absolute G-parity of the states being undetermined. Noting however that the trajectories with the intercepts equal to $\alpha_0^S(0, 0)$, $\alpha_0^S(0, \frac{1}{2})$, $\alpha_0^S(\frac{1}{2}, 0)$, and $\alpha_0^S(\frac{1}{2}, \frac{1}{2})$ contain a φ -meson isotriplet we may attribute a positive G-parity to the states lying on these trajectories. Then the G-parity of all the other states is fully defined. If we know the G-parity and isotopic

spins of the states belonging to the degenerated trajectories (87), (88) and their first daughter trajectories we may identify these states with the following lowlying scalar, pseudo-scalar, vector and axial-vector mesons (see Table I).

Table I.

Intercepts of trajectories corresponding to the $\kappa, \ell = 0, \frac{1}{2}$ quarks	Particles	μ^2 GeV	I^G	J
$\alpha_0^S(0,0), \alpha_0^S(0, \frac{1}{2}), \alpha_0^S(\frac{1}{2}, 0)$ $\alpha_0^S(\frac{1}{2}, \frac{1}{2})$	$\rho^0, \rho^+, \rho^-, \omega$	1	1^+ 0^+	1
$\alpha_0^A(0,0), \alpha_0^A(0, \frac{1}{2}), \alpha_0^A(\frac{1}{2}, 0)$ $\alpha_0^A(\frac{1}{2}, \frac{1}{2})$	$\pi^0, \pi^+, \pi^-, \eta$	$\frac{1}{2}$	1^- 0^-	0
$\alpha_0^V(0,0), \alpha_0^V(0, \frac{1}{2}), \alpha_0^V(\frac{1}{2}, 0)$ $\alpha_0^V(\frac{1}{2}, \frac{1}{2})$	$A_1^0, A_1^+, A_1^-, \omega$	$\frac{3}{2}$	1^- 0^-	1
$\alpha_0^S(0,0)-1, \alpha_0^S(0, \frac{1}{2})-1, \alpha_0^S(\frac{1}{2}, 0)-1$ $\alpha_0^S(\frac{1}{2}, \frac{1}{2})-1$	$B_1^+, B_1^-, B_1^0, \sigma$	1	1^+ 0^+	0
.

With the help of this table we may receive the well-known relations between the meson masses which are usually derived from the consideration of high energy sum rules or by the algebraic realization of chiral symmetry [5].

$$m_\rho = m_\sigma \tag{89}$$

$$m_\rho^2 + m_\sigma^2 = m_\pi^2 + m_{A_1}^2$$

$$m_\omega^2 = 3m_\rho^2 - m_\omega^2 - m_\pi^2$$

Studying the residues at the poles of the 4-point β_4^R and 5 point β_5^R functions corresponding to various internal and external states shown in Table I. We may receive analogous

relations between the coupling constants of these states. It makes possible to conclude that the symmetry appearing in the NSDM as a result of the SVT' is the $SU(2) \times SU(2) \times U(1) \times U(1) \times \dots$ symmetry.

Concluding we see that the resonance spectrum of the rearranged NSDM is far from reality and is spoiled by the presence of ghosts but it is worthwhile to note that its symmetry properties and quark structure bear surprising resemblance to those extracted from nature.

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**ВАКУУМНЫЕ ПЕРЕХОДЫ В ДУАЛЬНЫХ МОДЕЛЯХ. П.
КВАДРАТНАЯ СТРУКТУРА РЕЗОНАНСОВ В ДУАЛЬНОЙ МОДЕЛИ НЕВЬЕ-ШВАРЦА**

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