STAR FORMATION IN A DUSTY PLASMA CLOUD

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ABSTRACT

A dusty plasma cloud in space has a gravitational instability which allows star formation even orders of magnitude below the Jean's limit for gravitational collapse. This instability leads to a "stellessimal" accretion which is analogous to the planetesimal accretion. If the law of isochronism which holds for planetesimal accretion is applied to the formation of our sun, it gives an acceptable angular momentum to the early sun. Furthermore, the situation in the solar environment is reconcilable with the "initial condition" for formation of planets.
§ 1 The importance of dusty plasmas

The formation of stars is usually supposed to be due to a Jeans collapse of a cosmic cloud, which must be very massive \((10^4 M_\odot)\) in order to satisfy the Jeans criterion. Already long ago Spitzer (1941) and Whipple (1946) drew the attention to the possible importance of dust in star formation, but the effects they and other authors have considered \(\text{for surveys see Burbridge, Kahn, Ebert, v. Hoerner, Temesvary (1960)}\) still require very large masses for a collapse. Recently, however, Horedt (1976) has pointed out that the gravitation of the cloud causes dust to collect at its centre of gravity where a "protocore" is produced which may develop to a star by collecting the surrounding gas.

In spite of the obvious importance of dusty plasmas in cosmic physics there seems not yet to be any systematic study of their properties, including their instabilities. The Horedt mechanism is probably only one of many instabilities which may lead to star formation long before the Jeans condition is reached. Its basic properties can most easily be illustrated by the following simple model.

§ 2 Dust collection

At a distance \(R\) from the centre of gravity of a homogeneous dusty cloud with density \(\rho\), a spherical dust grain with radius \(r\), density \(\theta\), and mass \(\frac{4\pi\rho}{3} r^3\) is acted upon by gravitational force

\[
f_g = \frac{4\pi\theta}{3} r^3 \cdot G \cdot \frac{4\pi}{3} \rho \cdot R = \left(\frac{4\pi}{3}\right)^2 G \theta \rho r^3 \cdot R
\]

A grain moving with velocity \(v\) in a gas with thermal velocity \(u_{\text{th}} = (2kT/m)^{1/2}\) is acted upon by a viscous force

\[
f_v = \frac{4\pi r^2}{3} \cdot \rho \cdot \frac{2}{\eta} \cdot u_{\text{th}} v
\]
(See William and Crampin 1971), small sphere, low speed case.)

Putting \( f_g = f_v \) we find \( v = - \frac{2\pi}{3} \frac{8GrR}{u_{th}}^{3/2} \), which means that the grains sediment towards the centre of gravity with a time constant

\[
T = \frac{R}{v} = \frac{u_{th}}{3.7G\theta r}
\]

With \( G = 6.7 \times 10^{-8} \ \text{g}^{-1} \ \text{cm}^3 \ \text{s}^{-2} \), \( \theta = 1 \ \text{g} \ \text{cm}^{-3} \), \( u_{th} = 6 \times 10^4 \ \text{cm} \ \text{sec}^{-1} \) (for \( \text{H}_2 \) at \( 50^\circ \ \text{K} \)) we find \( T = 0.24 \times 10^{12} r^{-1} \ \text{sec} \) or

\[
T = \frac{10 \ 000 \ \text{years}}{r}
\]

**Time constant of sedimentation of dust particles with radius \( r \)**

<table>
<thead>
<tr>
<th>( r )</th>
<th>( 10^{-5} )</th>
<th>( 10^{-4} )</th>
<th>( 10^{-3} )</th>
<th>( 10^{-2} )</th>
<th>( \text{cm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( 10^9 )</td>
<td>( 10^8 )</td>
<td>( 10^7 )</td>
<td>( 10^6 )</td>
<td>( \text{years} )</td>
</tr>
</tbody>
</table>

In an initially homogeneous cloud a dust ball with a radius of \( 10^{-10} \) of the initial cloud will be formed in a time \( = 23 \ T \). As large particles accrete more rapidly, a ball consisting of the largest particles in the cloud will be found at the centre of gravity. This dust ball will exert a gravitational attraction in its close surroundings (see Fig 1) and will speed up the further accretion so that the real time of accretion for dust of a certain size may not be larger than \( T \). As the accretional time depends linearly on the size of the grains the time of accretion may be set essentially by the formation of large grains in the dusty plasma.

§ 3 Formation of stellesimals

If a dust ball is formed which has an escape velocity \( v_{es} \), it will accrete an atmosphere with a maximum density \( n \) from the surrounding cloud which has the density \( n_o \). If the temperature is constant we have

\[
\ln \frac{n}{n_o} = (v_{es}/u_{th})^2
\]
With \( n_0 = 10^5 \text{ cm}^{-3} \), we have \( n = 10^{20} \) for \( v_{es}/u_{th} = 6 \). This means that when the dust ball approaches lunar size, the increase in size becomes faster because of the formation of an atmosphere, in which there can be a condensation of solid and liquid substances.

A further increase in \( v_{es}/u_{th} \) cannot lead to an increase in the density of the atmosphere, because this will condense. The necessary result is a decrease in the density of the surrounding cloud, which is achieved through a run-away accretion. A homogeneous non-rotating, non-magnetized cloud will rapidly be completely accreted by the central body. Magnetization and rotation puts an outer limit to the region of accretion.

§ 4 Two-step accretion

If the cloud is inhomogeneous, so that it can be depicted as consisting of a large number of small local clouds, stelleisimals can form simultaneously in several of these clouds as a first step. The second step consists in the accretion of secondary stelleisimals by the biggest stelleisimal ("embryo"). The mass gain by the embryo may take place essentially through such a two-step accretional process. The intermediate stelleisimals may be \( 10^{-6} - 10^{-3} \, M_\odot \).

§ 5 Properties of the early sun and its environment

The process of star formation we have considered leads to a condensation of most of the cloud material which originally was located inside a certain limit. Outside this there remains a rather undisturbed dusty plasma hung up on the initial magnetic field. Between this and the star there is a region with very low density, through which dust and neutral gas fall inwards towards the star. This is a situation which is acceptable as "initial condition" for the formation of a planetary system (Alfvén and Arrhenius 1976).

A system of secondary bodies formed around a central body is determined by the mass and the spin of the central body (loc cit Chapter 23).

Hence the properties of the planetary system is initially determined by the mass and the spin of the primeval sun. The
solar mass is given by the mass of the condensing cloud. Its spin should be given by the accretional process which, as we have found, may be analogous to the planetesimal accretion, discussed in Chapter 13 (loc cit).

According to Chapter 9.7 there is an "isochronism of spins" over 12 orders of magnitude from asteroids of $10^{18}$ g up to Jupiter of $10^{30}$ g. If our sun were formed by a similar process, the isochronism might extend even three orders of magnitude more up to $10^{33}$ g (cf. 25.7).

According to Chapter 13, we should expect $\tau_{\theta}^{1/2}$ to be a constant and have a value around 8 hours ($g \text{ cm}^{-3}$)$^{1/2}$. Taking the radius of the early (D-burning) sun to be $10^{12}$ cm, (cf. loc cit Chapter 25) we find $\theta = 5 \cdot 10^{-4} \text{ g cm}^{-3}$. From the isochronism of spins we should expect a period of $8(5 \cdot 10^{-4})^{-1/2} = 360$ hours = 15 days. This value should be corrected for the non-homogeneous density distribution and for the possibly complicated evolutionary history of the sun. We should not expect such a correction to exceed a factor of 2.

According to 25.6 there is some indication that the period was about 20 days, which is reconcilable with the value we expected. The theory of planetesimal accretion is still in a very primitive state, and the isochronism is only an order-of-magnitude relation. However, the agreement makes it interesting to make a general analysis of the planetesimal - stellescimal accretion.

The stellescimal accretion does not conserve the magnetic flux of the initial dusty plasma. The primeval sun should become magnetized by the same kind of hydrmagnetic process in its interior as is causing the planets to be magnetized. (We need not here discuss the controversial theories of planetary magnetization, but simply take the magnetization as an empirical fact.)
When the magnetic field of the sun controls a region extending outside Pluto's orbit, the processes studied in Sp 345 will produce our solar system. The transplanetary region, studied in Chapter 19, may be identified with the rests of the dark cloud out of which the sun once was formed. Long period comets and meteoroids may be a very tiny rest from the stellesimal era.

No formation of planets could take place before the stellesimal accretion was finished, in the same way as the satellite systems could not be formed before the planetesimal accretion was over. The surprisingly high regularity of the main asteroidal belt shows that since it was formed there cannot have been any high eccentricity celestial body large enough to disturb it.

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Homogeneous cloud

Grav. pot.

Formation of star

After sedimentation of dust

Fig. 1
Gravitational instability in dusty plasma cloud.
Fig. 2. Density in cloud surrounding a sphere with radius R₀.
A dusty plasma cloud in space has a gravitational instability which allows star formation even orders of magnitude below the Jeans's limit for gravitational collapse. This instability leads to a "stellesimal" accretion which is analogous to the planetesimal accretion. If the law of isochronism which holds for planetesimal accretion is applied to the formation of our sun, it gives an acceptable angular momentum to the early sun. Furthermore, the situation in the solar environment is reconcilable with the "initial condition" for formation of planets.

**Key words:** Star formation, Dusty plasmas, Planetesimal accretion