7-14-77
25 NTIS

ORNL/TM-5974

Possible Effects of Drift Wave Turbulence on Magnetic Structure and Plasma Transport in Tokamaks

J. D. Callen

MASTER

OAK RIDGE NATIONAL LABORATORY
OPERATED BY UNION CARBIDE CORPORATION FOR THE ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

BLANK PAGE

Printed in the United States of America. Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road, Springfield, Virginia 22161
Price: Printed Copy $4.00; Microfiche $3.00

ORNL/TM-5974

Contract No. W-7405-eng-26

FUSION ENERGY DIVISION

POSSIBLE EFFECTS OF DRIFT WAVE TURBULENCE ON MAGNETIC STRUCTURE AND PLASMA TRANSPORT IN TOKAMAKS

J. D. Callen

Date Published: July 1977

NOTICE This document contains information of a preliminary nature. It is subject to revision or correction and therefore does not represent a final report.

Prepared by the
OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

ABSTRACT

A new mechanism is proposed by which low level, drift wave type fluctuations, such as those observed in the ATC and TFR experiments ($e\tilde{\phi}/T_e \lesssim 10^{-2}$), can cause anomalous radial electron heat transport in tokamaks. The model is based on the fact that since transport processes parallel to the magnetic field are many orders of magnitude more rapid than perpendicular ones, very small helically resonant magnetic perturbations (e.g., $\tilde{B}_r/B \gtrsim \sqrt{\chi_\perp/\chi_\parallel} \sim 10^{-4}$) that cause field lines to move radially allow the parallel transport process to contribute to radial electron heat transport. We hypothesize that the small magnetic perturbations ($\tilde{B}_r/B \lesssim 10^{-3}$) accompanying drift waves at any nonzero plasma β (ratio of plasma pressure to magnetic energy density) are large enough to produce significant effects in present tokamak experiments. The helical magnetic component of drift waves produces magnetic island structures whose spatial widths ($\delta \sim 0.5$ cm) can easily exceed the ion gyroradius. In a drift wave oscillation period, electrons circumnavigate a magnetic island, whereas the slower moving ions see only a tilt of the magnetic field lines. Thus, electrons try to diffuse radially more rapidly than ions; however, a radial potential builds up on a very short time scale to confine the electrons electrostatically and thereby keep the particle diffusion ambipolar. Nonetheless, this parallel electron diffusion process does cause net radial electron heat conduction through an ensemble of closely packed island structures. The heat conduction coefficient is estimated to be $\chi_e^\delta \sim (3/16)(\nu + \gamma)\delta^2$, where ν is the electron-ion collision frequency and γ is the drift wave growth rate, or inverse island correlation time. Other effects that these magnetic flutters may have on plasma transport and runaway electron processes are also discussed.

BLANK PAGE

1. INTRODUCTION

Small, resonant magnetic perturbations can cause significant distortions of the magnetic surfaces[1] in tokamaks. Magnetic perturbations can be caused by inhomogeneities in the external coils producing the confining magnetic field or by the currents induced by collective oscillations in the confined plasma. The magnetic islands induced by irregularities in the external coil system are typically narrow in width, and well separated in radius,[2] or at least can be designed to be so. Thus, even when they occur they are expected to have little effect on overall plasma transport. Similarly, in reasonably quiescent discharges, low mode number kink and tearing instabilities produce substantial distortions of the magnetic topology[3] and plasma transport[4] only in the near vicinity of low order rational surfaces. While internal disruptions, which are apparently due to a repetitive cycle of ohmic heating with consequent m = 1 tearing mode growth through a nonlinear flattening stage,[5] affect the entire central core region where the rotational transform exceeds 2π, they do not seem to have much direct influence on gross plasma confinement properties. The major disruptions, which can be avoided by keeping the rotational transform small enough, apparently result[6] from a coupling of two or more low mode number instabilities and seem to be due to a complete rearrangement of the magnetic topology in the plasma. Various ergodicity and magnetic braiding arguments[7] are often invoked to provide a mechanism for anomalous transport between low order rational surfaces; however, it seems[8] that in the absence of major disruptions, low mode number macroinstabilities do not have a significant effect on the overall plasma confinement in a tokamak. The hypothesis advanced in this paper is that the high mode number magnetic perturbations induced by the microscopic drift wave type turbulence that has been observed experimentally,[9] and which is probably not purely electrostatic,[10] can induce densely packed magnetic island structures and thereby strongly affect radial plasma transport in tokamaks. A similar model of effects of stochastic magnetic turbulence in a fluid model was proposed and found to be important in determining the electron energy confinement in the Zeta device.[11] Also, Rechester[12] has advanced a model of radial electron heat transport due to stochastic magnetic field lines in a collisional plasma.

This paper is organized as follows. In Sect. 2 we briefly discuss the physical mechanism through which small magnetic perturbations can affect radial transport processes. Then, in Sect. 3 we estimate the magnitude and spatial structure of the magnetic perturbations due to drift waves. The magnetic island structures induced by the drift wave magnetics are briefly discussed in Sect. 4. Next, in Sect. 5 the effects that these magnetic island structures have on particle and energy transport in a tokamak are derived in some detail. Our conclusions are summarized in Sect. 6.

2. PHYSICAL MECHANISM FOR MAGNETIC PERTURBATION EFFECTS ON TRANSPORT

The basic physical effect that allows very small magnetic perturbations to affect plasma transport so readily is that plasma transport is much faster along magnetic field lines than perpendicular to them. Thus, magnetic perturbations that cause field lines to flutter radially in a tokamak plasma can allow a portion of the very rapid, parallel transport processes to contribute to radial plasma transport. If the collisional mean free path (λ) in a collisional, magnetized plasma is so small that particles follow the radial motion of the magnetic field lines caused by the magnetic perturbations but do not go far enough along a field line to complete a significant fraction of a magnetic island, the effective radial heat transport is apparently[4,12]

$$\chi_{eff} \sim \chi_{\perp} + (\tilde{B}_r/B)^2 \chi_{\parallel} , \tag{1}$$

where $\chi_{\perp}$ and $\chi_{\parallel}$ are the heat transport coefficients perpendicular and parallel to the magnetic field, $\tilde{B}_r$ is the (stochastic[12]) radial component of the magnetic perturbation, and B is the strength of the confining magnetic field. In a collisional plasma $\chi_{\parallel}/\chi_{\perp} \sim \lambda^2/\rho^2$, where ρ is the particle gyroradius. Thus, magnetic perturbations may have a significant effect on radial heat transport processes in a collisional plasma if

$$\tilde{B}_r/B \gtrsim \sqrt{\chi_{\perp}/\chi_{\parallel}} \sim \rho/\lambda ; \tag{2}$$

the ratio of ρ to λ can easily be as small as 10^{-6} for electrons and 10^{-4} for ions. This is a miniscule level of magnetic fluctuations.

3. MAGNETIC PERTURBATIONS INDUCED BY DRIFT WAVES

While the magnetic perturbations produced by drift waves in a tokamak are small, they can easily be significant in the sense of Eqs. (1) and (2). Drift waves are usually analyzed in the electrostatic limit, which is valid in the limit of vanishingly small β, i.e., $\beta \ll m_e/m_i$, where β is the ratio of plasma pressure to magnetic energy density. However, for tokamak plasmas of present interest in which $\beta \sim 10^{-4} - 10^{-1}$, the non-electrostatic effects can be significant.[10] The magnetic perturbation component of drift waves is caused by the fluctuating electric current produced by the difference between the wave-induced perpendicular drifts

($\underline{V}_\perp$) of the ions and electrons.[13] Since for these low frequency oscillations the perturbed current must be divergence-free, the component of the perturbed current parallel to the magnetic field is given by $\tilde{J}_\parallel = (i/k_\parallel)\, \underline{\nabla} \cdot ne(\underline{V}_{\perp i} - \underline{V}_{\perp e})$. Taking into account the polarization and finite ion gyroradius drifts, one obtains[13] $\tilde{J}_\parallel \simeq -\,\omega(nm_i c^2/B^2)(k_\perp^2/k_\parallel)[1 + (T_i/T_e)(\omega_{*e}/\omega)]\,\tilde{\phi}$. Here $\tilde{\phi}$ is the perturbed potential, $\omega_{*e} \equiv k_\theta(cT_e/eB)(d\ell n\, n/dr)$ is the electron diamagnetic drift velocity, and in the sheared magnetic field characteristic of a tokamak $k_\parallel(x) = k_\theta\, x/L_s$, where $x \equiv r - r_o$ is the radial distance away from a rational surface and $L_s^{-1} = (r/Rq)(q^{-1}dq/dr)$ is the characteristic shear length of the magnetic field. The perturbed current induces, through Ampere's law, a perturbed magnetic field $\underline{\tilde{B}}_\perp$ that is perpendicular to the equilibrium magnetic field $\underline{B}$ but helically aligned with it at the rational surface:

$$\tilde{B}_\perp/B \simeq i(\omega/k_\parallel(x)V_A)(k_\perp\hat{\rho}_i)(V_s/V_A)(1+T_i\omega_{*e}/T_e\omega)(e\tilde{\phi}/T_e), \qquad (3)$$

where $V_A \equiv B/\sqrt{4\pi nm_i}$ is the Alfvén speed, $V_s \equiv \sqrt{T_e/m_i}$ is the ion sound speed, and $\hat{\rho}_i = V_s/\Omega_i$ is the ion gyroradius measured at the electron temperature. Note that: (1) $\tilde{B}_\perp$ bends or twists the magnetic field lines but does not compress them; (2) since $V_s/V_A = \sqrt{\beta_e/2}$, the strength of the magnetic perturbations is proportional to $\sqrt{\beta_e}$; and (3) this expression for $\tilde{B}_\perp$ is valid only for $k_\theta >> \partial/\partial x$ and $\omega < k_\parallel(x)V_A$, i.e., $x > x_A$ where $x_A \equiv \omega L_s/k_\theta V_A \simeq \hat{\rho}_i(L_s/r_n)\,(V_s/V_A)$. The equation governing the full radial or x dependence of $\tilde{B}_r$ is derived and discussed in Ref. 14. For typical parameters in present tokamak experiments ($k_\perp\hat{\rho}_i \leqslant 1$, $V_s/V_A \leqslant 1/20$, $T_i \leqslant T_e$, $e\tilde{\phi}/T_e \leqslant 10^{-2}$), we have $\tilde{B}_\perp/B \leqslant 10^{-3}(\omega/k_\parallel V_A) \leqslant 10^{-3}$;

comparison of this estimate with Eq. (2) indicates that these magnetic perturbations could indeed have significant effects on radial transport processes in tokamaks.

4. DRIFT WAVE MAGNETICS: EFFECTS ON MAGNETIC TOPOLOGY

These magnetic perturbations can change the magnetic topology of a tokamak by forming thin, high order magnetic islands[15] at surfaces where the pitch of the field lines and the pitch of the perturbations are the same. The radial variations of the perturbed potential $\tilde{\phi}$ and the magnetic field are shown in Figs. 1(a) and 1(b). In Fig. 1(c) we illustrate the magnetic islands formed by adding this magnetic perturbation to the helical component of the equilibrium magnetic field near a rational surface,[16] $B_\eta \simeq -[B_\theta - (r/R)(n/m)B_\phi] \simeq x\, B_\theta q^{-1} dq/dr$ where $\eta \equiv n\phi - m\theta$ is the helical angle variable for a given perturbation localized about the rational surface at which $q(r_s) = m/n$; here, ϕ and θ are respectively the toroidal and poloidal angle coordinates and n and m are respectively the toroidal and poloidal mode numbers. It can be shown that for the magnetic perturbation sketched in Fig. 1(b) where $\tilde{B}_r \simeq \tilde{B}(x_A/x)$ for large x, the full width of the induced magnetic island structure is given by[15]

$$\delta \simeq 2\,[(\tilde{B}/B_\theta)(2x_A r_s/nq')]^{1/3}. \tag{4}$$

For typical parameters in present tokamaks [$\tilde{B}/B_\theta \sim (B/B_\theta)(\tilde{B}/B) \sim 1/30$, $x_A \sim 0.1$ cm, $r_s \sim 10$ cm, $q' \sim 0.1$ cm^{-1}, and $n \sim 45$], we obtain $\delta \sim 0.5$ cm. The distance a magnetic field line travels in tracing out a particular magnetic island is given by $2\pi RN$, where[15]

$$N \sim q/n\delta q' \tag{5}$$

is the winding number, i.e., the number of times a field line circumnavigates the torus in one complete circumnavigation of an island. For typical parameters ($n \sim 45$, $\delta \sim 0.5$ cm, $q' \sim 0.1\ \mathrm{cm}^{-1}$, $q \sim 2$), we obtain $N \gg 1$. If the magnetic perturbation were nearly uniform in space, such as occurs in tearing modes or externally induced perturbations, then[15] in Eq. (4) the 1/3 power becomes 1/2, the x_A is replaced by 2, and the N in Eq. (5) becomes a factor of 4 larger.

5. DRIFT WAVE MAGNETICS: EFFECTS ON PLASMA TRANSPORT PROCESSES

The effects that drift wave magnetics have on electrons are quite different from the effects on ions. In one drift wave period, since $\omega \gg v_i/2\pi RN$, ions do not move far enough along the magnetic field lines to be aware of the magnetic island structure. Rather, they see only a small stochastic variation in the direction of the magnetic field lines. The perturbed ion distribution function response due to the magnetic perturbations can thus be calculated from the linearized Vlasov equation in the usual way. The quasilinear effect of this perturbation can be shown to be smaller by a factor β [because of the $V_s/V_A \sim \sqrt{\beta}$ factor in Eq. (3)] than the usual $\underset{\sim}{\underline{E}} \times \underline{B}$ electrostatic quasilinear effect of the drift waves on ion transport processes; hence it is negligible.

In contrast to the ions, electrons are aware of the full magnetic island structure since they circumnavigate the islands many times in a wave or growth period, i.e., $\omega \ll v_e/2\pi RN$. To compute the electron distribution function in the presence of the islands, we begin with the lowest order drift-kinetic equation

$$\frac{\partial f}{\partial t} + (v_\parallel \underline{n} + \underline{v}_E) \cdot \frac{\partial f}{\partial \underline{x}} + q v_\parallel \left(-\frac{1}{c}\frac{\partial A_\parallel}{\partial t}\right)\frac{\partial f}{\partial E} = C(f), \tag{6}$$

where we have neglected some finite Larmor radius effects because they have no significant effect on the analysis that follows. Magnetic perturbations come into Eq. (6) both directly through the $\partial A_{\parallel}/\partial t$ term and indirectly through the unit vector along $\underline{B}$, which becomes

$$\underline{n} \equiv \underline{B}/B \simeq \underline{n}_o + \underline{\tilde{B}}_{\perp}/B \tag{7}$$

in the presence of a magnetic perturbation $\underline{\tilde{B}}_{\perp}$. Here $\underline{n}_o = (B_{\phi}\,\hat{\phi} + B_{\theta}\,\hat{\theta})/B$ is the unit vector along the equilibrium magnetic field. Taking account of these magnetic perturbation effects, neglecting the curvature drift term since it will not contribute to the ultimate answers, linearizing Eq. (6) about a local Maxwellian distribution $\{f = f_m + \tilde{f}$, with $f_m = n_e(2\pi T/m)^{-3/2}$ $\exp[-(E - q\Phi)/T]$, and $E = mv^2/2 + q\Phi\}$, and assuming the lowest order $\tilde{f}$ is an adiabatic response $(\tilde{f} \equiv -q\tilde{\phi}f_m/T + g)$, we obtain for the equation governing the nonadiabatic electron response[10]

$$(\partial/\partial t + v_{\parallel}\,\underline{n}_o \cdot \underline{\nabla} - C)g = -i(\omega - \omega_*^T)q\tilde{\phi}f_m/T - v_{\parallel}(\tilde{B}_r/B)(1 - \omega/\omega_*^T)\partial f_m/\partial r. \tag{8}$$

Here, we have assumed that $\tilde{f}, \tilde{\phi}, \tilde{B}_r \sim \exp[i(n\phi - m\theta - \omega t)]$, used the fact that $\tilde{B}_r \simeq ik_{\theta}A_{\parallel}$, and defined in the $\underline{E} \times \underline{B}$ rest frame where $\Phi = 0$, $\omega_*^T \equiv -k_{\theta}(cT/qB)\partial \ell n\, f_m/\partial r = \omega_*[1 + \eta_e(E/T - 3/2)]$, with $\omega_* \equiv (ck_{\theta}T/eB)$ $\partial \ell n\, n/\partial r$, and $\eta_e \equiv \partial \ell n\, T/\partial \ell n\, n_e$. Integrating Eq. (8) over the characteristics of the left side of the equation, we obtain the usual perturbed electron response, which is responsible for the usual quasilinear particle and energy transport fluxes in the plasma. The $\tilde{B}_r^2$ contribution to Eq. (1) is obtained by assuming that the collision operator in Eq. (8) gives the dominant contribution to the left side of Eq. (8), (i.e., $C \sim \nu >> \omega, \omega_b$) and then computing the contribution to g and the heat flux due to the inhomogeneous $\tilde{B}_r$ term on the right of Eq. (8).

Here, we wish to determine the additional contribution (g_δ) to the perturbed electron distribution function due to the presence of the magnetic islands. This contribution is obtained by calculating the evolution of an initially Maxwellian distribution as it propagates along the magnetic island structure. Since the definition of a magnetic field line is $d\ell/B = dx/\tilde{B}_r$, we can identify $v_\parallel \tilde{B}_r/B = dx/dt$ as the radial velocity of a particle moving along a magnetic field line within the magnetic island structure. Thus, we see from Eq. (8) that the island contribution to the perturbed electron distribution function is governed by the equation

$$(\partial/\partial t + v_\parallel \underset{\sim}{n}_o \cdot \underset{\sim}{\nabla} + \nu)\, g_\delta = -\,(dx/dt)(1 - \omega/\omega_*^T)\, \partial f_m/\partial r, \tag{9}$$

where for simplicity we have taken the collision operator to be of the Krook type with an energy-independent collision frequency ν. Integrating over the characteristics of the left side of Eq. (9), we readily obtain for the time-asymptotic response (Im $\omega > 0$)

$$g_\delta \simeq -\sum_\omega e^{-i\omega t}(1 - \omega/\omega_*^T)(\partial f_m/\partial r) \int_{-\infty}^{t} dt'(dx/dt)' \exp[-i(\omega+i\nu)(t'-t)] \tag{10}$$

in which the primes denote integration along the electron trajectories, and $\tilde{B}_r \equiv \sum_\omega \hat{B}_r e^{-i\omega t}$ has been decomposed into its temporally and spatially varying parts such that $dx/dt = v_\parallel \hat{B}_r/B$.

To proceed further we must specify a trajectory for the radial motion of electrons as they circumnavigate the magnetic islands. Given the complexity of the island structures [cf. Fig. 1(c)], this is very difficult to do exactly. However, since the radial velocity dx/dt must be periodic in t with a period of $\tau_\delta = 2\pi RN/v_\parallel$, we can expand dx/dt in a

Fourier series in the island circumnavigation frequency: $dx/dt = \omega_\delta(\Delta/4)\times \sum_p C_p \exp(ip\omega_\delta t)$, where $\omega_\delta \equiv 2\pi/\tau_\delta = v_\parallel/RN$, $\Delta/4$ is the half-width of a given contour within a magnetic island. From the nature of electron motion within the islands, we observe that $C_p = 0$ if p is zero or an even integer. While this general form of dx/dt can be used, for simplicity in what follows, we assume that $C_1 = C_{-1} = 1/2$, $C_{n\neq\pm 1} = 0$ and then have for the electron trajectory simply

$$dx/dt = \omega_\delta\ (\Delta/4)\ \cos \omega_\delta t,\ x = x_o + (\Delta/4)\ \sin \omega_\delta t. \tag{11}$$

Performing the time-history integration in Eq. (10) along this trajectory, we obtain

$$g_\delta \simeq - \sum_\omega e^{-i\omega t}(1-\omega/\omega_*^T)(\Delta/4)(\partial f_m/\partial r)\{\sin \omega_\delta t - i[(\omega+i\nu)/\omega_\delta]\cos \omega_\delta t + \ldots\}, \tag{12}$$

in which we have made an expansion in $(\omega + i\nu)/\omega_\delta << 1$. It is readily seen from Eq. (11) that the $\sin \omega_\delta t$ term in Eq. (12) represents a flattening of the total electron distribution function $(f_m + \tilde{f})$ around x_o due to the magnetic island. This is, however, a reversible process, as evidenced by the $e^{-i\omega t}$ coefficient. The $[(\omega + i\nu)/\omega_\delta] \cos \omega_\delta t$ correction term to the flattening represents the irreversible processes of wave growth and collisional changes in the island-averaging period τ_δ through diffusion in $v_\parallel$, which will ultimately lead us to obtain net transport in the radial direction from this perturbation.

The induced particle transport flux is calculated from the $\int d^3v$ moment of Eq. (6), which can be written as $\partial n/\partial t + (1/r)(\partial/\partial r)\times [r(\Gamma_r^\delta + \Gamma_o)] = 0$ where

$$\Gamma_r^\delta = \langle \int d^3v\ (v_\parallel \tilde{B}_r/B) g_\delta \rangle \tag{13}$$

and Γ_o is the sum of the usual neoclassical and quasilinear particle fluxes. Here, the average is an island and wave period average:

$$\langle A \rangle \equiv (\pi N)^{-1} \int_{-\pi N/2}^{\pi N/2} d\phi \; (2\pi)^{-1} \int_{-\pi}^{\pi} d\theta \; (\omega/4\pi M) \int_{-M/\omega}^{M/\omega} dt \; A.$$

Again splitting $\tilde{B}_r$ into its spatially and temporally varying parts, and noting that $v_\parallel B_r/B = dx/dt$, we find

$$\Gamma_r^\delta \simeq - \sum_{\omega=\omega_{mn}} \frac{1}{2} (\Delta/4)^2 \int d^3v \; (1 - \omega/\omega_*^T)(\partial f_m/\partial r)(\gamma + \nu), \tag{14}$$

in which ω_{mn} are the set of rational surface drift frequencies over the radius of the plasma. Since the radial derivative of the Maxwellian distribution (at constant E) is given by $\partial f_m/\partial r = f_m[d\ell n\, n_e/dr - (e/T) d\Phi/dr + (E/T - 3/2) d\ell n\, T/dr]$, upon evaluating Eq. (14), we obtain

$$\Gamma_r^\delta \simeq - \sum_{\omega=\omega_{mn}} \frac{1}{2} (\Delta/4)^2 (\nu + \gamma) n_e [d\ell n\, n_e/dr - (e/T)d\Phi/dr - \omega eB/(ck_\theta T_e)]. \tag{15}$$

Now, since the ion particle flux is the much smaller quasilinear one, the radial electron particle flux just derived would cause a nonambipolar radial flow of electrons. This causes a radial potential to build up on a very short time scale ($<< \omega_\delta^{-1}$) just so as to make the particle flux ambipolar. From Eq. (15), a sufficient condition for having this occur is that the sum of the terms within the brackets in Eq. (15) vanish, thereby prescribing an ambipolar potential of the form

$$(e/T_e)\; d\Phi/dr \simeq (1 - \omega_{mn}/\omega_*)\; d\ell n\, n_e/dr. \tag{16}$$

Note that in the rest frame of the plasma the electrons are electrostatically confined in the radial direction. That is, they are no longer magnetically confined since they can flow radially along magnetic field lines; instead, the relatively immobile ions hold them back electrostatically. Nonetheless, the magnetic islands can produce a substantial enhancement of the radial electron heat flux.

Before calculating the radial electron heat flux, there is another effect we must take into account. We have calculated g_δ within only a single closed magnetic island of width $\Delta/2 < \delta/2$ [cf. Fig. 1(c)]; recall that $\delta/2$ is the full width of the island from one side of the separatrix to the rational surface. However, since the separatrices, which are isotherms, connect the island structures on opposite sides of the rational surface, on a very short time scale ($\lesssim \omega_\delta^{-1}$) the electron temperature profile will be flattened over the entire magnetic island structure. Thus, $\Delta/4$ in Eq. (12) should be replaced by $\delta/2$. Taking this and the specification of the ambipolar potential [Eq. (16)] into account, Eq. (12) becomes

$$g_\delta \simeq - \sum_\omega e^{-i\omega t}(1 - \omega/\omega_*^T)(\delta/2)(E/T-3/2)(f_m \, d\ln T/dr) \times \{\sin \omega_\delta t - i[(\omega + i\nu)/\omega_\delta]\cos \omega_\delta t + \dots\} \ . \tag{17}$$

Then, calculating the island-induced electron heat flux in the same manner as the particle flux, except for the insertion of the energy moment factor $mv^2/2$, we obtain

$$Q_r^\delta \simeq - n_e \, \chi_e^\delta \, dT_e/dr \ ,$$

where

$$\chi_e^\delta \simeq (3/4) \sum_{\omega \simeq \omega_{mn}} (\nu + \gamma)(\delta/2)^2 \, . \tag{18}$$

Note that this is essentially the sum of the irreversible radial electron heat conduction within each island belonging to the ensemble of magnetic islands present in the plasma. The physical implication of this "weak turbulence" result is that electron heat diffuses radially with a step size of half the magnetic island width ($\delta/2$), at a characteristic rate given by the sum of the drift wave growth rate and the (90^{o}) collisional scattering rate for changing the island circumnavigation period τ_δ, which is proportional to $v_\parallel$. In a fully developed turbulent state such as might be derived from strong turbulence theory,[17] the γ would apparently be replaced by the "birth and death rate" for the various magnetic islands. In an abstract sense, Eq. (18) would be replaced by $\chi_e^\delta \sim \int d\tau\ (dx/dt)_t\ (dx/dt)_{t+\tau}$, where dx/dt is the electron trajectory given by Eq. (11). This form illustrates the basic features of transport in the presence of the almost oscillatory radial motion. Namely, transport occurs only through the irreversible effects of collisions and island evolution, with net transport occuring only if the "death" phase of the islands is sufficiently different from their growth phase, i.e., the net radial heat transport is irreversible through an entire island life cycle. In order for there to be electron heat transport over the entire radius, the various magnetic islands represented by the ω_{mn} in Eq. (18) must be closely packed or perhaps even overlap. Since the spatial separation of rational surfaces having the same n is only $\Delta x \sim (nq')^{-1} \sim 0.2$ cm for typical parameters, such proximity is quite probable for the various m modes having the same n, and virtually certain for the doubly denumerable infinity of combinations of m and n admissible for the modes of interest.

Note that, with the exception of the general form of the spatial structure shown in Fig. 1, we have used little or no specific information about the particular types of drift instabilities involved. Thus, the drift wave growth rate in Eq. (18) can be that due to one or all of the destabilizing effects of trapped electrons, finite ion Larmor radius, plasma current, etc. on drift waves. In addition, our treatment of the effects of drift wave magnetics would also apply to the hypothesized high mode number drift-tearing modes,[18] with the island width δ and toroidal winding number N modified as indicated at the end of Sect. 4 for these $\tilde{B}_r \sim$ constant modes. Also, since the drift wave magnetics modify the magnetic topology so profoundly and stochastically, it seems that the highly ordered convection of wave energy away from a rational surface by a smoothly sheared magnetic field structure will no longer occur. Rather, the complicated, overlapping magnetic island structures should reflect a significant portion of the outgoing wave energy and thereby greatly diminish if not totally defeat the usual magnetic shear stabilization mechanism[10,15] for drift waves.

Since the electron heat transport due to the magnetic island formation scales as $\chi_e^\delta \sim \delta^2 \sim (e\tilde{\phi}/T_e)^{2/3}$, whereas the typical quasilinear estimate gives $\chi_e^{QL} \sim (e\tilde{\phi}/T_e)^2$, at low fluctuation levels the magnetics effects are likely to dominate. While a condition on $e\tilde{\phi}/T_e$ for which $\chi_e^\delta > \chi_e^{QL}$ can be written down, it is too complicated to be useful. As an illustration of the magnitudes involved, we note that for the fluctuations observed in ATC,[9] we use the parameters $\nu_{ei} \sim 2 \times 10^5\ \mathrm{sec}^{-1}$, $\gamma \sim 2 \times 10^5\ \mathrm{sec}^{-1}$, $k\rho_i \leq 1$, $\gamma/\omega_* \sim 1/2$, $V_s \sim 3 \times 10^7$ cm/sec, $r_n \sim 10$ cm, $e\tilde{\phi}/T_e \sim 5 \times 10^{-3}$, and obtain $\delta \sim 0.5$ cm; hence, $\chi_e^\delta \sim 2 \times 10^4\ \mathrm{cm}^2/\mathrm{sec}$ vs $\chi_e^{QL} \sim 4 \times 10^3\ \mathrm{cm}^2/\mathrm{sec}$.

Similarly, for the TFR experiment[9] we estimate that $\gamma \sim \nu_{ei} \sim 3 \times 10^5$, $\gamma/\omega_* \sim 1/2$, $V_s \sim 4 \times 10^7$ cm/sec, $r_n \sim 20$ cm, $e\tilde{\phi}/T_e \sim 10^{-3}$, and obtain $\delta \sim 0.25$ cm; hence $\chi_e^\delta \sim 7.5 \times 10^3$ cm^2/sec compared to $\chi_e^{QL} \sim 4 \times 10^2$ cm^2/sec. In both these cases $\chi_e^\delta \gg \chi_e^{QL}$ and the electron energy confinement time estimated from $\tau_{Ee} \sim a^2/4\chi_e^\delta$ is close to that observed experimentally.

Finally, we note that the magnetic flutter induced by the drift wave turbulence may explain some previously incomprehensible characteristics of runaway electrons in tokamaks. Moderate energy runaway electrons, which are not significantly affected by the ambipolar potential and whose radial drift off of a flux surface ($x_s = qv_e/\Omega_e$) is smaller than the island width (δ), should experience a radial diffusion rate of about χ_e^δ in magnitude. This diffusion mechanism might explain the observations on ST, Pulsator, and LT-3[19] on runaway electron production, from which it was inferred that the confinement time for the moderate energy runaways was comparable to the energy containment time of the plasma. For very high energy runaways whose radial excursions pass over many magnetic islands ($x_s \gg \delta$), we would expect the radial diffusion rate to be substantially reduced. This effect might explain the $D \sim 10^2$ cm^2/sec radial diffusion of very high energy ($\sim$ 5-10 MeV) runaway electrons observed[20] in the ORMAK experiment.

6. CONCLUSION

We have shown that the small magnetic fluctuations induced by the primarily electrostatic drift wave turbulence in tokamaks can easily split the flux surface topology into closely packed, high order magnetic island structures. These islands cause a flattening of the electron distribution function near a rational surface. The imposition of an ambipolar diffusion constraint leads to the determination of an ambipolar radial electric field as given by Eq. (16). This radial electric field provides electrostatic containment of the electrons. Then, except for some possible effects on runaway electrons, the only residual effect of the magnetic flutter is to enhance the radial electron heat conduction up to a value given by Eq. (18), which appears to be in reasonable agreement with available experimental data. These drift wave magnetics effects apparently simply add to the usual neoclassical and $\tilde{\underline{E}} \times \underline{B}$ quasilinear or strong turbulence transport processes. The important generic point of this study is the fact that very small magnetic perturbations indirectly induced by low frequency turbulence that is primarily electrostatic can have very substantial effects on the magnetic topology and plasma transport in magnetic confinement devices.

ACKNOWLEDGMENTS

First and foremost, the author gratefully acknowledges the encouragement and many useful discussions of the concepts developed in this work with K. T. Tsang and B. V. Waddell, who also rescued the author from many pitfalls along the way. The author is also grateful to the following people for helpful discussions and comments during the development of this model: L. A. Berry, P. J. Catto, J. F. Clarke, T. H. Dupree, K. Molvig, O. P. Manley, M. N. Rosenbluth, P. H. Rutherford, A. Samain, D. Tetrault, S. J. Zweben, and S. Yoshikawa.

As in the development of any new model, this work has a number of historical antecedents. In the interest of indicating how symbiotic such developments are in the international plasma fusion community, a brief historical account of the development of this particular model will be presented. The first direct stimulus for this work came from B. B. Kadomtsev, who, in an informal discussion with the author in Dubna in July 1975, emphasized the role that field errors or magnetic perturbations might have on plasma transport in tokamaks. In discussing this, Kadomtsev referred to a paper he had just recently published[21] as a "phenomenological" calculation and hoped it would serve as a "stimulus to young theorists" to find a more appropriate magnetic perturbation mechanism and concomitant effects. It might be said that the development

of this model is a response to this challenge. The next direct stimulus was provided by M. Cotsaftis, who, in a visit to ORNL in September of 1975, presented his work on and emphasized the role that the very anisotropic electron heat conductivity tensor could have on the magnetic perturbations (in this case tearing modes) in a resistive tokamak. An indirect stimulus at about the same time came from the work by Hazeltine and Strauss,[4] who showed that within the magnetic island produced by a tearing mode, one has $\chi_{er} \sim \chi_{e\perp} + (\tilde{B}_r/B)^2 \chi_{e\parallel}$. Other indirect stimuli came from the heat pulse propagation work by G. L. Jahns and the author,[22] who found that the radial electron heat transport process in a tokamak is a diffusive one, and from the development of the internal disruption model,[5] by Waddell, Jahns, Hicks, and the author, which reemphasized the fact that the magnetic flux surfaces are not rigid concentric tori in a tokamak. At the same time, work with K. T. Tsang and M. Murakami in trying to compare conventional trapped electron mode theory to experiment was producing progressively more discouraging results; the theoretical growth rates and quasilinearly estimated electron heat conduction coefficients seemed to be much smaller than those observed in the experiments. All of these stimuli came to a head at the Berchtesgaden IAEA meeting in October 1976. There, B. B. Kadomtsev, in trying to offer a possible explanation for the apparent (but since resolved[23]) anomaly between the χ_e determined by the heat pulse propagation[22] compared to that of the background plasma, strongly emphasized how fragile the magnetic field lines in tokamaks are to small helical perturbations--a fact that is most clearly seen in the helical magnetic coordinate systems.[16] Also, in informal discussions, P. Rebut and A. Samain emphatically stressed their feelings that somehow magnetic field lines in tokamaks were ergodic in nature, or at least not stationary in time but rather wiggling a bit. In addition, it appeared from the controversy at the meeting concerning the observed fluctuation levels and induced radial transport in ATC and TFR that a quasilinear type $\underline{\tilde{E}} \times \underline{B}$ effect of the fluctuations might not be sufficient to explain the experimentally observed radial electron heat transport. The hypothesis developed in this paper finally jelled as a result of an informal discussion with J. B. Taylor at the meeting, in which (in addition to emphasizing the possible effects of magnetic inhomogeneities on the shear stabilization of drift waves) he said: "it would seem that the anomalous transport is due to some indirect effect of drift waves on the magnetic configuration." While the author does not recall precisely what Taylor was thinking about in making this remark, it triggered in him the recollection (from some previous work on trapped electron drift instabilities of which he had been a part[10]) that all drift waves at any nonzero β have a magnetic component to them. Thus the present model was developed to illuminate indirect effects of drift waves.

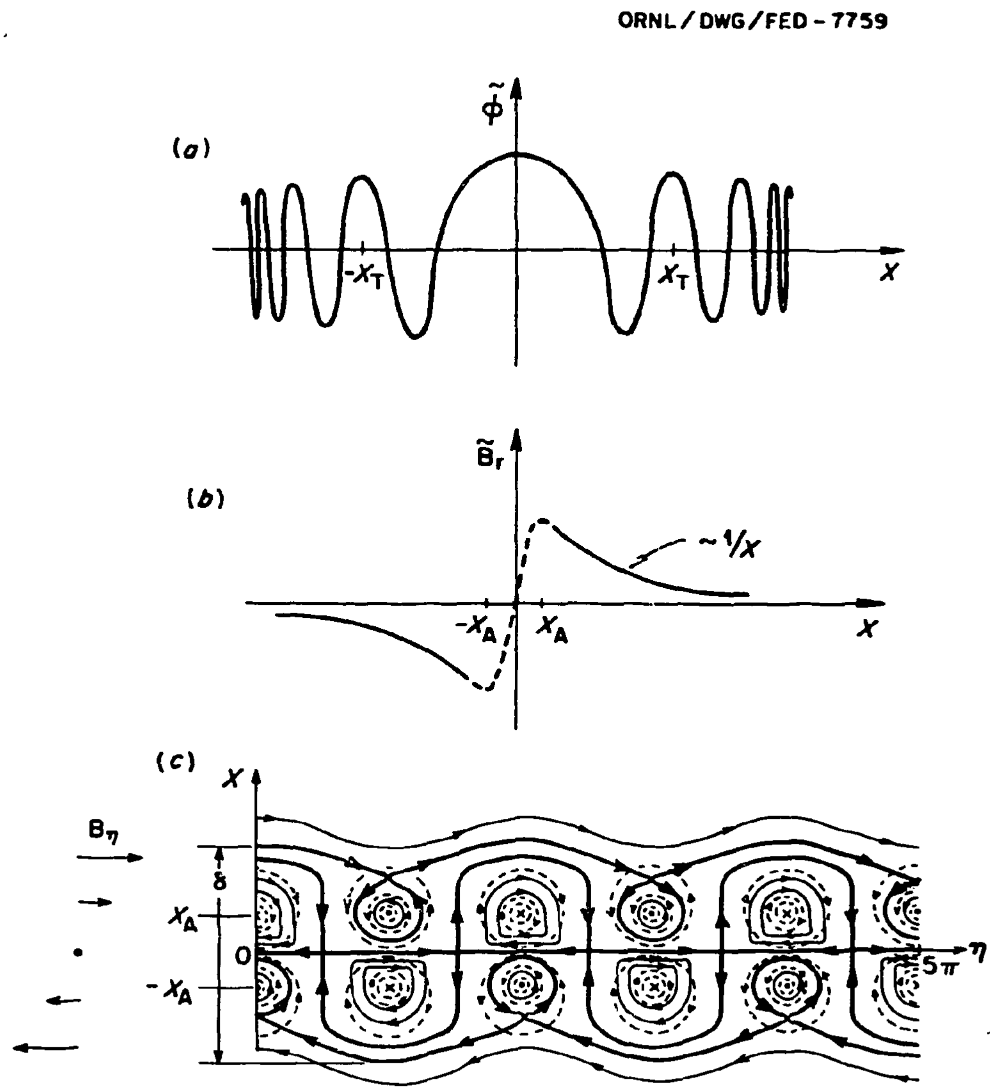

Fig. 1. Schematic illustration of the spatial structure of: (a) the perturbed potential $\tilde{\phi}$; (b) the magnetic field perturbation $\tilde{B}_r$; and (c) the magnetic island structure formed by the combination of $\tilde{B}_r$ and the equilibrium magnetic fields in the vicinity of a rational surface where the helical component of the equilibrium magnetic field is $B_\eta \simeq B_\theta' x q'/q$. The radial distance $x_A \equiv (L_s/r_n)\ (V_s/V_A)\hat{\rho}_i$ is the spatial point at which the phase velocity of the wave is equal to the Alfvén speed; i.e., $\omega/k_\parallel(x_A) = V_A$. The distance $x_T \sim \sqrt{T_e L_s/r_n T_i}\hat{\rho}_i$ is the turning point distance of the potential function eigenmode, which is of the form[10] $\tilde{\phi} \sim \exp(-ix^2/x_T^2)$.

REFERENCES

1. A. I. Morozov and L. S. Solov'ev, "The Structure of Magnetic Fields," p. 1 et seq in Reviews of Plasma Physics, Vol. 2, ed. M. A. Leontovich Consultants Bureau, New York, 1966.
2. S. Matsuda and M. Yoshikawa, Jap. J. Appl. Phys. 14, 87 (1975); P. Chrisman, J. F. Clarke, and J. A. Rome, Magnetic Island Formation in a Tokamak Plasma From Helical Perturbations of the Plasma Current, ORNL-TM-4501, Oak Ridge National Laboratory, Oak Ridge, Tennessee (March 1974).
3. B. B. Kadomtsev, Fiz. Plazmy 1, 710 (1975) [Sov. J. Plasma Phys. 1, 389 (1976)]; B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nucl. Fusion 16, 528 (1976).
4. J. A. Krommes and P. H. Rutherford, Nucl. Fusion 14, 695 (1974); R. D. Hazeltine and H. R. Strauss, Phys. Rev. Lett. 37, 102 (1976); M. Cotsaftis, Bull. Am. Phys. Soc. 20, 2160 (1975).
5. B. V. Waddell, G. L. Jahns, J. D. Callen, and H. R. Hicks, "Interpretation of Tokamak Sawtooth Oscillations" (submitted to Phys. Rev. Lett., February 1977); "Internal Disruptions in Tokamaks," ORNL/TM-5840, Oak Ridge National Laboratory, Oak Ridge, Tennessee (May 1977).
6. B. B. Kadomtsev, "Reconnection of Field Lines and Disruptive Instability in Tokamaks," paper B1 at 6th Intl. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, FRG, October 6-13, 1976; proceedings to be published.
7. A. Samain, Plasma Phys. 18, 551 (1976); A. B. Rechester and T. H. Stix, Phys. Rev. Lett. 36, 587 (1976).
8. J. L. Dunlap, J. H. Harris, and V. K. Paré, Bull. Am. Phys. Soc. 21, 1051 (1976).

9. E. Mazzucato, Phys. Rev. Lett. 36, 792 (1976); C. M. Surko and R. E. Slusher, Phys. Rev. Lett. 37, 1747 (1976); TFR Group, "An Energy Balance Study of a TFR Discharge," paper A3, at 6th Intl. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, FRG, October 6-13, 1976; proceedings to be published.

10. W. M. Tang, C. S. Liu, M. N. Rosenbluth, P. J. Catto, and J. D. Callen, Nucl. Fusion 16, 191 (1976). See also Ref. 14.

11. M. G. Rusbridge, Plasma Phys. 11, 35 (1969). The author is grateful to E. Apgar for pointing out the existence of this work on a similar hypothesis.

12. A. Rechester, "Electron Heat Transport in a Tokamak with Destroyed Magnetic Surfaces," paper E15 at Annual Controlled Fusion Theory Confe ence, San Diego, California, May 4-6, 1977.

13. B. B. Kadomtsev, pp. 82-83 in Plasma Turbulence, Academic Press, London, 1965.

14. P. J. Catto, A. M. El-Nadi, C. S. Liu, and M. N. Rosenbluth, Nucl. Fusion 14, 405 (1974).

15. J. D. Callen, G. G. Kelley, and B. V. Waddell, "Magnetic Island Characteristics in Tokamaks" (to be published as an ORNL/TM).

16. B. B. Kadomtsev and O. P. Pogutse, Zh. Eksp. Teor. Fiz. 65, 575 (1973) [Sov. Phys. - JETP 38, 283 (1974)].

17. T. H. Dupree, Phys. Fluids 10, 1049 (1967).

18. J. F. Drake and Y. C. Lee, "Kinetic Theory of Tearing Instabilities," Bull. Am. Phys. Soc. 21, 1091 (1976); paper PA5 at Annual Controlled Fusion Theory Conference, San Diego, California, May 4-6, 1977; Liu Chen, P. H. Rutherford, and W. M. Tang, "Drift-Tearing Instabilities Due to Trapped Electrons," paper PA4 at Annual Controlled Fusion Theory Conference, San Diego, California, May 4-6, 1977.

19. S. von Goeler, W. Stodiek, N. Sauthoff, and H. Selberg, "X-ray Spectra in the ST Tokamak," paper presented at the 3rd Intl. Symposium on Torodial Plasma Confinement, Garching, FRG, March 26-30, 1973; S. Sesnic and G. Fussmann, Runaway Bremsstrahlung Spectra in Pulsator Tokamak, Report IPP III/29, Max-Planck Institut für Plasmaphysik, Garching, FRG (August 1976); J. C. Strachan, Nucl. Fusion 16, 743 (1976).

20. S. J. Zweben, D. W. Swain, J. B. Wilgen, and H. H. Fleischman, Bull. Am. Phys. Soc. 22, 182 (1977).

21. B. B. Kadomtsev, Fiz. Plazmy 1, 938 (1975) [Sov. J. Plasma Phys. 1, 512 (1975)].

22. J. D. Callen and G. L. Jahns, Phys. Rev. Lett. 38, 491 (1977).

23. M. Soler (Junta de Energia Nuclear, Madrid, Spain), private communication, April 1976 at ORNL.